Equilibrium Unemployment
The matching models

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Empirical analyses reveal *very intense* job and workers reallocations... even in “sluggish” labor markets

“sluggish” ↔ markets with small variations in the (un)employment rate

⇒ This chapter develops a *dynamic* model that explicitly recognizes these flows...

... to yield a theory of (inefficient) unemployment in equilibrium
Outline

1. Facts
   - Definitions
   - Empirical regularities
   - Beveridge curve

2. The matching process

3. Static model

4. Dynamic model

5. Efficiency

6. Dynamics

7. Critique and Extensions

8. Note on the literature
Imagine a large data set of production units (firms or plants). In an country, a region or a sector, and for a given unit of time (often a year),

- Job creations (JC) = Σ of job gains in new units or in growing ones
- Job destructions (JD) = Σ of job losses due to closing or contractions
- Net employment changes = JC - JD
- Job reallocation (JR) = JC + JD
- Excess job reallocation
  = Job reallocation - | Net employment changes |
  = JC + JD - | JC - JD |
Worker flows

Imagine now a data set of movements of workers into jobs (hirings) and out of jobs (separations) over a specified period of time.

**Turnover:** it measures the gross number of labor market transitions *during a period of time*

= full counting of all events (i.e. every time a worker is hired or separates during the period) during that period;

**(Gross) worker reallocation:** it measures the number of persons who participate to transitions *between two discrete points in time*, say a month;

⇒ e.g. hirings equal the number of workers who are with the firm at time $t$, but were not with that employer at time $t-1$;

= a more limited counting than turnover;
If job creations & destructions, hirings $H$ & separations $S$ are all measured by a comparison between the same two points in times:

- **Net employment changes** = $JC - JD = H - S$
- **Excess worker reallocation or “churn”**
  - $= H + S - | \text{Net employment changes} |$
  - $= H + S - | H - S |$
- **Churning** (*definition different from churn above*)
  - $= \text{the difference between excess worker reallocation and excess job reallocation}$
  - $= H + S - JC - JD$

This difference “represents labor reallocation arising from firms churning workers through continuing jobs or employees quitting and being replaced on those jobs” (Bassanini and Marianna, 2009, p. 4)
These job and workers flows have been measured in a large number of countries ...

... on the basis of non homogenous definitions and country-specific data sets
⇒ lack of comparability is an issue;

Below, I produce some orders of magnitude for some of the flows defined above. Those interested by the cyclicality of these flows are referred to CCZ.
## Orders of Magnitude

### JC & JD

<table>
<thead>
<tr>
<th>Country (period)</th>
<th>JC</th>
<th>JD</th>
<th>JC+JD</th>
<th>Net Empl.</th>
<th>Excess job real.</th>
</tr>
</thead>
<tbody>
<tr>
<td>F* (99-00)</td>
<td>12.0</td>
<td>8.3</td>
<td>20.3</td>
<td>3.7</td>
<td>16.6</td>
</tr>
<tr>
<td>G* (77-99)</td>
<td>8.4</td>
<td>7.1</td>
<td>15.5</td>
<td>1.3</td>
<td>14.2</td>
</tr>
<tr>
<td>I* (86-94)</td>
<td>12.3</td>
<td>10.2</td>
<td>22.5</td>
<td>2.1</td>
<td>20.4</td>
</tr>
<tr>
<td>U.K.* (80-98)</td>
<td>11.5</td>
<td>12.6</td>
<td>24.2</td>
<td>-1.1</td>
<td>23.1</td>
</tr>
<tr>
<td>U.S. (88-97)</td>
<td>12.5</td>
<td>10.0</td>
<td>22.5</td>
<td>2.5</td>
<td>20.0</td>
</tr>
</tbody>
</table>

**Table:** Job creation and destruction flows. Annual average rate as a percentage of total employment, all sectors of the economy.

**Note:** * F=France, G =West Germany, I = Italy; Manufacturing only for the U.K. Mostly private sector for Germany and the U.S. Source: data from Haltiwanger et al. (2010); except for France where data is from Picart (2008, Table 2)
Job flows

Some conclusions

- Net employment growth is the (relatively small) difference between two large rates.
- Excess worker reallocation $> |$ Net employment changes $|.$
- Even with a narrow definition of the “sector”, job reallocation is to a large extent a within-sector phenomenon (a property not illustrated above).

Note: Job creation rates and job destruction rates are underestimated (job reallocations within firm are for instance ignored)

Order of Magnitude in levels:
Every day in France, *about* 10,000 jobs are created and 10,000 are destroyed.
In the US, 8 millions new jobs were created in the first quarter of 2000 and 7 millions were destroyed (Laing, 2011, p. 809).
## Worker flows

<table>
<thead>
<tr>
<th>Country</th>
<th>Entry rate (hirings)</th>
<th>Exit rate (separations)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Belgium</td>
<td>14</td>
<td>12</td>
</tr>
<tr>
<td>France</td>
<td>14</td>
<td>14</td>
</tr>
<tr>
<td>Germany</td>
<td>16</td>
<td>14</td>
</tr>
<tr>
<td>Italy</td>
<td>11</td>
<td>10</td>
</tr>
<tr>
<td>U.K.</td>
<td>15</td>
<td>14</td>
</tr>
<tr>
<td>U.S.</td>
<td>19</td>
<td>22</td>
</tr>
<tr>
<td>EU 15</td>
<td>15</td>
<td>14</td>
</tr>
</tbody>
</table>

**Table:** Annual employment inflows and outflows in percentages: Year 2011.

**Source:** OECD Labor Force Statistics Database. Note: 2010 for the United States. The entry rate is calculated as the ratio of persons employed for less than one year to the average stock of employment in $t$ and $t-1$ and the exit rate as the difference between the entry rate and the employment growth rate.
Job-to-job movements represent a substantial proportion of all manpower movements.

Exits from employment are the sum of

1. Quits,
2. The ending of short-term contracts,\(^1\)
3. Retirements,
4. Firing for cause,
5. Job loss through no fault of the employee = “Job displacement”. See next slide.

\(^1\)A big share of exits in countries where employment protection legislation is strong and coexists with a segment of the labor market which is much less protected.
Figure 9.12

Note: Percentage of employees aged 20–64 who are displaced from one year to the next, based on firms declarations.

Figure 9.10

Source: Davis et al. (2006, figure 1).
**Figure 9.13**
Unemployment inflow and outflow monthly rates in the OECD countries. The entry rate is the ratio between monthly entries into unemployment and the total number of employed persons during the month in question; the exit rate is the ratio between monthly exits from unemployment and the total number of unemployed persons during the month in question. The starting year for the available series varies between 1968 (for the United States) and 1986 (for New Zealand and Portugal). For all countries, the data end in 2009.

Source: Elsby et al. (2013).
**v and u coexist**

- Basic idea: A relationship between vacant jobs and unemployment to assess the importance of worker reallocations. More reallocation problems if a large vacancies-workforce ratio, $v$, coexists with a large unemployment rate $u$.

- The (descriptive) “Beveridge curve” = a plot of $(u, v)$ pairs measured at different points in time.

- On the two following slide, examples of Beveridge curves.

- In many countries, data about vacancies are rather poor because they are based on information collected by Public Employment Agencies (however, many vacant position are ignored by these agencies). Since the end of 2000, JOLTS data in the US are of good quality. Since 2008, European countries are supposed to develop specific surveys.
Shifts of the “Beveridge curve”
Source: Elsby, Michaels and Ratner (2015)
Comments

On the previous figure,
- Some movements where $v$ and $u$ move in opposite ways
- But also some marked inward and outward shifts

On the next figure,
- Outward shift of the US Beveridge Curve during the “Great Recession”.

For the US, Birchenall (2011) shows that the correlation between the trends in unemployment and vacancies is positive (+0.85) over 1951-2007 while the correlation between cyclical components is -0.90.
Beveridge curve in the US 2000-13
A recent outward shift associated to “long-term” unemployment

The EU commission and its statistical institute (EUROSTAT) propose “Beveridge curves” where the vertical axis does not display the vacancies-labor force ratio, but different measures such as:

- The “vacancy rate” understood as the number of job openings over the sum of employment and job openings. See http://ec.europa.eu/eurostat/web/labour-market/job-vacancies.

- The “labour shortage indicator”, i.e. the proportion of firms reporting labour shortage as a factor limiting production.

Note on Statistics in the EU
Beveridge curve: Summary of stylized facts

- “First, at cyclical frequencies unemployment and vacancies move in opposite directions, tracing out a negatively inclined Beveridge locus.”

- “Second, the position of this locus has shifted periodically in many developed economies, most notably during the persistent rise in European unemployment in the 1980s, and more recently in the wake of the Great Recession in the United States.” (Elsby, Michaels and Ratner, 2015, p. 572)
Central idea:
Trade in the labor marked is decentralized, uncoordinated, time-consuming and costly for both firms and workers (Pissarides, 2000, p.3)

Why? Because of heterogeneities, imperfect information and lack of coordination, that are generating “frictions” on the labor market.

- Jobs and workers are heterogeneous (in skills, location,...).
- Imperfection information about job and worker characteristics is prevalent.
- When deciding where to apply, job-seekers do not coordinate their choices.

Yet, in a macroeconomic perspective, modeling these heterogeneities in detail is not the aim and is out of scope.
Like the production function, the “matching function” is
“a modeling device that captures the implications of the
costly trading process without the need to make the
heterogeneities and other features that give rise to it
explicit.” (Pissarides, 2000, p.4)

So, formally, the agents who participate to trade in the labor
market are homogeneous. However, the fact that their “meeting” is
time-consuming is due to the above-mentioned “frictions”.

So, the basic framework intends to study “frictional” unemployment.
One more job-seeker of any type adds frictions that matter for all
other types. This a rather extreme assumption...
... Extension: Introduce explicit heterogeneities, e.g. in skill.  

Standard assumption: “Undirected search”: job seekers meet all
vacancies randomly.

2 The literature proposes different approaches here (see e.g. Mortensen and
Pissarides, 2003, Shimer, 2007, the assignment literature mentioned at the end of this
chapter).
Directed search as an alternative
Job seekers have here perfect information about the different wages offered for different jobs before they decide where to look for work.
The origin of frictions is then a lack of coordination among job-seekers because *trade is decentralized*.
The importance of directed search in the literature is growing steadily.

- Full specialization in either trade or production:
  jobs filled production
  jobs vacant search for applicants $\rightarrow$ matching
  worker employed production
  worker unemployed search for vacant jobs $\rightarrow$ matching
  $\Rightarrow$ No on-the-job search! (See extensions later however)

- Micro-foundation of the matching process: often an “urn-ball model” (See CCZ p. 584).
The matching function

Continuous-time setting

The *instantaneous* flow of hires, $H$, is assumed to be a function of the number (stock) of job-seekers, $U$, and the number (stock) of vacancies, $V$. This matching is assumed to be a random process (*all* vacancies and *all* job-seekers meet randomly$^3$). The matching function is defined as

$$H = M(V, U) \quad (1)$$

As it is standard now, let us assume that $M(V, U)$ is increasing, concave and homogeneous of degree 1.
Moreover: $M(0, U) = M(V, 0) = 0$.

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$^3$An interesting alternative is the so-called “stock-flow matching”.
Remarks:

- More generally, the function $H = M(V, U)$ describes the number of contacts between vacancies and job seekers: A contact does not necessarily lead to a new hire.

- Below, $U$ will designate the number of unemployed.
  
  - This number could be weighted to account for
    
    - Search intensity: $H = M(V, e \cdot U)$ with $e$ being search effort,
    
    - Heterogeneity in the unemployment pool: $H = M(V, c \cdot U)$ with $c$ being a time-dependent exogenous parameter, say, affected by the share of long-term unemployed,...

- With on-the-job search, employed job-seekers should be included as an argument of the matching function.
The matching function

Empirical support

- “The usefulness of the matching function depends on its empirical viability”. (Pissarides, 2000, p.4).
- See Petrongolo and Pissarides (2001) for a survey. Many studies support the above assumptions. In particular, constant returns to scale. Nevertheless, increasing returns to scale are sometimes advocated (in particular in Economic Geography).
- The Cobb-Douglas specification is often not rejected:

\[ H_t = A_t V_t^{1-\eta} U_t^\eta \]  \hspace{1cm} (2)

An order of magnitude for \( \eta \) being [0.4; 0.7].
- With firms and job-seekers posting vacancies on the internet, have matching frictions disappeared? No. Recent studies do not show a structural break due to the internet. A good match requires also information that cannot be easily revealed by a CV or description of a job vacancy on a website.
Tightness and various rates

Definition: Tightness on the labor market

\[ \theta \equiv \frac{V}{U} \]

The rate (= the probability per unit of time) at which a vacant job is filled is:

\[ \frac{M(V, U)}{V} = M(1, U/V) \equiv m(\theta) \]

Differentiating \( M(1, U/V) \equiv m(\theta) \) w.r.t. \( U \) yields:

\[ m'(\theta) < 0 \]

The following “Inada conditions” are useful to guarantee the existence of an equilibrium:

\[ \lim_{\theta \to 0} m(\theta) = +\infty \quad \text{and} \quad \lim_{\theta \to +\infty} m(\theta) = 0 \]
The matching process

The rate at which an unemployed finds a job:

\[ \frac{M(V, U)}{U} = \frac{V}{U} \frac{M(V, U)}{V} = \theta m(\theta) \]

Differentiating \( M(V, U)/U = \theta m(\theta) \) w. r. to \( V \) yields:

\[ [\theta m(\theta)]' > 0 \]

The following “Inada conditions” are useful to guarantee some properties of the model:

\[ \lim_{\theta \to 0} \theta m(\theta) = 0 \quad \text{and} \quad \lim_{\theta \to +\infty} \theta m(\theta) = +\infty \]  \hspace{1cm} (3)
An additional vacancy

reduces the rate at which job vacancies are filled
= the “congestion external effect”
increases the exit rate out of unemployment
= the “thick market (beneficial) externality”

Similarly for an additional unemployed in the queue for jobs:

“A worker deciding to join a queue or stay in one considers the probabilities of getting a job, but not the effects of his decision on the probabilities that others face...” (Tobin, 1972)
The mechanisms underlying the matching function are closely related to those at the root of the *job search model* considered earlier.

General equilibrium considerations clearly suggest that the job arrival rate and the level (or the distribution) of wages should be endogenous.

Both *Equilibrium Search* (previous chapter) and *Matching Models* put emphasis

1. on the role of employers (the demand side of the labor market)
2. on wage formation.
Equilibrium Search and Matching Models

There are two basic approaches:

**Wage posting models.** Take-it-or-leave-it wage offers are set ("posted") by employers in a non-cooperative setting while

- either workers search randomly like in Chapter 1, Sec. 2 (Equilibrium search model)
- or there is directed search (as in the so-called "Competitive Search Model" of Moen, 1997, briefly mentioned later on)

**Matching models (This chapter).** The number of job creations depends on the numbers of job-seekers and vacancies according to the "matching function". The **surplus** created by a match typically leads to a negotiation over the wage.

Relative importance of the two wage-settings

Surveys by Hall and Krueger (2012) and Brenzel, Gartner and Schnabel (2014) find similar shares. Quoting the latter:

*Both modes of wage determination coexist in the German labor market, with more than one-third of hirings being characterized by individual wage negotiations. Wage bargaining is more likely for more-educated applicants, for jobs with special requirements, and in tight regional labor markets. Wage posting (in the sense of a fixed offer) dominates in the public sector, in larger firms, in firms covered by collective bargaining agreements, and in jobs involving part-time and fixed-term contracts.*

Yet, Using data from the leading job board CareerBuilder.com, Marinescu and Wolthoff (2015) show that most vacancies do not post wages!
A static version of the model
Not included in CCZ

Assumptions:

- Risk-neutral workers and firm owners.
- At the beginning of the period, an exogenous number $N = U$ of individuals are jobless and search randomly for a job.
- Each firm is made of a single vacant or filled job.
- If a vacancy is filled, a given (real) amount of output $y$ is produced. The worker receives a (real) wage $w$ and the firm’s (real) profit level is $y - w - \kappa$, where $\kappa$ is a fixed cost incurred to open a vacancy (create a job slot).
- If the job remains vacant, the profit is equal to $-\kappa$. 
A static version of the model

Assumptions:

- \( M(V, N) < \min(V, N) \Rightarrow m(\theta) < \min(1, \theta^{-1}) \) where \( \theta \equiv V/N \).
- Wages are negotiated after efforts have been made on both sides of the labor market to create vacancies and search for a partner (sunk costs)
- Throughout this chapter, one neglects credit market frictions: Entrepreneurs have no problem financing their creation of vacancies and search of an applicant.

Questions that are raised:

1. How many vacancies are created?
2. How is the surplus of the match splitted?
3. How many are (un)employed?
A static version of the model
labor demand

There is free entry of vacancies. Firms open vacancies as long as the expected return is nonnegative.
In equilibrium:

\[ m(\theta)(y - w - \kappa) + (1 - m(\theta))(-\kappa) = 0 \text{ or } (4) \]

\[ m(\theta) = \frac{\kappa}{y - w} \text{ (5)} \]

Since \( m'(\theta) < 0 \), the higher \( y \) (the lower the wage and the cost of opening a vacancy), the higher \( \theta \): This means a ‘thick’ labor market with many vacancies per job-seeker.
Equation (5): A downward-sloping labor demand equation in a \((\theta, w)\) space.
Notice that \( 0 < \kappa/(y - w) < 1 \) is required, hence \( w < y \).
When an individual meets a vacancy, the total surplus created if they form a match is $y$ (since $\kappa$ is sunk + no income for the unemployed).

Assume the following non-cooperative two-stage game:

**Stage 1.** The firm-owner and the worker propose a wage contract.

**Stage 2.** If one of the two players (or both) does not accept the contract in stage 1, then

- with probability $\gamma$, $0 < \gamma < 1$, the worker makes a take-it-or-leave-it offer.
- with probability $1 - \gamma$ the firm owner makes such an offer.

If the offer is rejected, the job is destroyed.
A static version of the model
Characterization of the subgame perfect equilibrium

In stage 2
if the worker makes the offer, the pay-off for the employer
is zero and the total surplus, $y$, accrues to the worker.
The opposite holds if the firm owner makes the offer.

In stage 1
◊ the worker knows that at the end of stage 2, his
expected income will be $\gamma \cdot y + (1 - \gamma)0$.
◊ the employer knows that at the end of stage 2, his
expected pay-off will be equal to $\gamma \cdot 0 + (1 - \gamma)y$.
Therefore, in stage 1, indifference between
(i) signing a contract at stage 1 giving an expected income
to the worker $\gamma \cdot y$ and an expected profit $(1 - \gamma)y$ and
(ii) waiting until stage 2.
A static version of the model
The subgame perfect equilibrium

Assume a small cost of going to stage 2. The unique equilibrium consists in immediately signing a contract that shares the surplus by paying the wage:

\[ w = \gamma \cdot y. \]

Equivalently, one could maximize the following (asymmetric) Nash product with respect to \( w \):

\[ [w]^{\gamma} [y - w]^{1-\gamma} \quad (6) \]

The first-order condition is: \( w = \gamma y \).
A static version of the model

Characterization of the equilibrium

The equilibrium is the triple \((w^*, \theta^*, V^*)\) characterized by:

\[
\begin{align*}
w^* &= \gamma y, \\
m(\theta^*) &= \frac{\kappa}{(1 - \gamma)y} \Rightarrow V^* = N \cdot m^{-1} \left( \frac{\kappa}{(1 - \gamma)y} \right)
\end{align*}
\]

The higher \(\gamma\) or \(\kappa\) (the lower \(y\)), the smaller is \(\theta^*\).

The expected number of unemployed (and, by the law of large numbers, their actual number) is:

\[
N(1 - \theta^* m(\theta^*)) < N
\]

It is decreasing in \(\theta^*\).
Whatever the levels of the parameters \((\gamma, \kappa, y, N)\), there is some unemployment due to frictions (i.e. the exogenous function \(m(\cdot)\)).

This does not imply that the wage level does not matter: The intensity of the unemployment problem varies with the way total surplus \(y\) is splitted.

The mechanisms we have identified here hold true in the following dynamic setting.
Equilibrium of flows and the Beveridge Curve
in a dynamic environment

\[ N = \text{the size of the labor force. } N \text{ is large.} \]
\[ \dot{N} = \frac{dN}{dt} \text{ (exogenous! For an extension, see e.g. Sec. 7 of Elsby, Michaels and Ratner, 2015).} \]
Assume that those who enter the labor force begin by looking for a job.
Assume an exogenous separation rate \( q \) (extensions: endogenous \( q \)).
Dynamic model

Equilibrium of flows and the Beveridge Curve

\[ \dot{U} = \dot{N} + qL - \theta m(\theta)U \text{ (law of large numbers)} \]  
(10)

\[ \dot{u} = q + n - [q + n + \theta m(\theta)]u \text{ where } u = U/N, n = \dot{N}/N \]  
(11)

In a steady state \( \dot{u} = 0 \):

entries in = exit from unemployment.

The previous equation leads to

\[ u = \frac{q + n}{q + n + \theta m(\theta)}, \]  
(12)

which is strictly positive as soon as either \( q \) or \( n \) is > 0 and \( \theta m(\theta) < +\infty \).
Equilibrium of flows and the Beveridge Curve

Or, since the “vacancy rate” \( v = V/N \),

\[
u = \frac{q + n}{q + n + (\frac{v}{u})m(\frac{v}{u})}\]

= an implicit relationship between \( u \) and \( v \) that defines the “Beveridge curve” understood as

*the set of pairs* \((u, v)\) *such that the unemployment rate remains constant.*

To be distinguished from the plot of \((u, v)\) realizations seen before (except for years such that \( \dot{u} = 0 \)).

It can be shown that given the assumptions made before the “Beveridge curve” is decreasing and convex.
The Beveridge Curve as an equilibrium relationship
A rightward shift of the “Beveridge curve” can be the consequence of

- An increase in $q$ or in $n$ (the rate of growth of $N$!)
- A deterioration in the “efficiency of the matching process”

If we knew the equilibrium value of $\theta$ in steady state, then the equilibrium $u$ and $v$ would be determined.

The following is concerned by the determination of equilibrium tightness in steady state.
A dynamic model with ex-post wage bargaining

Assumptions

- There are two goods: a produced good (the *numeraire* sold in a competitive market) and labor.
- Each firm is made of a single vacant or filled job.
- Infinitely lived agents with perfect foresight + risk neutrality.
- $h$ is the (exogenous and deterministic) fixed cost of a vacant job *per unit of time*. ($h$ captures the cost involved in posting a vacancy, searching for applicants and selecting them).
- $r$ is the exogenous discount rate common to all agents. Firms can borrow and lend from perfect capital markets at the rate $r$ (Extension to imperfect credit markets: see Wasmer and Weil, 2004).
Expected profits

- One Euro invested at time $t$ yields $1 + r \, dt$ at time $t + \, dt$. So, the discount factor will be $\frac{1}{1 + r \, dt}$ on any interval of length $dt$.

- During any short time period $dt$, $(y - w) \, dt$ measures the current flow of return.
  - $y$ measures (exogenous) real output (sold),
  - $w$ is the real wage assumed to be lower than $y$,
  - working time is exogenous and normalized to 1. Hence, aggregate labor supply is $N \cdot 1$ if $w >$ the reservation wage defined later on.

- The worker and the firm separates with probability $q \, dt$.

- At any time $t$, a firm’s real discounted expected return from an occupied job is denoted $\Pi_e(t)$.
  A firm’s real discounted expected return from a vacant job is denoted $\Pi_v(t)$.
Expected return from an occupied job

With a perfect capital market and an infinite horizon, $\Pi_e(t)$ satisfies the following Bellman equation:

$$
\Pi_e(t) = \frac{(y - w)dt + qdt \Pi_v(t + dt) + (1 - qdt)\Pi_e(t + dt)}{1 + rdt}
$$

(13)

As CCZ, one here assumes that with probability $q \, dt$ the job becomes vacant.

Pissarides (2000) considers that at a rate $q$ the job is destroyed: Its value is then zero.

Under free-entry (see below), both approaches lead to the same conclusions.
Expected return from an occupied job

Multiply both sides of (13) by $1 + r \, dt$. Then divide both sides by $dt$ and take the limit for $dt \to 0$. This yields

$$r \Pi_e(t) - \frac{d \Pi_e(t)}{dt} = y - w + q(\Pi_v(t) - \Pi_e(t)).$$  

(14)

In a steady state, this Bellman equation becomes:

$$r \Pi_e = y - w + q(\Pi_v - \Pi_e).$$  

(15)

A filled vacancy can be seen as an asset owned by the firm.

- $r \Pi_e$ is the (instantaneous) rate of return on this asset.
- At any time $t$, this rate of return is the sum of
  - instantaneous profits $y - w$
  - and the expected net return due to a change of state is $q(\Pi_v - \Pi_e)$, which is actually negative.
Expected return from a vacant job

The same reasoning leads to the following Bellman equation for $\Pi_v(t)$. In an infinitesimal time interval, the probability of meeting more than one job-seeker can be neglected. *Implicit assumption: any contact with a job seeker leads to a match: $\Pi_e(t) > \Pi_v(t)$ and as all workers are identical the first one which is met is recruited.*

One ends up with:

$$r\Pi_v(t) - \frac{d\Pi_v(t)}{dt} = -h + m(\theta(t))(\Pi_e(t) - \Pi_v(t))$$  \hspace{1cm} (16)

or in a steady state

$$r\Pi_v = -h + m(\theta)(\Pi_e - \Pi_v).$$  \hspace{1cm} (17)

An unfilled job can also be seen as an asset owned by the firm (the interpretation is similar to the one on the previous slide).

The rest of this section is developed in a steady state.
The system of Bellman equations (15) and (17) can be solved to yield the following expression for $\Pi_v$:

$$
\Pi_v = \frac{-(r + q)h + m(\theta)(y - w)}{r(r + q + m(\theta))},
$$

which decreases with tightness $\theta$. Given the above-mentioned Inada conditions (3):

$$
\lim_{\theta \to 0} \Pi_v = \frac{(y - w)}{r} > 0 \text{ and } \lim_{\theta \to +\infty} \Pi_v = -\frac{h}{r} < 0.
$$

Whatever the unemployment level, the inflow of vacancies ends when the profit expected from an additional vacant job becomes zero:

$$
\Pi_v = 0
$$
Labor demand

Under *free entry* of vacancies (i.e. $\Pi_v = 0$),

\[
(17) \quad \Rightarrow \quad \Pi_e = \frac{h}{m(\theta)} \quad h \times \text{the expected length of time } 1/m(\theta)
\]

\[
(15) \quad \Rightarrow \quad \Pi_e = \frac{y - w}{r + q} \quad q \text{ is added to } r \text{ to discount } y - w
\]

So, the labor demand (or “*vacancy-supply curve*”) is the following downward-sloping relationship between $w$ and $\theta$:

\[
\frac{h}{m(\theta)} = \frac{y - w}{r + q} \Leftrightarrow w = y - \frac{(r + q)h}{m(\theta)}
\]
Labor demand

*If the wage level is exogenous* \((w < y)\), this demand side of the model leads to the following comparative statics (recall that \(m'(\theta) < 0\))
Note

The absence of physical capital

The absence of physical capital is noteworthy. An alternative, developed e.g. by Acemoglu (2001), consists in assuming that
(i) vacancy costs $h$ are negligible (actually, zero) and (ii) an equipment is required to produce, the acquisition cost being incurred before meeting applicants. Now, if $\tilde{k}$ designate a fixed cost of equipment per job, the free entry condition introduced above becomes

$$\Pi_v = \tilde{k}$$

This does not change the properties of the model in steady-state.

So, I stick to the standard presentation which emphasizes the role of vacancy costs.
The Behavior of workers

Keeping the assumption that workers are risk neutral, one could again look at a small interval of time $dt$ and then take the limit $dt \to 0$.

Let $V_e$ and $V_u$ be the (steady-state) real discounted expected value of the income stream respectively in employment and an unemployment. The model is written under the *implicit assumption that* $V_e > V_u$, which turns out to be true under Nash bargaining.

Utility = Instantaneous income in employment: $w$
(implicitly: hand-to-mouth agents; full-time job; leisure time ignored). At an exogenous rate $q$, the job is lost.

The expected utility of an employed person satisfies:

$$rV_e = w + q(V_u - V_e) \quad (19)$$
The Behavior of workers

An unemployed worker is always in search of a job (Job-search effort is here fixed and normalized to 1).

At each instant, this search procures him or her a net gain denoted by $z$ (by assumption $z < y$):

- the value of time (leisure, home production) minus whatever disutility, if any, comes from not having a job (stigmatization);
- $\oplus$ benefits linked to being unemployed (unemployment insurance or social welfare transfers, if any)
- $\ominus$ the various costs attached to searching for a job (commuting to the public employment agency, postage of applications,... costs that are presumably shrinking thanks to the Internet).
The Behavior of workers

Since the exit rate from unemployment is $\theta m(\theta)$, the expected utility of an unemployed person satisfies:

$$rV_u = z + \theta m(\theta)(V_e - V_u)$$

(20)

(the unemployed will not turn down job opportunities since $V_e > V_u$ under Nash bargaining)

Subtracting (20) from (19) implies that workers have no incentive to quit if $w > z$:

$$V_e - V_u = \frac{w - z}{r + q + \theta m(\theta)}.$$  

(21)

---

4Looking again at an interval of time of infinitesimal length, the probability of receiving more than one offer can be neglected.
Wage formation

The timing of events is essential here:

1. The (unemployed) workers and firms engage in a costly (time-consuming) search process (e.g. the firm incurs cost $h$ during a period of time in order to create a vacancy and recruit a worker);

2. Once they have sunk this cost, they bargain over the wage (ex-post individual bargaining over wages).

Implications can be phrased in two actually equivalent ways:

Once the two partners have met,

1. There is a range of wages at which both partners prefer to match rather than breakup or

2. There is a “match-specific surplus” (a rent) that has to be shared.
The surplus of a match

The (total) *surplus of a match* = the sum of the rents (of the firm and of the worker) that a filled job procures.

Rent = gain from the contractual relationship minus outside option

= $\Pi_e - \Pi_v$ in case of the employer

= $V_e - V_u$ in case of the employee

$\Rightarrow$ (total) surplus $S = V_e - V_u + \Pi_e - \Pi_v$.

As $V_e$ can be rewritten as $(w + q V_u) / (r + q)$ and $\Pi_e$ as $(y - w + q \Pi_v) / (r + q)$,\(^5\)

$$S = \frac{y - r (V_u + \Pi_v)}{r + q}$$

\(^5\)Note that both $V_e$ and $\Pi_e$ are linear in $w$. This is the so-called “transferable utility case”. See L’Haridon, Malherbet and Pérez-Duarte (2013).
How is the total surplus split?

One can define a worker’s reservation wage, \( w \) characterized by the indifference condition:

\[
V_e(w) - V_u = 0 \iff \frac{w - rV_u}{r + q} = 0
\]

Similarly, remembering that the cost of vacancy creation is sunk, the employer’s reservation wage, \( \bar{w} \), is such that:

\[
\Pi_e(\bar{w}) - \Pi_v = 0 \iff \frac{y - \bar{w} - r\Pi_v}{r + q} = 0
\]

So, the boundaries of the bargaining set in which the negotiated wage has to be is

\[
[rV_u, y - r\Pi_v]
\]

Any wage within the bargaining set could be an outcome of the bargain. So, there is an indeterminacy in matching models!
How is a wage chosen in \([rV_u, y - r\Pi_v]\)?

◊ The literature typically selects a specific wage in the bargaining set through either

   • An axiomatic Nash bargaining solution (Nash, 1953); See next slides.

   • Or a strategic bargaining game approach (Rubinstein, 1982): Two players alternate offers over many periods with an (in)finite horizon (An example of such a game has been introduced in the static model above).

   The precise environment of the game matters.

Under some assumptions both approaches lead to the same outcome. CCZ discuss all this on p.415-422. If needed, see Chapter 16 of Osborne (2004) and L’Haridon, Malherbet and Pérez-Duarte (2013).

◊ Alternative views are developed e.g. by Hall (2005) and Farmer (2011), who deal with this indeterminacy in matching models.
Wage bargaining

Surplus sharing

Wages are renegotiated continuously.
When they bargain over the current wage, the players take $V_u$ and $\Pi_v$ as well as tightness on the labor market as given.

The value of the wage negotiated at each moment is the solution of the maximization of the following “Nash product”:

$$\max_w (V_e - V_u)^\gamma (\Pi_e - \Pi_v)^{1-\gamma}, \ 0 \leq \gamma \leq 1$$

(22)

The first-order condition of this problem can be written as:

$$V_e - V_u = \gamma S \quad \text{and} \quad \Pi_e - \Pi_v = (1 - \gamma)S$$

(23)

So, the total surplus $S$ is split according to the shares $\gamma$ and $1 - \gamma$. 
To derive a wage equation from

\[ V_e - V_u = \gamma S = \gamma \frac{y - r(V_u + \Pi_v)}{r + q} \]

we do NOT exploit (21) but

\[ V_e - V_u = \frac{w - rV_u}{r + q} \]

\[ \Rightarrow w = rV_u + \gamma(y - r(V_u + \Pi_v)) \]

\[ 0 \text{ by free entry} \]

Expression \( w = rV_u + \gamma(y - rV_u) \) has an intuitive interpretation:

\[ \gamma = 0 \quad w = w = rV_u \text{ (workers get no rent)} \]

\[ \text{as } \gamma \uparrow \text{ an increasing share of the difference } \]

\( (y - rV_u) \text{ accrues to the worker} \]

**Note:** If \( \gamma = 1 \), \( \Pi_v \) is negative (see (15) and (17)).

So, firms do not open vacancies at all because they cannot recoup the sunk cost \( h \).
Wage bargaining
Getting rid of $rV_u$

To complete the analysis, one would like to relate the negotiated wage $w$ to $\theta$ and to the parameters of the model. So we need to reformulate $rV_u$ as a function of $\theta$.

This can be done in various ways.

1) The book follows one approach that leads to:

$$rV_u = \frac{z(r + q) + \gamma y\theta m(\theta)}{r + q + \gamma\theta m(\theta)}$$

Substituting this expression of $rV_u$, they get this ‘wage curve”:

$$w = z + (y - z)\Gamma(\theta) \quad \text{with} \quad \Gamma(\theta) = \frac{\gamma[r + q + \theta m(\theta)]}{r + q + \gamma\theta m(\theta)}, \quad \Gamma' > 0 \quad (24)$$
2) Consider now the approach of Pissarides (2000). The starting point is again

\[ w = rV_u + \gamma(y - rV_u) = (1 - \gamma)rV_u + \gamma \cdot y, \]  

(25)

where \( V_u \) solves

\[ rV_u = z + \theta m(\theta)(V_e - V_u) \]  

(26)

Under free entry, the solution to the game \( V_e - V_u = \gamma S \) can be rewritten:

\[ (1 - \gamma)(V_e - V_u) = \gamma \Pi_e \]

where \( \Pi_e = h/m(\theta) \). So,

\[ V_e - V_u = \frac{\gamma h}{1 - \gamma m(\theta)} \Rightarrow rV_u = z + \theta m(\theta) \frac{\gamma h}{1 - \gamma m(\theta)} \]
Wage bargaining

Plugging this expression in (25) yields this “wage curve” (WC)

\[ w = (1 - \gamma)z + \theta \gamma h + \gamma y \]  \hspace{1cm} (27)

or, if \( h \) is proportional to \( y \) (\( h = k \cdot y \)):

\[ w = (1 - \gamma)z + \gamma y (1 + \theta k) \]  \hspace{1cm} (28)

According to the problem studied, the wage curve (24) or (27) is more convenient.

Note: For any value of \( \theta \), the equilibrium wage is unique. Whether this holds true with on the job search is briefly discussed at the end (under the heading “extensions”).
The labor market equilibrium

\[ WC : w = (1 - \gamma)z + \theta \gamma h + \gamma y \quad LD : w = y - \frac{(r+q)h}{m(\theta)} \]

Because the matching function has C.R.S., equilibrium \((\theta, w) \perp (u, v)\).

Combining the wage curve (27) and the labor demand (18) it can be shown that the equilibrium \((\theta, w)\) pair is unique.
The labor market equilibrium

Eliminating the wage yields an implicit equation in (equilibrium) tightness. If $h$ is defined in level:

\[
\frac{(1 - \gamma)(y - z)}{r + q + \gamma \theta m(\theta)} = \frac{h}{m(\theta)}
\]  

(29)

where the LHS is decreasing and the RHS is increasing in $\theta$ (uniqueness thanks to the Inada conditions (3)).

If $h = k \cdot y$, then

\[
\frac{(1 - \gamma)(y - z)}{r + q + \gamma \theta m(\theta)} = \frac{k \cdot y}{m(\theta)}
\]

(30)

which can be made independent of $y$ if one assumes constant “replacement ratios” $z/w$ (check it; hint: Return to (28) $\Rightarrow w = \tilde{w}y$).

With the Cobb-Douglas matching function (2), Condition (29) becomes

\[
\frac{(1 - \gamma)(y - z)}{r + q + \gamma A \theta^{1-\eta}} = \frac{h}{A \theta^{-\eta}}
\]
Comparative statics

For comments, see CCZ p. 596-600. For a more nuanced view about the effect of unemployment benefits in matching models, see Landais, Michaillat and Saez (2017).
Dynamic model

Note
From the one-job-one-firm case to the “large firm” case

- Generalization to a large firm occupying a continuum of workers and possibly using capital:
  1. **Under individual wage negotiation:**
     - if labor and capital can be adjusted instantaneously so that returns to scale are constant, the large firm and the one-job-one-firm setting are equivalent (case discussed by CCZ).
     - When the marginal product of labor is decreasing, one has to think more at wage formation:
       - Under bilateral negotiation *without commitment about future wages*, an additional worker depresses the marginal product of labor and hence the wage of all existing workers (under the additional assumption of automatic renegotiation of wages); see Cahuc and Wasmer (2001), Cahuc, Marque and Wasmer (2008), and Elsby and Michaels (2013).
       - Kaas and Kircher (2015) discuss instead the case where firms *commit to long-term wage contracts*.
  2. **Other mechanisms of wage formation (collective bargaining,...):** see Mortensen and Pissarides (1999); Bauer and Lingens (2014).
Is the equilibrium (constrained) efficient?
Section 4 of the book

There are

- congestion effects within each category
- and positive externalities between the categories.

Are the search externalities internalized by the ex post Nash bargain? This is the question raised in this section.

Constrained efficiency means that the decentralized equilibrium is identical to the outcome chosen by a hypothetical social planner who maximizes social welfare given the fundamental frictions.

The social planner maximizes “social output” defined on the next slide. The social planner ignores distributional issues: (S)he only cares about aggregate net output created in this economy.
Social output

With risk neutral agents, social output at time $t$ (divided by the exogenous size of the labor force $N$) is denoted $\omega(t)$ and given by:

$$\omega(t) = y(1 - u(t)) + z \cdot u(t) - h \cdot v(t) = y + [z - y - h \cdot \theta(t)] u(t) \quad (31)$$

$\omega(t)$ is simply current output + the value of unemployment - the cost of opening vacancies.

One keeps the assumption $z < y$.

However, how should we interpret $z$?
If $z$ is the sum of UBs, say $b$, and the value of leisure net of search costs, consider that $b$ is financed through a tax $T$ on earnings and that the budget is balanced at each time $t$. Then, $\omega(t)$ values only the part of $z$ measuring the value of leisure net of search costs (possibly negative).
The choice made by the social planner
What should we expect?

Due to congestions, the impact of additional vacancies on the hiring rate is declining (keeping $u$ fixed).

Moreover, each additional vacancy costs $h$.

$\Rightarrow$ unlikely that creating more vacancies is always good from the point of view of net output $\omega$. 
The choice made by the social planner

Optimal control

Starting from an initial situation \( u(t = 0) = u_0 \), the social planner would solve:

\[
\max_{\theta(t), u(t)} \int_{0}^{+\infty} \omega(t) \cdot e^{-rt} \, dt
\]

subject to the equation of motion (taking \( n = 0 \)):

\[
\dot{u}(t) = q(1 - u(t)) - \theta(t) \cdot m(\theta(t)) \cdot u(t).
\]

This \textit{optimal control} approach is followed in Section 4.2.2. in the book (\( \theta \) is the control variable and \( u \) the state variable).
The choice made by the social planner

A simpler approach

Consider the steady-state value of $\omega(t)$ only
(ignoring the adjustment from one steady state to another $\Rightarrow$ the path of the economy is not discounted)
$\Rightarrow$ we can later compare the so-called “social optimum” in a steady state with the steady state emerging from decentralized decisions when $r \to 0$.

Maximizing (31) with respect to $\theta$ with $u$ defined by the Beveridge curve $u = \frac{q}{\theta m(\theta) + q}$ is equivalent to maximizing the following expression with respect to $\theta$ only:

$$y + \frac{q}{\theta m(\theta) + q} [z - y - h \cdot \theta].$$  \hspace{1cm} (32)

Interpret!
The choice made by the social planner

The optimal value of tightness has to be such that the sum of two effects becomes nil.

$$\frac{d}{d\theta} \left[ \frac{q}{\theta m(\theta) + q} \right] [z - y - h \cdot \theta] + \frac{q}{\theta m(\theta) + q} \cdot (-h) = 0$$

1. Increasing $\theta$ reduces the unemployment rate.
2. Conditional on the level of unemployment, a higher tightness entails more vacancy costs.

Through this maximization, the benevolent planner takes the consequences of his (her) choice of $\theta$ on the externalities due to frictions.
Is the equilibrium efficient?

This first-order condition of the planner’s problem with respect to $\theta$ leads to:

$$\frac{(y - z)(1 - \eta(\theta))}{q + \theta m(\theta)\eta(\theta)} = \frac{h}{m(\theta)},$$

where

- $\eta(\theta) = \left| \frac{d \log (m(\theta))}{d \log(\theta)} \right|$
- $1 - \eta(\theta) = \left| \frac{d \log (\theta m(\theta))}{d \log(\theta)} \right|$

In the decentralized equilibrium with ex post Nash bargaining, $\theta$ solves (see (29) above when $r \to 0$):

$$\frac{(y - z)(1 - \gamma)}{q + \theta m(\theta) \cdot \gamma} = \frac{h}{m(\theta)}$$

(34)

In general, the solution, say $\theta^e$, of (33) and the one, say $\theta^d$, of (34) are different.
Is the equilibrium efficient?

(33) $\iff$ (34) if

- the bargaining power of the worker $\gamma$ equals $\eta(\theta^e)$, the elasticity of the rate $m$ of filling a vacancy with respect to tightness $\theta$ (taken in absolute value).

- Because of the CRS assumption, $\eta(\theta^e)$ is also the elasticity of $M(V, U)$ with respect to unemployment.

Under the condition $\gamma = \eta(\theta^e)$, called the “Hosios condition” (Hosios, 1990), the search externalities are internalized by the ex-post Nash bargain.

Under the Hosios condition, the total surplus generated by a match is shared in such a way that the externalities are exactly balanced, so that efficiency is restored.
“Basically, the Hosios condition says that in order to maximize the aggregate gains from trade, less transaction costs, the traders’ bargaining shares must reflect their marginal contribution to the value of the aggregate transaction flow. This condition is satisfied in the case of a linearly homogeneous matching technology if and only if agents’ shares equal the elasticities of the matching function with respect to the stocks of buyers and sellers in the market.” (Mortensen and Wright, 2002)
Is the equilibrium efficient?

- Notice that the Hosios condition expresses that workers should have a positive bargaining power. (Very different from most union models where the underlying reference is the competitive labor market).
- Notice also that a certain level of unemployment is present when the outcome is efficient. (Very different from union models).
- If $\gamma > \eta(\theta^e)$, equilibrium unemployment is above its efficient level.
- Conversely, if $\gamma < \eta(\theta^e)$, equilibrium unemployment is inefficiently low.
- Too few (resp. too many) vacancies are created when $\gamma$ is too high (resp. too low).
Is the equilibrium efficient?

- There is no reason why the Hosios condition should be fulfilled. Hence, in general, search equilibria with ex post Nash bargaining are typically inefficient.

Other wage mechanisms than the sharing of the ex post surplus of the match can automatically lead to an efficient outcome (see e.g. the summary of the “Competitive Search Equilibrium” of Moen (1997) provided by CCZ, p. 603. Notice that search is directed and no more random).

- The laissez faire economy (without taxes and unemployment insurance) and under the Hosios condition is not the only efficient outcome.

  When the bargaining power does not fulfill the Hosios condition, taxation can restore efficiency because a positive marginal tax rate (resp. a negative one) decreases (resp. increases) the share of the surplus that accrues to the workers (Boone and Bovenberg, 2002).

Out-of-stationary-state dynamics
We skip most of this section (Section 6 in the book)

The study of out-of-stationary-state dynamics allows to diagnose the origin of the perturbations that affect movements in employment. Remedies adopted to reduce under-employment will vary with this diagnosis.

*aggregate* shocks Change in aggregate demand or supply of goods, and would *not* shift the Beveridge curve

*aggregate* shocks = A change in $y$, $r$, $z$, or $\gamma$

*reallocation* shocks A restructuring of production units, which would shift the Beveridge curve but not much aggregate supply or demand.

*reallocation* shocks A change in function $m(.)$ or in $q$
Out-of-stationary-state dynamics

Express value functions out of steady state (like Eq. (16) above).

All decisions of agents are directed toward the future ("forward-looking" decisions)
⇒ The number of vacancies (and hence, \( \theta \)) and the wage immediately "jump" to the stationary value (no inertia in wages unless the model is extended).

When labor market tightness has reached its stationary value \( \theta^* \), the differential equation (11) describing the evolution of the unemployment rate becomes:

\[
\dot{u} + [q + n + \theta^* m(\theta^*)]u = q + n
\]
Dynamic adjustment in the basic model

Source: Elsby, Michaels and Ratner (2015)

“Despite these qualitative successes, however, the standard search model faces challenges in explaining (...) crucial quantitative features of observed Beveridge curve dynamics” (Elsby, Michaels and Ratner, 2015, p. 586)
Out-of-stationary-state dynamics

Extensions: Real-Business-Cycle models with matching frictions

Standard approach since Merz (1995) and Andolfatto (1996):

- Output (of an aggregate good) can either be consumed, invested, or spent to cover the cost of vacant jobs.
- A (representative risk averse) household = a large extended family which contains a continuum of members. Members of the family perfectly insure each other against fluctuations in income due to employment and unemployment.
- The household has standard preferences over consumption and the fraction of their members who are not working (“leisure”).
- The household chooses consumption and savings given the law of motion of employment.
- The market for the aggregate good clears.
- Some parameters follow a random process with idiosyncratic and/or aggregate shocks.
Critique

1) Poor cyclical performances of the matching model?


The mechanism at work is well explained in the following quote: “(...) an increase in productivity increases the value of a match. As a consequence, firms post more vacancies which boosts workers’ job finding rate, raising their outside option (the value of being unemployed). The net result is that wages rise, eating up much of the gain received by firms associated with the increase in productivity, thereby lowering the response of vacancies.” (Gomme and Lkhagvasuren, 2015, p. 107)
Critique

The “Shimer’s critique” led to a major controversy. Main answers\(^6\):

a) **Introduce wage stickiness.**

   In the Nash product (22), the threat point for bargaining is the payoff of each partner if they separate (resp. \(V_u\) and \(\Pi_v\)). Plausible calibrations leads then to an elasticity of \(w\) with respect to \(y\) close to 1.

   According to Hall and Milgrom (2008), the realistic threat is to extend the bargaining (not to terminate it) as long as a solution can be found in \([w, \bar{w}]\). Under some assumptions, wages are fully rigid.

\(^6\)Many other propositions exist in the literature.
1) Reply to Shimer’s critique (Continued)

b) **Introduce a fixed cost of matching.**

Pissarides (2009) argues that wages in *new* matches are actually as cyclical as in the textbook model under Nash bargaining. Moreover, all what matters for job creation is the gap between (expected) productivity and wages in *new* matches.

The free-entry condition emphasizes that the firm’s cost of creating a vacancy is *proportional* to the expected duration of it.

“Other matching costs, such as training, negotiation, and one-off administrative costs of adding a worker on the payroll, are neglected by the model“ (Pissarides, 2009, p. 1363). Pissarides recommends to add such a fixed cost. Call it $H$. Then the left-hand side of (18) becomes $H + h/m(\theta)$.

Introducing this fixed cost of matching $H$ suffices to get good cyclical properties of the textbook model.
2) “Lagos (2000) emphasizes that if the matching function is a reduced-form relationship, one should be concerned about whether it is invariant to policy changes. Addressing this issue requires an explicit model of heterogeneity that gives rise to an empirically successful reduced-form matching function” (Shimer, 2007, p. 1077).

Large outward shifts in the locus of the Beveridge curve are mainly explained by “a deterioration in the matching process” (a “black box”). What explains the latter?

Efforts to address this critique in particular by Stevens (2007), Mortensen (2009), Ebrahimy and Shimer (2010), and Barnichon and Figura (2015).
3) “Random search is dumb search”: Each agent searches in “all directions” at random, nobody uses
- wages that firms commit to pay
- or a “match maker” / “market maker” (private or public employment agencies, head hunters, https://www.mturk.com/mturk/welcome,...) to “organize a less time-consuming search process”.

Opposite view: Directed search on the basis of posted wages (Moen, 1997) and/or creation of specific online platforms (CareerBuilder.com, www.craigslist.org,...). Now, frictions are due to a lack of coordination (as the labor market is decentralized, it would be very hard to coordinate search decisions. Hence some vacancies receive 0 applications, some others > 1).
4) The Internet has deeply modified the matching process (reinforcing Critique No 3).

Not so obvious!

- On the one hand, searching for a trade partner is now much simpler.

- On the other hand,
  - “Studies of the Internet’s effects on labour market matching (...), on balance, find little or no evidence of a friction-reducing effect” (Kuhn and Mansour, 2014, p. 1213). However, the latter paper find such an effect.

  - Kroft and Pope (2014) find no evidence that the rapid expansion of a major online job board (during the years 2005-2007) has affected city-level unemployment rates in the US.
Extensions

- **Endogenous effort to search for a job** $(e_u)$/for applicants $(e_f)$:
  \[ H = M(e_f \cdot V, e_u \cdot U) \] (see e.g. Chap. 5 of Pissarides, 2000, Lehmann and Van der Linden, 2007; for evidence of endogenous recruitment intensity, see Davis, Faberman and Haltiwanger, 2013).

- **Stochastic job matching**: Ex ante workers and firms are identical but the productivity of a job-worker pair is random (chap. 6 of Pissarides, 2000; with a learning process of job quality in Pries and Rogerson, 2005);

- **On-the-job-search** Important extension as job-to-job flows are observed to be important.

  This extension raises however several tricky issues in the presence of bargaining. See the seminal paper of Pissarides (1994) (or chap. 4 of Pissarides, 2000). While Shimer (2006) points out that the equilibrium wage distribution is no longer unique in wage bargaining models with on-the-job search, Cahuc, Postel-Vinay and Robin (2006) show that the introduction of renegotiation circumvents the multiple equilibria outcome.
Extensions

- **Endogenous size of the labor force and/or of the number of hours worked** (chap. 7 of Pissarides, 2000, Garibaldi and Wasmer, 2005, Pries and Rogerson, 2009);


- **Add explicitly a spatial dimension** (e.g. Wasmer and Zenou, 2006, Zenou, 2009 in the urban economics literature; Marimon and Zilibotti, 1999, and Decreuse, 2008, where agents are *ex ante* heterogeneous in a broader sense).

- **New insights on discrimination** (e.g. Rosen, 2003, and Masters, 2009, about statistical discrimination).

- **Introducing a life-cycle dimension** (heterogeneity in age, a finite age of retirement) (e.g. Chéron, Hairault and Langot, 2011, Menzio, Telyukova and Visschers, 2016).
Extensions

- Imperfect credit markets: “Traditional models [...] assume away credit market frictions. As a result, entrepreneurs have no problem whatsoever financing their search for a worker, whether they finance it on their own, or borrow the cost of posting vacancies on a perfect capital market. But if credit markets are imperfect, an entrepreneur with an idea but without any capital will encounter some impediments when he turns to credit markets to find the funds required to post a vacancy” (Wasmer and Weil, 2004)

- Asymmetric information and matching (Rosen and Moen, 2010).

- Felbermayr and Prat (2011) and Galenianos, Kircher and Virag (2011) introduce imperfect competition on the goods market.

Extensions

Endogenous job destruction

The job destruction rate $q$ can be made endogenous (see chap. 2 of Pissarides, 2000, or CCZ p. 862, and Chéron, Hairault and Langot, 2013, with finite horizon).

The productivity of filled vacancies is hit by random idiosyncratic shocks (often a Poisson process). Such a shock triggers an *automatic* renegotiation of the wage.\(^7\)

⇒ There is a “reservation” level of productivity below which it is preferable to separate and look for another partner.

⇒ The model can be expressed in terms of a “job creation” curve and a “job destruction” curve in a $(\theta, \text{reservation productivity})$ space.

\(^7\)Critique: It would be more sensible to assume renegotiation by mutual consent, i.e. neither party can force the other to renegotiate the wage. (See Postel-Vinay and Turon, 2010). Only a credible threat of ending the match can trigger renegotiation.
The expression “job matching” is used in other contexts than the “Matching models”:
Jovanovic (1979) developed a theory of separation (i.e. change of employer): when a match is formed, the quality of the match is uncertain and revealed through experience. This theory explains why the separation rate decreases with job-tenure and becomes close to constant after some point.

Search-Matching frictions appear also in literatures about:
- Goods and housing markets (starting with Stigler, 1961),
- Financial and money markets (e.g. Rocheteau and Wright, 2005),
- the “Marriage market” (e.g. Burdett and Coles, 1997),...

See Pries and Rogerson (2005) and Menzio, Telyukova and Visschers (2016) for the same idea in the framework developed in this chapter.
Assignment

The “assignment literature” studies how heterogeneous agents (say, job-seekers and job vacancies with different intrinsic productivity) find “appropriate” trade partners when there are complementarities in production.

- “Positive Assortative Matching” (PAM)
  = “better-qualified job-seekers match with better jobs”
  This property comes out in a Walrasian static matching economy (Becker, 1973). Unemployment and vacant jobs cannot coexist.

- In the presence of frictions on the labor market, unemployment and vacant jobs can coexist for all types (productivity levels). Despite a certain degree of PAM, there can be some mismatch: “some high-productivity firms are forced to hire low-productivity workers whereas some low-productivity firms are able to hire higher productivity workers.” (Shimer, 2005b, p. 999).

Diamond-Mortensen-Pissarides search-matching approach has been honored by the Nobel price in 2010. Albrecht (2011) provides a brief overview of this literature.

The website of the international network of researchers in the field is: http://sam.univ-lemans.fr/
References I


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References VIII


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References XI


Note on the literature

References XVI


References XVII


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References XIX


