Job Reallocation and Unemployment
Equilibrium matching models

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Empirical analyses reveal *very intense* job and workers reallocations... even in “sluggish” labor markets

“sluggish” $\iff$ markets with small variations in the (un)employment rate

$\Rightarrow$ This chapter develops a *dynamic* model that explicitly recognizes these flows...

... to yield a theory of (inefficient) unemployment in equilibrium
Outline

1. Facts
   - Definitions
   - Empirical regularities
   - Beveridge curve

2. The matching process

3. Static model

4. Dynamic model

5. Efficiency

6. “Hold-up”

7. Dynamics

8. Critiques and Extensions

9. Note on the literature

10. References
Imagine a large data set of production units (firms or plants). In a country, a region or a sector, and for a given unit of time (often a year),

- **Job creations (JC)** = $\Sigma$ of job gains in new units or in growing ones
- **Job destructions (JD)** = $\Sigma$ of job losses due to closing or contractions
- **Net employment changes** = JC - JD
- **Job reallocation (JR)** = JC + JD
- **Excess job reallocation**
  = Job reallocation - | Net employment changes |
  = JC + JD - | JC - JD |
Worker flows

Imagine now a data set of movements of workers into jobs (hirings) and out of jobs (separations) over a specified period of time.

*turnover:* it measures the gross number of labour market transitions *during a period of time*  
= full counting of all events (i.e. every time a worker is hired or separates during the period) during that period;

*(gross) worker reallocation:* it measures the number of persons who participate to transitions *between two discrete points in time*, say a year;
⇒ e.g. hirings equal the number of workers who are with the firm at time t, but were not with that employer at time t-1;
= a more limited counting than turnover;
= the measure used below.
If job creations & destructions, hirings $H$ and separations $S$ are all measured by a comparison between the same two points in times:

- Net employment changes = $JC - JD = H - S$
- Excess worker reallocation
  = $H + S - |Net employment changes|$
  = $H + S - |H - S|$
- Churning
  = the difference between excess worker reallocation and excess job reallocation
  = $H + S - JC - JD$

This difference “represents labour reallocation arising from firms churning workers through continuing jobs or employees quitting and being replaced on those jobs” (Bassanini and Marianna, 2009, p. 4)
These job and workers flows have been measured in a large number of countries ...

... on the basis of non homogenous definitions and country-specific data sets
⇒ lack of comparability;

An important message was until recently standard: “worker turnover in Europe is much less than in the United States, whereas job turnover is roughly the same” (Pries and Rogerson, 2005, abstract)

By using the same data collection protocol, Bassanini and Marianna (2009) question this conclusion...
“we find that both idiosyncratic firm (industry, age and size) and worker (age, gender, education) characteristics are key factor affecting gross job and worker flows in all countries. Nevertheless, even controlling for these idiosyncratic factors, cross-country differences concerning both gross job and worker flows appear large and of a similar magnitude. Both job and worker flows in countries such as the United States and the United Kingdom exceed those in certain continental European countries by a factor of two. Moreover, the variation of worker flows across different dimensions is well explained by the variation of job flows, [...]. Consistently, churning flows, that is flows originating from firms churning workers and employees quitting and being replaced, display much less variation across countries.” (Bassanini and Marianna, 2009, p. 1)
Data used by Bassanini and Marianna (2009):

- Firms (not plants) as unit of observation (i.e. an organisational unit of production with a certain degree of autonomy in decision-making)
- Some data limitations (explained on p. 4 of their discussion paper)

Worker flows (among wage and salary employees aged 15 years or more) based on data from

- the European Labour Force Survey
- the Displaced workers/Job tenure supplement of the Current Population Survey (CPS)

⇒ A sample of 22 countries (Norway, Switzerland, Turkey, the United States and some European Union countries)

Unit of time: a year.

Rates obtained by dividing flow totals by average employment in years $t - 1$ and $t$. 
There is a significant country effect shaping gross job reallocation rates.
Job flows

Other conclusions

- Net employment growth is the (relatively small) difference between two large rates.
- Job creation rates and job destruction rates are underestimated (job reallocations within firm are for instance ignored)
- Even with a narrow definition of the “sector”, job reallocation is to a large extent a within-sector phenomenon.

Order of Magnitude in levels: Every day in France, about 10,000 jobs are created and 10,000 are destroyed.
“Not surprisingly, the turnover of workers appears to be greater than that of jobs. Annual worker reallocation (i.e. the sum of hirings and separations) averaged across industries, was about 33% of dependent employment during 2000-2005. Of this, industry-level excess worker reallocation (i.e. the difference between total worker reallocation in each industry and the absolute value of industry-level net employment growth) was about 30% of dependent employment at our level of aggregation.” (Bassanini and Marianna, 2009, p. 13)
### Table 8. Worker flows, by country

Worker reallocation and excess worker reallocation, 2000-05

Country averages of worker reallocation rates expressed in percentages and adjusted by industry composition

<table>
<thead>
<tr>
<th></th>
<th>Worker reallocation</th>
<th>Excess worker reallocation</th>
<th>Hirings</th>
<th>Separations</th>
</tr>
</thead>
<tbody>
<tr>
<td>Austria</td>
<td>27.0</td>
<td>24.8</td>
<td>13.6</td>
<td>13.4</td>
</tr>
<tr>
<td>Belgium</td>
<td>32.8</td>
<td>30.5</td>
<td>16.6</td>
<td>16.1</td>
</tr>
<tr>
<td>Czech Republic</td>
<td>29.5</td>
<td>26.3</td>
<td>14.9</td>
<td>14.6</td>
</tr>
<tr>
<td>Denmark</td>
<td>50.0</td>
<td>47.4</td>
<td>24.9</td>
<td>25.1</td>
</tr>
<tr>
<td>Finland</td>
<td>46.3</td>
<td>43.7</td>
<td>23.6</td>
<td>22.7</td>
</tr>
<tr>
<td>France</td>
<td>36.7</td>
<td>34.4</td>
<td>18.7</td>
<td>18.0</td>
</tr>
<tr>
<td>Germany</td>
<td>34.0</td>
<td>31.4</td>
<td>16.7</td>
<td>17.3</td>
</tr>
<tr>
<td>Greece</td>
<td>26.3</td>
<td>22.1</td>
<td>13.9</td>
<td>12.5</td>
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<tr>
<td>Hungary</td>
<td>28.6</td>
<td>23.1</td>
<td>15.2</td>
<td>13.4</td>
</tr>
<tr>
<td>Ireland</td>
<td>41.7</td>
<td>36.4</td>
<td>22.1</td>
<td>19.5</td>
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<tr>
<td>Italy</td>
<td>28.5</td>
<td>25.5</td>
<td>15.3</td>
<td>13.2</td>
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<tr>
<td>Norway</td>
<td>34.4</td>
<td>28.4</td>
<td>16.5</td>
<td>17.9</td>
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<td>Poland</td>
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<td>38.8</td>
<td>21.6</td>
<td>21.3</td>
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<tr>
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<td>30.8</td>
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<td>42.0</td>
<td>22.4</td>
<td>22.9</td>
</tr>
<tr>
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<td>49.7</td>
<td>47.0</td>
<td>24.6</td>
<td>25.1</td>
</tr>
</tbody>
</table>

Note: Estimated average rates that would be observed in each country if it had the same industry composition of the average country. Data refer to: Czech Republic: 2002-05; Ireland: 2000-03; Norway: 2000-04; Poland: 2004-05; Slovak Republic: 2003-05; Switzerland: 2002-07; Turkey: 2007; United States, 2000, 2002 and 2004; other countries: 2000-05.
Worker flows
Main lessons from Bassanini and Marianna (2009).

“Total worker reallocation is at or above 40% in countries with [...] large shares of temporary workers (Finland, Poland, Spain) or relatively flexible regulations for open-ended contracts (Denmark, Ireland, the United Kingdom, the United States). In these countries more than 20% of the employees separate, at least once, from their employer each year. However, this high rate of mobility out of jobs is matched by comparably high flows of new hires. By contrast, countries with both annual hiring and separation rates below 15% (such as Austria, the Czech Republic, and Greece) have all low shares of temporary contracts or moderate-to-rigid dismissal regulations.

[...] both gross job and worker flows in the most flexible countries in our samples appear to exceed those in the least flexible by at least a factor of two." (p. 19)
Churning

Comparison of job and worker flows for the same countries, industries and years ⇒ a rough measure of churning.

After correcting for industry composition, churning flows turn out to be rather similar across Finland, Germany, Portugal,..., Sweden, the UK and the US (1997 - 2004):
Order of magnitude: [12%, 17%]
v and u coexist

- Basic idea: A relationship between vacant jobs and unemployment to assess the importance of worker reallocations. More reallocation problems if a large vacancies-workforce ratio, v, coexists with a large unemployment rate u.
- “Beveridge curve” = a plot of \((u, v)\) pairs measured at different points in time.
- On the two following slides, examples of Beveridge curves until 2001.
- Note: In many countries, data about vacancies are rather poor!
Shifts of the “Beveridge curve”

Chart 6.1: Beveridge curves for UK, US, Germany and France

Sources: OECD; Eurostat
On the previous figure,

- The inwards shift of the UK curve in the mid to late 1990s,
- and the marked inward shift in the US throughout the past decade,
- contrast with the outward shifts in both France and Germany (though with suggestions of “recent” improvement in each of the latter cases).
“Beveridge curve” in the US 2000-10
A recent outward shift

Central idea:
Trade in the labor market is decentralized, uncoordinated, time-consuming and costly for both firms and workers (Pissarides, 2000, p.3)

Why? Because of heterogeneities, frictions and imperfect information.

Jobs and workers are heterogeneous (in skills, location,...). Yet, in a macroeconomic perspective, modeling these heterogeneities in detail is not the aim and is out of scope.
Like the production function, the “matching function” is

“a modeling device that captures the implications of the costly trading process without the need to make the heterogeneities and other features that give rise to it explicit.” (Pissarides, 2000, p.4)

So, formally, the “agents” who participate to trade in the labor market are homogeneous. However, the fact that their “meeting” is time-consuming is due to implicit heterogeneities.

- Extension: Introduce explicit heterogeneities, say in skill.¹

Standard assumption: “Undirected search”: job seekers meet all vacancies randomly.

¹The literature proposes different approaches here (see e.g. Mortensen and Pissarides, 2003, Shimer, 2007, the assignment literature mentioned at the end of this chapter).
The matching process

- Full specialization in either trade or production:
  - jobs filled → production
  - jobs vacant → search for applicants → matching
  - worker employed → production
  - worker unemployed → search for vacant jobs → matching

⇒ No on-the-job search (see extensions later).

- Micro-foundation of the matching process: often an “urn-ball model” (have a quick look at CZ p. 518-20).
The matching function
Continuous-time setting

The \textit{instantaneous} flow of hires, $H$, is assumed to be a function of the number (stock) of job-seekers, $U$, and the number (stock) of vacancies, $V$. This matching is assumed to be random process (vacancies and job-seekers meet randomly).

The matching function is defined as

$$H = M(V, U) \quad (1)$$

As it is standard now, let us assume that $M(V, U)$ is increasing, concave and homogeneous of degree 1.

Moreover: $M(0, U) = M(V, 0) = 0$.

Note: In discrete time, one would impose that $M(V, U) < \text{Min}(V, U)$. 
The matching function

Empirical support

- “The usefulness of the matching function depends on its empirical viability” (Pissarides, 2000, p.4).
- Matching functions have been estimated for a number of countries.
- See Petrongolo and Pissarides (2001) for a survey. Many studies support the above assumptions.

Nevertheless,

- See CZ p. 520 for an interesting alternative: the so-called “stock-flow matching”.
- Increasing returns to scale are sometimes advocated.
Tightness and various rates

Definition: Tightness on the labor market

\[ \theta \equiv \frac{V}{U} \]

The rate (= the probability per unit of time) at which a vacant job is filled is:

\[ M(V, U) = M(1, U/V) \equiv m(\theta) \]

Differentiate \( M(1, U/V) \equiv m(\theta) \) w.r.t. \( U \): \( m'(\theta) < 0 \).

The rate at which an unemployed finds a job:

\[ \frac{M(V, U)}{U} = \frac{V}{U} \frac{M(V, U)}{V} = \theta m(\theta) \]

Differentiate \( M(V, U)/U = \theta m(\theta) \) w.r. to \( V \): \( [\theta m(\theta)]' > 0 \).
The matching process

Search or congestion externalities

An additional vacancy

reduces the rate at which vacancies (=vacant jobs) are filled = the “congestion external effect”
increases the exit rate out of unemployment = the “thick market (beneficial) externality”

Similarly for an additional unemployed in the queue for jobs.

“A worker deciding to join a queue or stay in one considers the probabilities of getting a job, but not the effects of his decision on the probabilities that others face...” (Tobin, 1972)
Links with the job-search literature

- The mechanisms underlying the matching function are closely related to those at the root of the *Job search model* considered earlier.
- General equilibrium considerations clearly suggest that the job arrival rate and the level (or the distribution) of wages should be endogenous.
- *Equilibrium Search Models* put emphasis
  1. on the role of employers (the demand side of the labor market)
  2. on wage formation.
Links with the job-search literature

There are two approaches:

**Wage posting models (Chapter 1, Sec. 2).** Take-it-or-leave-it wage offers are set ("posted") by employers in a non-cooperative setting while workers search for the best offer (the supply side is modeled as in "Job search models").

**Matching models (This chapter).** The number of job creations depends on the numbers of job-seekers and vacancies according to the "matching function". The surplus created by a match typically leads to a negotiation over the wage. Seminal papers: Diamond (1981), Diamond (1982), and Mortensen (1982).
A static version of the model
not included in the book of CZ

Assumptions:

- Risk-neutral workers and firm owners.
- At the beginning of the period, an exogenous number $N = U$ of individuals are jobless and search randomly for a job.
- Each firm is made of a single vacant or filled job.
- If a vacancy is filled, a given (real) amount of output $y$ is produced. The worker receives a (real) wage $w$ and the firm’s (real) profit level is $y - w - \kappa$, where $\kappa$ is a fixed cost incurred to open a vacancy (create a job slot).
- If the job remains vacant, the profit is equal to $-\kappa$. 
A static version of the model

Assumptions:

- \( M(V, N) < \min(V, N) \Rightarrow m(\theta) < \min(1, \theta^{-1}) \) where \( \theta \equiv \frac{V}{N} \).
- Wages are negotiated after efforts have been made on both sides of the labour market to create vacancies and search for a partner (sunk costs).
- Throughout this chapter, one neglects credit market frictions: Entrepreneurs have no problem financing their creation of vacancies and search of an applicant.

Questions that are raised:

1. How many vacancies are created?
2. How is the surplus of the match splitted?
3. How many are (un)employed?
A static version of the model

Labour demand

There is free entry of vacancies. Firms open vacancies as long as the expected return is nonnegative.

In equilibrium:

\[ m(\theta)(y - w - \kappa) + (1 - m(\theta))(-\kappa) = 0 \quad \text{or} \quad (2) \]

\[ m(\theta) = \frac{\kappa}{y - w} \quad \text{(3)} \]

Since \( m'(\theta) < 0 \), the higher \( y \) (the lower the wage and the cost of opening a vacancy), the higher \( \theta \): This means a ‘thick’ labor market with many vacancies per job-seeker.

Equation (3): A downward-sloping labour demand equation in a \((\theta, w)\) space.

Notice that \( 0 < \kappa/(y - w) < 1 \) is required, hence \( w < y \).
A static version of the model

The wage bargain

When an individual meets a vacancy, the total surplus created if they form a match is $y$
(since $\kappa$ is sunk + no income for the unemployed).

Assume the following non-cooperative two-stage game:

Stage 1. The firm-owner and the worker propose a wage contract.

Stage 2. If one of the two players (or both) does not accept the contract in stage 1, then
- with probability $\gamma$, $0 < \gamma < 1$, the worker makes a take-it-or-leave-it offer
- with probability $1 - \gamma$ the firm owner makes such an offer.
- If the offer is rejected, the job is destroyed.
A static version of the model
Characterization of the subgame perfect equilibrium

In stage 2 if the worker makes the offer, the pay-off for the employer is zero and the total surplus, \( y \), accrues to the worker. The opposite holds if the firm owner makes the offer.

In stage 1 ◊ the worker knows that at the end of stage 2, his expected income will be \( \gamma \cdot y + (1 - \gamma)0 \).
◊ the employer knows that at the end of stage 2, his expected pay-off will be equal to \( \gamma \cdot 0 + (1 - \gamma)y \).

Therefore, in stage 1, indifference between (i) signing a contract at stage 1 giving an expected income to the worker \( \gamma \cdot y \) and an expected profit \( (1 - \gamma)y \) and (ii) waiting until stage 2.
Assume a small cost of going to stage 2. The unique equilibrium consists in immediately signing a contract that shares the surplus by paying the wage:

\[ w = \gamma \cdot y. \]

Equivalently, one could maximize the following (asymmetric) Nash product with respect to \( w \):

\[ [w]^\gamma [y - w]^{1-\gamma} \]  \hspace{1cm} (4)

The first-order condition is: \( w = \gamma y \).
A static version of the model

Characterization of the equilibrium

The equilibrium is the triple \((w^*, \theta^*, V^*)\) characterized by:

\[
\begin{align*}
  \quad w^* &= \gamma y, \\
  m(\theta^*) &= \frac{\kappa}{(1 - \gamma)y} \Rightarrow V^* = N \cdot m^{-1} \left( \frac{\kappa}{(1 - \gamma)y} \right)
\end{align*}
\]

The higher \(\gamma\) or \(\kappa\) (the lower \(y\)), the smaller is \(\theta^*\).

The expected number of unemployed (and, by the law of large numbers, their actual number) is:

\[
N(1 - \theta^* m(\theta^*)) < N
\]

It is decreasing in \(\theta^*\).
A static version of the model

Message

- Whatever the levels of the parameters \((\gamma, \kappa, y, N)\), there is some unemployment due to frictions (i.e. the exogenous function \(m(\cdot)\)).

- This does not imply that the wage level does not matter: The intensity of the unemployment problem varies with the way total surplus \(y\) is splitted.

The mechanisms we have identified here hold true in the following dynamic setting.
Equilibrium of flows and the Beveridge Curve
in a dynamic environment


\[ N = \text{the size of the labor force. } N \text{ is large.} \]
\[ \dot{N} = \frac{dN}{dt} \text{ (exogenous!).} \]
Assume that those who enter the labor force begin by looking for a job.
Assume an exogenous separation rate \( q \) (extensions: endogenous \( q \)).
Equilibrium of flows and the Beveridge Curve

\[
\dot{U} = \dot{N} + qL - \theta m(\theta)U \text{ (law of large numbers)} \tag{8}
\]

\[
\dot{u} = q + n - [q + n + \theta m(\theta)] u \text{ where } u = U/N, n = \dot{N}/N \tag{9}
\]

In a steady state \( \dot{u} = 0 \):
entries in = exit from unemployment.

The previous equation leads to

\[
u = \frac{q + n}{q + n + \theta m(\theta)} \tag{10}\]
Equilibrium of flows and the Beveridge Curve

Or, since the “vacancy rate” $v = V/N$,

$$u = \frac{q + n}{q + n + (\frac{v}{u})m(\frac{v}{u})}$$

= an implicit relationship between $u$ and $v$ that defines the “Beveridge curve” understood as

the set of pairs $(u, v)$ such that the unemployment rate remains constant.

Quite different from the plot of $(u, v)$ realizations seen before (unless $\dot{u} = 0$ for some years).

It can be shown that given the assumptions made before the “Beveridge curve” is decreasing and convex.
The Beveridge Curve as an equilibrium relationship
A rightward shift of the “Beveridge curve” can be the consequence of
- An increase in $q$ or in $n$ (the rate of growth of $N$!)
- A deterioration in the “efficiency of the matching process”

If we knew the equilibrium value of $\theta$ in steady state, then the equilibrium $u$ and $v$ would be determined.

The following is concerned by the determination of equilibrium tightness in steady state.
A dynamic model with ex-post wage bargaining

Assumptions

- There are two goods: a produced good (the numeraire sold in a competitive market) and labour.
- Each firm is made of a single vacant or filled job.\(^2\)
- Infinitely lived agents with perfect foresight + risk neutrality.
- \(h\) is the (exogenous and deterministic) fixed cost of a vacant job \(\textit{per unit of time}\).
  \((h\) captures the cost involved in posting a vacancy, searching for applicants and selecting them).\)
- \(r\) is the exogenous discount rate common to all agents. Perfect capital markets (Extension: see Wasmer and Weil, 2004).

\(^2\)Can easily be generalized to a large firm with capital and labour if labour can be adjusted instantaneously and returns to scale are constant (Section 4.1 of the book that can be skipped). When the marginal product of labour is decreasing and under \textit{individual} wage negotiation, the formula derived below are no more correct (see Cahuc and Wasmer, 2001, and Cahuc, Marque and Wasmer, 2008).
Expected profits

One Euro invested at time $t$ yields $1 + r \, dt$ at time $t + dt$. So, the discount factor will be $\frac{1}{1 + r \, dt}$ on any interval of length $dt$.

During any short time period $dt$, $(y - w) \, dt$ measures the current flow of return.

- $y$ measures (exogenous) real output (sold),
- $w$ is the real wage,
- working time is exogenous and normalized to 1.

The worker and the firm separates with probability $q \, dt$.

At any time $t$, a firm’s real discounted expected return from an occupied job is denoted $\Pi_e(t)$. A firm’s real discounted expected return from a vacant job is denoted $\Pi_v(t)$. 

The presentation follows CZ2004, p. 523 - 537).
Dynamic model

Expected return from an occupied job

With a perfect capital market and an infinite horizon, \( \Pi_e(t) \) satisfies the following Bellman equation:

\[
\Pi_e(t) = \frac{(y - w)dt + qdt \Pi_v(t + dt) + (1 - qdt)\Pi_e(t + dt)}{1 + rdt}
\]  

As Cahuc and Zylberberg (2004), one here assumes that with probability \( q \, dt \) the job becomes vacant. Pissarides (2000) considers that at a rate \( q \) the job is destroyed: Its value is then zero. Under free-entry (see below), both approaches lead to the same conclusions.
Expected return from an occupied job

Multiply both sides of (11) by $1 + r \, dt$. Then divide both sides by $dt$ and take the limit for $dt \to 0$. This yields

$$r \Pi_e(t) - \frac{\partial \Pi_e(t)}{\partial t} = y - w + q(\Pi_V(t) - \Pi_e(t)).$$

(12)

In a steady state, this Bellman equation becomes:

$$r \Pi_e = y - w + q(\Pi_V - \Pi_e).$$

(13)

A filled vacancy can be seen as an asset owned by the firm.

- $r \Pi_e$ is the (instantaneous) rate of return on this asset.
- At any time $t$, this rate of return is the sum of
  - instantaneous profits $y - w$
  - and the expected net return due to a change of state is $q(\Pi_V - \Pi_e)$, which is actually negative.
Expected return from from a vacant job

The same reasoning leads to the following Bellman equation for $\Pi_v(t)$

$$r \Pi_v(t) - \frac{\partial \Pi_v(t)}{\partial t} = -h + m(\theta(t))(\Pi_e(t) - \Pi_v(t))$$  \hspace{1cm} (14)

or in a steady state

$$r \Pi_v = -h + m(\theta)(\Pi_e - \Pi_v).$$  \hspace{1cm} (15)

A unfilled job can also be seen as an asset owned by the firm (the interpretation is similar to the one on the previous slide). The rest of this section is developed in a steady state.
Labor demand

The inflow of vacancies ends when the profit expected from a vacant job becomes zero:

\[ \Pi_v = 0 \]

Under free entry,

\[ \Pi_e = \frac{h}{m(\theta)} \cdot \text{the expected length of time } 1/m(\theta) \]

\[ \Rightarrow \Pi_e = \frac{y-w}{r+q} \quad q \text{ is added to } r \text{ to discount } y - w \]

So, the labor demand (or “vacancy-supply curve”) is the following downward-sloping relationship between \( w \) and \( \theta \):

\[ \frac{h}{m(\theta)} = \frac{y-w}{r+q} \Leftrightarrow m(\theta) = \frac{(r+q)h}{y-w} \]
Labor demand

*If the wage level is exogenous*, this demand side of the model leads to the following comparative statics (recall that $m'(\theta) < 0$)
Note

The absence of physical capital

The absence of physical capital is noteworthy. An alternative, developed e.g. by Acemoglu (2001), consists in assuming that

(i) vacancy costs $h$ are negligible (actually, zero) and
(ii) an equipment is required to produce, the acquisition cost being incurred before meeting applicants.

Now, if $\tilde{k}$ designate a fixed cost of equipment per job, the free entry condition introduced below becomes

$$\Pi_v = \tilde{k}$$

This does not change the properties of the model.

So, I stick to the standard presentation which emphasizes the role of vacancy costs.
The Behavior of workers

Keeping the assumption that workers are risk neutral, one could again look at a small interval of time \(dt\) and then take the limit \(dt \to 0\). (Useful as an exercise)

Let \(V_e\) and \(V_u\) be the (steady-state) real discounted expected value of the income stream respectively in employment and an unemployment.

Instantaneous income in employment: \(w\)
At a rate \(q\), the job is lost.

So, the expected utility of an employed person satisfies:

\[
 rV_e = w + q(V_u - V_e) \tag{17}
\]
The Behavior of workers

An unemployed worker is always in search of a job (Job-search effort is here fixed and normalized to 1).

At each instant, this search procures him or her a net gain denoted by $z$ (by assumption $z < y$):

- benefits linked to being unemployed (unemployment insurance, social welfare transfers, and also whatever utility comes from not having to work)
- minus the various costs attached to searching for a job (transportation, postage of applications, extra training, etc.).
The Behavior of workers

Since the exit rate from unemployment is $\theta m(\theta)$, the expected utility of an unemployed person satisfies:

$$rV_u = z + \theta m(\theta)(V_e - V_u)$$  \hfill (18)

This is written under the implicit assumption that the unemployed will not turn down job opportunities (i.e. assuming $V_e > V_u$). It turns out that this property holds under Nash bargaining.

Subtracting (18) from (17) implies that workers have no incentive to quit if $w > z$:

$$V_e - V_u = \frac{w - z}{r + q + \theta m(\theta)}.$$  \hfill (19)
Wage formation

The timing of events is essential here:

1. The (unemployed) workers and firms engage in a costly (time-consuming) search process (e.g. the firm incurs cost $h$ during a period of time in order to create a vacancy and recruit a worker);

2. Once they have sunk this cost, they bargain over the wage (ex-post bargaining over wages).

Implications can be phrased in two actually equivalent ways:

1. Once the two partners have met, there is a range of wages at which both partners prefer to match rather than breakup or

2. there is a “match-specific surplus” (a rent) that has to be shared.
How is this surplus split?

Typically through an individual negotiation over the wage. More specifically, the literature adopts either

- An axiomatic Nash bargaining solution is standard (Nash, 1953; CZ2004, p. 382-3);
- or a strategic bargaining game approach (Rubinstein, 1982; CZ2004, p. 387-8):
  Two players alternate offers over many periods with an (in)finite horizon (An example of such a game has been introduced in the static model above and can also be read on p. 527).

The precise environment of the game matters.

Under some assumptions both approaches lead to the same outcome (CZ2004, p. 388-90). If needed, see Chapter 16 of Osborne (2004) and L’Haridon, Malherbet and Pérez-Duarte (2010) for alternatives.

---

3 Individual meaning between the worker and the employer. Unions and collective bargaining can be added but this setting is deemed less natural in a one-job-one-firm setting at least.
Wage bargaining

Surplus sharing

The (total) *surplus of a match* = the sum of the rents (of the firm and of the worker) that a filled job procures.

Rent = gain from the contractual relationship minus outside option

= $\Pi_e - \Pi_v$ in case of the employer
= $V_e - V_u$ in case of the employee

$\Rightarrow$ (total) surplus $S = V_e - V_u + \Pi_e - \Pi_v$.

Wages are renegotiated continuously.
When they bargain over the *current* wage, the players take $V_u$ and $\Pi_v$ as well as tightness on the labor market as given.
Wage bargaining

The value of the wage negotiated at each moment is the solution of the maximization of the following “Nash product”:

$$\underset{w}{\text{Max}} \ (V_e - V_u)^{\gamma} (\Pi_e - \Pi_v)^{1-\gamma}, \ 0 \leq \gamma \leq 1$$  \hspace{1cm} (20)

The result of the negotiation is written:

$$V_e - V_u = \gamma S \quad \text{and} \quad \Pi_e - \Pi_v = (1 - \gamma) S$$  \hspace{1cm} (21)

Now we need a convenient expression for $S$ as a function of the parameters and of $V_u$ and $\Pi_v$ that are taken as given.
Wage bargaining

From $V_e - V_u = \gamma S$ to a wage equation

1. An expression for $S$

\[
\begin{align*}
  r\Pi_e &= y - w + q(\Pi_v - \Pi_e) \\
  \oplus \left( rV_e &= w + q(V_u - V_e) \right) \\
  \Rightarrow r(\Pi_e + V_e) - r(V_u + \Pi_v) &= y - q \cdot S - r(V_u + \Pi_v) \\
  \Rightarrow S &= \frac{y - r(V_u + \Pi_v)}{r + q}
\end{align*}
\]

2. An expression for $V_e - V_u$

\[
\begin{align*}
  rV_e &= w + q(V_u - V_e) \Rightarrow V_e = \frac{w + q V_u}{r + q} \Rightarrow V_e - V_u &= \frac{w - rV_u}{r + q}
\end{align*}
\]
Wage bargaining

From $V_e - V_u = \gamma S$ to a wage equation

$$V_e - V_u = \gamma S \Rightarrow \frac{w - rV_u}{r + q} = \gamma \frac{y - r(V_u + \Pi_v)}{r + q}$$

$$\Rightarrow w = rV_u + \gamma (y - r(V_u + \Pi_v))$$

Expression $w = rV_u + \gamma (y - rV_u)$ has an intuitive interpretation:

- $\gamma = 0 \quad w = rV_u$ (workers get no rent)
- as $\gamma \uparrow$ an increasing share of the difference $(y - rV_u)$ accrues to the worker

Note: If $\gamma = 1$, $\Pi_v$ cannot be non negative (see (13) and (14)).
So, firms do not open vacancies at all because they cannot recoup the sunk cost $h$. 
Wage bargaining

Getting rid of $rV_u$

To complete the analysis, one would like to relate the negotiated wage $w$ to $\theta$ and to the parameters of the model $\Rightarrow$ such a relationship between $rV_u$ and $\theta$.

This can be done in various ways.

1) The book follows one approach that leads to:

$$rV_u = \frac{z(r + q) + \gamma y \theta m(\theta)}{r + q + \gamma \theta m(\theta)}$$

Substituting this expression of $rV_u$, they get this “wage curve”:

$$w = z + (y - z)\Gamma(\theta) \text{ with } \Gamma(\theta) = \frac{\gamma[r + q + \theta m(\theta)]}{r + q + \gamma \theta m(\theta)}, \Gamma' > 0 \quad (22)$$
2) Consider now the approach of Pissarides (2000). The starting point is again

\[ w = rV_u + \gamma(y - rV_u) = (1 - \gamma)rV_u + \gamma \cdot y, \]  

(23)

where \( V_u \) solves

\[ rV_u = z + \theta m(\theta)(V_e - V_u) \]  

(24)

Under free entry, the solution to the game \( V_e - V_u = \gamma S \) can be rewritten:

\[(1 - \gamma)(V_e - V_u) = \gamma \Pi_e\]

where \( \Pi_e = h/m(\theta) \). So,

\[ V_e - V_u = \frac{\gamma}{1 - \gamma} \frac{h}{m(\theta)} \Rightarrow rV_u = z + \theta m(\theta) \frac{\gamma}{1 - \gamma} \frac{h}{m(\theta)} \]
Wage bargaining

Plugging this expression in (23) yields this “wage curve” $(WC)$

$$w = (1 - \gamma)z + \theta \gamma h + \gamma y$$  \hspace{1cm} (25)

or, if $h$ is proportional to $y$ ($h = k \cdot y$):

$$w = (1 - \gamma)z + \gamma y (1 + \theta k)$$ \hspace{1cm} (26)

According to the problem studied, the wage curve (22) or (25) is more convenient.
The labour market equilibrium

\[ WC : w = (1 - \gamma)z + \theta \gamma h + \gamma y \]

\[ LD : m(\theta) = \frac{(r+q)h}{y-w} \]

Because the matching function has C.R.S., equilibrium \((\theta, w) \perp (u, v)\). Combining the wage curve (25) and the labor demand (16) (see also Fig. 9.4 in the book):

\[ d y > 0, \quad d h > 0, \quad d \gamma > 0, \quad d z > 0 \]

\[ d q > 0, \quad d h > 0, \quad d r > 0 \]

\[ d y > 0, \quad d m(.) > 0 \]
The labour market equilibrium

Eliminating the wage yields an implicit equation in (equilibrium) tightness. If $h$ is defined in level:

\[
\frac{(1 - \gamma)(y - z)}{r + q + \gamma \theta m(\theta)} = \frac{h}{m(\theta)}
\]  

(27)

If $h = k \cdot y$:

\[
\frac{(1 - \gamma)(y - z)}{r + q + \gamma \theta m(\theta)} = \frac{k \cdot y}{m(\theta)}
\]  

(28)

which can be made independent of $y$ if one assumes constant “replacement ratios” $z/w$ (check it; hint: Return to (26) $\Rightarrow w = \tilde{\rho}y$).

In Section 3.5.2, CZ develop a comparative static analysis. You should read this.

Finally, CZ conduct a numerical analysis. Message: The model is “too simple” to explain differences in unemployment across the OECD.
Note 1 on the role of unemployment benefits

A comparative static analysis confirms:

Search and matching theory [...] states that more generous benefits bring about higher unemployment [...].


The way the Nash product (20) is written is however essential. In the alternative approach of Hall and Milgrom (2008) (“H&M”), the role of unemployment benefits is downplayed. Explanation:

- Above, the threat point for bargaining is the payoff of each partner if they separate (resp. $V_u$ and $\Pi_v$).
- According to H&M, the realistic threat is to extend the bargaining (not to terminate it) ⇒ less connection between the wage and outside options (in particular $V_u$!) & less reaction of wages to tightness (real wages are more rigid... linked to Shimer (2005)’s critique).
Is the equilibrium efficient?
Section 6 of the book

There are

- *congestion effects* within each category
- and *positive externalities* between the categories.

Are the search externalities internalized by the ex post Nash bargain? This is the question raised in this section.

Ignoring taxes and social security contributions, interpret $z$ as the value of leisure net of search costs.
Is the equilibrium efficient?

With risk neutral agents, social output at time $t$ (divided by the exogenous size of the labour force $N$) is given by

$$\omega(t) = y(1 - u(t)) + z \cdot u(t) - h \cdot v(t) = y + [z - y - h \cdot \theta(t)] \cdot u(t) \quad (29)$$

$\omega$ is simply output + the (possibly negative) value of unemployment - the cost of opening vacancies.

Due to congestions, the impact of additional vacancies on the hiring rate is declining (keeping $u$ fixed).

Moreover, each additional vacancy costs $h$.

$\Rightarrow$ unlikely that creating more vacancies is always good from the point of view of net output $\omega$. 
Is the equilibrium efficient?

Optimal control

Starting from an initial situation $u(t = 0) = u_0$, a social planner who ignores distributional issues would solve:

$$\max_{\theta(t), u(t)} \int_0^{+\infty} \omega(t) \cdot e^{-rt} dt$$

subject to the equation of motion (taking $n = 0$):

$$\dot{u}(t) = q(1 - u(t)) - \theta(t) \cdot m(\theta(t)) \cdot u(t).$$

This optimal control approach is followed in Section 6.2.2. in the book ($\theta$ is the control variable and $u$ the state variable).
Is the equilibrium efficient?
A simpler approach

Consider the steady-state value of $\omega(t)$ only (ignoring the adjustment from one steady state to another $\Rightarrow$ the path of the economy is not discounted) $\Rightarrow$ we can later compare the so-called “social optimum” in a steady state with the steady state emerging from decentralized decisions when $r \to 0$.

Maximizing (29) with respect to $\theta$ with $u$ defined by the Beveridge curve $u = \frac{q}{\theta m(\theta) + q}$ is equivalent to maximizing the following expression with respect to $\theta$ only:

$$y + \frac{q}{\theta m(\theta) + q} [z - y - h \cdot \theta].$$

(30)

Interpret!
Is the equilibrium efficient?

The optimal value of tightness has to be such that the sum of two effects becomes nil.

\[
\frac{d}{d\theta} \left[ \frac{q}{\theta m(\theta) + q} \right] [z - y - h \cdot \theta] + u \cdot (-h) = 0
\]

1. increasing $\theta$ reduces the unemployment rate.
2. conditional on the level of unemployment, a higher tightness entails more vacancy costs.

Through this maximization, the benevolent planner takes the consequences of his (her) choice of $\theta$ on the externalities due to frictions.
Is the equilibrium efficient?

This first-order condition with respect to $\theta$ leads to:

$$\frac{(y - z)(1 - \eta(\theta))}{q + \theta m(\theta) \eta(\theta)} = \frac{h}{m(\theta)},$$

(31)

where

- $\eta(\theta) = \left| \frac{d \log(m(\theta))}{d \log(\theta)} \right|$
- $(1 - \eta(\theta)) = \left| \frac{d \log(\theta m(\theta))}{d \log(\theta)} \right|

In the decentralized economy with ex post Nash bargaining, $\theta$ solves (see (27) above when $r \to 0$):

$$\frac{(y - z)(1 - \gamma)}{q + \theta m(\theta) \cdot \gamma} = \frac{h}{m(\theta)}$$

(32)
Is the equilibrium efficient?

(31) ⇔ (32) if

- the bargaining power of the worker $\gamma$ equals $\eta(\theta)$, the elasticity of the rate $m$ of filling a vacancy with respect to tightness $\theta$ (taken in absolute value).
- Because of the CRS assumption, $\eta$ is also the elasticity of $M(V, U)$ with respect to unemployment.

Under the condition $\gamma = \eta(\theta)$, called the “Hosios condition” (Hosios, 1990), the search externalities are internalized by the ex-post Nash bargain.

Under the Hosios condition, the total surplus generated by a match is shared in such a way that the externalities are exactly balanced, so that efficiency is restored.
Is the equilibrium efficient?

“Basically, the Hosios condition says that in order to maximize the aggregate gains from trade, less transaction costs, the traders’ bargaining shares must reflect their marginal contribution to the value of the aggregate transaction flow. This condition is satisfied in the case of a linearly homogeneous matching technology if and only if agents’ shares equal the elasticities of the matching function with respect to the stocks of buyers and sellers in the market.” (Mortensen and Wright, 2002)
Is the equilibrium efficient?

- Notice that the Hosios condition expresses that workers should have a positive bargaining power. (Very different from most union models where the underlying reference is the competitive labor market)
- Notice also that a certain level of unemployment is present when the outcome is efficient. (Very different from union models)
- If $\gamma > \eta(\theta)$, equilibrium unemployment is above its efficient level.
- Conversely, if $\gamma < \eta(\theta)$, equilibrium unemployment is inefficiently low.
- Too few (resp. too many) vacancies are created when $\gamma$ is too high (resp. too low).
Is the equilibrium efficient?

- There is no reason why the Hosios condition should be fulfilled. Hence, in general, search equilibria with ex post Nash bargaining are inefficient.

  Other wage mechanisms than the sharing of the ex post surplus of the match can automatically lead to an efficient outcome (see e.g. the summary of the “Competitive Search Equilibrium” of Moen (1997) provided by Cahuc and Zylberberg (2004), Section 6.3. Notice that search is **directed** and no more random).

- The *laissez faire* economy (without taxes and unemployment insurance) and under the Hosios condition is not the only efficient outcome.

  When the bargaining power does not fulfill the Hosios condition, taxation can restore efficiency because a positive marginal tax rate (resp. a negative one) decreases (resp. increases) the share of the surplus that accrues to the workers (Boone and Bovenberg, 2002).

- Lehmann and Van der Linden (2007) deal with the efficiency of an economy with search-matching frictions when workers are risk averse.
Note 2 on the role of unemployment benefits

In a setting where (i) workers bear a setup cost (need to relocate or train to take a job), (ii) through bargaining both the output flow and the setup costs are shared and (iii) the matching rate for an unemployed is a parameter, Diamond (1981) concludes "welfare [but also unemployment] is increased by the introduction of unemployment compensation even though all agents are risk neutral" as the unemployed become more selective.

"unemployment benefits shift the composition of employment toward high-wage jobs. Because the composition of jobs in the laissez-faire equilibrium is inefficiently biased toward low-wage jobs, the labor regulations [...] may improve welfare" (Acemoglu, 2001).

On the same topic, see also Marimon and Zilibotti (1999) and Acemoglu and Shimer (1999).

\(^4\)Capital-intensive and hence more productive jobs are more costly to create; this cost is sunk before the bargaining take places. = A "hold-up" story (see below).
Investment in specific capital

Some investments are specific to each particular match between a firm and a worker and will be lost if the match is broken. Examples:

- formal firm-specific training (including the effort made to allocate the worker to the most appropriate occupation)
- other firm-specific investments such as buying a house next to your workplace, learning the rules of the firm, learning how to operate their computer equipment,...

On the other hand, there are also general investments (i.e. non-specific to a particular match; in the case of workers’ human capital: training that is valued equally by many (all) firms).
The “hold-up” problem

The question

Generally, it is claimed that training which is neither perfectly general nor purely specific can be regarded as the sum of a general and a specific component.

Stevens (1996) and Gathmann and Schönberg (2010) argue that most training, rather, is useful to a limited number of firms (the notion of ‘transferable training’).

**Question:** Are decentralized decisions efficient *from the point of view of the worker-firm pair*?

Whether or not the employers and the workers invest in an efficient way is an important question. Think e.g. about specific training. If the choice is inefficient, scope for public interventions?

**Note:** A similar question is raised in the case of union-firm bargaining.
Assumptions

- I here focus on the case where the firm invests. What follows can be adapted to the case where the employee makes a specific investment.

- The marginal product of a filled vacancy, $y$, is a function of a specific investment ($i$ is the real level of money invested in specific training).

  $\frac{\partial y(i)}{\partial i} > 0$

  $\frac{\partial^2 y(i)}{\partial i^2} < 0$.

- Everything is still done in a steady state.
Efficient investment

The net (ex-post) total surplus of a match is

\[ S_n(i) \equiv \Pi_e + V_e - \Pi_v - V_u - i. \]

The firm’s expected return from a filled vacancy verifies:

\[ r\Pi_e(i) = y(i) - w(i) + q(\Pi_v - \Pi_e(i)). \]  

(33)

Therefore:

\[ \Pi_e(i) = \frac{y(i) - w(i) + q\Pi_v}{r + q}, \]

(34)

where \( \Pi_v \) is independent of \( i \) since the investment is specific to a particular match.
Efficient investment

Similarly,

$$V_e(i) = \frac{w(i) + qV_u}{r + q},$$  \hspace{1cm} (35)$$

where $V_u$ is for the same reason also independent of $i$. So, the total net surplus can be written as:

$$S_n(i) = \frac{y(i) - r(\Pi_v + V_u)}{r + q} - i.$$  \hspace{1cm} (36)$$

The investment level that maximizes $S_n(i)$ can be said to be efficient. Let $i^*$ denote this efficient level. It is easily seen that $i^*$ solves

$$y_i(i^*) = r + q.$$  \hspace{1cm} (37)$$
Complete contracts

The timing of events is as follows:

**Stage 1:** A wage contract is bargained *that cannot be renegotiated*; this wage is a function of \( i \): \( w(i) \);

**Stage 2:** Investment is chosen.

It will be shown that the optimal investment level is efficient.

The basic idea of subgame perfect equilibrium is to require that the players’ behavior be optimal in each situation. To obtain the perfect equilibrium, we move backwards:
Complete contracts

Stage 2. The optimal investment level, $\bar{i}$, conditional on $w(i)$ solves:

$$\max_i \left[ \frac{y(i) - w(i) + q\Pi_v}{r + q} \right] - i \Rightarrow y_i(\bar{i}) - w_i(\bar{i}) = r + q. \quad (38)$$

Stage 1. The component of the Nash product that captures the employer side is critical here.

- If there is an agreement, the investment will take place in stage 2. So, the payoff in case of an agreement is $\Pi_e(i) - i$.
- If there is no agreement, the worker and the firm separate. Then, nothing is produced but the firm has not to invest either.

So, the wage contingent on each realization of $i$ solves:

$$\max_{w(i)} (V_e(i) - V_u)^{\gamma}(\Pi_e(i) - i - \Pi_v)^{1-\gamma} \quad (39)$$
Complete contracts

With a complete contract that cannot be renegotiated, the equilibrium level of investment is efficient.

The solution to that problem can be written as:

$$\Pi_e(i) - i - \Pi_v = (1 - \gamma)S_n(i).$$  \hfill (40)

From (34) and (36), this expression can be rewritten as:

$$w(i) = (1 - \gamma)rV_u + \gamma[y(i) - (r + q)i] - \gamma r\Pi_v.$$  \hfill (41)

This equality implies that $w$ varies with $i$ according to:

$$w_i(i) = \gamma[y_i(i) - (r + q)].$$  \hfill (42)

This holds for any $i$. Now, at this stage, the players anticipate the investment behavior characterized by (38). Combining (42) and (38) yields:

$$y_i(\bar{i}) = r + q.$$  \hfill (43)

Put another way, $\bar{i} = i^*$. 

Incomplete contracts

If the wage is bargained over (or renegotiated) when the firm has already invested, the employee can reap part of the payoff generated by the investment.

Remember that this analysis can also be conducted in the case where the worker invests in specific capital.

This phenomenon, the “hold-up” problem, is easily captured by the following two-stage game:

- **Stage 1**: The firm decides over investment.
- **Stage 2**: The wage is bargained afterwards.

Backward induction is used again.
Incomplete contracts

Stage 2

Since the investment has already been made, the net gain for the employer is $\Pi_e(i) - i$ if the negotiation is successful and $\Pi_v - i$ if it fails. So the bargained wage, denoted by $\omega(i)$, solves:

$$\max_{\omega(i)} (V_e(i) - V_u) \gamma (\Pi_e(i) - \Pi_v)^{1-\gamma}$$

(44)

It is easily checked that $\omega(i)$ verifies:

$$\omega(i) = (1 - \gamma) rV_u + \gamma [y(i) - r\Pi_v]$$

(45)

*Compare with (41)!*

Differentiating (45) with respect to $i$ yields:

$$\omega_i(i) = \gamma y_i(i).$$

(46)
Incomplete contracts
Stage 1

Anticipating this outcome, the firm will reconsider her investment. Compared to the complete contract case, the investment problem is now:

$$\max \limits_i \frac{y(i) - \omega(i) + q\Pi_v}{r + q} - i.$$  (47)

Let \(\bar{i}\) denote the optimal investment level in this setting. The first-order optimal condition is:

$$y_i(\bar{i}) - \omega_i(\bar{i}) = r + q.$$  (48)

Remembering (46), this expression can also be rewritten as:

$$y_i(\bar{i}) = \frac{r + q}{1 - \gamma} > r + q \text{ if } 0 < \gamma < 1.$$  (49)

Under the assumptions about \(y(i)\), \(\bar{i} < i^*\).
Incomplete contracts

\[ \bar{i} < i^* \Rightarrow \text{a lower productivity per worker, } y, \text{ and therefore a lower equilibrium value of } \theta \text{ and of equilibrium employment.} \]

If \( \gamma = 0 \), then \( \bar{i} \) would be efficient. However, then

1. an inefficiently low level of unemployment (too many vacancies).
2. as workers also invest in match-specific capital, \( \gamma = 0 \Rightarrow \) no rent from their investment \( \Rightarrow \) zero investment

Note: In the basic static and dynamic models introduced in previous sections, there is already a hold-up problem since the cost of opening a vacancy has to be paid before each match and is sunk before the bargain over wages take place.
Is the “hold-up” problem a real one?

One can imagine more complex contracts with clauses that protect parties against the consequences of the holdup. So, it is not certain that the incompleteness of labor contracts necessarily leads to under-investment in specific capital.

Empirical analyses have focussed on the role of unionization on investment (not necessarily match-specific):
- direct effect (“hold-up” above) and
- indirect effect (unions $\rightarrow$ profit level $\rightarrow$ investment).

The empirical literature often concludes that the direct effect is negative. But, opposite conclusions are not rare. See e.g. the synthesis in Dobbelnaere (2005).
Out-of-stationary-state dynamics

We skip most of this section (Section 5 in the book)

The study of out-of-stationary-state dynamics allows to diagnose the origin of the perturbations that affect movements in employment. Remedies adopted to reduce under-employment will vary with this diagnosis.

**aggregate shocks**

- change in aggregate demand or supply of goods, and would *not* shift the Beveridge curve
- = a change in $y$, $r$, $z$, or $\gamma$
- a restructuring of production units, which would shift the Beveridge curve but not (much) aggregate supply or demand.
- changes in function $m(.)$ or in $q$
Out-of-stationary-state dynamics

Environment is stationary, initial unemployment rate out of steady state

Express value functions out of steady state (like Eq. (14) above).

All decisions of agents are directed toward the future ("forward looking" decisions)
⇒ The number of vacancies (and hence, $\theta$) and the wage immediately "jump" to the stationary value (no inertia unless the model is extended).

When labor market tightness has reached its stationary value $\theta^*$, the differential equation (9) describing the evolution of the unemployment rate becomes:

$$\dot{u} + \left[ q + n + \theta^* m(\theta^*) \right] u = q + n$$

The propagation of shocks is too simple (Extensions needed).
Critiques

1) Poor cyclical performances of the matching model? According to Shimer (2005), “the textbook search and matching model cannot generate the observed business-cycle-frequency fluctuations in unemployment and job vacancies in response to [productivity] shocks of a plausible magnitude.”

This paper led to a major controversy. Main features of this debate:

- Hall and Milgrom (2008): Replace Nash bargaining by another mechanism that delivers more wage stickiness.
- Pissarides (2009) argues that wages in new matches are actually as cyclical as in the textbook model under Nash bargaining. Moreover, all what matters for job creation is the gap between (expected) productivity and wages in new matches. Introducing a fixed cost of matching suffices to get good cyclical properties of the textbook model. More details on next slide.
Note

The “matching cost”

“Matching costs in the model are of two kinds: the worker’s foregone leisure and unemployment income, and the vacancy posting costs of the firm.” (Pissarides, 2009, p. 1362)

The free-entry condition emphasizes that the firm’s cost of creating a vacancy is proportional to the expected duration of it.

“Other matching costs, such as training, negotiation, and one-off administrative costs of adding a worker on the payroll, are neglected by the model” (Pissarides, 2009, p. 1363). Pissarides recommends to add such a fixed cost. Call it $H$. Then the left-hand side of (16) becomes $H + h/m(\theta)$. This has deep implications on the cyclical properties of the model.

In contrast, in Blanchard and Galí (2010), vacancies are assumed to be filled immediately by paying a hiring cost that increases with tightness on the labour market.
Critiques

2) “Lagos (2000) emphasizes that if the matching function is a reduced-form relationship, one should be concerned about whether it is invariant to policy changes. Addressing this issue requires an explicit model of heterogeneity that gives rise to an empirically successful reduced-form matching function.” (Shimer, 2007, p. 1077). Stevens (2007) deals with the same issue.

3) “Random search is dumb search”: Each agent searches in “all directions” at random, nobody uses wages that firms commit to pay or a “match maker” for instance to “organize a less time-consuming search process”. Opposite view: directed search on the basis of posted wages (Moen, 1997); frictions are due to a lack of coordination (as the labour market is decentralized, it would be very hard to coordinate search decisions. Hence some vacancies receive 0 applications, some others > 1).
Extensions
Keeping $q$ exogenous

- **Endogenous effort to search for a job/for applicants** (see e.g. Lehmann and Van der Linden, 2007; endogenous effort on the job (working hours), see Sec 3.2.1 of Chap. 12 of CZ.

- **Stochastic job matching: Ex ante workers and firms are identical but the productivity of a job-worker pair is random** (chap. 6 of Pissarides, 2000; with a learning process of job quality in Pries and Rogerson, 2005);

- **On-the-job-search** Important extension as job-to-job flows are observed to be important.
  This extension raises however several tricky issues in the presence of bargaining. See the seminal paper of Pissarides (1994) (or chap. 4 of Pissarides) and the more recent contributions of Shimer (2006), Cahuc, Postel-Vinay and Robin (2006) and Bonilla and Burdett (2010). Under some conditions, a non degenerate distribution of wages can be observed in equilibrium.
Extensions

Keeping $q$ exogenous

- **Endogenous size of the labour force and/or of the number of hours worked** (chap. 7 of Pissarides, 2000, Garibaldi and Wasmer, 2005);


- **Add explicitly a spatial dimension** (e.g. Wasmer and Zenou, 2006, Zenou, 2009).

Extensions
Keeping $q$ exogenous

- Kolm (2005) introduces social norms on the basis of a simple idea:
  - In normal times (unemployment is low), there is a normative pressure to earn one’s own living
  - The more common it is to be unemployed, the weaker the normative pressure.

Formally, $z$ above is replaced by $z - c(u)$, where $c(u)$ captures a psychological cost of being unemployed, with $c'(\cdot) < 0$.

Multiplicity of equilibria:
Low unemployment $\rightarrow$ high psy. cost of being unemployed $\rightarrow$ low wage demands that sustain low unemployment.
And conversely when unemployment is important.
Extensions
Endogenous job destruction

The job destruction rate $q$ can be made endogenous (see chap. 2 of Pissarides, 2000, or Section 4.3 of Chapter 11 in CZ).

The productivity of filled vacancies is hit by random shocks (often a Poisson process).

⇒ There is a “reservation” level of productivity below which it is preferable to separate and look for another partner.

⇒ The model can be expressed in terms of a “job creation” curve and a “job destruction” curve in a $(\theta, \text{reservation productivity})$ space.
The expression “job matching” is used in other contexts than the “Matching models”: Jovanovic (1979) developed a theory of separation (i.e. change of employer): when a match is formed, the quality of the match is uncertain and revealed though experience.\textsuperscript{5} This theory explains why the separation rate decreases with job-tenure and becomes close to constant after some point.

Matching functions appear also in literatures about:

- goods and in particular housing markets
- Financial markets
- the “marriage market”,...

\textsuperscript{5}See Pries and Rogerson (2005) for the same idea in the framework developed in this chapter.
Assignment

The “assignment literature” studies how heterogeneous agents (say, job-seekers and job vacancies with different intrinsic productivity) find “appropriate” trade partners when there are complementarities in production.

- “Positive Assortative Matching” (PAM) = “better-qualified job-seekers match with better jobs”
  This property comes out in a Walrasian static matching economy (Becker, 1973). Unemployment and vacant jobs cannot coexist.

- In the presence of frictions on the labour market, unemployment and vacant jobs can coexist for all types (productivity levels). Despite a certain degree of PAM, there can be some mismatch: “some high-productivity firms are forced to hire low-productivity workers whereas some low-productivity firms are able to hire higher productivity workers.” (Shimer, 2005, p. 999).


Garibaldi, P. and E. Wasmer (2005) “Equilibrium search unemployment, endogenous participation, and labor market flows”.


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