Specific investment, the ‘hold-up problem’ and
wage contracts
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1 Introduction

Some investments are specific to each particular match between a firm and a worker
and will be lost if the match is broken. Examples:
- formal firm-specific training (including the effort made to allocate the worker to
  the most appropriate occupation)
- other firm-specific investments such as buying a house next to your workplace,
  learning the rules of the firm, learning how to operate their computer equipment,...

On the other hand, there are also general investments (i.e. non-specific to a
particular match; in the case of workers’ human capital: training that is valued
equally by many (all) firms).

Generally, it is claimed that training which is neither perfectly general nor purely
specific can be regarded as the sum of a general and a specific component. Stevens
(1996) challenges this view and argues that most training, rather, is useful to a
limited number of firms (the notion of ‘transferable training’).

In addition, in many countries, firms have to make redundancy payments, seek
approval from public agencies when they fire workers. In addition, there are hiring
costs (advertising, interviewing, screening...). These are again payments specific to

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a given match. So, the role played by these specific costs is related to the one of specific investments. However, the focus will be here on specific investments.\(^1\)

Are the costs mentioned above important? The answer is yes but they are not easily measured in a correct way (see Malcomson, 1999, p. 2312, Hamermesh, 1993, p. 208, Abowd and Kramarz, 2003).\(^2\)

| Some data about the importance of these costs. Abowd and Kramarz (2003) analyze a sample of French establishments in manufacturing, construction and (some) service industries, with at least 20 employees. All costs are reported in 1992 French Francs (FF). The hiring cost per hire was on average 5560 FF (advertising, searching for and screening of applicants). In addition to that, training costs divided by total employment was on average 3025 FF. Termination costs per termination (all motives) amounted on average to 95,531 FF. These numbers can be compared to the average labor cost 171,022 FF. |

What are the implications for the labor market of investments specific to the match between a firm and a work? Do private agents invest efficiently in specific training or are these investments an additional reason why the labor market does not behave like a competitive one? If yes, why do public authorities feel important to enhance these investments (see e.g. the European Employment Strategy). These questions are raised for specific investments only. Yet, similar questions can be raised for general investments, too (see Malcomson, 1999, or Cahuc and Zylberberg, 2004, Chapter 11, Section 2.2.1).

2 A quick look at the literature


\(^1\)Notice that the so-called ‘insiders-outsiders’ literature (Lindbeck and Snower, 1988) is based on the idea that turnover costs give (ex post) bargaining power to the employees (whether they are unionized or not!). However, this literature does not emphasize the implications upon the level of investment and is not sufficiently rigorous (often static models with an underlying dynamic story). For the latter reason, it has been rather easy to challenge their main message (namely, “turnover costs are an explanation of (involuntary) unemployment”); see Vetter and Andersen (1994) and Booth (1997). There is also a large literature about labour demand when the firm faces turnover costs and random shocks. A summary can be found in Bertola (1999) or in Cahuc and Zylberberg (2004) (Chapter 4). According to the type of assumption made about the shape of turnover costs (linear vs convex; symmetric vs asymmetric), their effect on the dynamic adjustment of labour demand is very different. A general conclusion is that higher turnover costs reduce the cyclical variability of labour demand. On the contrary, their impact on the average level of labour demand is specific to each model.

\(^2\)This is also true for firing costs since the legislation is often complicate (see e.g. Commission, 1997).
2.1 Pre- and post training wages and investment levels are specified in the initial contract

This is the world of Becker and Mincer (who, in addition, assume perfect information and also perfect competition for general skills). At the start of the match, a contract is signed that specifies the pre- and post training wages and the investment levels. Moreover, this contract is enforceable (the information included in the contract is verifiable by a court in case of a disagreement between the worker and the firm). The conclusions of this literature can be found in any textbook and are summarized in Gibbons and Waldman (1999) (p. 2378-2381).

However, human-capital investments and many other investments specific to a given match are often not stipulated in contracts. Nor are they easily observed by a third party (courts). In addition, post-training wages are not often specified in contracts and they can often be renegotiated ex post (i.e. after the training has taken place). These observations led to a recent literature where contracts are incomplete.

2.2 Common wisdom revisited

The recent literature has relaxed the Beckerian framework through the introduction of asymmetric information (see Gibbons and Waldman, 1999, p. 2380-2381): the investment choice is not observed by other employers, the current employer observes the worker’s ability but potential employers do not, etc. Among other things, this literature leads to revised conclusions about the incentive firms have to pay for investment in general human capital.

Another branch of the recent literature is based on the following characteristics:
- symmetric information;
- because of turnover costs or specific investments, there are rents to be shared if the relationship continues;
- contracts are incomplete in the sense that they cannot be contingent on all the events that affect the payoffs of the parties;
- any contract can be renegotiated by mutual consent.

The next section discusses one of the implications of these assumptions (the so-called ‘hold-up’ problem) and some of the answers found in the literature.

3 Specific investment and the “hold-up problem”

Malcomson (1999) (p. 2311-2337) is an excellent reference. However, I develop here a simplified presentation based on Cahuc and Zylberberg (2004) (Chapter 11, Section 2.2.2; see also Chapter 9, Section 4.2). This presentation uses the equilibrium search approach. I here focus on the case where the firm invests. What follows can be adapted to the case where the employee makes a specific investment. This investment is now subject to hold-up initiated by the firm! Neither below nor in Malcomson’s survey, is the discussion concerned with trade union bargaining. This does not mean that hold-up is not an issue in a unionized setting. On the contrary!
But dealing with unions adds an employment dimension (and employment is linked to investment). Notice that the literature on the “hold-up problem” is closely related to the so-called ‘poaching externality’ in the training literature (see e.g. Stevens, 1996, and Moen and Rosen, 2004). It is also linked to a recent macroeconomic literature about the consequences of appropriability of quasi-rents (Caballero and Hammour, 1998).

To simplify the exposition, let us ignore taxes ($\tau = 0$). Moreover, search effectiveness $s$ is exogenous here and normalized to one. This is not crucial for the main arguments. Notations and formulas are simply adapted from the chapter “Equilibrium search models”. As in this chapter, we consider a steady-state equilibrium.

As in this chapter, information is symmetric (meaning in particular that the employers know $V_u$).\footnote{This assumption can be relaxed. See section 4.7 in Malcomson’s survey. It is based on Hall and Lazear (1984).} The following is new. The marginal product of a filled vacancy, $y$, is a function of a specific investment ($i$ is the real level of money invested in specific training) and of the “state of the world” denoted $S$ ($S = 1, \ldots, S$). $S$ can for instance capture technological shocks. The investment takes place before the realisation of state $S$ is known. It is assumed that the relationship between $y$ and $i$ is concave ($y_i(i, S) \equiv \frac{\partial y(i, S)}{\partial i} > 0$, $y_{ii}(i, S) \equiv \frac{\partial^2 y(i, S)}{\partial i^2} < 0$). To avoid the introduction of risk-sharing issues, let us assume risk-neutral agents.

### 3.1 Efficient investment

The (ex-post) total surplus of a match $\Sigma(i, S) \equiv J_e + V_e - J_v - V_u - i$. Extending formulas of Subsection 4 in the chapter entitled “Equilibrium search models”, it is easily seen that for each values of $S$ and $i$ the firm’s discounted expected return from a filled vacancy is in a steady state:

$$rJ_e(i, S) = y(i, S) - w(i, S) + \phi(J_v - J_e(i, S)).$$  \hfill (1)

Therefore:

$$J_e(i, S) = \frac{y(i, S) - w(i, S) + \phi J_v}{r + \phi},$$  \hfill (2)

where $J_v$ is independent of $i$ since the investment is specific to a particular match. Similarly,

$$V_e(i, S) = \frac{w(i, S) + \phi V_u}{r + \phi},$$  \hfill (3)

where $V_u$ is for the same reason also independent of $i$. So, the total surplus, $\Sigma$, can be written as:

$$\Sigma(i, S) = \frac{y(i, S) - r(J_v + V_u)}{r + \phi} - i.$$  \hfill (4)

The investment level that maximizes the expected surplus $E_S[\Sigma(i, S)]$ can be said to be efficient. Let $i^*$ denote this efficient level. It is easily seen that $i^*$ solves

$$E_S[y_i(i^*, S)] = r + \phi.$$  \hfill (5)
3.2 Complete contracts

The timing of events is as follows:
- stage 1: A wage contract is bargained that cannot be renegotiated; this wage is a function of \( i \) and \( S \) and is denoted by \( w(i, S) \);
- stage 2: Investment is chosen;
- stage 3: The state \( S \) is revealed.

It will be shown that the optimal investment level will be efficient. The basic idea of subgame perfect equilibrium is to require that the players' behavior be optimal in each situation. To obtain the perfect equilibrium, we move backwards:
- Stage 3. When \( S \) is known, taking \( i \) and \( w(i, S) \) as given, the firm’s discounted expected return from a filled vacancy is in a steady state given by Expression (2):

\[
J_e(i, S) = y(i, S) - w(i, S) + \phi J_v.
\]

(6)

- Stage 2. The optimal investment level, \( \tilde{i} \), conditional on \( w(i, S) \) solves:

\[
\max_i E_S \left[ \frac{y(i, S) - w(i, S) + \phi J_v}{r + \phi} \right] - i.
\]

(7)

The first-order optimal condition is:

\[
E_S [y(i, S) - w(i, S)] = r + \phi.
\]

(8)

- Stage 1. Following the chapter entitled “Equilibrium search model”, individual ex-post Nash bargaining is assumed. The component of the Nash product that captures the employer side is critical here. If there is an agreement, the investment will take place in stage 2 and production will take place in stage 3. So, the payoff in case of an agreement is \( J_e(i, S) - i \). If there is no agreement, the worker and the firm separate. Then, nothing is produced but the firm has not to invest either. So, the wage contingent on each realization of \( i \) and of the state \( S \) solves the following problem:

\[
\max_{w(i, S)} \left( V_e(i, S) - V_u \right)^{\beta} (J_e(i, S) - i - J_v)^{1-\beta}
\]

(9)

The solution (first-order condition) to that problem can be written in various ways. In particular, it can be shown that the firm receives a share \( 1 - \beta \) of the surplus created by the match:

\[
J_e(i, S) - i - J_v = (1 - \beta) \Sigma(i, S).
\]

(10)

From (4) and (6), this expression can be rewritten as:

\[
w(i, S) = (1 - \beta) r V_u + \beta [y(i, S) - (r + \phi) i] - \beta r J_v.
\]

(11)

This equality implies that \( w \) varies with \( i \) according to:

\[
w(i, S) = \beta [y(i, S) - (r + \phi)].
\]

(12)
This holds for any \((i, S)\) pair. Now, at this stage, the players anticipate the investment behavior characterized by (8). Combining (12) and (8) yields:

\[
E_S \left[ y_i(i, S) \right] = r + \phi. \tag{13}
\]

Put another way, \(\bar{I} = i^*\): With a complete contract that cannot be renegotiated, the equilibrium level of investment is efficient. So, in this complete contract setting, the contingent equilibrium wage is \(w(i^*, S), S = 1, ..., S\).

### 3.3 Incomplete contracts: The “hold-up” problem

If the wage is bargained over (or renegotiated) when the firm has already invested, the employee can reap part of the payoff generated by the investment.\(^4\) This phenomenon, the “hold-up” problem, is easily captured by the following two-stage game:

- **Stage 1**: The firm decides over investment.
- **Stage 2**: The wage is bargained afterwards at a moment when the state \(S\) is known.

Backward induction is used again.

- **Stage 2**. Since the investment has already been made, the net gain for the employer is \(J_e(i, S) - i\) if the negotiation is successful and \(J_v - i\) if it fails. So the bargained wage, denoted by \(\omega(i, S)\), solves:

\[
\max_w (V_e(i, S) - V_u)\beta (J_e(i, S) - J_v)^{1-\beta} \tag{14}
\]

It easily checked that \(\omega(i, S)\) verifies:

\[
\omega(i, S) = (1 - \beta) r V_u + \beta [y(i, S) - r J_v] \tag{15}
\]

*Compare with (11)!* Differentiating (15) with respect to \(i\) yields:

\[
\omega_i(i, S) = \beta y_i(i, S). \tag{16}
\]

- **Stage 1**. Anticipating this outcome, the firm will reconsider her investment. Compared to the complete contract case, the investment problem is now:

\[
\max_i E_S \left[ \frac{y(i, S) - \omega(i, S) + \phi J_v}{r + \phi} \right] - i. \tag{17}
\]

Let \(\bar{i}\) denote the optimal investment level in this setting. The first-order optimal condition is:

\[
E_S \left[ y_i(\bar{i}, S) - \omega_i(\bar{i}, S) \right] = r + \phi. \tag{18}
\]

\(^4\)Remember that this analysis can also be conducted in the case where the worker invests in specific capital.
Remembering (16), this expression can also be rewritten as:

$$E_S \left[ y_i(\tilde{i}, S) \right] = \frac{r + \phi}{1 - \beta} > r + \phi \text{ if } 0 < \beta < 1.$$  \hfill (19)

Under the hypotheses that $y_i(\tilde{i}, S) > 0, y_{ii}(\tilde{i}, S) < 0$, the conclusion is immediate: $\tilde{i} < i^*$. This implies a lower productivity per worker, $y_i$, and therefore a lower equilibrium value of $\theta$. So, equilibrium employment is lower. If $\beta$ was put to zero, then the investment level $\tilde{i}$ would be efficient. However, the so-called Hosios condition would not be met then because the latter requires that workers have an appropriate positive bargaining worker. Moreover, if workers also invest in match-specific capital (see the introduction), then $\beta = 0$ would imply that they get no rent from their investment. Hence, their level of investment will obviously be zero.

3.4 Empirical evidence of hold-up

Hirsch (1992) presents evidence that in the US hold up is a real phenomenon: Investment in physical capital (= rather general investment) and R&D (= at least to some extent a specific investment) are substantially lower in the presence of unions. The analysis is based on individual firm data. For a summary, see Teulings and Hartog (1998).

4 Contracts to protect investments under symmetric information

If the hold-up problem is a matter of concern, it is plausible that economic agents already have imagined answers to this problem. Moreover, the scientific literature has suggested some responses to this problem.

4.1 The ‘firm sets wage’

Still assuming that the firm (not the worker) is investing, Malcomson (1999) shows in section 4.2 that hold-up will be avoided if the firm is explicitly given the right to set the wage (say, by the law). But he recognizes that this is not standard in many countries. Moreover, this is not an answer to the hold-up problem when workers invest in a specific relationship.

When there are frictions on the labour market, the efficiency problem is more complex. For both investment and the level of tightness should be efficient. Acemoglu and Shimer (1999) show that it is impossible to reconcile these two requirements in the standard equilibrium search model with ex-post wage bargaining and random matching (i.e. the case of Section 3.3). They also show that these two requirements are met in the following setting. Firms choose the level of match-specific capital and they commit to and post wages (i.e. ‘firms set wages’) before meeting workers in order to attract applicants. Workers have a fairly good information
about posted wages and decide which job (i.e. which level of wage) to seek. Their chance of having such a job depends on the number of job-seekers queuing for the same vacancy. This is an extension of the so-called “competitive search model” of Moen (1997) to the case where there is ex-ante investment. Acemoglu and Shimer (1999) also show that “the key feature is the ability of workers to direct their search toward firms with different levels of capital, not toward those offering higher wages per se” (p.828). Both firms’ investment and tightness are efficient if wages are determined by ex-post Nash bargaining provided that workers are able to observe firms’ investment and direct their search appropriately.

4.2 Fixed wage contracts

In a frictionless economy, Malcomson (1999) considers the case of a fixed wage defined before the investment is chosen. He also provides empirical evidence of the relevance of fixed wage contracts. A fixed wage is clearly not a sufficient condition to avoid hold-up since the wage can be renegotiated after the investment is made. However, an offer to change the wage will actually lead to a modification only if both parties agree. Otherwise, rejection of an offer simply leaves the wage unchanged (‘renegotiation by mutual consent’). Then, either the match continues at unchanged conditions or the firm and the worker separates.

The timing of events is as follows:

- stage 0: A fixed wage contract is bargained;
- stage 1: Investment is chosen;
- stage 2: The state $S$ is revealed;
- stage 3: Renegotiation of the wage is possible (by mutual consent); if the firm and the worker do not separate, production and employment occur.

In Section 4.3, Malcomson (1999) shows that fixed wage contracts do not perfectly protect investments against hold-up but they can do a reasonable job.

5 Final remarks and extensions

- The formal presentation has been based on the “equilibrium job search model”. In this model, the generalized Nash bargaining outcome can also be interpreted as the solution to a particular strategic bilateral bargaining (under complete information) - see e.g. Section 4.3 of Mortensen and Pissarides (1999). The selected strategic game implies that the fall-back levels in the Nash product measure outside opportunities. Other games can also be envisaged. An example can be found in Section 4 of Malcomson (1999). In his setting the

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5 Notice that this can be extended to a multiperiod setting where this timing of events is repeated in each ‘period’.

6 Outside opportunities can be understood as alternatives if the match ends and the worker and the firm incur search costs in order to find another partner.
fall-back levels are the inside options\(^7\) and the outside options play the role of constraints (the outcome of the game has to yield to each player at least what they could get if the worker and the firm separate).

- Malcomson (1999) also discusses hold up of general investment.
- The hold-up problem has also been analyzed in theoretical settings where the players have private information. For example, each player ignores the value of the outside option of the other player (only the distribution is known). Section 4.7 of Malcomson (1999) provides a survey. Further analyses in an asymmetric information setting are however needed.

6 Conclusion

Actual contracts are incomplete in the sense that they cannot be contingent on all the events that affect the payoffs of the parties. In this lecture, one reason of incompleteness has been put forward: Investments are often not easily observed by a court. Another source of incompleteness is that a court generally cannot verify which exogenous shocks (\(S\)) occurred (even, if one restricts attention to those that are relevant for the employment relationship). Under symmetric information, a fixed contract combined with the opportunity to renegotiate (when the investment has taken place and the ‘state of the nature’ \(S\) is known) does a reasonable job of protecting investment (compared to what happens in the absence of a contract). Moreover, this type of contract is compatible with some stylized facts (see Malcomson, p. 2300).

There is a need for empirical work to test the predictions of these theories... in the US and the UK but also in Continental Europe!

7 Exercise

This question is about the choice of match-specific investment in a frictional economy (using equilibrium search models).

1.1 Consider that the firm invests and that contracts are complete. In this case, express the Nash product that is maximized with respect to the wage. Explain and justify this expression.

1.2 Consider that the worker invests and that contracts are complete. Explain the timing of events in this case. Solve then the problem (i.e. characterize the level of wages and the level of investment). Is the level of investment efficient? Why (not)?

\(^7\)While the bargaining takes place, the inside option is for instance the employee’s utility from staying at home unpaid and working for nobody and the profit derived when nobody works but the job is maintained available in case of an agreement.
1.3 Consider that the worker invests and that contracts are incomplete. More precisely, let first the worker decide over investment, the wage being bargained over afterwards. **You do not have to solve this problem.** I give you the condition that characterizes the level of investment, say $\bar{i}$:

$$E_S \left[ y_i(\bar{i}, S) \right] = \frac{r + \phi}{\beta}.$$  \hfill (20)

Under the hypotheses that $y_i(i, S) > 0$, $y_{ii}(i, S) < 0$, is this level $\bar{i}$ efficient? Why/When (not)?

References


