Local Network Identifiability: Path Conditions

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1 Introduction

We consider the identifiability of a network matrix $G(q)$, where the network is made up of n node signals stacked in the vector $w(t) = [w_1(t) \quad w_2(t) \ \cdots \ w_n(t)]^\top$, known external excitation signals $r(t)$, measured node signals $y(t)$ and unmeasured noise $v(t)$ related to each other by:

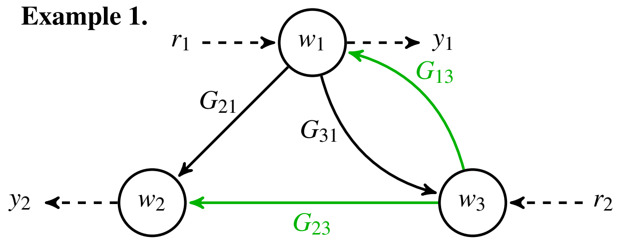
$$\begin{aligned} w(t) &= G(q)w(t) + Br(t) + v(t) \\ y(t) &= Cw(t), \end{aligned} \tag{1}$$

where matrices B and C are binary selections indicating respectively the n_B excited nodes and n_C measured nodes, forming sets $\mathscr{B}$ and $\mathscr{C}$ respectively.

The nonzero entries of the transfer matrix $G(q)$ define the network topology: $G_{ij}(q)$ is the transfer function from node j to node i, represented by an edge (j,i) in the graph. Some of those transfer functions are known and collected in matrix $G^\bullet(q)$, and the others are unknown, collected in matrix $G^\circ(q)$, such that $G(q) = G^\bullet(q) + G^\circ(q)$.

We assume that *the input-output relations between the excitations r and measurements y have been identified*, and that the network topology is known. From this knowledge, we aim at recovering an entry of $G^\circ(q)$, or a subset of them.

Example 1.



$$G^\circ = \begin{bmatrix} 0 & 0 & G_{13} \\ 0 & 0 & G_{23} \\ 0 & 0 & 0 \end{bmatrix}, \quad G^\bullet = \begin{bmatrix} 0 & 0 & 0 \\ G_{21} & 0 & 0 \\ G_{31} & 0 & 0 \end{bmatrix}, B = \begin{bmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 1 \end{bmatrix},$$

$$C = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$

G_{13} and G_{23} are identifiable.

A first line of work extends the classical closed-loop identification techniques to identify a single module, see e.g. [1]. A recent approach employs graph-theoretical tools to derive identifiability conditions on the graph of the network. Using this approach, [2] addresses the particular case where all nodes are excited/measured. In the general case of partial measurement *and* excitation, [3] introduces a local version of identifiability and derives algebraic necessary and sufficient conditions. In this work, we consider local identifiability with partial excitation and measurement. From the conditions of [3], we derive a path conditions which generalize the results of [2] when not all nodes are excited/measured.

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2 Problem reformulation

Starting from the definition of a network system in (1), we first define $T(q) \triangleq (I - G(q))^{-1}$, which is assumed to be proper and stable. The input-output model of (1) is:

$$y(t) = C\,T(q)\,B\,r(t) + \tilde{v}(t),$$

where $\tilde{v}(t) \triangleq CT(q)v(t)$. We assume that $r(t)$ is sufficiently rich so that, for any B and C, $CT(q)B$ can be consistently estimated from $\{y(t), r(t)\}$ data. From the knowledge of $CT(q)B$, the aim is to identify $G^\circ(q)$. This motivates the following definition, which restricts the usual generic identifiability from [2] to non-discrete sets of solutions.

Definition 1. *The module G_{ij} is* locally identifiable *at G from excitations $\mathscr{B}$ and measurements $\mathscr{C}$ if there exists $\varepsilon > 0$ such that for any $\tilde{G}$ with same zero and known entries $G^\bullet$ as G satisfying $||\tilde{G} - G|| < \varepsilon$, there holds*

$$C\tilde{T}(q)B = CT(q)B \Rightarrow \tilde{G}_{ij} = G_{ij}, \tag{2}$$

where $\tilde{T}(q) = (I - \tilde{G})^{-1}$. G_{ij} is generically locally identifiable *if it is locally identifiable at* almost all G.

3 Results

In [3], a linearization of (2) yields a necessary and sufficient condition for generic local identifiability, based on the generic rank of a matrix K constructed from B, C and T.

In this work, we show how the generic rank of K relies on the generic rank of certain particular transfer matrices. Besides, we know from [2] that the generic rank of a transfer matrix can be characterized as paths in the network graph.

Combining those results allows to derive a necessary condition and a sufficient one for generic local identifiability in terms of paths in the network, which will pave the way for further developments in the subject.

References

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