Abstract

Active labor market policies (ALMPs) have intricate effects in general equilibrium. A general equilibrium matching model is built where workers are heterogeneous. Heterogeneity allows to look at the distribution of effects on labor market indicators and on welfare. Job search effort and wages are endogenous. The net effect of short-duration ALMPs appears to be gloomy. However, their impact on employment can be deeply affected by the design of unemployment insurance. Performance indicators of the labor market and welfare criteria often vary in opposite directions. This questions the widespread focus on labor market indicators to guide the design of ALMPs.

Keywords: training; social programs; evaluation; policy complementarities; wage bargaining; equilibrium unemployment; equilibrium search.

JEL classification: J63, J64, J65, J68.
1 Introduction

Active labour market policies (ALMPs) play a very important role in the European Employment Strategy. The conclusions of academic evaluations are rather mixed however. Most macroeconomic evaluations are empirical exercises based on a simple or even vague theoretical framework (see e.g. Boeri and Burda, 1996, Dor, Van der Linden and Lopez-Novella, 1997, de Koning and Mosley, 2001). These analyses are plagued with problems such as a short period of observations and the endogeneity of programs. When they reach clear-cut conclusions, a deep understanding of the mechanisms at work is often missing. ALMPs have complicate and yet not well-understood effects on employment, wages and welfare (see Calmfors, Forslund and Hemström, 2002). This is in particular true because passive and active programs coexist (Coe and Snower, 1997). The contribution of this paper is to explain the complex mechanisms that are at work in a simple and yet fairly general setting. To do so, I develop a general equilibrium model based on a matching framework where individuals are risk averse, job-search effort and participation are endogenous and wages are bargained over (Pissarides, 2000). Workers are heterogeneous in two dimensions: Their skill and their utility when inactive. The purpose of this model is to deal with the effects of short-duration ALMPs (such as counseling or retraining policies). Since the profile and not only an average level of unemployment benefits is taken into account, the present setting offers new insights on the interactions between so-called ‘passive’ and ‘active’ labor market policies. The paper develops both analytical results and a simulation exercise. The latter stresses the possible conflict between various evaluation criteria.

Heckman (2001) urgently calls for evaluation approaches that combine three requirements. They should deal with the general equilibrium consequences of programs, they should care about distributional consequences and they should cover a range of evaluation criteria. The present paper adopts this perspective. The literature instead typically considers a representative agent or focuses on a single criterion (typically, the (un)employment rate). Holmlund and Lindén (1993) highlight the wage-push effect of direct job creation schemes for the unemployed (also called ‘relief jobs’). Calmfors and Lang (1995) add that ALMPs can counteract true duration dependence. Calmfors (1994) proposes a fairly general framework based upon Layard, Nickell and Jackman (1991). It has recently been updated by Calmfors, Forslund and Hemström (2002). Among other things, these authors point to the need of a better un-
derstanding of an omitted relationship, namely the one between policies and job-search effort. The present paper investigates this relationship. The role of ALMPs in sectoral reallocation is considered by Calmfors and Skedinger (1995). This dimension is not taken into account here. Masters (2000) studies the role of retraining programs whose aim is to enlarge the set of jobs a worker can do. Albrecht, van den Berg and Vroman (2002) evaluate a one-year program that intends to enhance the level of skills of the unemployed. The effects of taxation on schooling and training decisions are studied in perfectly competitive OLG model by Heckman, Lochner and Taber (1998). The general-equilibrium literature about income supplement programs and about wage, employment and hiring subsidies is quite large (see in particular Davidson and Woodbury, 1993, Mortensen and Pissarides, 2001, and Smith, Lise and Seitz, 2003).

Theoretical analyses of complementarities can be found in Coe and Snower (1997), L’Haridon (2001) and Burda and Weder (2002). To the best of my knowledge, these articles have not looked at the possible complementarities between the sequencing of unemployment benefits and ALMPs. Furthermore, except in L’Haridon (2001), the impact on welfare criteria has been neglected.

The rest of the paper is organized as follows. Section 2 develops the model. Section 3 disentangles the equilibrium effect of ALMPs. A numerical analysis is conducted in Section 4. Section 5 concludes the paper.

2 The Model

2.1 The range of ALMPs considered

The model is adapted for publicly-provided short-duration active programs organized for the unemployed. The policies considered here do not intend to enhance skills. However, they pursue (some of) the following objectives. They intend to enhance the matching effectiveness of the beneficiaries (e.g. through job-search assistance i.e. individual counseling, job clubs and the like). By raising motivation or thanks to a (brief) training period, they can also reduce hiring costs (in particular firm-specific training). Finally, a side-motivation can be to better compensate participants. The focus on interventions that last a few weeks or months imply that the model will neglect the “locking-in” effect.\footnote{This effect occurs when participants do not flow out of the programs before they are completed.}
There is no room for cross-country comparisons of public spending on ALMPs nor for an overview of evaluation results (see Martin and Grubb, 2001). Let me simply mention that the interest for counseling and job-search assistance has been growing during the nineties in many European countries. Short training schemes are widespread. They are typically organized by the Public Employment Service (‘PES’) or by specialized agencies. Heckman, LaLonde and Smith (1999) summarize the conclusions of European evaluations of training programs. They conclude that their impact on participants’ wages is negligible. Therefore, it is sound to assume that participation into such schemes does not modify the productivity of the worker. Heckman, LaLonde and Smith (1999) also conclude that the case for positive impacts on employment is stronger.

The approach is the following. I take for granted that the ALMP I consider “does work” at the individual level and I look for its induced or indirect effects through various channels that will shortly be explained. In the model, the active intervention nevertheless fails at a given rate. Furthermore, it has no long-lasting effect (once back in unemployment, the participants are no more different from non-participants).\(^2\)

2.2 Basic assumptions

Differences in skill, denoted by \(n\), and in utility levels while inactive are the two sources of heterogeneity in this economy. The model features a homogeneous good (the numeraire) and labor. The good market is perfectly competitive. Returns to scale are constant. Each firm uses one and only one type of skill. The labor market is therefore by assumption segmented in the skill dimension. For simplicity, a representative firm will be modeled for each skill. Each firm is composed by filled and vacant occupations. In the simulation exercise of Section 4, an aggregate budget constraint of the State will introduce a link between the labor markets.

A markovian model is developed in a continuous-time setting and in steady state (see Figure 1). In accordance with institutions in many OECD countries, a two-tired benefit system is assumed to prevail. An insured unemployed whose ‘high’ benefits has expired enters a state where (s)he indefinitely can benefit from a lower or equal unemployment benefit. The latter could be an assistance benefit. High benefits expire at a rate \(\pi\). For jobless individuals, three states are identified: Insured unemployment with high benefits \((U_n)\), insured unemployment with low benefits \((X_n)\) and participation in an ALMP \((T_n)\). ‘Employment’

\(^2\)On the absence of evidence about long-run effects of ALMPs, see Martin and Grubb (2001).
should be understood as salaried employment in private firms. For reasons that will shortly be clear, a distinction has to be made according to the origin of those employed: \( E_n \) when coming directly from unemployment and \( E_{T,n} \) when coming from the ALMP. Inactivity, \( I_n \), is the sixth state. These upper-case symbols will designate both the states and the number of individuals occupying them in steady state.

Coles and Masters (2000) deal with labor market policies in the presence of true duration dependence. A growing literature shows that duration dependence is largely spurious in Continental Europe (see Machin and Manning, 1999). True duration dependence is therefore assumed to be a negligible phenomenon in this economy.

Due to various imperfections that are not explicitly introduced in the model, the matching process is not instantaneous. A model of directed search is therefore built where firms open skill-specific vacancies that are accessible either to participants or to the other job-seekers.\(^3\) This assumption requires that participation to the ALMP is costlessly observed by employers. The flows of hires, \( M_n \) and \( M_{T,n} \), are a function of an indicator of the number of job-seekers, \( S_n \) and \( S_{T,n} \), and of the number of vacancies, \( V_n \) and \( V_{T,n} \). The matching functions are by assumption identical in all markets and they are written respectively \( M_n = m(S_n, V_n) \) and \( M_{T,n} = m(S_{T,n}, V_{T,n}) \). The function \( m(.,.) \) is assumed to be increasing, concave and homogeneous of degree 1.

At each moment, the timing of decisions is by assumption the following:

1. Firms post vacancies and this costs a fixed amount \( K_n \) per unit of time. Jobless workers search for a job or stay out of the labor force.

2. The firm incurs a fixed cost \( H_{T,n} \) if the recruited worker has benefited from an ALMP and \( H_n \) otherwise (\( H_{T,n} < H_n \)). These match-specific fixed costs include training expenses.

3. Having Continental Europe in mind where collective bargaining is widespread and following Cahuc and Lehmann (2000), it is assumed that the current wage is bargained over by incumbent employees on behalf of all workers. At this stage, \( H_{T,n}, H_n \) are a sunk cost.\(^4\) The fall-back level for these ‘insiders’ is the intertemporal discounted

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\(^3\)Undirected search could be more plausible if the focus is on ALMPs such as job-search assistance.

\(^4\)This creates a ‘hold-up’ problem. Since \( H_{T,n} < H_n \), the latter is less acute when entrants exit from the ALMP.
utility of an unemployed entering state $U_n$, $V_{U,n}$.

4. If an agreement is reached, production occurs and the total surplus is shared between the worker and the firm.

5. An exogenous fraction $\phi_n$ of the matches is destroyed. The workers who occupied these jobs enter insured unemployment and these jobs become vacant. As will soon be clear, workers have no incentive to quit.

Search intensity is *endogenous*. Let $s_{U,n}$, $s_{X,n}$ and $s_{T,n}$ denote search intensities in the various states. A unique *exogenous* matching effectiveness parameter $c_n$ will be associated to states $U_n$ and $X_n$. For ALMP participants, this parameter can be different and will be denoted $c_{T,n}$. It is assumed that $c_{T,n} \geq c_n > 0$. The ALMP can intrinsically improve the effectiveness of search effort. Other explanations can be suggested, too. As job-entry rates are often used in the assessment of labor programs, the PES can for instance give priority to participants to these programs, in particular in the case of a closed treatment of job offers.\(^5\) A signalling effect of ALMPs could also be invoked. So, in the matching function, $S_n \equiv c_n (s_{U,n} U_n + s_{X,n} X_n)$ and $S_{T,n} \equiv c_{T,n} s_{T,n} T_n$.

Due to the constant returns to scale in the matching process, the model can be developed in terms of tightness indicators *measured in efficiency units*, namely $\theta_n \equiv \frac{V_n}{S_n}$ and $\theta_{T,n} \equiv \frac{V_{T,n}}{S_{T,n}}$. The rate at which vacant jobs become filled is $q(\theta_n) \equiv M_n/V_n = m(1/\theta_n, 1)$, $q'(\theta_n) < 0$.

An ‘efficient job-seeker’ moves into employment according to a Poisson process with rate $\alpha(\theta_n) \equiv M_n S_n = \theta_n q(\theta_n)$, with $\alpha'(\theta_n) > 0$. $q(\theta_{T,n})$ and $\alpha(\theta_{T,n})$ are defined similarly. An insured unemployed $i$ endowed with skill $n$ and searching with a search intensity $s_{U,n}^i$ exits to employment at a rate to $c_n s_{U,n}^i \alpha(\theta_n)$.

The unemployed enter the ALMP at a rate $\gamma_n$.\(^6\) A program ends at an exogenous rate

\(^5\)This refers to the case where the PES identifies those who are suitable for vacancies in their register. Cockx (2000) and Heckman, Heinrich and Smith (2002) emphasize the role of incentives on the behavior of programs’ administrators.

\(^6\)It will turn out that entering a program implies a gain for the unemployed. However, waiting for a job offer could be more advantageous. Nevertheless, if an unemployed receives an offer to enter an ALMP, the model assumes that it will be accepted. This simplifying assumption is plausible since refusing an active programs is more and more a motive for being sanctioned. Conditioning the access to an ALMP on the level of unemployment benefit would be considered as discriminatory. So, this possibility is ruled out here. As it is observed in several countries, participation to active programs is a sufficient condition to become eligible to high benefits again.
This parameter can be interpreted as the rate of failure of the public intervention. It is quite natural to assume that \( \lambda_n \geq \phi_n, \forall n \).

### 2.3 Preferences and search effort

Individuals are risk averse and have no access to capital markets. Equilibrium search model with risk averse workers are notoriously difficult to handle. So, a simple separable instantaneous utility function is adopted, namely \( \ln(C) - \psi_n \frac{s^i_n}{\xi_n}, \) with \( C \) denoting consumption and \( s \) effort while \( \psi_n > 0 \) and \( \xi_n > 1 \) are parameters. Effort in employment is fixed and normalized to zero.\(^7\)

Let \( W_n \) denote the net wage. The wage of a worker endowed with skill \( n \) is written \( W_n = w_{T,n} \) if (s)he holds a job after participation in an ALMP and \( W_n = w_n \) otherwise. Let \( b_{\iota,n} \) be the level of benefit (\( \iota = U, X, T \)).\(^8\) The following very plausible ranking is assumed: \( W_n > b_{T,n} > b_{U,n} > b_{X,n} > 0 \). Let \( v_{\iota,n} = \ln(b_{\iota,n}) - \psi_n \frac{(s_{\iota,n})\xi_n}{\xi_n}, \iota \in\{U, X, T\} \).

Let \( r \) be the discount rate assumed to be common to all agents. Holding a job yields an intertemporal utility \( V_{E,n} \) (respectively \( V_{E,n|T} \) after a program) defined by:

\[
rv_{E,n} = \ln(w_n) + \phi_n(V_{U,n} - V_{E,n}) \quad \text{and} \quad rv_{E,n|T} = \ln(w_{T,n}) + \phi_n(V_{U,n} - V_{E,n|T})
\]

The level of job-search is optimized at any point in time. For an individual \( i \) endowed with skill \( n \), the intertemporal utility levels solve the following state-dependent Bellman equations:

\[
rV_{U,n}^i = \max_{s^i_{U,n}} \{v_{U,n} + c_n s^i_{U,n} \alpha(\theta_n)(V_{E,n} - V_{U,n}^i) + \gamma_n(V_{T,n}^i - V_{U,n}^i) + \pi(V_{X,n} - V_{U,n}^i)\}; \quad (2)
\]

\[
rV_{T,n}^i = \max_{s^i_{T,n}} \{v_{T,n} + c_T s^i_{T,n} \alpha(\theta_{T,n})(V_{E,n|T} - V_{T,n}^i) + \lambda_n(V_{U,n}^i - V_{T,n}^i)\}; \quad (3)
\]

\[
rV_{X,n}^i = \max_{s^i_{X,n}} \{v_{X,n} + c_n s^i_{X,n} \alpha(\theta_n)(V_{E,n} - V_{X,n}^i) + \gamma_n(V_{T,n}^i - V_{X,n}^i)\}. \quad (4)
\]

Only symmetric equilibria are considered, where all agents have the same level of search.

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\(^7\)It could equally well be normalized to any other value without changing the results. Van der Linden (2003) summarizes the major changes when an isoelastic function of consumption is used instead of \( \ln(C) \).

\(^8\)As such, levels of unemployment benefits are not a function of the skill. However, when they are (to some extent) indexed on wages, a dependency with \( n \) appears via the wage.
effort. Henceforth, superscript $i$ will be dropped. Let

$$\Delta_{1,n} \equiv (r + c_n s_{X,n} \alpha(\theta_n) + \gamma_n) [(r + c_n s_{U,n} \alpha(\theta_n) + \phi_n) [r + c_{T,n} s_{T,n} \alpha(\theta_{T,n}) + \lambda_n] + \gamma_n [r + c_{T,n} s_{T,n} \alpha(\theta_{T,n}) + \phi_n]] + \pi [r + c_{T,n} s_{T,n} \alpha(\theta_{T,n}) + \lambda_n] [r + c_n s_{X,n} \alpha(\theta_n) + \phi_n] + \gamma_n [r + c_{T,n} s_{T,n} \alpha(\theta_{T,n}) + \phi_n],$$

$$\Delta_{2,n} \equiv r + \pi + c_n s_{X,n} \alpha(\theta_n) + \gamma_n,$$

$$\Delta_{3,n} \equiv r + c_{T,n} s_{T,n} \alpha(\theta_{T,n}) + \lambda_n + \gamma_n,$$

$$\Delta_{4,n} \equiv r + c_{T,n} s_{T,n} \alpha(\theta_{T,n}) + \phi_n.$$

Let $\delta_{ET,n} \equiv \ln(w_{T,n}) - v_{T,n}$ and $\delta_{t',n} \equiv v_{t,n} - v_{t',n}$, $t, t' \in \{U, X, T\}$, $t \neq t'$. The following differences can be derived from Equations (1) to (4):

$$V_{E,n} - V_{U,n} = [(r + c_{T,n} s_{T,n} \alpha(\theta_{T,n}) + \lambda_n) [(r + c_n s_{X,n} \alpha(\theta_n) + \gamma_n) (\ln(w_n) - v_{U,n})] + \gamma_n (r + \pi + c_n s_{X,n} \alpha(\theta_n) + \gamma_n) (\ln(w_n) - v_{T,n})] \Delta_{1,n}^{-1},$$

$$V_{U,n} - V_{X,n} = [\delta_{UX,n} + c_n (s_{U,n} - s_{X,n}) \alpha(\theta_n) (V_{E,n} - V_{U,n})] \Delta_{2,n}^{-1},$$

$$V_{T,n} - V_{U,n} = [(r + c_n s_{X,n} \alpha(\theta_n) + \gamma_n) (\delta_{TU,n} + (c_{T,n} s_{T,n} \alpha(\theta_{T,n}) - c_n s_{U,n} \alpha(\theta_n)) (V_{E,n} - V_{U,n})) + \pi (\delta_{TX,n} + (c_{T,n} s_{T,n} \alpha(\theta_{T,n}) - c_n s_{X,n} \alpha(\theta_n)) (V_{E,n} - V_{U,n}))] \Delta_{3,n}^{-1},$$

if $w_{T,n} = w_n$,

$$V_{E,n | T} - V_{T,n} = [\delta_{ET,n} + (\lambda_n - \phi_n) (V_{T,n} - V_{U,n})] \Delta_{4,n}^{-1}. \quad (8)$$

The flows in Figure 1 require that jobless people have an incentive to accept job offers. The following proposition establishes the ranking of intertemporal utilities. Some of its assumptions introduce a hierarchy between endogenous variables that will be checked later.

**Proposition 1.** $\forall n$, if $w_{T,n} \geq w_n > b_{T,n} > b_{U,n} > b_{X,n} > 0$, $\theta_{T,n} \geq \theta_n$, $c_{T,n} \geq c_n$ and $\phi_n < \lambda_n$, then $V_{E,n} > V_{U,n} > V_{X,n}$ and $V_{E,n | T} > V_{T,n} > V_{U,n}$.

**Proof.** By (5) (respectively, (8)), it is obvious that $V_{E,n} > V_{U,n}$ (respectively, $V_{E,n | T} > V_{T,n}$). The proof that $V_{U,n} > V_{X,n}$ and $V_{T,n} > V_{U,n}$ is left to Appendix 1. $\blacksquare$

The optimal levels of search effort $s_{U,n}$, $s_{X,n}$ and $s_{T,n}$ are respectively solution to the following (sufficient) first-order conditions:

$$\psi_n(s_{U,n})^{\xi_n - 1} = c_n \alpha(\theta_n) (V_{E,n} - V_{U,n}), \quad (9)$$

$$\psi_n(s_{X,n})^{\xi_n - 1} = c_n \alpha(\theta_n) (V_{E,n} - V_{X,n}), \quad (10)$$

$$\psi_n(s_{T,n})^{\xi_n - 1} = c_{T,n} \alpha(\theta_{T,n}) (V_{E,n | T} - V_{T,n}). \quad (11)$$
Proposition 1 implies that $s_{X,n} > s_{U,n}$. From (9) and (11), $s_{T,n} \geq s_{U,n}$ if and only if

$$c_{T,n} \alpha(\theta_{T,n})(V_{E,n|T} - V_{E,n}) + (c_{T,n} \alpha(\theta_{T,n}) - c_n \alpha(\theta_n))(V_{E,n} - V_{U,n})$$

$$\geq c_{T,n} \alpha(\theta_{T,n})(V_{T,n} - V_{U,n})$$

(12)

If the difference between $V_{T,n}$ and $V_{U,n}$ is sufficiently small, $s_{T,n}$ can be higher than $s_{U,n}$.

### 2.4 Job creation

For simplicity, taxation is linear. The tax rate is denoted $\tau_n$. The firm’s discounted expected return from an occupied job is denoted $\Pi_{E,n}$ if this firm operates in the skill segment $n$ (respectively, $\Pi_{E,n|T}$ if a former participant is occupied). The discounted expected return of a vacant job is $\Pi_{V,n}$ (respectively, $\Pi_{V,n|T}$). Since, conditional on their skill, workers are equally productive, let $y_n$ be the constant marginal product of a filled vacancy. Consider that $K_n$, the cost of posting a vacancy and of selecting applicants, is proportional to $y_n$: $K_n \equiv k_n y_n$. Similarly, let us assume that the fixed hiring costs are proportional to $y_n$: $H_n \equiv \kappa_n y_n$, $H_{T,n} \equiv \kappa_{T,n} y_n$ ($\kappa_n > \kappa_{T,n}$). For each skill $n$, the discounted expected returns satisfy the following conditions:

$$r \Pi_{E,n} = y_n - (1 + \tau_n)w_n + \phi_n \left(\max \left[\Pi_{V,n}, \Pi_{V,n|T}\right] - \Pi_{E,n}\right),$$

$$r \Pi_{V,n} = -k_n y_n + q(\theta_n) \left(\Pi_{E,n} - \kappa_n y_n - \Pi_{V,n}\right).$$

(13)  (14)

$r \Pi_{E,n|T}$ and $r \Pi_{V,n|T}$ are defined in a similar way.

In equilibrium, vacancies are opened as long as they yield a positive expected return. Therefore, in equilibrium, $\Pi_{V,n|T} = \Pi_{V,n} = 0$. These properties combined with (13), (14), and their equivalent yield two ‘vacancy-supply curves’ for each $n$:

$$\left(\frac{k_n}{q(\theta_n)} + \kappa_n\right) y_n = \frac{y_n - (1 + \tau_n)w_n}{r + \phi_n}, \quad \left(\frac{k_n}{q(\theta_{T,n})} + \kappa_{T,n}\right) y_n = \frac{y_n - (1 + \tau_n)w_{T,n}}{r + \phi_n}.$$  

(15)

It is easily checked that the ‘vacancy-supply curves’ establish a decreasing relationship between the net wage and the corresponding indicator of tightness. In particular,

$$w_n = VS(\theta_n | \kappa_n, k_n, y_n, r, \phi_n, \tau_n) \equiv \frac{y_n \left(1 - (r + \phi_n) \left(\frac{k_n}{q(\theta_n)} + \kappa_n\right)\right)}{1 + \tau_n},$$

(16)

with $\frac{\partial VS}{\partial \theta_n} < 0$ and $\frac{\partial VS}{\partial \tau_n} < 0$.

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9 Tax rates are typically not skill-specific. This is however a convenient way of capturing the idea that marginal and average taxation can vary with the level of earnings.
2.5 The wage bargain

It is assumed that wages are collectively bargained over in each segment (firm) \( n \) by insiders whose inter-temporal utility is \( V_{U,n} \) in case of layoff.\(^{10}\) In order to show that the wage is unique in each firm, let us imagine that the insiders of type \( n \) bargain over two wages \( w_n \) and \( w_{n,T} \). Nash bargaining is assumed. This assumption per se is not essential, though rent sharing is essential. For each \( n \), the Nash maximization program can be written as:

\[
\begin{align*}
\max_{w_n} & \ (V_{E,n} - V_{U,n})^{\beta_n} (\Pi_{E,n} - \max \left[ \Pi_{V,n}, \Pi_{V,n|T} \right])^{1-\beta_n}, \\
\max_{w_{n,T}} & \ (V_{E,n|T} - V_{U,n})^{\beta_n} (\Pi_{E,n|T} - \max \left[ \Pi_{V,n}, \Pi_{V,n|T} \right])^{1-\beta_n},
\end{align*}
\]

with \( 0 < \beta_n < 1 \). The assumption of a single representative firm has been made for the sake of simplicity. It does not imply that the wage bargain is centralized in such a way that insiders and the representative firm take care of the equilibrium effect of wages on tightness. The first-order condition can be written as:

\[
\begin{align*}
w_n &= \frac{1}{1 + \tau_n} \frac{\beta_n \Pi_{E,n} - \max \left[ \Pi_{V,n}, \Pi_{V,n|T} \right]}{1 - \beta_n} \\
w_{n,T} &= \frac{1}{1 + \tau_n} \frac{\beta_n \Pi_{E,n|T} - \max \left[ \Pi_{V,n}, \Pi_{V,n|T} \right]}{1 - \beta_n}
\end{align*}
\]

2.6 Wages and tightness in a symmetric equilibrium

Taking (1), (13), and free entry into account, the first-order conditions (19) and (20) can be rewritten as:

\[
\begin{align*}
\ln(w_n) &= rV_{U,n} + \frac{\beta_n}{1 - \beta_n} \left( \frac{y_n}{w_n(1 + \tau_n)} - 1 \right), \\
\ln(w_{n,T}) &= rV_{U,n} + \frac{\beta_n}{1 - \beta_n} \left( \frac{y_n}{w_{n,T}(1 + \tau_n)} - 1 \right).
\end{align*}
\]

Therefore, \( w_n = w_{n,T} \). These equations have an intuitive interpretation. If \( \beta_n \) was equal to zero, the instantaneous utility in employment would be equal to the minimum compensation that an unemployed worker requires to stop searching. As \( \beta_n \) increases, a growing share of the relative difference \( y_n - w_n(1 + \tau_n) \) (scaled by \( w_n(1 + \tau_n) \)) accrues to the worker. The fact that \( w_n = w_{n,T} \) in each segment \( n \) has two implications. First, \( V_{E,n|T} = V_{E,n} \). Second, from

\(^{10}\)The existence of a minimum wage will be taken into account in the numerical analysis below.
(15), one has \( \kappa_n - \kappa_T, n = k_n \left( \frac{1}{q(\theta, n)} - \frac{1}{q(\theta_n)} \right) \). Therefore, \( \theta_{T, n} > \theta_n \). The labor market for participants is therefore more tight, implying that vacancies open for these job-seekers are filled at a lower rate. The last equality implicitly defines a positive relationship between \( \theta_{T, n} \) and \( \theta_n \):

\[
\theta_{T, n} = T(\theta_n | \kappa_T, n, \kappa_n, k_n) \text{ with } \frac{\partial T}{\partial \theta_n} > 0 \text{ and } \frac{\partial T}{\partial \kappa_T, n} < 0.
\]

The next step consists in replacing \( rV_{U, n} \) in (21) by a function of \( \theta_n, \theta_{T, n}, s_{U, n}, s_{X, n}, s_{T, n} \) and the parameters of the model. Substituting expressions (6) and (7) in (2) allows to write \( rV_{U, n} \) as a function of \( V_{E, n} - V_{U, n} \). Combining (13), (15), (19) and the free-entry conditions allows to rewrite \( V_{E, n} - V_{U, n} \) as:

\[
\mathcal{V}(\theta_n | \beta_n, k_n, \kappa_n, r, \phi_n, y_n) = \frac{\beta_n}{1 - \beta_n (1 - (r + \phi_n)(\frac{k_n}{q(\theta_n)} + \kappa_n))} \text{ with } \frac{\partial \mathcal{V}}{\partial \theta_n} > 0.
\]

Henceforth, \( \mathcal{V}(\cdot) \) will designate \( \mathcal{V}(\theta_n | \beta_n, k_n, \kappa_n, r, \phi_n, y_n) \). Let \( B_n = (b_{T, n}, b_{U, n}, b_{X, n}) \) and \( Z_n = (c_n, c_{T, n}, \phi_n, \gamma_n, \lambda_n, \pi, \kappa_n, \kappa_{T, n}, k_n, y_n, r, \beta_n) \). Using (24) and (15) yields then an explicit (net) 'wage-setting curve':

\[
\ln(w_n) = WS(\theta_n, \theta_{T, n}, s_{T, n}, s_{U, n}, s_{X, n} | Z_n, B_n) \equiv \frac{\left[ r + c_{T, n}s_{T, n}\alpha(\theta_{T, n}) + \lambda_n \right] [r + \gamma_n + c_n s_{X, n}\alpha(\theta_n)]}{\Delta_2, n \Delta_3, n} [v_{U, n} + c_n s_{U, n}\alpha(\theta_n)] \mathcal{V}(\cdot) + \frac{\gamma_n}{\Delta_1, n} [v_{T, n} + c_{T, n}s_{T, n}\alpha(\theta_{T, n}) \mathcal{V}(\cdot)] + \frac{\pi [r + c_{T, n}s_{T, n}\alpha(\theta_{T, n}) + \lambda_n]}{\Delta_2, n \Delta_3, n} [v_{X, n} + c_n s_{X, n}\alpha(\theta_n)] \mathcal{V}(\cdot) + (r + \phi_n)\mathcal{V}(\cdot).
\]

For each skill \( n \), the 'wage-setting curve' \( WS(\theta_n, \theta_{T, n}, s_{T, n}, s_{U, n}, s_{X, n} | Z_n, B_n) \) is upward-sloping in a \((\theta_n, w_n)\) space. It is easily seen that an increase in \( \theta_{T, n} \) shifts the wage-setting curve upwards. It can be shown that this curve is not affected by marginal changes in search effort levels (see also Fredriksson and Holmlund, 2001, and Lehmann and Van der Linden, 2002). Equality (23) can be substituted in (25) to yield another wage setting equation \( WS(\theta_n, s_{T, n}, s_{U, n}, s_{X, n} | Z_n, B_n) \) equal to:

\[
WS(\theta_n, T(\theta_n | \kappa_T, n, \kappa_n), s_{T, n}, s_{U, n}, s_{X, n} | Z_n, B_n).
\]

The downward-sloping 'vacancy-supply' curve (16) and the upward-sloping wage-setting equation \( \ln(w_n) = WS(\theta_n, s_{T, n}, s_{U, n}, s_{X, n} | Z_n, B_n) \) define the equilibrium value of \( w_n \) and
Taking the ln of (16) yields an implicit equation for \( \theta_n \), namely \( F(\theta_n, s_{T,n}, s_{U,n}, s_{X,n} \mid Z_n, \tau_n, B_n) = 0 \) with :

\[
F(\cdot) \equiv \ln(VS(\theta_n \mid \kappa_n, k_n, y_n, r, \phi_n, \tau_n)) - WS(\theta_n, s_{T,n}, s_{U,n}, s_{X,n} \mid Z_n, B_n)
\]

(27)

Marginal changes in search effort do not affect function \( F \).

### 2.7 Search effort as a function of tightness

In a symmetric equilibrium, Expression (24) can be substituted for \( V_E - V_U \) in the first-order conditions (9), (10) and (11) in which \( V_U - V_X \) has first been replaced by (6) and \( V_T - V_U \) by (7). After some manipulation, this leads for each \( n \) to:

\[
\Sigma_U(\theta_n, s_{U,n} \mid Z_n, B_n) \equiv \psi_n s_{U,n}^{\xi_{n}^{w}} - c_n \mathcal{V}(\cdot) = 0, \tag{28}
\]

\[
\Sigma_X(\theta_n, s_{U,n}, s_{X,n} \mid Z_n, B_n) = 0 \tag{29}
\]

with

\[
\Sigma_X \equiv \Delta_{2,n} \psi_n s_{X,n}^{\xi_{n}^{w}} - c_n s_{X,n}^{\xi_{n}^{w}} \alpha(\theta_n) \left[ \delta_{U,3,n} + (\Delta_{2,n} + c_n s_{U,n} - s_{X,n} \alpha(\theta_n)) \mathcal{V}(\cdot) \right],
\]

\[
\Sigma_T(\theta_n, \theta_T,n, s_{U,n}, s_{X,n}, s_{T,n} \mid Z_n, B_n) = 0 \tag{30}
\]

with

\[
\Sigma_T \equiv \Delta_{2,n} \Delta_{3,n} \psi_n s_{T,n}^{\xi_{n}^{w}} - c_T s_{T,n} \alpha(\theta_T,n) \left[ (\Delta_{2,n} \Delta_{3,n} - [r + c_n s_{X,n} \alpha(\theta_n) + \gamma_n]) \right.
\]

\[
\left. \left[ c_T s_{T,n} \alpha(\theta_T,n) - c_n s_{U,n} \alpha(\theta_n) \right] - \pi [c_T s_{T,n} \alpha(\theta_T,n) - c_n s_{X,n} \alpha(\theta_n)] \right] \mathcal{V}(\cdot)
\]

\[
- (c_T s_{X,n} \alpha(\theta_n) + \gamma_n) \delta_{U,n} - \pi \delta_{T,X,n}.
\]

Totally differentiating equations (28), (29) and (30), it can be checked that \( \frac{\partial \Sigma_i}{\partial \tau_i} = 0 \quad \forall i, i' \in \{ \{T, n\}, \{X, n\}, \{U, n\} \}, i \neq i' \). Moreover, levels of search effort increase with tightness.

### 2.8 Extending the model

Conditional on \( (Z_n, \tau_n, B_n) \), it is easily seen that the equilibrium, if any, is unique. Up to now, the budget constraint of the State has been ignored. Let \( P_n \) denote the exogenous size of the working age population endowed with skill \( n \) and \( P \equiv \sum_n P_n \). The participation rate \( p_n \) is \( L_n/P_n \). Let lower case letters \( e_n, u_n, x_n, t_n, v_n \) and \( v_{T,n} \) be the rates obtained by dividing the absolute numbers by the corresponding size of the labor force \( L_n \) (e.g. \( e_n \equiv E_n / L_n \)). The budget of the State scaled by \( P \) can be written as follows:

\[
\frac{Q}{P} + \sum_n (b_{U,n} u_n + b_{X,n} x_n + (b_{T,n} + C) t_n) p_n \frac{P_n}{P} = \sum_n \tau_n w_n (e_n + e_{T,n}) p_n \frac{P_n}{P}, \tag{31}
\]
where $Q$ is an exogenous level of net expenses and $C$ is the average cost per ALMP participant.\footnote{More complex cost functions could here be introduced.} Constraint (31) establishes the unique direct link between the two labor markets. To meet this constraint, one could either adjust the level of allowances $(B_n)$ or the one of taxes. Adjusting taxes is the most standard approach. However, Rocheteau (1999) has shown that this can lead to multiple equilibria. Appendix 2 shows that the uniqueness of equilibrium can be preserved if the replacement ratios are constant.

Participation is modeled in a very simple way (see Pissarides, 2000). Inactive people have an arbitrage condition: Staying inactive or entering state $X_n$.\footnote{Alternatively, they could enter uninsured unemployment (i.e. start an unemployment spell without any benefit). However, in many OECD countries, people who are ready to take a job and have no income are eligible to a minimum income guarantee. The latter is typically related to the lowest level of unemployment benefits. So, the simplifying assumption made here is not a substantial limitation.} Let $[V_{1,n}, V_{2,n}]$ be the finite support of the distribution of intertemporal utility levels in inactivity, $V_{I,n}$. With a uniform distribution, the participation rate is simply defined as

$$p_n = \frac{V_{X,n} - V_{1,n}}{V_{2,n} - V_{1,n}}. \quad (32)$$

### 3 Decomposing the effects of the ALMP

Calmfors, Forslund and Hemström (2002) enumerate a wide range of effects of ALMPs. This complex picture should in principle become more intricate since search effort is endogenous. The above model actually allows to simplify the presentation. The focus will be on the “policy parameters” characterizing the ALMP, namely $\gamma_n, \lambda_n, c_{T,n}$ and $\kappa_{T,n}$.

#### 3.1 Direct effects

Direct effects can be defined as impacts of $c_{T,n}, \gamma_n$ and $\lambda_n$ conditional on $\theta_n, \theta_{T,n}, s_{U,n}, s_{X,n}$ and $s_{T,n}$. In steady state, the stocks of individuals in each position ($U_n, X_n,...$) are constant. Equalities between entries and exits in each state and the identity $1 \equiv e_n + c_{T,n} + u_n + x_n + t_n$ determine $e_n, c_{T,n}, u_n, x_n$ and $t_n$. If

$$\Delta_{5,n} \equiv [c_{T,n} s_{T,n} \alpha(\theta_{T,n}) + \lambda_n] ([c_n s_{U,n} \alpha(\theta_n) + \phi_n] [c_n s_{X,n} \alpha(\theta_n) + \gamma_n] \\
+ \pi [c_n s_{X,n} \alpha(\theta_n) + \phi_n] + \gamma_n [c_{T,n} s_{T,n} \alpha(\theta_{T,n}) + \phi_n] [\pi + c_n s_{X,n} \alpha(\theta_n) + \gamma_n], \quad (33)$$
the employment rate is:

\[ e_n + e_{T,n} = \left[ c_{T,n} s_{T,n} \alpha(\theta_{T,n}) + \lambda_n \right] \left( c_n s_{U,n} \alpha(\theta_n) \right) \left[ c_n s_{X,n} \alpha(\theta_n) + \gamma_n \right] \\
+ \pi c_n s_{X,n} \alpha(\theta_n) + \gamma_n c_{T,n} s_{T,n} \alpha(\theta_{T,n}) \left[ \pi + c_n s_{X,n} \alpha(\theta_n) + \gamma_n \right] \Delta_{\theta_n}^{-1}. \] 

(34)

**Proposition 2.** For each skill \( n \),

1. The employment rate \( e_n + e_{T,n} \) increases with \( \theta_n, \theta_{T,n}, s_{U,n}, s_{X,n}, s_{T,n} \) and the parameter \( c_{T,n} \).

2. \( \frac{\partial e_n + e_{T,n}}{\partial \gamma_n} \) can be positive if \( c_{T,n} s_{T,n} \alpha(\theta_{T,n}) \) is sufficiently larger than \( c_n s_{U,n} \alpha(\theta_n) \) and \( c_n s_{X,n} \alpha(\theta_n) \).

3. \( \frac{\partial e_n + e_{T,n}}{\partial \lambda_n} \leq 0 \) if \( c_{T,n} s_{T,n} \alpha(\theta_{T,n}) \geq c_n s_{U,n} \alpha(\theta_n) \) and \( c_{T,n} s_{T,n} \alpha(\theta_{T,n}) \geq c_n s_{X,n} \alpha(\theta) \) and \( c_{T,n} s_{T,n} \alpha(\theta_{T,n}) \) and \( c_{T,n} s_{T,n} \alpha(\theta_{T,n}) < c_n s_{U,n} \alpha(\theta_n) \) and \( c_{T,n} s_{T,n} \alpha(\theta_{T,n}) < c_n s_{X,n} \alpha(\theta) \).

**Proof.** From (33) and (34), the marginal effects of \( s_{T,n}, s_{U,n}, s_{X,n}, c_{T,n}, \theta_n \) and \( \theta_{T,n} \) are clear. Moreover,

\[
\frac{\partial e_n + e_{T,n}}{\partial \gamma_n} = \frac{\phi_n(c_{T,n} s_{T,n} \alpha(\theta_{T,n}) + \lambda_n)}{\Delta_{5,n}^2} \left[ \pi(c_{T,n} s_{T,n} \alpha(\theta_{T,n}) + \lambda_n + \gamma_n) c_n s_{U,n} - s_{X,n} \alpha(\theta_n) \right] \\
+ (\pi + c_n s_{X,n} \alpha(\theta_n) + \gamma_n)(c_n s_{X,n} \alpha(\theta_n) + \gamma_n)(c_{T,n} s_{T,n} \alpha(\theta_{T,n}) - c_n s_{U,n} \alpha(\theta_n)) \\
+ \pi(c_{T,n} s_{T,n} \alpha(\theta_{T,n}) - c_n s_{X,n} \alpha(\theta_n))).
\]

Since \( s_{U,n} < s_{X,n} \), this derivative is only positive if \( c_{T,n} s_{T,n} \alpha(\theta_{T,n}) \) is sufficiently larger than the two other exit rates.

\[
\frac{\partial e_n + e_{T,n}}{\partial \lambda_n} = \frac{\phi_n \gamma_n(\pi + c_n s_{X,n} \alpha(\theta_n) + \gamma_n)}{\Delta_{5,n}^2} \left[ (c_n s_{X,n} \alpha(\theta_n) + \gamma_n)(c_n s_{U,n} \alpha(\theta_n) - c_{T,n} s_{T,n} \alpha(\theta_{T,n})) \\
+ \pi(c_n s_{X,n} \alpha(\theta_n) - c_{T,n} s_{T,n} \alpha(\theta_{T,n}))).
\]

Corollary 1. If \( \pi = 0 \), \( c_{T,n} s_{T,n} \alpha(\theta_{T,n}) > c_n s_{U,n} \alpha(\theta_n) \) is a sufficient condition for \( \frac{\partial e_n + e_{T,n}}{\partial \gamma_n} \) to be positive.
Proof. Substitute \( \pi = 0 \) in the proof of the previous proposition.

Everything else constant, raising the efficiency of the ALMP (increasing \( c_{T,n} \)) is, as expected, favorable to employment. The direct effect of the rate of entry into programs (\( \gamma_n \)) strongly depends however on the relative values of the hiring rates. If the exit rate of participants is the highest among the three groups of job seekers, the effect of an increase in \( \gamma_n \) can still be ambiguous for each additional unemployed who flows from state \( U_n \) into training would instead have increased his (her) search effort at the moment of entry in state \( X_n \) (if \( \pi > 0 \)). The corollary implies that the conditions needed to get a positive direct effect of \( \gamma_n \) on the employment rate are less numerous when unemployment benefits are constant. One cannot say more since the level of the endogenous variables present in the above conditions vary whether unemployment benefits are constant or not.

3.2 Effects on vacancy supply

Nor the vacancy-supply relationship (16) nor the relationship between \( \theta_{T,n} \) and \( \theta_n \) (23) shift if \( c_{T,n}, \gamma_n \) or \( \lambda_n \) are changed. Having expressed the model in terms of tightness in efficiency units explains the lack of effect of \( c_{T,n} \) on the position of the vacancy-supply curve. From (23), for each value of \( \theta_n \), equilibrium tightness \( \theta_{T,n} \) increases if the ALMP becomes more efficient in the sense of requiring a lower fixed recruitment cost \( H_{T,n} \) (or \( \kappa_{T,n} \)). The intuition is straightforward. Since the ALMP has to be financed, it is also useful to notice that the vacancy-supply curve shifts downwards if the marginal tax rates are augmented.

3.3 Effects on wage setting

Recall that marginal changes in search effort do not affect the position of the wage-setting curve (26). The wage-push effect of active labor market policies, a major point made by Holmlund and Lindén (1993), remains here. Put another way, increasing the size (\( \gamma_n \)) or the efficiency (\( c_{T,n} \)) of the ALMP increases wage pressure. This should be understood at constant tightness and tax levels. By the way, notice that the framework developed above allows to sign the net impact of the variety of effects on wage formation enumerated in Calmfors, Forslund and Hemström (2002). The wage-setting curve (26) does not simply reflect bargaining, however. This curve is also shaped by the free entry conditions. This explains in particular the role of \( \kappa_{T,n} \). We have just seen that increasing this parameter
shifts the relationship between tightness levels (23) downwards. A less tight market for ALMP participants reduces wage pressure. To sum up,

**Proposition 3.** For any $\theta_n$, the wage increases (resp. decreases) with $\gamma_n$ and $c_{T,n}$ (resp., $\lambda_n$ and $\kappa_{T,n}$).

**Proof.** Remembering Proposition 1, it can be checked that

$$\frac{\partial WS}{\partial \theta_n} = \frac{r + c_{T,n}\theta_n}{\Delta_{3,n}} + \lambda_n \Delta_{3,n},$$

$$\frac{\partial WS}{\partial \lambda_n} = -\gamma_n \Delta_{3,n},$$

$$\frac{\partial WS}{\partial c_{T,n}} = \gamma_n \psi_n \theta_{n}\Delta_{3,n},$$

and

$$\frac{\partial WS}{\partial \kappa_{T,n}} = \frac{\partial WS}{\partial \theta_{T,n}} \frac{\partial \theta_{T,n}}{\partial \kappa_{T,n}} < 0.$$ 

3.4 Effects on equilibrium tightness $\theta_n$

The net effect on tightness is clearly key. The policy parameters do not affect the vacancy-supply curve (16). So, their impact on equilibrium tightness $\theta_n$ is an immediate consequence of their impact on the wage-setting curve. Any variation that has a wage-push effect leads to a higher equilibrium net wage and lower tightness $\theta_n$ and conversely:

**Proposition 4** For each skill $n$, the equilibrium net wage $w_n$ (respectively, the level of tightness $\theta_n$) increases (respectively, decreases) with $\gamma_n$ and $c_{T,n}$. The equilibrium net wage $w_n$ (respectively, the level of tightness $\theta_n$) decreases (respectively, increases) with $\lambda_n$ and $\kappa_{T,n}$. The marginal tax rate $\tau_n$ has a negative effect on the equilibrium wage and on tightness.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>$s_{U,n}$</th>
<th>$\theta_n$</th>
<th>$s_{U,n}$</th>
<th>$\theta_n$</th>
<th>$s_{X,n}$</th>
<th>$\theta_n$</th>
<th>$s_{X,n}$</th>
<th>$\theta_n$, $\theta_{T,n}$</th>
<th>$s_{T,n}$</th>
<th>$\theta_n$</th>
</tr>
</thead>
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<tr>
<td>$\gamma_n$</td>
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<td>-</td>
<td>-</td>
<td>+</td>
<td>-</td>
<td>+</td>
<td>+</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>$\lambda_n$</td>
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<td>+</td>
<td>0</td>
<td>+</td>
<td>+</td>
<td>+</td>
<td>+</td>
<td>+</td>
<td>+</td>
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</tr>
<tr>
<td>$c_{T,n}$</td>
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<td>0</td>
<td>-</td>
<td>+</td>
<td>0</td>
<td>+</td>
<td>*</td>
<td>+</td>
<td>-</td>
</tr>
<tr>
<td>$\kappa_{T,n}$</td>
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<td>0</td>
<td>+</td>
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<td>+</td>
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<td>+</td>
</tr>
<tr>
<td>$\tau_n$</td>
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<td>-</td>
<td>0</td>
<td>-</td>
<td>0</td>
<td>*</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>

* i.e. $s_{U,n}$ solving $\Sigma_U(\theta_n, s_{U,n} \mid Z_n) = 0$.
\[\dagger\] i.e. $s_{X,n}$ solving $\Sigma_X(\theta_n, s_{U,n}, s_{X,n} \mid Z_n) = 0$.
\[\ddagger\] i.e. $s_{T,n}$ solving $\Sigma_T(\theta_n, \theta_{T,n}, s_{U,n}, s_{X,n}, s_{T,n} \mid Z_n) = 0$.

* means that the adjustment of $\theta_n$ (and $\theta_{T,n}$) in equilibrium is taken into account.

Table 1. Search effort levels: Comparative statics.
3.5 Effects on search effort

Keeping $\theta_n, \theta_{T,n}$ and intertemporal utility levels as fixed, a more effective ALMP (higher $c_{T,n}$) stimulate search effort among the participants (see (11)). From Proposition 2, this raises the employment rate. Moreover, looking at (9), (10) and (11), it is obvious that a more tight labor market stimulates search effort for given values of the intertemporal utility levels. Taking the adjustment of these levels into account complicate the reasoning. The effects of the parameters of interest can be derived from (28), (29) and (30) (conditional on $\theta_n$ and $\theta_{T,n}$).\footnote{See Table 1. This table also presents the comparative static properties when the adjustment of $\theta_n$ and $\theta_{T,n}$ is taken into account according to (27) and (23) (see columns $s^*_{U,n}, s^*_{X,n}$ and $s^*_{T,n}$). Table 1 indicates that in addition to its wage-push effect (Proposition 4), $\gamma_n$ also reduces the equilibrium level of job-search effort in states $U_n$ and $X_n$. The prospect of entering more rapidly in an ALMP gives an incentive to search less. This effect can be related to the so-called “Ashenfelter dip” (Ashenfelter, 1978). Ashenfelter came to the conclusion that the earnings of participants fell before they enroll in a training program.\footnote{See also Smith, Lise and Seitz (2003) who conclude that some welfare recipients delay exit in order to qualify for an income supplement.} The effect of $\gamma_n$ on $s^*_{T,n}$ is ambiguous. Increasing the efficiency parameter $c_{T,n}$ has an ambiguous effect on $s^*_{T,n}$ and a negative impact n $s^*_{U,n}$ and $s^*_{X,n}$. Reducing the relative training cost $\kappa_{T,n}$ has favorable effects on search effort through the adjustment of tightness. The tax parameters affect equilibrium search efforts via their impacts on tightness, too.} The effectsoftheparametersofinterestcanbederivedfrom(28),(29)and(30)(conditionalon

3.6 Effects on participation to the labor market and on tax rates

From (32), policy parameters that improve the intertemporal utility of job searchers $V_{X,n}$ will raise participation. $V_{X,n}$ can be written as:\footnote{This expression can be computed by exploiting $V_{X,n} = V_{E,n} - (V_{U,n} - V_{X,n})$ and (6). $V_{U,n}$ can then be replaced by $V_{E,n} - (V_{E,n} - V_{U,n})$ with $V_{E,n}$ defined by (1). Finally, $V_{E,n} - V_{U,n} = V(\cdot)$ is substituted everywhere.} 

$$V_{X,n} = \frac{\ln(w_n) - (r + \phi_n)\mathcal{V}(\cdot)}{r} - \frac{\delta_{U,n} + c_n(s_{U,n} - s_{X,n})\alpha(\theta_n)\mathcal{V}(\cdot)}{\Delta_{2,n}}.$$ (35)
The impact of the policy parameters on $V_{X,n}$ is hard to sign analytically. The participation rate mainly influences the budget constraint of the State (31). From Propositions 2 and 4 and Table 1, any parameter that affects positively the marginal tax rate $\tau_n$ will have a negative indirect effect on equilibrium tightness and on the employment rate.

### 3.7 Summary of the analytical properties

I focus here on one evaluation criterion, namely employment. With respect to the rate of entry into the ALMP ($\gamma_n$), one knows that $\theta_{T,n} > \theta_n$, $\forall n$, in equilibrium. This and the plausible assumption that $c_{T,n} \geq c_n$ can imply that the hiring rate of participants will be greater than the hiring rates of those in states $U_n$ and $X_n$. From Proposition 2, it is then plausible (but not sure) that $\gamma_n$ has a positive direct effect on the employment rate. The marginal effect of $\gamma_n$ is however negative on tightness and on search effort levels in states $U_n$ and $X_n$. In sum, even if one ignores the financing of the policy, increasing the scale of an ALMP (of the type considered here) has a lot of negative induced effects.

Increasing the matching effectiveness of participants ($c_{T,n}$) has a clear positive direct effect on the employment rate but tightness and search effort in states $U_n$ and $X_n$ decrease because of a wage-push effect. If the efficiency of the ALMP improves through a decrease in the relative fixed cost of recruiting a worker, $\kappa_{T,n}$, there is a direct positive effect on the indicator of tightness $\theta_{T,n}$ relevant for the participants but there are also indirect negative impacts because tightness $\theta_n$ declines.

### 4 A numerical analysis

#### 4.1 Calibration

The model is calibrated for Belgium with the month as unit of time. Various surveys\(^{16}\) and published statistics have been used to calibrate the model. The period 1997-1998 has been used as a reference.\(^{17}\) Belgium is a country plagued with long-term unemployment. For a very long time, more than 60% of the stock is unemployed for more than a year. In

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\(^{17}\)The years close to 1993 were deeply affected by the major recession of the nineties. The last years of this decade were clearly a boom. During the period 1997-1998 the unemployment rate was fairly stable.
Belgium, negative duration dependence is very strong but Cockx and Dejemeppe (2002) and Dejemeppe (2003) have shown that it is largely spurious. Due to statistical availability, only two levels of skill are distinguished. It is assumed that holding at most a lower-secondary degree captures relatively well the notion of ‘low skill’. Low-skilled workers then represent about 34% of the labor force and 64% of the stock of unemployed.

There is first a period of one year where unemployment benefits stay constant. For the calibration, \( \pi \) is therefore equal to 0.083. For about two thirds of the insured unemployed, the level of benefits decreases afterwards. In 1998, less than 2% of the unemployed have lost their entitlement (after a very long spell of unemployment). This phenomenon is therefore neglected. Short-duration vocational training for the unemployed is the ALMP considered here. A microeconometrical evaluation by Cockx and Bardoulat (1999) concludes that Belgian vocational training programs enhance the exit rate of the participants. In accordance with the model, programs that put the unemployed back to school are ignored. In 1998, according to EUROSTAT data, the average stock of jobless people participating in training programs amounted to 0.67% of the active population and the average cost of training programs per worker amounted to 669 EURO/month (net of transfers to beneficiaries of these programs).

The calibration procedure is explained in details in Van der Linden (2003). Here, I focus on the essentials. The calibrated parameters are shown in Table 2. The discount rate is fixed at 0.004 (5% on an annual basis). Annual reports of the PES allow to fix parameters \( \lambda_n \) and \( \gamma_n \) (see Table 2).

As many other papers, let us assume the following Cobb-Douglas matching function (see e.g. Broersma and van Ours, 1999): 
\[
m(S_n, V_n) \equiv m_0 S_n^{0.5} V_n^{0.5} \text{ and } m(S_{T,n}, V_{T,n}) \equiv m_0 S_{T,n}^{0.5} V_{T,n}^{0.5}.
\]
Parameter \( m_0 \) is a scaling factor for the various \( c_\iota \)'s. Assuming that \( m_0 = 0.5 \) yields reasonable values.

The expected duration of a vacancy (2.5 month) and the share of the low-skilled in the total number of recruitments (0.38) is used to calibrate the \( \theta \)'s. The ‘vacancy-supply curves’ (15) are then used to calibrate the \( k \)'s. An assumption about \( \kappa_{T,n} \) is needed in order to calibrate \( \theta_{T,n}, n \in \{h,l\} \). There is no evidence against the assumption that \( \kappa_{T,l}/\kappa_l = \kappa_{T,h}/\kappa_h \).

For various values of this ratio, the flow equilibrium conditions are used to fix the products \( c_\iota s_\iota, \ i = \{T, n\}, \{X, n\}, \{U, n\}, n \in \{l, h\} \). Conditional on these products, the calibration then fixes the \( c_\iota \)'s, the \( s_\iota \)'s, \( \xi_n \), \( \psi_n \) and the bargaining power of the workers \( \beta_n \). This part of the calibration is based on equations (25), (28), (29), (30) and on additional equations.
stipulating a value for the elasticity of unemployment duration with respect to the level of unemployment benefits. One adopts the lowest ratio $\kappa_{T,n}/\kappa_n$ compatible with $c_{T,n}/c_n \geq 1 \forall n$. So, $\kappa_{T,n}/\kappa_n = 0.85, \forall n$. As expected, the labor market is more tight for trainees than for other job-seekers (see Table 2). It turns out that skilled workers search more intensively. As expected, they have higher matching effectiveness parameters. The calibrated values also imply that the wage elasticity of salaried employment amounts to reasonable values, namely -0.72 for low-skilled workers and -0.25 for skilled ones. Finally, as far as participation is concerned, the elasticity of $p_n$ with respect to $w_n$ is fixed to 0.25 for both skill groups.

4.2 Simulation results

In the following simulations, the rates of entry $\gamma_n$ are the same for both skill groups and vary from 0 to 0.1. Attention will be paid to the certainty equivalents, $\exp[rV]$, of the skill- and state-specific $V$’s and to a utilitarian criterion for the active population $r\Psi \equiv \sum_n r\Psi_n \frac{L_n}{L}$ with $L \equiv \sum_n L_n$ and $r\Psi_n \equiv (\exp[rV_{E,n}](e_n + e_{T,n}) + \exp[rV_{U,n}]u_n + \exp[rV_{X,n}]x_n + \exp[rV_{T,n}]t_n)$. Let us first keep all the other parameters at their calibrated values. Focusing on the low-skilled, Figure 2 summarizes a simulation where taxation and the level of unemployment benefits are fixed. Since $b_{T,n}$ is only slightly higher than $b_{U,n}$, the wage-push effect of training programs mainly comes through better employment prospects for trained individuals. Figure 2 highlights a moderate positive effect on $w_l$ and a more substantial negative impact on tightness $\theta_l$ (and $\theta_{T,l}$). Search effort levels and the employment rate are strongly decreasing. The aggregate unemployment rate $u + x$ is declining but ‘open unemployment’ $(u + x + t)$ is strongly increasing. Nevertheless, increasing $\gamma$ improves the intertemporal utility levels of all groups in the active population (Figure 2 only displays the average).

Corollary 1 suggests that the direct effect of $\gamma_n$ on $e_n + e_{T,n}$ depends on the level of $\pi$. To illustrate that point, let us keep the assumptions of the previous simulation except that $\pi$ is now set equal to zero. The relationship between $e_l + e_{T,l}$ and $\gamma$ is now (slightly) positive. Since a benefit system where $\pi = 0$ is more generous than the one where $\pi = 0.083$, this example illustrates the lack of generality of the assertion of Coe and Snower (1997) according to which “the more generous are passive unemployment policies, the less effective will be active unemployment policies” (p. 22). However, unreported simulation results show that this sentence holds true as far as the level of benefits is concerned.

Training schemes entail a cost in addition to the transfer to the beneficiaries. This cost
is now taken into account in a crude way (see (31)). To avoid multiple equilibria, the replacement ratios are now constant. $\pi$ is also back to its calibrated value (0.083). Figure 4 shows that the wage-push effect of training schemes is now more than compensated by the depressing effect of higher tax rates. With constant replacement ratios, the level of benefits is declining, too. These effects would lead to the expectation that the profile of tightness will be more favorable compared to the case where taxes are fixed (Figure 2). This expectation is not verified for higher taxes are detrimental to the creation of vacancies. Eventually, the net effect on the employment rates is negative. In the inter-temporal utility functions, the decrease in search effort and the more probable entry in training schemes are more than compensated by the depressed employment perspectives for the unemployed and the lower levels of income. Hence, increasing $\gamma$ harms each component of the workforce.

Are these pessimistic conclusions robust? When $\pi = 0$, unreported simulation results show that $e_l + e_{T,l}$ is now again increasing with $\gamma$. However, the welfare analysis leads exactly to the same qualitative conclusions. Another sensitivity analysis would consist in lowering the ratio $\frac{\kappa_T\kappa_n}{\kappa_n}$. Figure 5 shows the impact of $\gamma$ when the ratio $\frac{\kappa_T\kappa_n}{\kappa_n} = 0.5$ (instead of 0.85) and no other parameter is changed. As far as labor market indicators are concerned, Figure 5 is qualitatively similar to but quantitatively different from Figure 4. Furthermore, even if it is still declining with $\gamma$, the level of tightness $\theta_{T,n}$ is now much higher for both skill groups. This really boosts search effort levels $s_{T,n}$. When $\gamma$ is increasing, all tightness indicators are declining at a similar pace but search effort levels are more rapidly declining for the low skilled. This difference explains why intertemporal utility levels of the low and the high skilled people are now vary in opposite directions. In sum, if the ratio $\frac{\kappa_T\kappa_n}{\kappa_n}$ was much lower, labor market indicators would still be better without them. In addition, the favorable effects on the welfare of the low skilled would come from differences in the pace of decline of search effort when the rate of entry into training rises.

5 Conclusion

This paper has developed an equilibrium matching model that is well-suited to conduct evaluations of short-duration ALMPs. In this model, workers are risk averse and heterogeneous in skill, job-search is endogenous and wages are bargained over. ALMPs do not only improve the fall-back position of the workers. They also improve the matching effectiveness of
participants and reduce the cost that firms incur when they recruit workers.

ALMPs have quite complex effects in general equilibrium (see Calmfors, Forslund and Hemström 2002). This paper has introduced new relationships (in particular the role of programs on search effort) and yet many clear-cut effects have been analytically shown. Under certain conditions that are more stringent in the presence of a two-tiered benefit system, increasing the rate of entry in ALMPs has a positive direct effect on the employment rate. However, the indirect effects are detrimental to employment. This certainly questions the rationale of a massive use of short-duration ALMPs.

This paper has also developed a simulation exercise. The calibration has been based on an extensive and well-informed use of statistics and studies for Belgium. Short-duration vocational training programs have been evaluated. The results strongly emphasize the importance of the choice of the evaluation criterion. Indeed, performance indicators of the labor market and welfare criteria quite often lead to opposite conclusions, in particular because the latter takes care of the disutility of job-search effort. Despite its microeconomic favorable effect on the hiring rate, vocational training for the unemployed appears to have harmful net effects in Belgium when its financing is taken into account. It should be emphasized that this paper has not dealt with programs covering possibly several years and intending to lift the productivity of the unemployed.

Appendix 1. Proof of Proposition 1

Let us prove that \( V_{U,n} > V_{X,n} \). If \( V_{U,n} \) was lower or equal to \( V_{X,n} \) and \( s_{U,n} \) was optimally chosen by each unemployed, the following inequalities would hold:

\[
rv_{U,n} = \ln(b_{U,n}) - \psi_n(s_{U,n})\xi_n + c_n s_{U,n} \alpha(\theta_n)(V_{E,n} - V_{U,n}) + \gamma_n(V_{T,n} - V_{U,n}) + \pi(V_{X,n} - V_{U,n})
\]

\[
\geq \ln(b_{U,n}) - \psi_n(s_{X,n})\xi_n + c_n s_{X,n} \alpha(\theta_n)(V_{E,n} - V_{U,n}) + \gamma_n(V_{T,n} - V_{U,n}) + \pi(V_{X,n} - V_{U,n})
\]

\[
> \ln(b_{X,n}) - \psi_n(s_{X,n})\xi_n + c_n s_{X,n} \alpha(\theta_n)(V_{E,n} - V_{X,n}) + \gamma_n(V_{T,n} - V_{X,n})
\]

\[
= rv_{X,n},
\]

which leads to a contradiction. Therefore, \( V_{U,n} > V_{X,n} \).

Similarly, if \( V_{U,n} \) was higher than \( V_{T,n} \) and \( s_{T,n} \) was optimally chosen by the trainee, the
Following inequalities would be verified:

\[ rV_{T,n} = \ln(b_{T,n}) - \psi_n \frac{(s_{T,n})^\xi_n}{\xi_n} + c_{T,n} s_{T,n} \alpha(\theta_{T,n})(V_{E,n}|T - V_{T,n}) + \lambda_n(V_{U,n} - V_{T,n}) \]

\[ \geq \ln(b_{T,n}) - \psi_n \frac{(s_{U,n})^\xi_n}{\xi_n} + c_n s_{U,n} \alpha(\theta_{T,n})(V_{E,n}|T - V_{E,n} + V_{E,n} - V_{U,n} + V_{U,n} - V_{T,n}) + (\lambda_n - \gamma_n + \gamma_n)(V_{U,n} - V_{T,n}) \]

\[ > \ln(b_{U,n}) - \psi_n \frac{(s_{U,n})^\xi_n}{\xi_n} + c_n s_{U,n} \alpha(\theta_{T,n})(V_{E,n}|T - V_{E,n} - V_{E,n} - V_{U,n}) + \gamma_n(V_{T,n} - V_{U,n}). \]

From (1) and the assumption that \( w_{T,n} \geq w_n \), one has \( V_{E,n}|T - V_{E,n} = \frac{\ln(w_{T,n}) - \ln(w_n)}{\tau + \phi_n} \geq 0 \). Therefore, assuming that \( \theta_{T,n} \geq \theta_n \) and knowing that \( V_{U,n} > V_{X,n} \), Expression (36) leads to:

\[ rV_{T,n} > \ln(b_{U,n}) - \psi_n \frac{(s_{U,n})^\xi_n}{\xi_n} + c_n s_{U,n} \alpha(\theta_n)(V_{E,n} - V_{U,n}) + \gamma_n(V_{T,n} - V_{U,n}) + \pi(V_{X,n} - V_{U,n}) \]

\[ = rV_{U,n}, \]

which leads to a contradiction. So, \( V_{T,n} > V_{U,n} \).

**Appendix 2. The budget constraint of the State and the uniqueness of equilibrium**

Assume that the net replacement ratios are constant (e.g. \( \rho_{U,n} = b_{U,n}/w_n \)). Let \( R_n \) designate the vector of net replacement ratios with \( R = (R_h, R_l) \). Let also \( L = (L_h, L_l) \). Equation (31) implies that at least one of the marginal tax rates has to become endogenous. In Section 4, all tax rates are assumed to vary proportionately. Rearranging (31), the marginal tax rates can then be written as functions of the rate of individuals in the various states, say

\[ \tau_n = g_n(\epsilon_n + e_{T,n}, u_n, x_n, t_n | R, L, Q, C). \]  

(36)

Obviously, \( \tau_n \) increases with the rates \( u_n, x_n, t_n \) and decreases with the employment rate.

Adapting (27), equilibrium tightness is now potentially a function of these rates:

\[ F(\theta_n, s_{T,n}, s_{U,n}, s_{X,n} | Z_n, g_n(\epsilon_n + e_{T,n}, u_n, x_n, t_n | R, L, Q, C), R_n, w_n) = 0, \]  

(37)

with \( w_n = V S(\theta_n | \kappa_n, k_n, y_n, \tau_n, \phi_n, g_n(\epsilon_n + e_{T,n}, u_n, x_n, t_n | R, L, Q, C)) \). Totally differentiating (37) and making use of (25) and (27) lead to \( \frac{dF}{d\theta_n} = -\frac{dS}{d\theta_n} \) and \( \frac{dF}{dQ} = \frac{dF}{du_n} = \frac{dF}{dx_n} = \frac{dF}{dt_n} = 0 \). Hence, equilibrium tightness is not a function of \( \epsilon_n + e_{T,n}, u_n, x_n, t_n \). Furthermore, the functions \( \Sigma \) are independent of \( \tau_n \).
References


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Table 2. Stocks and parameters.
†This average wage is above minimum wages. The latter are taken into account during the simulations.
‡The replacement ratios are lower than reported values by the OECD on the basis of a range of earnings and family situations (see e.g. Table A.1 of OECD, 1999). However, for Belgium, these OECD statistics exclude some groups whose replacement ratio is quite low.
* Since workers are risk averse, the Hosios conditions $\beta_n = 0.5$ does not necessarily guarantee that a laissez-faire economy is optimal (see Lehmann and Van der Linden, 2002). One could wonder why $\beta_l > \beta_h$. In Belgium, unionization is a widespread phenomenon, especially among blue-collar workers. This can explain why the bargaining power of low-skilled workers is higher.
Figure 1: Labor market flows.
Figure 2: Simulations of changes in $\gamma$ keeping benefits and tax rates unchanged; $\pi = 0.083$. Scale on the horizontal axis: $100 \times \gamma$.

Figure 3: Simulations of changes in $\gamma$ keeping benefits and tax rates unchanged; $\pi = 0$. Scale on the horizontal axis: $100 \times \gamma$. 

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Figure 4: Simulations of changes in $\gamma$ when the tax rates $\tau_n$ are adjusted to keep the budget of the State balanced; $\pi = 0.083$. Scale on the horizontal axis: $100 \times \gamma$. 
Figure 5: Sensitivity analysis: Simulations of changes in $\gamma$ if $\frac{\kappa_T n}{\kappa_n}$ equals 0.5 instead of 0.85. The tax rates $\tau_n$ are adjusted to keep the budget of the State balanced and $\pi = 0.083$. Scale on the horizontal axis: $100 * \gamma$. 