Job Search
Section 1. Basic Job Search Theory

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1. Introduction
   - Motivation
   - Preliminary technical issues
   - Outline

2. Job-Search Theory
   - The Basic Model
   - Extensions
Why Job Search Theory?

- Neoclassical labor supply theory
  - Perfect information
  - No room for unemployment\(^1\): people are either employed or out of the labor force\(^2\) ( = “non-participant”)
- In the data, unemployment duration is often non negligible (Evidence of “long-term unemployment”: see below)
- Idea: Introduce imperfect information on who offers suitable job vacancies and what wage it pays.
  Individual behavior in such a setting? Can this explain unemployment (duration)?

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\(^1\) ILO definition: An unemployed is someone who at the same time has no job, is searching for a job and available to take a job opportunity (e.g. not long-term ill).  
\(^2\) Identity: The Labor Force or the Workforce \(\equiv\) Employed + Unemployed
Facts

Figure: Average Monthly Inflow rate into and exit rate out of unemployment.

Source: Elsby, Hobijn and Sahin (2013). The starting year for the available series varies between 1968 (for the United States) and 1986 (for New Zealand and Portugal). For all countries, the data end in 2009.
Facts

Long-term unemployment
Persons unemployed for 12 months or more as a percentage of total unemployed

Figure: Long-term Unemployment. Source: OECD Factbook 2013
**Facts**

Bachmann and Baumgarten (2012)

**Figure**: Average number of active job-search methods during the previous 4 weeks (EU Labour Force Survey 2006-2008): Maximum = 7. Examples: Direct applications to employers; Studying advertisements.
Facts
Krueger and Mueller (2012)

How do the unemployed use their time the day before the survey?

<table>
<thead>
<tr>
<th>Country</th>
<th>Period</th>
<th>Participation rate in job search (%)</th>
<th>Average job search (min./day)</th>
</tr>
</thead>
<tbody>
<tr>
<td>France</td>
<td>1998-9</td>
<td>19</td>
<td>21</td>
</tr>
<tr>
<td>Germany</td>
<td>2001-2</td>
<td>10</td>
<td>9</td>
</tr>
<tr>
<td>Spain</td>
<td>2002-3</td>
<td>11</td>
<td>18</td>
</tr>
<tr>
<td>US</td>
<td>2003-6</td>
<td>20</td>
<td>32</td>
</tr>
</tbody>
</table>

Among those who search, median search time: 115 min./day (US), 120 min./day (Spain).
In Cahuc, Carcillo and Zylberberg (2014) (henceforth ‘CCZ’), the analysis is conducted in continuous time:
Meaning: No natural times (e.g. weeks) at which labor events always occur.
Note that a presentation in a discrete-time setting is not rare (an example will be provided when I discuss some extensions of the basic framework).

In this continuous time setting, at any moment \( t \), we will consider a small “decision period” \([t, t + dt]\).
Meaning: The job-seeker decides over one or many actions that each take a constant value all along this “decision period”. Later we let \( dt \to 0 \).
The Poisson Process

Poisson processes are often used in these lecture notes. E.g. in assumption A2 (resp., A6) below, the arrival of a job offer (resp., job destruction) is a Poisson process with parameter $\lambda$ (resp., $q$). The probability of receiving an offer in time interval $dt$ is $\lambda dt + o(dt)$, where $o(dt)$ is defined as a term such that $\lim_{dt \to 0} o(dt)/dt \to 0$. The probability of receiving more than 1 offer during $dt$ is negligible.

Poisson process in continuous time (Appendix D of CCZ)

Given a series of parameters $\lambda(t) \geq 0$ defined for $t \in [0, +\infty)$, an event $X$ (e.g. the occurrence of a shock) follows a Poisson process with parameters $\{\lambda(t)\}$ if the duration $T(t)$ to wait as of time $t$ for $X$ to occur has:

A CDF: $F_t(y) \equiv \Pr \{ T(t) \leq y \} = 1 - e^{-\int_t^{t+y} \lambda(\xi)d\xi}$

A probability density: $f_t(y) = \lambda(t + y)e^{-\int_t^{t+y} \lambda(\xi)d\xi}$
Bellman equations

These lecture notes often study *intertemporal problems* where agents optimize over a certain (typically infinite) horizon. *Bellman equations* (or *Dynamic Programming*) are often used to express and solve these optimization problems. To get Bellman’s intuition, consider a deterministic setup in discrete time. At time $t$, an individual maximizes a discounted sum of utility levels:

$$U_t \equiv \sum_{\tau=t}^{\tau=+\infty} \beta^{\tau-t} u(c_{\tau}) \text{ i.e. } u(c_t) + \beta u(c_{t+1}) + \beta^2 u(c_{t+2}) + \ldots,$$

(1)

with $\beta$ a discount factor $\in (0, 1)$ and $c_{\tau}$ designates consumption during $\tau$, subject to the evolution of a variable $x$ that evolves according to:

$$x_{t+1} = f(x_t, c_t) \quad \text{ (“law of motion”)}$$

(2)

$x$ could designate for instance wealth. Constraints on $x$ and $c$ can be added.
Bellman equations (continued)

- $c$ is under the control of the individual
- while $x$ is not and is called a “state variable”.

At time $t$, the individual chooses the stream $\{c_t\} \equiv (c_t, c_{t+1}, ...)$, given the current state $x_t$.

Let us define $V(x_t)$ the optimal value as of time $t$:

$$V(x_t) \equiv \max_{\{c_t\}} U_t \text{ subject to } x_{t+1} = f(x_t, c_t). \quad (3)$$

Since, by additivity of (1)

$$U_t = u(c_t) + \beta U_{t+1} \quad (4)$$

Bellman had the idea of rewriting Problem (3) as:

$$V(x_t) \equiv \max_{c_t} u(c_t) + \beta V(x_{t+1}) \text{ subject to } x_{t+1} = f(x_t, c_t). \quad (5)$$
Bellman equations (continued)

\[ V(x_t) \equiv \max_{c_t} u(c_t) + \beta V(x_{t+1}) \] is the Bellman equation corresponding to Problem (3).

In the next slides, Bellman equations will be used

- in a continuous time setting,
- and in a stochastic environment where shocks occur according to Poisson processes.

The construction of the Bellman equations will be intuitive. A more rigorous treatment can be found e.g. in Appendix D of the book (CCZ) or in the textbook of K. Wälde (free download at http://www.waelde.com/aio.html) where examples and solutions can be found.
The Leibniz rule

Let $f(\cdot), g(\cdot)$ and $h(\cdot)$ be any differentiable functions and define:

$$I(x) = \int_{g(x)}^{h(x)} f(y; x) dy$$

where $x$ and $t$ are real numbers ($x$ above is not the reservation wage). Then

$$\frac{dl(x)}{dx} = \int_{g(x)}^{h(x)} \frac{\partial f(y; x)}{\partial x} dy + f(h(x); x) \frac{dh}{dx} - f(g(x); x) \frac{dg}{dx}$$
Three sections:

1. **Job Search Theory** (the rest of this file Job-Search1.pdf)
   
   1.1 The Basic Model: choice of reservation wage
   
   1.2 Extensions (among which):
   
   - Eligibility to unemployment benefits
   - On-the-Job Search
   - Job Search Intensity
   - Benefit Exhaustion

2. The equilibrium search model (File Job-Search2.pdf)

3. Empirical aspects of job search (File Job-Search3.pdf)

Main reference for these slides: Chap. 5 of CCZ.
Sequential or Non-Sequential Search?

1. **Non-Sequential**: The job-seeker decides *ex ante* how many job offers she will collect (at a fixed unit cost) before choosing the best one.
   
   Seminal paper: Stigler (1961)
   
   Problem: What if the first offer proposes the best possible wage?

2. **Sequential**: Each job offer is screened upon arrival; if it exceeds a chosen threshold, the offer is accepted and the job-search process stops.
   

   We only consider the sequential approach.
1.1 A simple initial framework

Assumptions

A1 Rational forward-looking and risk-neutral\(^3\) homogeneous agents who only care about their income and consume all their instantaneous income (hand-to-mouth consumers). They discount the future in a standard rational way. All unemployed are entitled to a flat unemployment benefit ("UB"), \(b\), with no time limit. No taxes in the model.

A2 Job search intensity is fixed. Job offers arrive randomly. The arrival rate of job offers, \(\lambda\), is exogenous and constant. On a small interval of time, an unemployed can only get a single offer (with probability \(\lambda \cdot dt\)). A job offer = a wage offer for a full-time job (working time not modelled; no disutility of work)

\(^3\)Shimer and Werning (2007) consider risk-averse workers.
Assumptions (continued)

A3 At each $t$, job-seekers choose to reject or accept a job offer, if any ($\Rightarrow$ no bargaining). Rejected offers cannot be recalled. No sanction (i.e. loss of benefit) if an offer is rejected.

A4 No on-the-job search.

A5 There is an *exogenous true distribution* of wage offers on a support $[0, +\infty)$ (could also be $[\underline{w}, \overline{w}] \in [0, +\infty)$)\(^4\); If accepted, the wage stays constant.

Known and constant cumulative distribution of wage offers: $H(\cdot)$, density function $h(\cdot)$.

A6 Constant exogenous job destruction rate, $q > 0$.

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\(^4\)Where do people get information about this distribution? In addition to unions, networks and individual past experience, there are now specialized web sites (see for the US: [http://www.salarysite.com](http://www.salarysite.com); [http://www.vault.com](http://www.vault.com); ...).
1.1.1 Search and the Reservation Wage

The Discounted Expected Utility at time $t$ of a job paid $w$: $V_e(t)$

Consider a small interval of time $[t, t + dt]$. If $w$ is high enough (no quits), $V_e(t)$ satisfies (according to dynamic programming techniques and already neglecting the $o(\Delta t)$ term):

$$V_e(t) = \frac{1}{1 + r dt} \left[ w \; dt + q \; dt \; V_u(t + dt) + (1 - q \; dt) \; V_e(t + dt) \right]$$

(6)

with $r$ the discount rate, $V_u(\cdot)$ the expected discounted utility in case of a return in unemployment. Multiplying by $1 + r dt$ and dividing by $dt$:

$$rV_e(t) = w + q[V_u(t + dt) - V_e(t + dt)] + \frac{V_e(t + dt) - V_e(t)}{dt}$$

For $dt \to 0$:

$$rV_e(t) = w + q [V_u(t) - V_e(t)] + \lim_{dt \to 0} \frac{V_e(t + dt) - V_e(t)}{dt}$$
Discounted Expected Utility of a job

Interpretation of the so-called “Bellman equation” for $V_e$:

$$rV_e(t) = w + q [V_u(t) - V_e(t)] + \lim_{dt \to 0} \frac{V_e(t + dt) - V_e(t)}{dt}$$

- $rV_e(t)$: discounted expected flow utility (or income)
- $w = \text{wage} = \text{instantaneous return for an employed}$
- $q [V_u(t) - V_e(t)]$: rate of job-loss, $q$, times the expected change in discounted income $[V_u(t) - V_e(t)]$
- $\dot{V}_e \equiv \lim_{dt \to 0} \frac{V_e(t + dt) - V_e(t)}{dt}$ = the change in value of the discounted lifetime earnings = “the capital gains from changes in the value of the job seen as an asset”.
Discounted Expected Utility of a job

In steady state

The assumptions made guarantee a stationary environment. Then, $\dot{V}_e(t) = 0$. Getting rid of the time index and emphasizing the role of $w$:

$$rV_e(w) = w + q(V_u - V_e(w)) \quad (7)$$

Hence, making the link between $V_e$ and $w$ explicit:

$$V_e(w) - V_u = \frac{w - rV_u}{r + q} \quad (8)$$

Equation (8) reveals that the gain of accepting a job is monotonically increasing in $w$. Whatever the value of $V_u$, there exists some $x = rV_u$ such that

$$V_e(w) \geq V_u \iff w \geq x \quad (9)$$

Hence, $x = rV_e(x)$. 
The decision in unemployment

Let

- $c =$ the out-of-pocket costs of job search + opportunity cost of time devoted to search
- $b =$ the monetary value of domestic production and “leisure” net of losses due to unemployment *per se* (stigma, low self-esteem) + unemployment benefits (if any)
- The *net* instantaneous income in unemployment is $z \equiv b - c$.

Then, the intertemporal discounted value in unemployment at $t$ verifies:

$$V_u(t) = \frac{1}{1 + r d t} \left[ z \frac{d t}{dt} + \lambda d t V_\lambda(t + d t) + (1 - \lambda d t) V_u(t + d t) \right]$$

where $V_\lambda$ denotes the discounted expected utility *conditional* on being offered a job.
The Optimal Search Strategy of the unemployed

As the environment is stationary, the optimal strategy is constant all along the unemployment spell. So, getting rid of the time index, the choice of the unemployed is captured by

\[ V_\lambda \equiv \mathbb{E}_w \max\{ V_e(w), V_u \} \]  

(11)

where \( \mathbb{E}_w \) designates the expectation taken over the (random) wage \( w \).

Remembering (9), the choice in (11) can be summarized as follows: If an offer is received, apply the following stopping rule:

1. accept job offer \( \iff V_e(w) > V_u \iff w > x \equiv rV_u \)
2. reject job offer \( \iff V_e(w) \leq V_u \iff w \leq x \equiv rV_u \)

where \( x \) is called the reservation wage defined above by \( rV_u = rV_e(x) \).
If a job-seeker follows the stopping rule, his discounted expected utility, conditional on being offered a job, $V_\lambda$ can be rewritten as:

$$V_\lambda = \int_0^x V_u dH(w) + \int_x^{+\infty} V_e(w) dH(w)$$

where $dH(w) \equiv h(w)dw$

$$= V_u H(x) + \int_x^{+\infty} V_e(w) dH(w)$$

Unconditional on a job offer, $V_u$ solves (in a stationary state):

$$V_u = \frac{1}{1 + r dt} [z dt + \lambda dt \: V_\lambda + (1 - \lambda dt) \: V_u]$$
Discounted expected utility in unemployment

Rearranging as before and letting $dt \to 0$, we obtain:

$$rV_u = z + \lambda (V_\lambda - V_u)$$

Using the definition of $V_\lambda$ this can be rewritten:

$$rV_u = z + \lambda \left[ V_u H(x) + \int_x^{+\infty} V_e(w) dH(w) - (H(x) + 1 - H(x)) V_u \right]$$

Since $(1 - H(x)) V_u = \int_x^{+\infty} V_u dH(w)$, we have (Interpret!):

$$rV_u = z + \lambda \int_x^{+\infty} [V_e(w) - V_u] dH(w)$$

(13)
Reservation wage, hazard rate, expected duration

Using relation (8) and $x \equiv rV_u$ we obtain an implicit characterization of the reservation wage as a function of the parameters of the model:

$$x = z + \frac{\lambda}{r + q} \int_x^{+\infty} (w - x) dH(w)$$  \hspace{1cm} (14)

By a fix-point argument, this implicit equation has a unique solution $x$. 
To interpret (14), let us define

- The *hazard rate* (exit rate) $\phi$ from unemployment
  
  $$\phi(x) \equiv \lambda \overline{H}(x),$$

  where $\overline{H}(x) \equiv (1 - H(x))$ is the *acceptance rate*. The hazard rate is here constant.

- The *average unemployment duration* $\equiv T_u$:
  
  $$T_u = \frac{1}{\phi(x)} = \frac{1}{\lambda \overline{H}(x)}$$  \hfill (15)
Interpretation of (14)

Since \( z = b - c \) and the conditional expectation

\[
\mathbb{E}_w(w - x|w > x) \equiv \int_x^{+\infty} (w - x) \frac{h(w)}{H(x)} \, dw,
\]

we can rewrite Eq. (14) as:

\[
c + x - b = \phi(x) \int_0^{+\infty} \mathbb{E}_w(w - x|w > x) e^{-(r+q)t} \, dt \tag{16}
\]

Interpretation: Assume an offer is available paying \( x \):

LHS = instantaneous expected cost of continuing search

= direct \((c)\) + net opportunity cost of search \((x - b)\)

RHS = instantaneous expected return to continuing search

= the exit rate out of unemployment times the discounted sum of expected instantaneous income gains \((w - x)\) when continuing search, knowing that only wage offers above the reservation wage are accepted.
Graphical representation

Replace $x$ by any level of wage offer $W$, 

$$W-(b-c)$$ 

$$\phi(W)E(w-W \mid w>W)/(r+q)$$ 

continue  Stop searching
1.1.2 Comparative Statics of the Basic Model

Relation (14) is an **implicit equation** in the reservation wage $F(x, z, r, \lambda, q) = 0$ where

$$F(x, z, r, \lambda, q) \equiv x - z - \frac{\lambda}{r + q} \int_{x}^{+\infty} (w - x) dH(w)$$

The direction of the derivative of $x$ as a function of the parameters is obtained by **totally differentiating** $F = 0$:

$$\frac{\partial x}{\partial i} = -\frac{F_i}{F_x} \quad \text{if} \quad F_x \neq 0,$$  \hspace{1cm} (17)

where $i$ denotes any of the parameters $\{z, r, \lambda, q\}$ and $F_i = \frac{\partial F}{\partial i}$.

From relation (15), the main comparative statics properties of the average duration in unemployment are derived (see CCZ and Comparative statics Note.pdf on Moodle).
Exercise

Take an exponential density function \( h(w) = \gamma \exp[-\gamma \cdot w] \) (with \( \gamma > 0 \)) and use (14) and integration by part to verify that the reservation wage solves the implicit equation

\[
x = z + \frac{\lambda}{r + q} \frac{\exp[-\gamma \cdot x]}{\gamma}
\]

Is \( x \) unique? Interpret the role of the parameters.

Exercise

Apply (17) and the Leibnitz rule to calculate \( F_x \) to check that

\[
\frac{\partial x}{\partial z} > 0, \quad \frac{\partial x}{\partial \lambda} > 0, \quad \frac{\partial x}{\partial r} < 0 \quad \text{and} \quad \frac{\partial x}{\partial q} < 0
\]

(18)

Provide intuition for these properties.
Exercise

Assume that any unemployed person who finds a job is paid the net wage $w$ plus an untaxed subsidy $s$ paid by the government. Assume that the subsidy does neither affect the exogenous wage offer distribution $H$, nor the job arrival rate $\lambda$, nor the job separation rate $q$. You can also assume that search effort is exogenous and that there is no on-the-job search. The environment is stationary.

1. Write the Bellman equation solved by the intertemporal value of having a job, $V_e$, and from there characterize the reservation wage $x$. Is there a stopping rule? Explain why.

2. Compute by how much the reservation wage $x$ changes after a marginal increase in the subsidy $s$.

3. Compute the impact on the exit rate out of unemployment of a subsidy $s$ paid to unemployed workers who find a job?
1.1.3 Nonparticipation, Job-Seeking, and Employment

Some definitions:

- The *working age population* is generally considered to be the population aged between 15 and 64 years (74 years in EU stat.).
- The *participants* to the labor market = the labor force = Employed (≥ 1 hour in the week of reference) + unemployed populations
- The *participation rate*  
  \[
  \text{Participation rate} = \frac{\text{Labor force}}{\text{Working age population}}
  \]
- The *unemployment rate*  
  \[
  \text{Unemployment rate} = \frac{\text{Unemployed population}}{\text{Labor force}}
  \]
- The *employment rate*  
  \[
  \text{Employment rate} = \frac{\text{Employed population}}{\text{Working age population}}
  \]
Nonparticipation, Job-Seeking, and Employment
The Reservation Wage and Alternative Income

Neoclassical theory of labor supply  Let $w_A$ denote the “reservation wage” in Labour Supply theory.

$$\begin{cases} w > w_A \implies \text{employee} \\ w \leq w_A \implies \text{non-participant} \end{cases}$$  \hspace{1cm} (19)

Job Search Theory
Let $\Omega = \{H(\cdot), z, q, \lambda, r\}$ and $V_I = R_I/r$ the intertemporal value of a non-participant. To the extent that participation is a decision,

$$\begin{cases} x(\Omega) > R_I \implies \text{participant} \\ x(\Omega) \leq R_I \implies \text{non-participant} \end{cases}$$  \hspace{1cm} (20)

$$\begin{cases} \text{if job offer and } w > x(\Omega) \implies \text{employee} \\ \text{if no job offer or } \{ \text{a job offer and } x(\Omega) \geq w \} \implies \text{unemployed} \end{cases}$$  \hspace{1cm} (21)
Nonparticipation, Job-Seeking, and Employment

Consequence:

Consider that $R_I$ is exogenously distributed in the population. Parameters such as $b$, $\lambda$, $q$ that influence $x$ affect both

- Participation and
- Unemployment

Example 1: $\uparrow b \Rightarrow \uparrow x \Rightarrow$ The following indicators rise:

- The intertemporal value in unemployment ($x = rV_u$)
- The level of unemployment (because the acceptance rate shrinks)
- The participation rate $P[R_I \leq x]$

Example 2: $\uparrow \lambda \Rightarrow \uparrow x \Rightarrow \uparrow P[R_I \leq x]$
1.2 Extensions to the Basic Model

1.2.1 Eligibility and Unemployment

The coverage rate of social benefits is actually larger for those unemployed for a year or more than those unemployed for a shorter period. This is also the case in Germany, where those exhausting entitlement to insurance benefits have unemployment assistance to fall back on for an unlimited period.

In practice, the proportion of the unemployed receiving benefits tends to decline with the duration of the spell out of work, at least for spells of three months or more. On average, only 30% of those unemployed for 12 months or more were in receipt of benefits in 2011 in the EU as opposed to over 40% of those unemployed for between 3 and 5 months (Table 5).

In 8 Member States, 7 of them EU13 countries—the other being Italy—less than 5% of those unemployed for 12 months or more received benefits and in another two (Greece and Lithuania), the proportion was less than 10%. By contrast in 5 countries (Denmark, Finland, Belgium, Germany and Malta), a larger proportion of the long-term unemployed were in receipt of benefit.

Table 5 Proportion (%) of those unemployed for 3 months or more in receipt of benefits by duration of unemployment, 2011

<table>
<thead>
<tr>
<th></th>
<th>3-5 months</th>
<th>6-11 months</th>
<th>12+ months</th>
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</thead>
<tbody>
<tr>
<td>Italy</td>
<td>11.5</td>
<td>9.7</td>
<td>1.6</td>
</tr>
<tr>
<td>Slovakia</td>
<td>29.1</td>
<td>11.7</td>
<td>3.3</td>
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<tr>
<td>Bulgaria</td>
<td>22.9</td>
<td>16.0</td>
<td>1.5</td>
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<tr>
<td>Poland</td>
<td>18.2</td>
<td>13.6</td>
<td>2.7</td>
</tr>
<tr>
<td>Latvia</td>
<td>31.2</td>
<td>18.5</td>
<td>1.8</td>
</tr>
<tr>
<td>Croatia</td>
<td>26.8</td>
<td>18.6</td>
<td>12.0</td>
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<td>Lithuania</td>
<td>32.1</td>
<td>21.4</td>
<td>9.5</td>
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<tr>
<td>Estonia</td>
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<td>3.1</td>
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<td>Romania</td>
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<td>Cyprus</td>
<td>36.8</td>
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<td>2.4</td>
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<td>Czech Republic</td>
<td>54.7</td>
<td>13.9</td>
<td>2.7</td>
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<tr>
<td>Greece</td>
<td>38.4</td>
<td>31.6</td>
<td>9.1</td>
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<td>Sweden</td>
<td>24.7</td>
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<td>23.4</td>
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<td>Malta</td>
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<td>40.9</td>
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<tr>
<td>EU</td>
<td>40.7</td>
<td>37.8</td>
<td>30.1</td>
</tr>
</tbody>
</table>

Note: No data available for IE, NL and UK. EU is defined to exclude these.
Source: Eurostat, LFS

A1’ Net income for unemployed is higher if eligible to UB than if ineligible: \( z > z_n \);

Once employed, a worker becomes *immediately* eligible to UB ⇒ Equation (7) determines \( V_e(w) \).

The reservation wage of non-eligible job-seekers, \( x_n \), can be shown to be (CCZ p. 270):

\[
    r(x_n - z_n) + q(x - z_n) = \lambda \int_{x_n}^{+\infty} (w - x_n)dH(w)
\]

If UB increases, the incentive to work is enhanced for ineligible workers, since employment now entitles them to higher future UB ("entitlement effect"):

\[
    \frac{\partial x_n}{\partial z} = -\frac{q}{(r + \phi(x_n))} \frac{\partial x}{\partial z} < 0
\]
1.2.2 On-the-Job Search

A4’ Workers continue searching when on the job. For simplicity, cost of on-the-job search = 0 ⇒ search always on-the-job: no threshold wage above which searching no longer pays. The current employer does not revise the wage upwards to avoid a quit.

\[ rV_e(w) = w + q [V_u - V_e(w)] + \lambda_e \int_{w}^{+\infty} [V_e(\xi) - V_e(w)] dH(\xi) \]  

(22)

\[ V'_e(w) > 0 \Rightarrow \text{satisfies a stopping rule. The reservation wage then satisfies the following equation (for derivation see CCZ):} \]

\[ x = z + (\lambda_u - \lambda_e) \int_{x}^{\infty} \frac{\overline{H}(\xi)}{r + q + \lambda_e \overline{H}(\xi)} d\xi \quad \text{with} \quad \overline{H}(\xi) \equiv 1 - H(\xi) \]  

(23)
Extensions to the Basic Model

On-the-Job Search

Four Cases

1. $\lambda_e = 0 \Rightarrow$ no on-the-job search $\Rightarrow$ back to basic model: $x > z$

2. Increasing $\lambda_e$ lowers reservation wage (as long as $\lambda_e < \lambda_u$) $\Rightarrow x > z$;

3. $\lambda_e = \lambda_u \Rightarrow x = z$;

4. $\lambda_e > \lambda_u \Rightarrow x < z$: interesting to accept low-paid as it is a powerful steppingstone to better paid jobs.

The empirical literature generally finds $\lambda_u \geq \lambda_e$.
For instance, Robin (2011) finds that $\lambda_e = 0.12 \cdot \lambda_u$.
However, van den Berg and Ridder (1998) find that $\lambda_e$ was slightly larger than $\lambda_u$ in the 80’s in The Netherlands.
Extensions to the Basic Model

1.2.3 Choosing How Hard to Look

Introduce search effort $e$ (e.g. time spent on searching)

\[ \lambda = \alpha \lambda(e) \text{ with } \lambda' > 0 \text{ and } \lambda'' < 0. \]

The parameter $\alpha > 0$ is interpreted as an indicator of the state of the labor market (can also depend on characteristics like education), independent of individual efforts.

- $c(e)$ is the cost arising from the search effort $e$, with $c' > 0$ and $c'' > 0$.\(^5\)

- Still, working time not modelled; no disutility of work.


---

\(^5\)This cost of effort can include direct expenses, the opportunity cost of time, and psychological costs of searching (see e.g. Krueger and Mueller, 2011; Schwartz, 2015). Mortensen (1977) deals with a non-separable utility function of consumption and effort.
Extensions to the Basic Model
Choosing How Hard to Look

The reservation wage $x$ is defined, as before, by the equation (14):

$$rV_u = x = b - c(e) + \frac{\alpha \lambda(e)}{r + q} \int_x^{+\infty} (w - x) dH(w)$$  \hspace{1cm} (24)

We maximize $rV_u$ wrt $e$ and set to zero to obtain the optimal search effort:\(^6\)

$$c'(e) = \frac{\alpha \lambda'(e)}{r + q} \int_x^{+\infty} (w - x) dH(w)$$  \hspace{1cm} (25)

System (24) and (25) implicitly defines $e$ and $x$ as a function of $\alpha, b, r, q$.

---

\(^6\)Second-order condition satisfied since $c'' > 0$ and $\lambda'' < 0$.
Extensions to the Basic Model
Choosing How Hard to Look

Totally differentiating this system wrt $e, x, \alpha, b, r, q$ leads to:

\[
\frac{\partial x}{\partial \alpha} > 0 \quad \text{and} \quad \frac{\partial e}{\partial \alpha} > 0
\]

\[
\frac{\partial x}{\partial b} > 0 \quad \text{and} \quad \frac{\partial e}{\partial b} < 0
\]

Messages:

- A higher rate of arrival of job offers raises the reservation wage and job-search effort.
- Higher unemployment benefits raises the reservation wage and lowers job-search effort.
- A simultaneous $\nearrow$ of $\alpha$ and $b$ has an ambiguous effect on search effort.
An improvement in business conditions (i.e. $\alpha \uparrow$), has therefore an ambiguous net effect on the exit rate:

$$\phi = \alpha \lambda(e)(1 - H(x))$$

From empirical analyses however, $\phi$ increases with $\alpha$.

Similarly, from total differentiation, it can be checked that

$$\frac{\partial x}{\partial (r + q)} < 0 \quad \text{and} \quad \frac{\partial e}{\partial (r + q)} < 0.$$

Hence, more patient job-seekers (lower $r$) are more choosy ($x$ increases) and search harder.

$\Rightarrow$ ambiguous impact on the exit rate.\(^7\)

\(^7\)In a more complex setting, Proposition 4 of Della Vigna and Paserman (2005) shows that the relationship between $\phi$ and $r$ is U-shaped.
Determinants of job-search effort
What do other sciences tell us?

Up to now, search effort is a function of a few parameters \((\alpha, b, r, q)\), the shape of the wage offer distribution \(H(\cdot)\) and functions \((\lambda(\cdot), c(\cdot))\).

Economists and sociologists have added other factors such as the social network of the job-seeker.

Psychology lists a wider set of determinants, namely:

1. “motives to search” (employment commitment, financial hardship);
2. “job-search competencies” (self-efficacy, emotion control,...);
3. “job search constraints” (ill-health, child-care obligations, ...)

Exercise

This is an exercise about the monitoring of search effort inspired by Manning (2009).

1. From (24) and (25), consider the two implicit equations in $(e, x)$:

$$f(e, x) \equiv x - b + c(e) - \frac{\alpha \lambda(e)}{r + q} Q(x) = 0 \quad (26)$$

$$g(e, x) \equiv c'(e) - \frac{\alpha \lambda'(e)}{r + q} Q(x) = 0, \quad (27)$$

where $Q(x) = \int_x^{+\infty}(w - x)dH(w)$. Check that $Q'(x) < 0$.

2. Draw these two curves $f(e, x) = g(e, x) = 0$ in a $(e, x)$ space.

3. In unemployment insurance (UI) $b = b_0$. In unemployment assistance (UA), $b = b_1 < b_0$. Draw on the same graph the curves in the cases of UI and UA.

4. Imagine now that search effort is no more chosen without constraint in UI, but well in UA. To do so, introduce a minimum search effort level $e$ such that if the chosen effort level $e \geq e$ the unemployment remains entitled to UI. Otherwise, (s)he gets UA. For what levels of $e$ is the claimant adapting search effort upwards? (what happen to $x$?) When does the claimant drop out of the register of insured unemployed (UI) and collect UA?
Exercise

Consider the job-search model with endogenous search effort in steady state. Start from \( f(e, x) = 0 \) and \( g(e, x) = 0 \) as in the previous exercise. Assume that 
\[
c(e) = c \cdot e, \quad \text{with } c > 0 \text{ and } \lambda'(e) > 0, \lambda''(e) < 0.
\]
Assume also that to survive the job-seeker needs a minimal exogenous amount of income (hence of consumption) denoted \( C_0 > 0 \). So, the following constraint is imposed:

\[
b - c \cdot e \geq C_0 \quad \text{with } b > C_0
\]  

(28)

There exists only an UI with a flat benefit level \( b \) and no unemployment assistance scheme. Let \( e^* \) denote the optimal search effort when (28) is ignored. Consider henceforth an unemployed individual for whom Constraint (28) is binding, so that her effort verifies \( e \leq e^* \). For this individual only:

a) Produce the system of equations solved by the levels of search effort and of the reservation wage.

b) What are the implications of a marginal rise in \( b \) on search effort and on the reservation wage? Are the signs of these effects clear?

c) Are the two implications found in Sub-question [b] standard in the theoretical literature?

d) Explain how the exit rate out of unemployment is related to the levels of search effort and of the reservation wage. Are there clear-cut implications of a rise in \( b \) on this exit rate?
Extensions to the Basic Model

1.2.4 Nonstationarity

Sources of nonstationarity:
A change in any of the parameters \((b, c, \lambda, r)\) or in the distribution of wage offers \(\text{after entry into unemployment}\). E.g.: Stigma effect of duration so that \(d\lambda/dt < 0\).

a) If changes in parameters are \(\text{unanticipated}\) random shocks, the individual might think that all the parameters will remain at their current values.

Consider the case of random macro shocks that affect \(\lambda\). Then, by analogy with the basic stationary model:

\[
x(t) = z + \frac{\lambda(t)}{r + q} \int_{x(t)}^{+\infty} (w - x(t))dH(w)
\]
Extensions to the Basic Model

Nonstationarity under perfect foresight

b) If parameters change in a deterministic way. Here, assume one modification:

\[ A1'' \quad z(t) \leq z(t') \text{ for all } t \geq t'. \]

E.g. declining unemployment benefits while \( c \) is constant.

Perfect foresight: \textit{changes in the values of the parameters are correctly anticipated.}

The inter-temporal value at the time of entry in unemployment, \( V_u(0) \), should differ from \( V_u(t) \) at later times.

However, \( V_e(w) \) is still stationary:

\[ rV_e(w) = w + q[V_u(0) - V_e(w)] \]  \hspace{1cm} (29)

Hence, \( V_e(w) \) is still increasing in \( w \Rightarrow \text{reservation wage property.} \)
Extensions to the Basic Model
Nonstationarity under perfect foresight

In this case the lifetime utility of the unemployed, \( V_u(t) \), maximizes wrt \( s \):

\[
\frac{z(t)dt + \lambda dt}{1 + rdt} \left[ \int_0^{+\infty} V_e(w) dH(w) + V_u(t + dt)H(s) \right] + (1 - \lambda dt)V_u(t + dt)
\]

\[ (30) \]

divided by \( 1 + rdt \). The optimal reservation wage, \( x(t) \), is obtained by setting to zero the derivative with respect to \( s \) of the term between brackets (considering \( V_u(t + dt) \) as given):

\[
V_e[x(t)] = V_u(t + dt) \quad \text{and if} \quad dt \rightarrow 0: \quad V_e[x(t)] = V_u(t)
\]

As \( V_e(\cdot) \) is increasing, \( x(t) \) and \( V_u(t) \) vary in the same direction.
Extensions to the Basic Model
Nonstationarity under perfect foresight

Intuitively, since $z(t)$ decreases over time, $V_u(t) \leq V_u(t')$ necessarily obtains for every $t \geq t'$.

If so, since $x(t)$ and $V_u(t)$ vary in the same direction, we can deduce that $x(t) \leq x(t')$ for every $t \geq t'$.

⇒ the exit rate $\phi(x(t))$ should *increase* with unemployment duration!

Counterfactual?

Not necessarily if,

- The job arrival rate $\lambda$ declines simultaneously with duration;
- Unobserved heterogeneity: the more productive workers leave first, so that over time those still unemployment are less productive and hence have a lower $\lambda$. 
Extensions to the Basic Model

Nonstationarity under perfect foresight

Illustration (van den Berg, 1990): Assume:

\( h_1 \) \( \exists T < +\infty \mid z(t) = \text{some constant } z \text{ for } t \geq T \text{ and } z(t) \leq z(t') \text{ for all } T \geq t \geq t' \);

\( h_2 \) \( q = 0 \iff \text{jobs last forever} \Rightarrow V_e(w) = w/r; \)

\( h_3 \) Keep \( H(\cdot), \lambda, r \) constant.

From (30) and from \( V_e(x(t)) = x(t)/r = V_u(t) \), one easily derives:

\[
\begin{align*}
    rV_u(t) &= z(t) + \frac{\lambda}{r} \int_{x(t)}^{+\infty} (w - rV_u(t))dH(w) + \dot{V}_u(t) \\
    \text{so, } \dot{x}(t) &= r \cdot x(t) - r \cdot z(t) - \lambda \int_{x(t)}^{+\infty} (w - x(t))dH(w)
\end{align*}
\]

This differential equation has a unique solution for \( x(t) \), given the boundary condition that follows from the assumption that the model is stationary for \( t \geq T \).
Illustration

Net Replacement ratio(*) as a function of unemployment duration

(*) Adjust gross replacement rates for income tax and social contributions paid, and for other benefits that jobseekers and wage earners receive, such as family benefits, housing benefits, social assistance and in-work benefits.
Extensions to the basic model

1.2.5. Nonwage characteristics

Rupert, Stancanelli and Wasmer (2009) exploit survey data in the EU (1994-2001). A sample of individuals has received a job offer during the previous 4 weeks.

Table 1: Reasons for Rejecting Offers

<table>
<thead>
<tr>
<th>Reason</th>
<th>%</th>
<th>% excl. last 3</th>
<th>% excl. last 3 &amp; hrs</th>
<th>% compared to wage rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. rate of pay</td>
<td>12.1</td>
<td>21.8</td>
<td>24.7</td>
<td>59.7</td>
</tr>
<tr>
<td>2. temporary/insecure job</td>
<td>6.65</td>
<td>12.0</td>
<td>13.6</td>
<td>-</td>
</tr>
<tr>
<td>3. type of work</td>
<td>12.9</td>
<td>23.3</td>
<td>26.4</td>
<td>-</td>
</tr>
<tr>
<td>4. number of working hours</td>
<td>6.05</td>
<td>11.0</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>5. working time (day time, night time, shifts...)</td>
<td>6.42</td>
<td>11.6</td>
<td>12.4</td>
<td>-</td>
</tr>
<tr>
<td>6. working conditions / environment</td>
<td>3.06</td>
<td>5.54</td>
<td>6.27</td>
<td>-</td>
</tr>
<tr>
<td>7. distance to job / commuting</td>
<td>8.14</td>
<td>14.7</td>
<td>16.7</td>
<td>40.3</td>
</tr>
<tr>
<td>8. could not start the job at required time</td>
<td>4.82</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>9. other reasons for not accepting</td>
<td>20.99</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>10. not yet decided</td>
<td>18.93</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Sum</td>
<td>100</td>
<td>100</td>
<td>100</td>
<td>100</td>
</tr>
</tbody>
</table>
Up to now, job-seekers do not value non-wage characteristics of jobs such as

- Hours worked,
- Job amenities (including commuting time).

Some papers have extended the analysis to include those features.
⇒ Reservation wage replaced by a reservation utility level.

Examples: Blau (1991); Section 4.3 of van den Berg and Ridder (1998); Bloemen (2008); Sullivan and To (2014); Hall and Mueller (2018); Guglielminetti, Lalivez, Ruh and Wasmer (2017).
Extensions to the basic model

1.2.6. Many extensions have not been covered here.

Examples:

  Note: Liquidity constraints are prevalent among job losers (Card, Chetty and Weber, 2007; Chetty, 2008; Basten, Fagereng and Telle, 2014).

- Household job-search: Couples may make joint decisions about job-search (Guler, Guvenen and Violante, 2012; Mankart and Oikonomou, 2016).

- The role of the social network (see e.g. Zenou, 2013) and referrals (see e.g. Dustmann, Glitz and Schönberg, 2016).
Extensions to the basic model
1.2.7. Questioning the quality of the information of the unemployed.

The basic model and the extensions assume that the unemployed are well informed and in particular have correct beliefs.

- Altmann, Falk, Jäger and Zimmermann (2018) show that German unemployed at risk of long-term unemployment are in need of “information about the current labor market situation, the non-pecuniary consequences of (un)employment and effective job search strategies” (p. 34).

- There is also some evidence that (some) job-seekers have wrong beliefs (e.g. are too optimistic about job opportunities)? See Spinnewyn (2015), Krueger and Mueller (2016), and Mueller, Spinnewyn and Topa (2018).
Extensions to the basic model

1.2.8. Behavioral Economics and Job-search

Behavioral economics: The research area that studies the link between psychology and economics.

Some relevant topics:

- What if the job-seeker is “impatient”? See next slides.
- What if the job-seeker has reference-dependent preferences, where the reference point is given by recent income. See Della Vigna, Lindner, Reizer and Schmieder (2015).

These non-standard assumptions allow to explain phenomena that are difficult to explain with standard assumptions. E.g. spikes at the Unemployment Insurance exhaustion point in the case of the last paper.
Hyperbolic discounting
Della Vigna and Paserman (2005)

Low search intensity compatible with high unit cost of search, low returns from search ... or points to the need for an alternative theoretical setting?

Up to now, job-seekers discount the future in a “standard way” (exponential discounting). However, there is empirical evidence of a higher discount rate in the short run than in the long run (creating procrastination).

“Hyperbolic discounting”: Let $u_t$ be the utility level at time $t$ (time will be discrete here). The present ($t = 0$) value of the stream $(u_t)_{t \geq 0}$ is

$$u_0 + \beta \sum_{t=1} \delta^t \cdot u_t, \quad 0 < \beta, \delta \leq 1.$$  

(31)

$\beta$ is the parameter of short-run impatience and $\delta$ the one of long-run impatience.
These preferences are dynamically inconsistent (compare the stream $(u_t)_{t \geq \tau}$ from today’s perspective and from the perspective of time $\tau > 0$).

Put another way, it is as if the agent has two selves:

- The agent who is making a decision in the current period, the current self, is impatient: discounting the payoffs of search by $\beta \cdot \delta$
- The future self (who benefits from past search effort) who discount the future in a standard way: The discount factor is $\delta$. 
Extensions to the basic model

Hyperbolic discounting

Consider a stationary framework where a job accepted at duration $t$ starts in $t+1$.

Simplify Assumption (A2'): $\alpha \lambda(e) = e$.

In period $t$, the unemployed chooses the search effort $e_t$ and the reservation wage $x_t$ to max. $V_{u,t}$:

$$
\max_{e_t} b - c(e_t) + \beta \delta \left[ e_t E_H \{ \max (V_{e,t+1}(w), V_{u,t+1}) \} + (1 - e_t) V_{u,t+1} \right],
$$

where from the perspective of the individual in period $t$:

$$
V_{e,t+1}(w) = w + \delta \left[ q V_{u,t+2} + (1 - q) V_{e,t+2} \right] \quad (32)
$$

and the reservation wage $x_t$ chosen in period $t$ by the current self is such that:

$$
V_{u,t+1} = V_{e,t+1}(x_t). \quad (33)
$$
Extensions to the basic model

Hyperbolic discounting

The “sophisticated” hyperbolic discounter has rational expectations: (s)he knows that future preferences will be hyperbolic as well (while the “naive” hyperbolic agent believes (s)he will discount exponentially later on).

For brevity, consider the “sophisticated” agent only. Della Vigna and Paserman (2005) show that:

- A more impatient (lower $\beta$) agent searches less intensively but sets lower reservation wages.
- In general, the impact of short-run impatience on the exit rate is therefore ambiguous. Under reasonable assumptions, a more impatient (lower $\beta$) agent exits less rapidly from unemployment.
- An empirical analysis confirms the latter property.
The partial-equilibrium job-search model explains how imperfect information about job offers generate frictions on the labor market.

Some clear-cut properties emerge in a stationary and a non-stationary setting.

The job-search model is the framework on the basis of which a lot of empirical research has been developed (see Section 3, i.e. job-search3.pdf).

“Frictional unemployment” appears under the assumption that wage dispersion for a homogenous type of worker is a pervasive phenomenon. This has for long time been considered as an odd assumption (see Section 2, i.e. job-search2.pdf).
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References II


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