Job Search Theory

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Outline

1. Introduction
   - A bit of vocabulary
   - Motivation

2. The Basic Model

3. Extensions

Thanks to Bart Cockx (Ghent U.) and Koen Declercq (USL-B) for their contributions to these slides.
In class, please do not open Teams.

If you are not in the class room, please

- Shut down your microphone and your camera.
- Turn your microphone on only when you have a question or a reaction. Then turn it off again.
- If you have no microphone, you can also use the chat.
INTRODUCTION
Basic notions

- The *working age population* is generally considered to be the population aged between 15 and 64 years (74 years in EU stat.).
- Identity: *Active population* or workforce or labor force $\equiv$ Employed Population + Unemployed Population
- Total population - Active population $\equiv$ inactive (or out-of-the-labor-force) population

An unemployed has no job, searches for a job, and is available to take job offers (International Labour Organization’s definition)

Complex measurement issues:
- How to measure “job-search effort” and “availability”? Difficult...
- When is someone “employed”? Answer: worked $\geq 1$ hour in the week of reference (where “working” needs to be made precise)
Some rates

- The *unemployment rate*

\[
\frac{\text{Unemployed population}}{\text{Active population}}
\]

- The “participation rate” is defined as:

\[
\frac{\text{Active population}}{\text{Working age population}}
\]

- The “employment rate” (or “employment-to-population ratio”) defined as

\[
\frac{\text{Employment Population}}{\text{Working age population}} \equiv (1 - \text{unemployment rate}) \times \text{participation rate}
\]
Why Job Search Theory?

- Neoclassical labor supply theory
  - Assumes perfect information
  - No room for unemployment: People are either employed or out of the labor force

- In the data, unemployment duration is often non-negligible (Evidence of "long-term unemployment": see below)

- Job search theory: Introduce imperfect information on who offers suitable job vacancies and what wage it pays. Two main questions are addressed:
  - How does an individual job-seeker behave in such a setting?
  - Can this explain unemployment (duration)?
Introduction

Preliminary comments

1. Job search theory is a “partial equilibrium analysis”
   - Taking as given the behavior of (potential) employers;
   - Taking for granted that imperfect information leads to a non-degenerate distribution of wages (more generally, working conditions) for homogeneous workers.

2. Job search theory was created when the Internet did not yet exist. (In current [“developed”] economies,) is there still imperfect information on who offers suitable job vacancies and what wage it pays?
   - Online job boards have removed many “search frictions”.
   - However, a lot of relevant information about vacant jobs (resp., applicants) is arguably not revealed by a vacancy (resp., a CV) posted on a job board.
   - Looking at the US from 1948 to 2018, Martellini and Menzio (2020) observe that “the rate which unemployed workers become employed (UE rate) [...] do not have an over-riding secular trend”. See the figure on next slide.
Facts

Figure: UE rate (rate at which unemployed workers become employed) and EU rate (rate at which employed workers become unemployed): USA 1948-2018. Source: Martellini and Menzio (2020)
Facts

**Figure:** Average Monthly Inflow rate into and exit rate out of unemployment.

Source: Elsby, Hobijn and Sahin (2013). The starting year for the available series varies between 1968 (for the United States) and 1986 (for New Zealand and Portugal). For all countries, the data end in 2009.
Facts

Long-term unemployment

*Persons unemployed for 12 months or more as a percentage of total unemployed*

**Figure:** Long-term Unemployment. Source: OECD Factbook 2013
**Figure:** Average number of active job-search methods during the previous 4 weeks (EU Labour Force Survey 2006-2008): Maximum = 7. Examples: Direct applications to employers; Studying advertisements. Source: Bachmann and Baumgarten (2012).
How do the unemployed use their time the day before the survey?

<table>
<thead>
<tr>
<th>Country</th>
<th>Period</th>
<th>Participation rate in job search (%)</th>
<th>Average job search (min./day)</th>
</tr>
</thead>
<tbody>
<tr>
<td>France</td>
<td>1998-9</td>
<td>19</td>
<td>21</td>
</tr>
<tr>
<td>Germany</td>
<td>2001-2</td>
<td>10</td>
<td>9</td>
</tr>
<tr>
<td>Spain</td>
<td>2002-3</td>
<td>11</td>
<td>18</td>
</tr>
<tr>
<td>US</td>
<td>2003-6</td>
<td>20</td>
<td>32</td>
</tr>
</tbody>
</table>

Among those who search, median search time: 115 min./day (US), 120 min./day (Spain). Source: Krueger and Mueller (2012).
This set of slides develops mainly theoretical results. It delivers some important predictions. Later, another set of slides will focus on empirical analyses inspired by Job Search Theory.
The Basic Job-search Model

Main reference:
Chap. 5 of Cahuc, Carcillo and Zylberberg (2014), henceforth ‘CCZ’.
Sequential or Non-Sequential Search?

1. **Non-Sequential:** The job-seeker decides *ex ante* how many job offers she will collect (at a fixed unit cost) before choosing the best one.
   Problem: What if the first offer proposes the best possible wage?

2. **Sequential:** Each job offer is screened upon arrival; if it exceeds a chosen threshold, the offer is accepted and the job-search process stops.

We only consider the **sequential** approach.
Assumptions of the basic Job Search Model

A1 Rational forward-looking and risk-neutral\(^1\) homogeneous agents who only care about their income and consume all their instantaneous income (hand-to-mouth consumers). They discount the future in a standard rational way. All unemployed are entitled to a flat unemployment benefit ("UB"), \(b\), with no time limit. No taxes in the model.

A2 Job search intensity is fixed. *Job offers arrive randomly.* The arrival rate of job offers, \(\lambda\), is exogenous and constant. On a small interval of time, an unemployed can only get a single offer (with probability \(\lambda \cdot dt\)). A job offer = a wage offer for a full-time job (working time not modeled; no disutility of work)

\(^1\)Shimer and Werning (2007) consider risk-averse workers.
A3 At each $t$, job-seekers choose to reject or accept a job offer, if any ($\Rightarrow$ no bargaining). Rejected offers cannot be recalled. There is no sanction (i.e. no loss of UB) if an offer is rejected.

A4 No on-the-job search.

A5 There is an *exogenous true distribution of wage offers* on a support $[0, +\infty)$ (could also be $[\underline{w}, \overline{w}] \in [0, +\infty))$; If accepted, the wage stays constant. *Known and constant* cumulative distribution of wage offers: $H(\cdot)$, density function $h(\cdot)$.

A6 Constant exogenous job destruction rate, $q > 0$.

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Where do people get information about this distribution? In addition to unions, networks and individual past experience, there are now specialized web sites (see for the US: [http://www.vault.com](http://www.vault.com); ...).
The Basic Model

Search and the Reservation Wage

The Discounted Expected Utility at time \( t \) of a job paid \( w \): \( V_e(t) \)

Consider a small interval of time \( [t, t + dt] \).
If \( w \) is high enough (no quits), \( V_e(t) \) satisfies (according to dynamic programming techniques and already neglecting the \( o(dt) \) term):

\[
V_e(t) = \frac{1}{1 + rdt} \left[ w \, dt + q \, dt \, V_u(t + dt) + (1 - q \, dt) \, V_e(t + dt) \right]
\]

(1)

with \( r \) the discount rate, \( V_u(\cdot) \) the expected discounted utility in case of a return in unemployment. Multiplying by \( 1 + rdt \) and dividing by \( dt \):

\[
rV_e(t) = w + q[V_u(t + dt) - V_e(t + dt)] + \frac{V_e(t + dt) - V_e(t)}{dt}
\]

For \( dt \to 0 \):

\[
rV_e(t) = w + q \left[ V_u(t) - V_e(t) \right] + \lim_{dt \to 0} \frac{V_e(t + dt) - V_e(t)}{dt}
\]
Discounted Expected Utility of a job

Interpretation of the so-called “Bellman equation” solved by $V_e$:

$$r V_e(t) = w + q [V_u(t) - V_e(t)] + \lim_{dt \to 0} \frac{V_e(t + dt) - V_e(t)}{dt}$$

- $r V_e(t)$: discounted expected flow utility (or income)
- $w =$ wage = instantaneous return for an employed
- $q [V_u(t) - V_e(t)] =$ rate of job-loss, $q$, times the expected change in discounted income $[V_u(t) - V_e(t)]$
- $\dot{V}_e \equiv \lim_{dt \to 0} \frac{V_e(t + dt) - V_e(t)}{dt} =$ the change in value of the discounted lifetime earnings = “the capital gains from changes in the value of the job seen as an asset”.
The assumptions made guarantee a stationary environment. Then, $\dot{V}_e(t) = 0$. Getting rid of the time index and emphasizing the role of $w$:

$$rV_e(w) = w + q(V_u - V_e(w))$$

(2)

Hence, making the link between $V_e$ and $w$ explicit:

$$V_e(w) - V_u = \frac{w - rV_u}{r + q}$$

(3)

Equation (3) reveals that the gain of accepting a job is monotonically increasing in $w$.

Whatever the value of $V_u$, there exists some $x = rV_u$ such that

$$V_e(w) \gtrless V_u \iff w \gtrless x$$

(4)

Hence, $x = rV_e(x)$. 
The decision in unemployment

Let

- $c = \text{the out-of-pocket costs of job search + opportunity cost of time devoted to search}$
- $b = \text{the monetary value of domestic production and “leisure” net of losses due to unemployment per se (stigma, low self-esteem) + unemployment benefits (if any)}$
- The net instantaneous income in unemployment is $z \equiv b - c$.

Then, the intertemporal discounted value in unemployment at $t$ verifies:

$$V_u(t) = \frac{1}{1 + r dt} \left[ z \frac{dt}{dt} + \lambda dt V_\lambda(t + dt) + (1 - \lambda dt) V_u(t + dt) \right]$$

where $V_\lambda$ denotes the discounted expected utility conditional on being offered a job.
The Optimal Search Strategy of the unemployed

As the environment is stationary, the optimal strategy is constant all along the unemployment spell. So, getting rid of the time index, the choice of the unemployed is captured by

$$V_\lambda \equiv \mathbb{E}_w \max\{V_e(w), V_u\}$$ (6)

where $\mathbb{E}_w$ designates the expectation taken over the (random) wage $w$.

Remembering (4), the choice in (6) can be summarized as follows: If an offer is received, apply the following stopping rule:

1. accept job offer $\iff V_e(w) > V_u \iff w > x \equiv rV_u$
2. reject job offer $\iff V_e(w) \leq V_u \iff w \leq x \equiv rV_u$

where $x$ is called the reservation wage defined above by $rV_u = rV_e(x)$. 
If a job-seeker follows the stopping rule, his discounted expected utility, \emph{conditional} on being offered a job, $V_{\lambda}$ can be rewritten as:

$$
V_{\lambda} = \int_{0}^{x} V_{u}dH(w) + \int_{x}^{+\infty} V_{e}(w)dH(w) \text{ where } dH(w) \equiv h(w)dw
$$

$$
= V_{u} H(x) + \int_{x}^{+\infty} V_{e}(w)dH(w)
$$

\emph{Unconditional} on a job offer, $V_{u}$ solves (in a stationary state):

$$
V_{u} = \frac{1}{1 + rdt} \left[ z \ dt + \lambda \ dt \ V_{\lambda} + (1 - \lambda \ dt) \ V_{u} \right]
$$
Discounted expected utility in unemployment

Rearranging as before and letting \( dt \to 0 \), we obtain:

\[
rV_u = z + \lambda (V_\lambda - V_u)
\]

Using the definition of \( V_\lambda \) this can be rewritten:

\[
rV_u = z + \lambda \left[ V_u H(x) + \int_x^{+\infty} V_e(w) dH(w) \right.
\]

\[
- \left( H(x) + 1 - H(x) \right) V_u \left] \right.
\]

Since \((1 - H(x)) V_u = \int_x^{+\infty} V_u dH(w)\), we have (Interpret!):

\[
rV_u = z + \lambda \int_x^{+\infty} [V_e(w) - V_u] dH(w)
\] (8)
Using relation (3) and $x \equiv rV_u$ we obtain an implicit characterization of the reservation wage as a function of the parameters of the model:

$$x = z + \frac{\lambda}{r + q} \int_{x}^{+\infty} (w - x) \, dH(w)$$

(9)

By a fix-point argument, this implicit equation has a unique solution $x$:

- The left-hand side (‘LHS’) is the 45 degree line;

- (By the Leibnitz rule,) the right-hand side (‘RHS’) is decreasing in $x$. 
To interpret (9), let us define

- The *hazard rate* (or exit rate) $\phi$ from unemployment

$$\phi(x) \equiv \lambda \overline{H}(x),$$

where $\overline{H}(x) \equiv (1 - H(x))$ is the *acceptance rate*.

The hazard rate is here constant!

- The *average unemployment duration* $\equiv T_u$:

$$T_u = \frac{1}{\phi(x)} = \frac{1}{\lambda \overline{H}(x)} \quad (10)$$

with a clear-cut effect of $x$ on this duration.
Interpretation of (9)

Since \( z = b - c \) and the conditional expectation

\[
\mathbb{E}_w(w - x|w > x) \equiv \int_x^{+\infty} (w - x) \frac{h(w)}{H(x)} \, dw,
\]

we can rewrite Eq. (9) as:

\[
c + x - b = \phi(x) \int_0^{+\infty} \mathbb{E}_w(w - x|w > x) e^{-(r+q)t} \, dt \tag{11}
\]

Interpretation: Assume an offer is available paying \( x \):

LHS = instantaneous expected cost of continuing search
  = direct \((c)\) + net opportunity cost of search \((x - b)\)

RHS = instantaneous expected return to continuing search
  = the exit rate out of unemployment times the discounted sum of expected instantaneous income gains \((w - x)\) when continuing search, knowing that only wage offers above the reservation wage are accepted.
Graphical representation

Replace $x$ by any level of wage offer $W$, 

$$\phi(W)E(w-W \mid w>W)/(r+q)$$
Comparative Statics of the Basic Model

Relation (9) is an implicit equation in the reservation wage $x$, denoted $F(x, z, r, \lambda, q) = 0$ where

$$F(x, z, r, \lambda, q) \equiv x - z - \frac{\lambda}{r + q} \int_{x}^{+\infty} (w - x) dH(w)$$

Let $i$ denote any of the parameters $\{z, r, \lambda, q\}$ and $F_i \equiv \frac{\partial F}{\partial i}$. The direction of the derivative of $x$ as a function of any of the parameters $i$ is obtained by totally differentiating $F(x, z, r, \lambda, q) = 0$:

$$F_x dx + F_i di = 0 \iff \frac{dx}{di} = -\frac{F_i}{F_x} \quad \text{if} \quad F_x \neq 0, \quad (12)$$

From relation (10), the main comparative statics properties of the average duration in unemployment are derived (see CCZ and the note Comparative statics Note.pdf on Moodle). See also one the next exercises.
Exercise

Take an exponential density function \( h(w) = \gamma \exp[-\gamma \cdot w] \) (with \( \gamma > 0 \)) and use (9) and integration by part to verify that the reservation wage solves the implicit equation

\[
x = z + \frac{\lambda}{r + q} \exp[-\gamma \cdot x] \gamma
\]

Is \( x \) unique? Interpret the role of the parameters.

Exercise

Apply (12) and the Leibnitz rule to calculate \( F_x \) to check that

\[
\frac{\partial x}{\partial z} > 0, \quad \frac{\partial x}{\partial \lambda} > 0, \quad \frac{\partial x}{\partial r} < 0 \quad \text{and} \quad \frac{\partial x}{\partial q} < 0 \quad (13)
\]

Look then at the induced effect on the expected unemployment duration \( T_u \). Provide intuition for these properties.
Exercise

Assume that any unemployed person who finds a job is paid the net wage $w$ plus an untaxed subsidy $s$ paid by the government. Assume that the subsidy does neither affect the exogenous wage offer distribution $H$, nor the job arrival rate $\lambda$, nor the job separation rate $q$. You can also assume that search effort is exogenous and that there is no on-the-job search. The environment is stationary.

1. Write the Bellman equation solved by the intertemporal value of having a job, $V_e$, and from there characterize the reservation wage $x$. Is there a stopping rule? Explain why.

2. Compute by how much the reservation wage $x$ changes after a marginal increase in the subsidy $s$.

3. Compute the impact on the exit rate out of unemployment of a subsidy $s$ paid to unemployed workers who find a job?
Neoclassical theory of labor supply Let $w_A$ denote the “reservation wage” in Labour Supply theory.

\[
\begin{align*}
  w > w_A & \implies \text{employee} \\
  w \leq w_A & \implies \text{non-participant}
\end{align*}
\] (14)

Job Search Theory
Let $\Omega = \{H(\cdot), z, q, \lambda, r\}$ and $V_I = R_I/r$ the intertemporal value of a non-participant. To the extent that participation is a decision,

\[
\begin{align*}
  x(\Omega) > R_I & \implies \text{participant} \\
  x(\Omega) \leq R_I & \implies \text{non-participant}
\end{align*}
\] (15)

\[
\begin{align*}
  \text{if job offer and } w > x(\Omega) & \implies \text{employee} \\
  \text{if no job offer or } \{\text{a job offer and } x(\Omega) \geq w\} & \implies \text{unemployed}
\end{align*}
\] (16)
The Basic Model

Non-participation, Job-Seeking, and Employment

Consequence:

Consider that $R_I$ is exogenously distributed in the population. Parameters such as $b, \lambda, q$ that influence $x$ affect both

- Participation and
- Unemployment

Example 1: $\uparrow b \Rightarrow \uparrow x \Rightarrow$ The following indicators rise:
  - The intertemporal value in unemployment ($x = rV_u$)
  - The level of unemployment (because the acceptance rate shrinks)
  - The participation rate $P[R_I \leq x]$

Example 2: $\uparrow \lambda \Rightarrow \uparrow x \Rightarrow \uparrow P[R_I \leq x]$
Extensions

We consider one extension at the time!

Main reference: Chap. 5 of CCZ and more recent papers
1. Coverage of unemployment benefits

Table 5 Proportion (%) of those unemployed for 3 months or more in receipt of benefits by duration of unemployment, 2011

<table>
<thead>
<tr>
<th></th>
<th>3-5 months</th>
<th>6-11 months</th>
<th>12+ months</th>
</tr>
</thead>
<tbody>
<tr>
<td>Italy</td>
<td>11.5</td>
<td>9.7</td>
<td>1.6</td>
</tr>
<tr>
<td>Slovakia</td>
<td>29.1</td>
<td>11.7</td>
<td>3.3</td>
</tr>
<tr>
<td>Bulgaria</td>
<td>22.9</td>
<td>16.0</td>
<td>1.5</td>
</tr>
<tr>
<td>Poland</td>
<td>18.2</td>
<td>13.6</td>
<td>2.7</td>
</tr>
<tr>
<td>Latvia</td>
<td>31.2</td>
<td>18.5</td>
<td>1.8</td>
</tr>
<tr>
<td>Croatia</td>
<td>26.8</td>
<td>18.6</td>
<td>12.0</td>
</tr>
<tr>
<td>Lithuania</td>
<td>32.1</td>
<td>21.4</td>
<td>9.5</td>
</tr>
<tr>
<td>Estonia</td>
<td>45.7</td>
<td>33.6</td>
<td>3.1</td>
</tr>
<tr>
<td>Romania</td>
<td>25.1</td>
<td>26.5</td>
<td>13.0</td>
</tr>
<tr>
<td>Cyprus</td>
<td>36.8</td>
<td>14.4</td>
<td>2.4</td>
</tr>
<tr>
<td>Czech Republic</td>
<td>54.7</td>
<td>13.9</td>
<td>2.7</td>
</tr>
<tr>
<td>Greece</td>
<td>38.4</td>
<td>31.6</td>
<td>9.1</td>
</tr>
<tr>
<td>Sweden</td>
<td>24.7</td>
<td>27.0</td>
<td>23.4</td>
</tr>
<tr>
<td>Malta</td>
<td>15.0</td>
<td>24.6</td>
<td>40.6</td>
</tr>
<tr>
<td>Portugal</td>
<td>37.4</td>
<td>42.9</td>
<td>23.2</td>
</tr>
<tr>
<td>Slovenia</td>
<td>36.9</td>
<td>36.2</td>
<td>30.2</td>
</tr>
<tr>
<td>Luxembourg</td>
<td>31.2</td>
<td>41.5</td>
<td>33.8</td>
</tr>
<tr>
<td>Spain</td>
<td>44.6</td>
<td>40.9</td>
<td>31.3</td>
</tr>
<tr>
<td>Hungary</td>
<td>54.9</td>
<td>47.0</td>
<td>33.5</td>
</tr>
<tr>
<td>France</td>
<td>52.2</td>
<td>52.0</td>
<td>35.7</td>
</tr>
<tr>
<td>Denmark</td>
<td>53.6</td>
<td>56.2</td>
<td>60.2</td>
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<td>Austria</td>
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<td>Finland</td>
<td>58.3</td>
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<td>Belgium</td>
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<tr>
<td>Germany</td>
<td>81.4</td>
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<tr>
<td>EU</td>
<td>40.7</td>
<td>37.8</td>
<td>30.1</td>
</tr>
</tbody>
</table>

Note: No data available for IE, NL and UK. EU is defined to exclude these. Source: Eurostat, LFS

A1’ Net income for unemployed is higher if eligible to UB than if ineligible: \( z > z_n \). So, distinction \( V_u \) vs \( V_{un} \) (\( x \) vs \( x_n \)); Once employed, a worker becomes immediately eligible to UB \( \Rightarrow rV_e(w) = w + q(V_u - V_e(w)) \)

The reservation wage of non-eligible job-seekers, \( x_n \), can be shown to solve (interested by a proof? See CCZ p. 270):

\[
r(x_n - z_n) + q(x - z_n) = \lambda \int_{x_n}^{+\infty} (w - x_n) dH(w)
\]

If \( z \) increases, the incentive to work is *enhanced* for ineligible job-seekers, since employment now entitles them to higher flow income in case the job is lost (this is called an “entitlement effect”):

\[
\frac{dx_n}{dz} = \frac{dx_n}{dx} \frac{dx}{dz} = -\frac{q}{(r + \phi(x_n))} \frac{dx}{dz} < 0
\]

(the term in green by the Leibnitz rule again).
2. On-the-Job Search

A4’ Workers continue searching when on the job. For simplicity, cost of on-the-job search $= 0 \Rightarrow$ search always on-the-job: no threshold wage above which searching no longer pays. The current employer does not revise the wage upwards to avoid a quit.

$$rV_e(w) = w + q[V_u - V_e(w)] + \lambda_e \int_{w}^{+\infty} [V_e(\xi) - V_e(w)] dH(\xi) \quad (17)$$

The derivative of $V_e(w)$ w.r.t. $w$: 

$$V_e'(w) = \frac{1}{r + q + \lambda_e(1 - H(w))} > 0$$

$V_e'(w) > 0 \Rightarrow$ satisfies a stopping rule. The optimal strategy of a job seeker is characterized by a reservation wage ($V_e(x) = V_u$).
The discounted expected flow utility of being unemployed:

\[ rV_u = z + \lambda_u \int_{x}^{+\infty} [V_e(\xi) - V_u] \, dH(\xi) \]  \hspace{1em} (18)

Equate now (18) and (17) in which \( w = x \). Recalling that \( V_u = V_e(x) \):

\[ z + \lambda_u \int_{x}^{+\infty} [V_e(\xi) - V_u] \, dH(\xi) = x + \lambda_e \int_{x}^{+\infty} [V_e(\xi) - V_e(x)] \, dH(\xi) \]

The reservation wage then satisfies the following equation:

\[ x = z + (\lambda_u - \lambda_e) \int_{x}^{+\infty} [V_e(\xi) - V_u] \, dH(\xi) \]

Which can lead to (interested by a proof? See CCZ p. 272)

\[ x = z + (\lambda_u - \lambda_e) \int_{x}^{\infty} \frac{\overline{H}(\xi)}{r + q + \lambda_e \overline{H}(\xi)} \, d\xi \quad \text{with} \quad \overline{H}(\xi) \equiv 1 - H(\xi) \]  \hspace{1em} (19)
Four Cases from looking at \( x = z + (\lambda_u - \lambda_e) \int_{x}^{+\infty} [V_e(\xi) - V_u] \, dH(\xi) \):

1. \( \lambda_e = 0 \Rightarrow \) no on-the-job search \( \Rightarrow \) back to basic model: \( x > z \)

2. Increasing \( \lambda_e \) lowers reservation wage (as long as \( \lambda_e < \lambda_u \Rightarrow x > z \));

3. \( \lambda_e = \lambda_u \Rightarrow x = z \);

4. \( \lambda_e > \lambda_u \Rightarrow x < z \): Interesting to accept low-paid as it is a powerful steppingstone to better paid jobs.

The empirical literature generally finds \( \lambda_u \geq \lambda_e \).
For instance, Robin (2011) finds that \( \lambda_e = 0.12 \cdot \lambda_u \).
However, van den Berg and Ridder (1998) find that \( \lambda_e \) was slightly larger than \( \lambda_u \) in the 80’s in The Netherlands.
3. Choosing (Freely!) How Hard to Look for a job

Introduce search effort \( e \) (e.g. time spent on searching)

\[ \lambda = \alpha \lambda(e) \] with \( \lambda' > 0 \) and \( \lambda'' < 0 \). The parameter \( \alpha > 0 \) is independent of individual effort and interpreted as an indicator of the state of the labor market (could also depend on characteristics like education of the job-seeker).

\[ c(e) \] is the cost arising from the search effort \( e \), with \( c' > 0 \) and \( c'' > 0 \).

Still, in-work effort (e.g. time) not modeled; no disutility of work.

---

\(^3\)This cost of effort can include direct expenses, the opportunity cost of time, and psychological costs of searching (see e.g. Krueger and Mueller, 2011; Schwartz, 2015). The seminal paper of Mortensen (1977) deals with a non-separable utility function of consumption and effort.
The reservation wage $x$ is defined, as before, by the equation (9):

$$rV_u = x = b - c(e) + \frac{\alpha \lambda'(e)}{r + q} \int_x^{+\infty} (w - x) dH(w),$$

(20)

in which $e$ should be understood as the best chosen level of effort.

This level maximizes $rV_u$. The first-order optimality condition (“FOC”) writes:

$$c'(e) = \frac{\alpha \lambda'(e)}{r + q} \int_x^{+\infty} (w - x) dH(w)$$

(21)

System (20) and (21) is a system of non-linear equations that implicitly defines the two unknowns, $e$ and $x$, as a function of parameters $\alpha, b, r, q$ and of the shapes of the CDF $H(\cdot)$ and of the functions $\lambda \cdot$ and $c(\cdot)$.

---

4 Second-order condition is satisfied since $c'' > 0$ and $\lambda'' < 0$. 

---
We can write this as system of two equations (see Comparative statics Note.pdf on Moodle):

\[
f(e, x) \equiv x - b + c(e) - \frac{\alpha \lambda(e)}{r + q} \int_x^{+\infty} (w - x) dH(w) = 0, \quad (22)
\]

\[
g(e, x) \equiv c'(e) - \frac{\alpha \lambda'(e)}{r + q} \int_x^{+\infty} (w - x) dH(w) = 0, \quad (23)
\]

being conscious that the parameters appear in these two equalities as well.

Totally differentiating this system wrt the unknowns \{e, x\} and parameter \(b\) yields:

\[
\frac{\partial f}{\partial e} \, de + \frac{\partial f}{\partial x} \, dx = - \frac{\partial f}{\partial b} \, db \quad (24)
\]

\[
\frac{\partial g}{\partial e} \, de + \frac{\partial g}{\partial x} \, dx = 0 \quad (25)
\]
Consider the first equality:

By the optimality condition for effort: \( \frac{\partial f}{\partial e} = 0, \)

Moreover: \( \frac{\partial f}{\partial x} > 0 \) and \( \frac{\partial f}{\partial b} < 0 \)

So, from equation (24) we obtain:

\[
\frac{dx}{db} = -\frac{\partial f}{\partial b} \frac{\partial f}{\partial x} > 0
\]

Next, in (25),

\[
\frac{\partial g}{\partial e} > 0 \quad \text{and} \quad \frac{\partial g}{\partial x} > 0 \quad \Rightarrow \quad \frac{de}{dx} = -\frac{\partial g}{\partial x} \frac{\partial g}{\partial e} < 0
\]

Finally, a rise in \( b \) affects \( e \) via \( x \):

\[
\frac{de}{db} = \frac{de}{dx} \frac{dx}{db} < 0.
\]
Clear-cut conclusions

- Higher unemployment benefits raises the reservation wage and lowers job-search effort.
- So, higher unemployment benefits lower the unemployment exit rate:

$$\phi = \alpha \lambda(e)(1 - H(x))$$

Note: These conclusions can be questioned when $b$ is low enough and the model is generalized: See Mesén Vargas and Van der Linden (2019) (and the references mentioned in this paper) and Ferraro, Jaimovich, Molinari and Young (2020).
Next, totally differentiating System (24)-(25) wrt $e$, $x$, $\alpha$ leads to:

\[
\frac{\partial x}{\partial \alpha} > 0 \quad \text{and} \quad \frac{\partial e}{\partial \alpha} > 0
\]

Message:

- A positive shift in the rate of arrival of job offers (i.e. $\alpha \uparrow$) raises the reservation wage and job-search effort.
- An improvement in business conditions (i.e. $\alpha \uparrow$), has therefore an ambiguous net effect on the exit rate:

\[
\phi = \alpha \lambda(e)(1 - H(x))
\]

From empirical analyses however, $\phi$ increases with $\alpha$.

Note: Properties with respect to $r$ and $q$ can also be derived.
The insured unemployed are monitored

Up to here,
- Whether an unemployed accepts or rejects a job offer and
- How much an unemployed searches for a job
are the outcome of a free arbitrage between private costs and benefits.

However, the Unemployment Insurance agency exerts a certain degree of monitoring on the behavior of the “insured unemployed” (= an unemployed worker who gets an unemployment insurance benefit). The same can also be true in case of an assistance benefit.

Detected rejections of a “suitable” job offer and “insufficient” job-search effort can lead to a sanction, i.e. a temporary or permanent, complete or partial, loss of benefit.

About the impact of monitoring and sanction schemes, see e.g. Cockx, Dejemeppe, Launov and Van der Linden (2018) and the references therein. See also an exercise below.
Exercise

This is an exercise about the monitoring of search effort inspired by Manning (2009).

1. From (20) and (21), consider the two implicit equations in $(e, x)$:

\[ f(e, x) \equiv x - b + c(e) - \frac{\alpha \lambda(e)}{r + q} Q(x) = 0 \]  

\[ g(e, x) \equiv c'(e) - \frac{\alpha \lambda'(e)}{r + q} Q(x) = 0, \]

where $Q(x) = \int_x^{+\infty} (w - x) dH(w)$. Check that $Q'(x) < 0$.

2. Draw these two curves $f(e, x) = g(e, x) = 0$ in a $(e, x)$ space.

3. In unemployment insurance (UI) $b = b_0$. In unemployment assistance (UA), $b = b_1 < b_0$. Draw on the same graph the curves in the cases of UI and UA.

4. Imagine now that search effort is no more chosen without constraint in UI, but well in UA. To do so, introduce a minimum search effort level $e$ such that if the chosen effort level $e \geq e$ the unemployment remains entitled to UI. Otherwise, (s)he gets UA. For what levels of $e$ is the claimant adapting search effort upwards? (what happen to $x$?) When does the claimant drop out of the register of insured unemployed (UI) and collect UA?
Exercise

Consider the job-search model with endogenous search effort in steady state. Start from \( f(e, x) = 0 \) and \( g(e, x) = 0 \) as in the previous exercise. Assume that \( c(e) = c \cdot e \), with \( c > 0 \) and \( \lambda'(e) > 0, \lambda''(e) < 0 \). Assume also that to survive the job-seeker needs a minimal exogenous amount of income (hence of consumption) denoted \( C_0 > 0 \). So, the following constraint is imposed:

\[
    b - c \cdot e \geq C_0 \quad \text{with } b > C_0
\]  

(28)

There exists only an UI with a flat benefit level \( b \) and no unemployment assistance scheme. Let \( e^* \) denote the optimal search effort when (28) is ignored. Consider henceforth an unemployed individual for whom Constraint (28) is binding, so that her effort verifies \( e \leq e^* \). For this individual only:

a) Produce the system of equations solved by the levels of search effort and of the reservation wage.

b) What are the implications of a marginal rise in \( b \) on search effort and on the reservation wage? Are the signs of these effects clear?

c) Are the two implications found in Sub-question [b] standard in the theoretical literature?

d) Explain how the exit rate out of unemployment is related to the levels of search effort and of the reservation wage. Are there clear-cut implications of a rise in \( b \) on this exit rate?
4. Non-stationarity

Caused by a change in any of the parameters \((b, \lambda, \ldots)\) or in the distribution of wage offers *after entry into unemployment*. Example 1: Stigma effect of duration so that \(d\lambda/dt < 0\).
Example 2: Post taxes and transfers “Replacement ratio” \((b/w)\) as a function of unemployment duration

![Chart 4: Net replacement rates over a five-year period (2014) for a one-earner married couple with two children and previous in-work earnings equal to the average wage](image)

a) Unanticipated shocks

If changes in parameters are *unanticipated* random shocks, the individual *might* think that all the parameters will remain at their current values.

Consider the case of random macro shocks that affect $\lambda$. Then, by analogy with the basic stationary model:

$$x(t) = z + \frac{\lambda(t)}{r + q} \int_{x(t)}^{+\infty} (w - x(t))dH(w)$$
b) Non-stationarity under perfect foresight

If instead parameters change in a deterministic way.

Let us here assume only one modification:

\[ A1' \quad z(t) \leq z(t') \text{ for all } t \geq t'. \]

E.g. declining unemployment benefits while \( c \) is constant.

Perfect foresight: *Changes in the values of the parameters are by assumption correctly anticipated!*

The inter-temporal value at the time of entry in unemployment, \( V_u(0) \), should differ from its value \( V_u(t) \) at later times \( (t > 0) \). However, \( V_e(w) \) is still stationary:

\[ rV_e(w) = w + q[V_u(0) - V_e(w)] \quad (29) \]

Hence, \( V_e(w) \) is still increasing in \( w \) \( \Rightarrow \) reservation wage property.
Non-stationarity under perfect foresight

In this case the lifetime utility of the unemployed at duration \( t \), \( V_u(t) \), maximizes wrt \( s \):

\[
\begin{align*}
    z(t)dt + \lambda dt \left[ \int_s^{+\infty} V_e(w)dH(w) + V_u(t + dt)H(s) \right] \\
    + (1 - \lambda dt)V_u(t + dt)
\end{align*}
\]

(30)

divided by \( 1 + rdt \). The optimal reservation wage at duration \( t \), \( x(t) \), is obtained by setting to zero the derivative with respect to \( s \) of the term between brackets (considering \( V_u(t + dt) \) as given):

\[
V_e[x(t)] = V_u(t + dt) \quad \text{and if } dt \to 0: \quad V_e[x(t)] = V_u(t)
\]

As \( V_e(\cdot) \) is increasing, \( x(t) \) and \( V_u(t) \) vary in the same direction.
Intuitively, since $z(t)$ decreases over time, $V_u(t) \leq V_u(t')$ necessarily obtains for every $t \geq t'$.

If so, since $x(t)$ and $V_u(t)$ vary in the same direction, we can deduce that $x(t) \leq x(t')$ for every $t \geq t'$.

⇒ the exit rate $\phi(x(t))$ should \textit{increase} with unemployment duration!

Isn’t this counterfactual? There is evidence of a \textit{negative} relationship between the unemployment exit rate and unemployment duration.

The above prediction of positive relationship can be more than compensated if,

- The job arrival rate $\lambda$ declines simultaneously with duration;
- Unobserved heterogeneity: The more productive workers leave first, so that over time those still unemployment are less productive and hence have a lower $\lambda$. 
Illustration of above theory (van den Berg, 1990)

Assume:

h1 \( \exists T < +\infty \mid z(t) = \) some constant \( z \) for \( t \geq T \) and \( z(t) \leq z(t') \) for all \( T \geq t \geq t' \);

h2 \( q = 0 \Leftrightarrow \) jobs last forever \( \Rightarrow V_e(w) = w/r; \)

h3 Keep \( H(\cdot), \lambda, r \) constant.

From (30) and from \( V_e(x(t)) = x(t)/r = V_u(t) \), one easily derives:

\[
\begin{align*}
rV_u(t) &= z(t) + \frac{\lambda}{r} \int_{x(t)}^{+\infty} (w - rV_u(t))dH(w) + \dot{V}_u(t) \\
so, \dot{x}(t) &= r \cdot x(t) - r \cdot z(t) - \lambda \int_{x(t)}^{+\infty} (w - x(t))dH(w)
\end{align*}
\]

This differential equation has a unique solution for \( x(t) \), given the boundary condition that follows from the assumption that the model is stationary for \( t \geq T \).
5. Behavioral Economics and Job-search

“Behavioral economics”: Loosely speaking, the research area that studies the link between psychology and economics.

Why behavioral economics? Non-standard assumptions (about preferences) can allow to explain phenomena that are difficult to explain with standard assumptions.

Two examples are briefly introduced below. They exploit

5.1 The so-called “reference-dependent preferences”, where the reference point is given by recent income. See Della Vigna, Lindner, Reizer and Schmieder (2017).

5.2 A non-standard way of discounting the future. What if the job-seeker is “impatient”? See Della Vigna and Paserman (2005).
5.1 Explaining “spikes” in the exit rate out of unemployment

This presentation is an intuitive introduction (see Della Vigna, Lindner, Reizer and Schmieder, 2017, for an exposition)

What is the problem?

“(…) in most Western countries (…) The benefits are set at a constant replacement rate for a fixed period, typically followed by lower benefits under unemployment assistance. In such systems, the hazard rate [= exit rate] from unemployment typically declines from an initial peak the longer workers are unemployed, surges at unemployment exhaustion, and declines thereafter [→ creating a spike] (…) It is well known that a basic job search model à la Mortensen (1986) and van den Berg (1990) is unable to match this pattern.” (Della Vigna et al., 2017, p. 1970)
A reform in Hungary

- Consider the following two unemployment benefit regimes.
  - Constant UB for 280 days followed by lower UB (in blue)
  - Step-down system with higher UB for the first 100 days (in red).
- Discounted UBs are the same under both regimes at the start of unemployment and after 280 days in unemployment.
The standard non-stationary job-search model à la van den Berg (1990) leads to the following predictions for changes in the exit rates out of unemployment induced by the reform:

Hence, no spike!
Reference-dependent preferences in a nutshell

- Workers are loss-averse with respect to consumption below a “reference point”.
- This reference point is given by recent income levels.
- Flow utility from consumption $c_t$ in period $t$ with $r_t$ the reference point:

$$u(c_t|r_t) = v(c_t) + \eta[v(c_t) - v(r_t)] \text{ if } c_t \geq r_t, \; \eta \geq 0 \quad (31)$$

$$u(c_t|r_t) = v(c_t) + \eta \lambda[v(c_t) - v(r_t)] \text{ if } c_t < r_t \quad (32)$$

- The utility consists of consumption utility $v(c_t)$ and gain-loss utility $v(c_t) - v(r_t)$.
- The parameter $\lambda > 1$ captures loss-aversion.
Without proof, the job-search model with reference-dependent preferences leads to the following predictions for changes in the exit rates out of unemployment induced by the reform:

Without proof, this model provides a better fit of the observed hazard rates.
5.2. Another explanation of low job-search effort

What is the problem? Low search intensity compatible with high unit cost of job-search, low returns from search ... It could also be due to a non-standard comparison of immediate costs and future returns.

Up to now, job-seekers discount the future in a “standard way” (exponential discounting).

However, there is empirical evidence of a higher discount rate in the short run than in the long run (creating procrastination).

“Hyperbolic discounting”: Let \( u_t \) be the utility level at time \( t \) (time will be discrete here). The present (\( t = 0 \)) value of the stream \((u_t)_{t \geq 0}\) is

\[
  u_0 + \beta \sum_{t=1}^{\infty} \delta^t \cdot u_t, \quad 0 < \beta, \delta \leq 1.
\]  

\( \beta \) is the parameter of short-run impatience and \( \delta \) the one of long-run impatience.
These preferences are dynamically inconsistent (compare the stream \((u_t)_{t \geq \tau}\) from today’s perspective and from the perspective of time \(\tau > 0\)).

Put another way, it is as if the agent has two selves:

- The agent who is making a decision in the current period, the current self, is impatient: discounting the payoffs of search by \(\beta \cdot \delta\)
- The future self (who benefits from past search effort) who discount the future in a standard way: The discount factor is \(\delta\).
Consider a stationary framework where a job accepted at duration $t$ starts in $t+1$.

Simplify Assumption (A2'): $\alpha \lambda(e) = e$.

In period $t$, the unemployed chooses the search effort $e_t$ and the reservation wage $x_t$ to max. $V_{u,t}$:

$$
\max_{e_t} \quad b - c(e_t) + \beta \delta \left[ e_t E_H \{ \max \left( V_{e,t+1}(w), V_{u,t+1} \right) \} + (1 - e_t) V_{u,t+1} \right],
$$

where from the perspective of the individual in period $t$:

$$
V_{e,t+1}(w) = w + \delta \left[ q V_{u,t+2} + (1 - q) V_{e,t+2} \right] \quad (34)
$$

and the reservation wage $x_t$ chosen in period $t$ by the current self is such that:

$$
V_{u,t+1} = V_{e,t+1}(x_t). \quad (35)
$$
The “sophisticated” hyperbolic discounter has rational expectations: (s)he knows that future preferences will be hyperbolic as well (while the “naive” hyperbolic agent believes (s)he will discount exponentially later on).

For brevity, consider the “sophisticated” agent only. Della Vigna and Paserman (2005) show that:

- A more impatient (lower $\beta$) agent searches less intensively but sets lower reservation wages.

- In general, the impact of short-run impatience on the exit rate is therefore ambiguous. Under reasonable assumptions, a more impatient (lower $\beta$) agent exits less rapidly from unemployment.

- An empirical analysis confirms the latter property.
6. Numerous other extensions

a) Up to now, job-seekers do not value non-wage characteristics of jobs such as
   - Hours worked;
   - Commuting time / distances (see e.g. Rupert, Stancanelli and Wasmer, 2009);
   - Job amenities.

Some papers have extended the analysis to include those features.

⇒ Reservation wage replaced by a reservation utility level.

Examples: Blau (1991); Section 4.3 of van den Berg and Ridder (1998); Bloemen (2008); Sullivan and To (2014); Hall and Mueller (2018); Guglielminetti, Lalive, Ruh and Wasmer (2017).

Note: Liquidity constraints are prevalent among job losers (Card, Chetty and Weber, 2007; Chetty, 2008; Basten, Fagereng and Telle, 2014).

c) Household job-search: Couples make joint decisions about job-search (Guler, Guvenen and Violante, 2012; Mankart and Oikonomou, 2016).

d) The role of the social network (see e.g. Zenou, 2013) and referrals (see e.g. Dustmann, Glitz and Schönberg, 2016).
e) Questioning the quality of the information of the unemployed. The basic model and most of the extensions assume that the unemployed are well informed and in particular have correct beliefs. However,

- There is also evidence that (some) job-seekers have wrong beliefs (e.g. are too optimistic about job opportunities). See Spinnewyn (2015), Krueger and Mueller (2016), and Mueller, Spinnewyn and Topa (2018).

- Altmann, Falk, Jäger and Zimmermann (2018) show that German unemployed at risk of long-term unemployment are in need of “information about the current labor market situation, the non-pecuniary consequences of (un)employment and effective job search strategies” (p. 34). See also Belot, Kircher and Muller (2019).
General conclusions

- The partial-equilibrium job-search model explains how imperfect information about job offers generate “search frictions” on the labor market.

- Some clear-cut properties emerge in a stationary and a non-stationary setting.

- The job-search model is the framework on the basis of which a lot of empirical research has been developed.
What do other sciences tell us?
The determinants of job-search effort

According to the basic job-search model, search effort is a function of a few parameters \((\alpha, b, r, q)\), the shape of the wage offer distribution \(H(\cdot)\) and functions \((\lambda(\cdot), c(\cdot))\).

Psychology lists a wider set of determinants, namely:

1. “motives to search” (employment commitment, financial hardship);
2. “job-search competencies” (self-efficacy, emotion control, ...);
3. “job search constraints” (ill-health, child-care obligations, ...)

Wanberg, Kanfer and Rotundo (1999) find that motives and competencies affect job search intensity, but no effect of constraints. A higher job search intensity increases reemployment.
Impact of psychology on economists’ research

Illustration

Economists now look at job-search competencies (the locus of control).

- Individuals with an external locus of control believe that outcomes are primarily matters of fate or chance.
- Individuals with an internal locus of control believe outcomes depend primarily on their own efforts.

Individuals with an internal locus of control search more and have lower reservation wages.

- McGee and McGee (2016): Laboratory experiment


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Extensions

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