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Conditions for Local Network Identifiability

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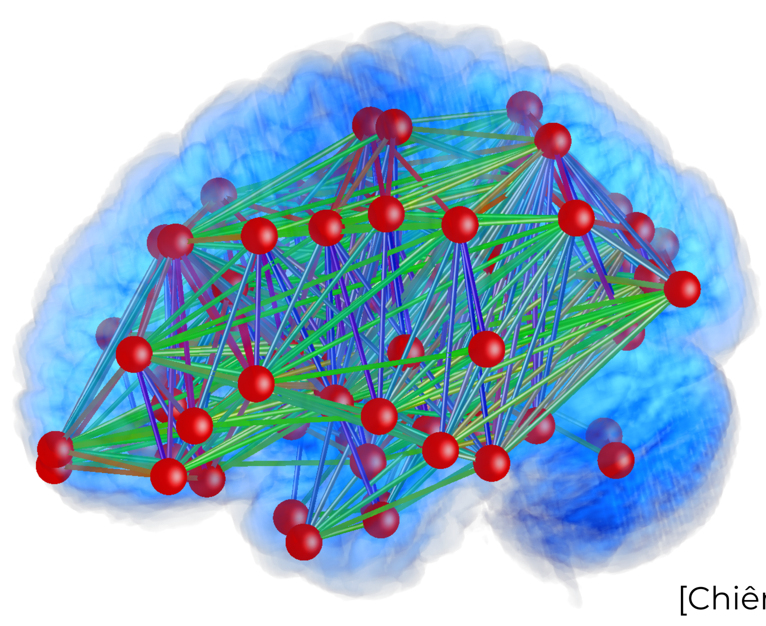
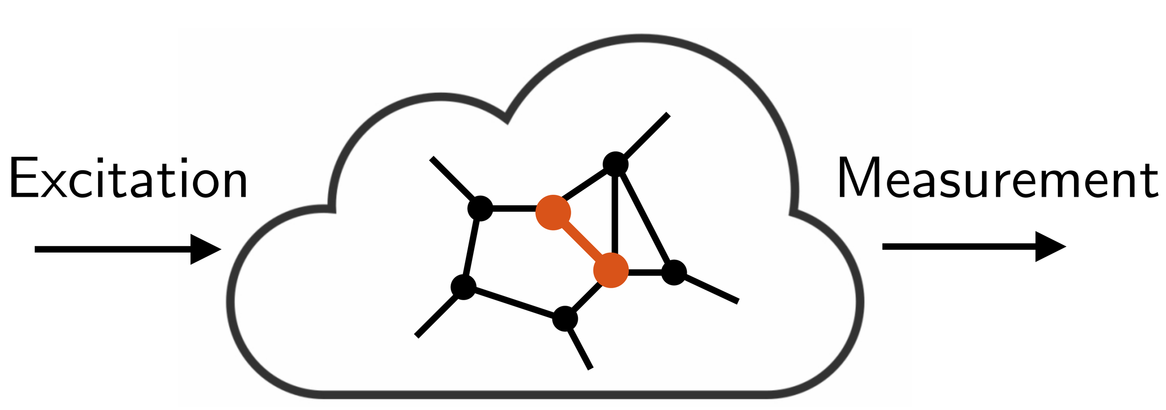
Motivation



- Neuroscience
- Social networks
- Smart grid
- ...



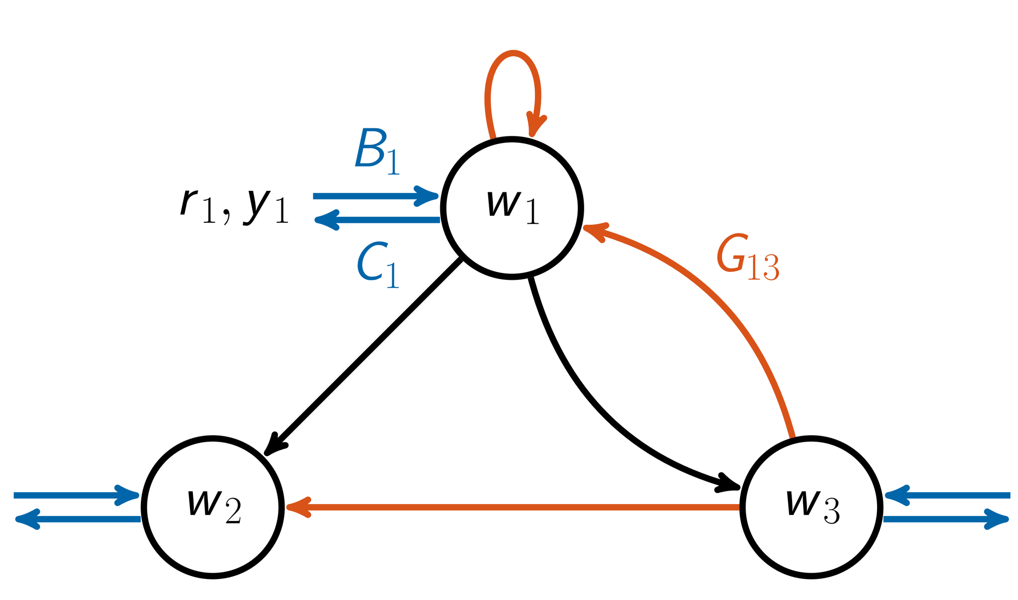
Model

state ⟵ $w = G\,w + B\,r$ ⟶ excitation

measure ⟵ $y = C\,w$

From given excitations/measurements, which unknown transfer functions can be recovered?

i.e. From r at B and y at C, which unknown G_{ij} can be recovered?



Transfer functions that can be recovered are *identifiable.*

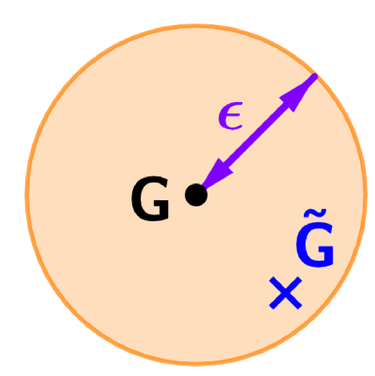
Assumptions:

- Network topology is known
- Global matrix $CT(G)B = C(I-G)^{-1}B$ known
- *Not all* nodes are excited/measured

Local identifiability

Definition: Transfer fct (i,j) *locally* identifiable at G if for all compatible $\tilde{G}$ *on an ϵ–ball,*

$$\underbrace{C\,T(G)\,B}_{f(G)} = C\,T(\tilde{G})\,B \Rightarrow G_{ij} = \tilde{G}_{ij}$$



Generically if it holds for almost all G.

- Necessary for generic identifiability
- No counter-ex to sufficiency known

Identifiability as the injectivity of f:

$$f(G) = f(\tilde{G}) \Rightarrow G_{ij} = \tilde{G}_{ij}$$

Algebraic edge condition

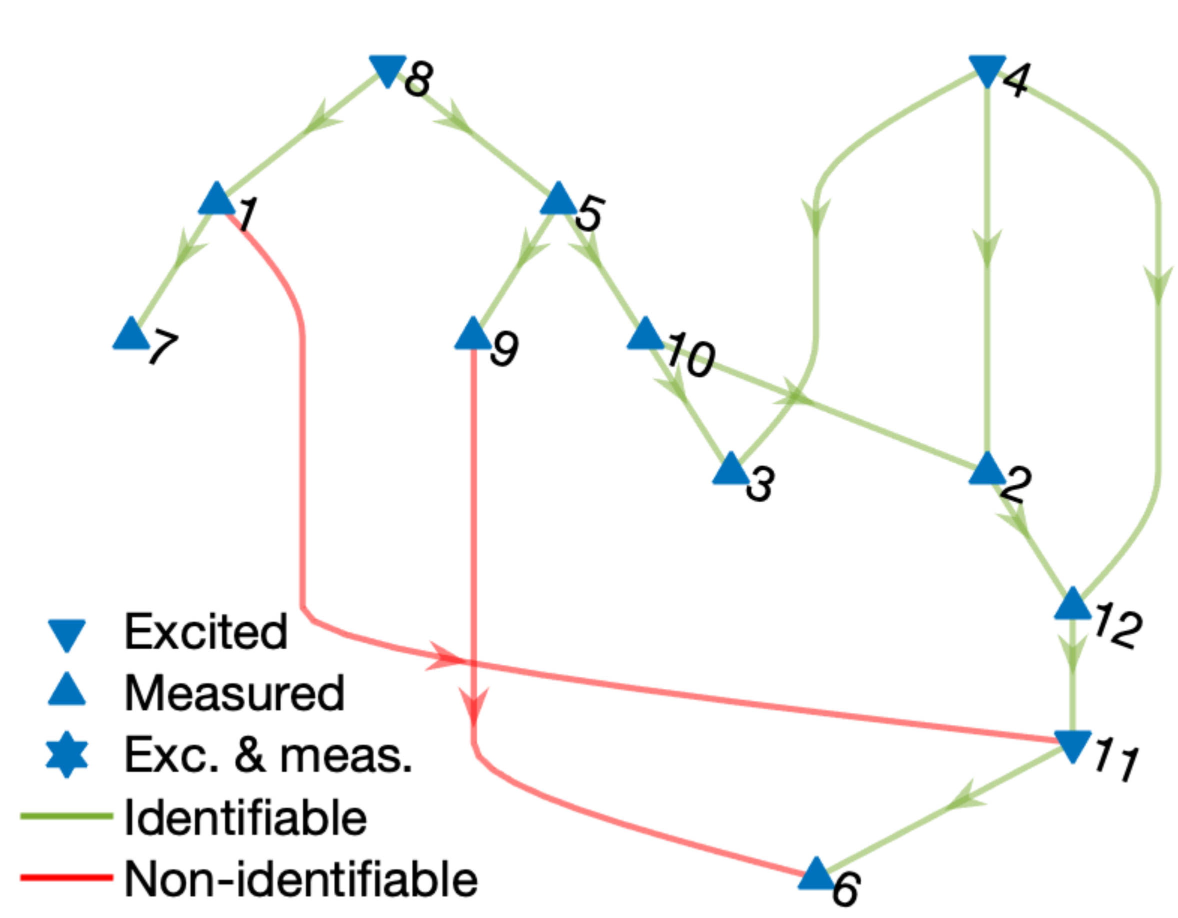
Intuition Local injectivity should rely on ∇f

Correct if ∇f has constant rank, which is the case generically.

Theorem [1]

G_{ij} generically locally identif	⇔	$\ker \nabla f \perp \mathbf{e}_{ij}$ for almost all G
		⇕
		$CT\Delta TB = 0 \Rightarrow \Delta = 0$ for almost all G

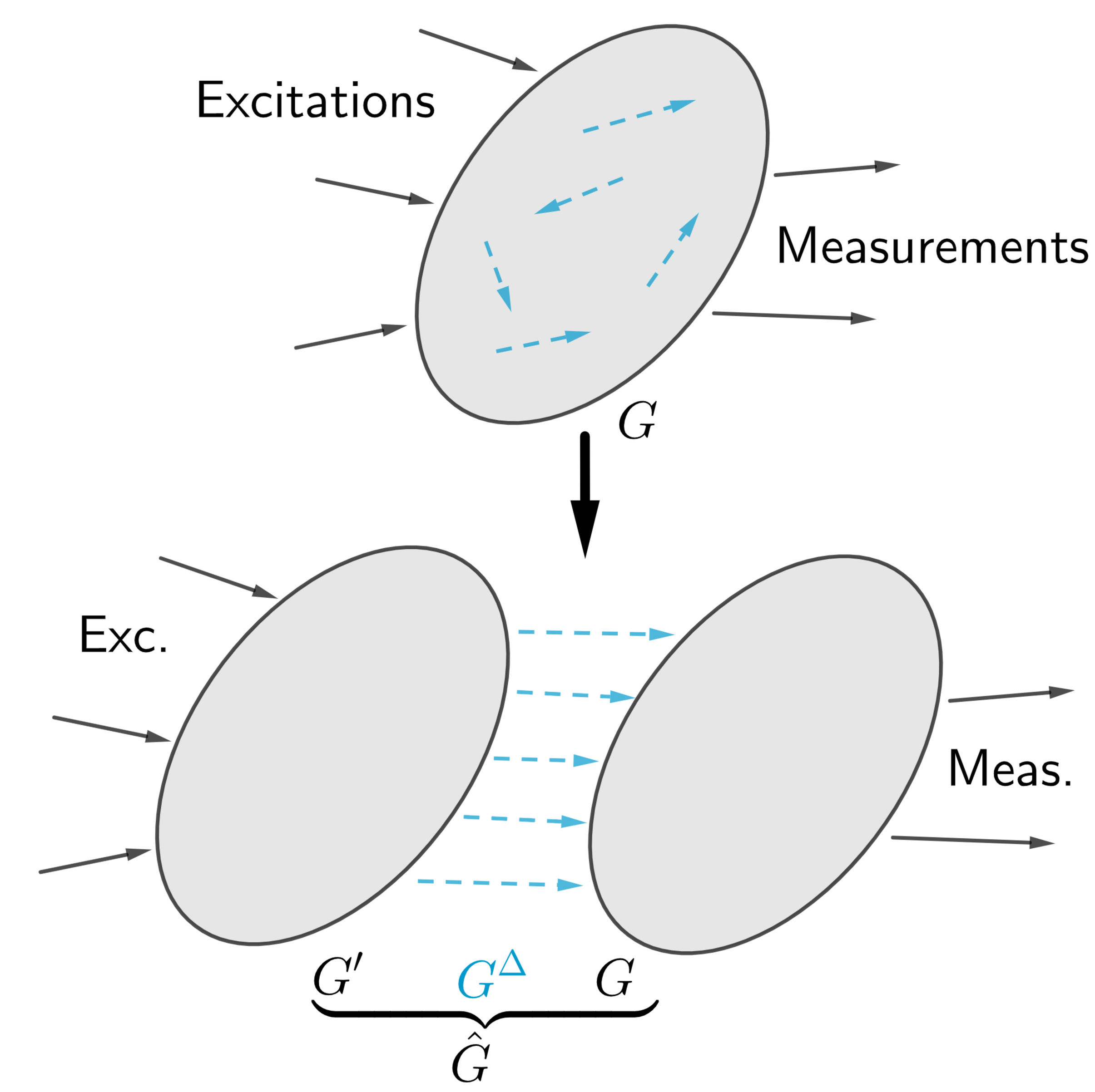
⟶ Randomized probability-1 algorithm:
`github.com/alegat/identifiable`



Decoupled identifiability

Definition: Network decoupled-identifiable at (G,G') if for all compatible Δ,

$$CT\Delta T'B = 0 \Rightarrow \Delta = 0$$



Generic decoupled-identifiability is:

- Necessary for generic (local) identif.
- No counter-ex to sufficiency known
- Equivalent to generic identifiability of decoupled network $\hat{G}$

Graph network condition

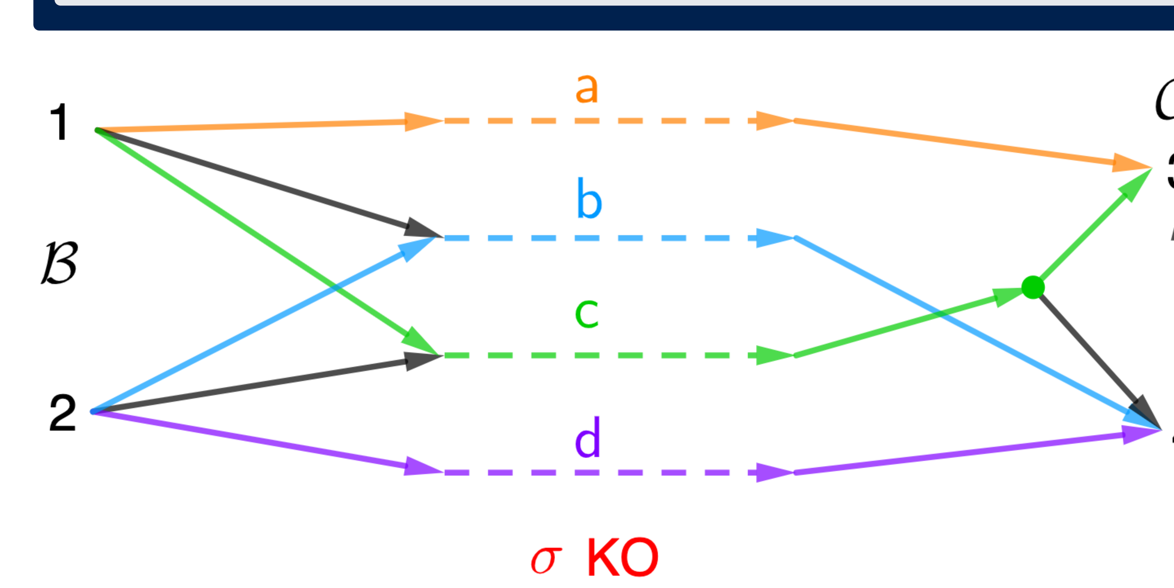
An assignation σ assigns to each unknown edge an (excitation, measurement) pair.

Theorem [2]

If G is generic decoupled-identifiable, there is at least one assignation σ s.t:

1. $|\mathcal{C}|$ edges assigned to each excitation
2. $|\mathcal{B}|$ edges assigned to each measure
3. σ is connected
4. for each excitation b, there are $|\mathcal{C}|$ vertex-disjoint paths between the edges assigned to b and measures $\mathcal{C}$
5. for each measure c, there are $|\mathcal{B}|$ vertex-disjoint paths between the edges assigned to c and measures $\mathcal{B}$

If there is only one such assignation, then this condition is also sufficient.



References

[1] Antoine Legat and Julien M Hendrickx. "Local network identifiability with partial excitation and measurement". In: *2020 59th IEEE CDC.*

[2] Antoine Legat and Julien M Hendrickx. "Path-based conditions for local network identifiability". In: *2021 60th IEEE CDC.*