II Risk-sharing on the labour market and wage rigidities

Bruno Van der Linden*

February 11, 2003

Fonds National de la Recherche Scientifique and
Department of Economics,
Université Catholique de Louvain

1 Introduction

• General Equilibrium Theory (GET) under uncertainty takes as a starting point a set of possible (mutually exclusive) “states of the world”, say

\[ s = 1, \ldots, S \]

If there exists a full set of insurance markets, one for each commodity contingent on each state,
GET extends standard results obtained in a deterministic setting.

• The model of an economy with a complete set of insurance markets in an abstract idealization!

\[ \leftrightarrow \] for most consumers, the major source of uncertainty originates in HUMAN CAPITAL. (Private) insurance opportunities are almost non-existent for labour income AND diversification of human capital is severely restricted:

A system of spot markets for labour services would leave individuals to bear themselves the risks attached to their human capital -risks which are notoriously difficult to diversify. (Drèze and Gollier, 1993, p. 1458)
“Under incomplete [insurance] markets, the degree of risk aversion varies across agents, often in a systematic way; for instance, it is higher for workers or consumers than for firms” (Drèze, 2001, p. 10). From there, the following basic issues:

- If firms are less risk averse than workers, couldn’t labour contracts be a partial substitute for missing insurance markets covering human capital?
- If indeed they are a partial substitute for these missing insurance markets, what are the implications for (real) wages and employment? Are (real) wages rigid? Does this explain inefficient allocation of labour?

⇒ the basic intuition:

Assume 2 states: a good one $s = 1$, a bad one $s = 2$

$LD_s(w) =$ the labour demand curve $(s = 1, 2)$

$LS$ vertical if $w > m$ ($m$ is exogenous and has the same meaning as ‘R’ in the lectures notes about unions$^1$)

Let $w_s$ denote competitive wages

→ production efficiency but a high variability of workers’ income.

EX ANTE, risk averse workers will prefer a contract that guarantees $w^*$ whatever $s$ ($w_2 < w^* < w_1$). If $w^* < E_s MVP_L(s)$, the firm agrees.

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{figure1.png}
\caption{Basic intuition.}
\end{figure}

$^1$This change of notation is motivated by the notation used in the compulsory reading for the present topic: Sections 1 to 3 of Drèze and Gollier (1993).
• Basic distinctions :
  – a COMPLETE CONTRACT specifies each party’s obligation in every conceiviable state of the world : the assumption made here. In practice : incomplete contacts.
  – SYMMETRIC information : the proba. distribution of s and the actual ex post realization of s are costlessly observed and agreed upon by ALL contracting parties.
  – ASYMMETRIC information : in these lectures, only the case where the firm but not the workers can EX POST observe the state s. See also the lectures notes for topics 6 and 7.

Table of contents

2. Risk-sharing through labour contracts when the utility is fairly general and information is symmetric (+ complete contracts)

3. Enforcement problems

4. Risk-sharing on the labour market and second-best wage rigidities
2 Risk-sharing through labour contracts when the utility is fairly general and information is symmetric (+ complete contracts)

This presentation is close to Malcomson (1999), p 2300-2303. As far as the vocabulary is concerned, this literature (that started at the beginning of the seventies) has often been called the ‘implicit contract’ literature. ‘Implicit’ because we do not see actual labor contracts that specify the wage contingent on every possible ‘state of the world’.

- Utility Function \( U = U(C(s), L(s)) \)

  Where \( C \) is consumption \( (C = w.L) \)
  \( L \) is labour supply per worker
  \( s \) is a random shock with a known distribution

  \[ p(s) \quad s = 1, \ldots, S \quad : \sum_s p(s) = 1. \]

  Assumptions: Risk-averse workers. \( U = U(C(s), L(s)) \) is assumed to be such that:

  \[ U_C > 0 \quad U_{CC} < 0 \]
  \[ U_L < 0 \quad U_{LL} < 0 \]

  There is a given number \( n \) of identical workers objective function : EX ANTE, i.e. before \( s \) is known

  \[ \max_{C,L} E_S \left[ U(C(s), L(s)) \right] = \sum_{s=1}^{S} p(s)U(C(s), L(s)) \]

- The firm

  Production Function \( Y = s.F\left( N(s) \right) \quad F' > 0, F'' < 0 \)
  where \( N \) is utilized labour\(^2\)

  \[ N(s) = n.L(s) \]

  (real) profit function

  \[ \pi(s) = sF(n.L(s)) - n.C(s) \]

  The managers of the firm have a utility function \( v(\pi) \)

\(^2\)Writing “\( N(s) = n.L(s) \)” implies work-sharing among the \( n \) employees.
The ex-ante Pareto optimal contract we are looking for maximizes (1) s. to a profit constraint

\[ \sum_{s=1}^{S} p(s) v\left(\pi(s)\right) \geq \bar{\pi} \]

where \( \bar{\pi} \) is exogenous.

Denote the optimum by \( \left\{ (C^*(s), L^*(s)), s = 1, \ldots, S \right\} \)

First-order Conditions

Assuming \( C^*(s) \) and \( L^*(s) > 0 \), \( \exists \lambda > 0 \) such that

\[
\begin{align*}
\frac{\partial U\left(C^*(s), L^*(s)\right)}{\partial C(s)} &= \lambda v'\left(\pi^*(s)\right).n \quad s = 1, \ldots, S \\
\frac{\partial U\left(C^*(s), L^*(s)\right)}{\partial L(s)} &= -\lambda v'\left(\pi^*(s)\right).n F'\left(n.L^*(s)\right) \quad s = 1, \ldots, S
\end{align*}
\]

and

\[ \sum_{s=1}^{S} p(s) v\left(\pi^*(s)\right) \geq \bar{v} \]

where \( \pi^*(s) \equiv s F'[n.L^*(s)] - n C^*(s) \).

Analysis of the F.O.C.

Let \( s \) and \( t \) be two different states. Equality (4) implies

\[
\begin{align*}
\frac{\partial U\left(C^*(s), L^*(s)\right)}{\partial C(s)} &= \frac{\partial U\left(C^*(t), L^*(t)\right)}{\partial C(t)} = \frac{v'\left(\pi^*(s)\right)}{v'\left(\pi^*(t)\right)} \quad \text{“BORCH-CONDITION”}
\end{align*}
\]

Pareto optimal risk-sharing among risk-averse workers and firm owners is obtained if the marginal rate of substitution between any pair of state contingent payoffs are equal for the workers and the firm

Example : risk-neutral firm, i.e. \( v' = \text{a constant.} \)

Then

\[ U_C\left(C^*(s), L^*(s)\right) = U_C\left(C^*(t), L^*(t)\right) \quad \forall s, \forall t \]

(\text{where } U_C \equiv \frac{\partial U}{\partial C})
(4) in (5) implies

\[
\frac{-U_L(C^*(s), L^*(s))}{U_C(C^*(s), L^*(s))} = sF'(n.L^*(s)) \quad \forall s
\]

The marginal rate of substitution between consumption and labour equals the marginal productivity of labour

\[\iff\] efficiency in production!!

Further developments assuming risk-neutral firms \((v'\text{ constant})\) when \(s\) is taken as a continuous variable.

→ Comparative statics on (4) and (5) :
  Differentiating (4) and (5) w.r. to \(s\) yields :
  (Note : the superscript “\(s\)" is not added)

\[
U_{CC} \frac{C'}{ds} + U_{CL} L' = 0
\]

\[
U_{CL}C' + U_{LL}L' = -\lambda v'n[F' + sF''nL'] \tag{10}
\]

or

\[
U_{CL}C' + [U_{LL} + \gamma sF''n] L' = -\gamma F' \tag{10'}
\]

The solution of this system is given by

\[
C' = \frac{\gamma F'U_{CL}}{\Delta} \tag{11}
\]

\[
L' = -\frac{\gamma F'U_{CC}}{\Delta} \tag{12}
\]

where

\[
\Delta = U_{CC}U_{LL} - U_{CL}^2 \geq 0 \text{ by risk-aversion} + U_{CC}\gamma sF''n > 0
\]

Implications :

\[
(11) \Rightarrow (A) \ C' \text{ is zero (i.e. } C(s), \text{ or } w(s).L(s), \text{ is completely smoothed) if } U_{CL} = 0
\]

\( (= \text{ preferences are strongly separable in } C \text{ and } L)\)\(^3\)


\(^3\)Also, if \(L\) is fixed (independent of \(s\) ⇒ cond. (5) does not apply), then with risk-neutral employers, \(C'(s)\) is zero. This is easily seen from cond. (4). Here, \(C'(s) = 0 \Rightarrow v'(s) = 0\) : the wage rate is fixed!
(12) ⇒ (B) \( L' > 0 \). (In a “better state”, each worker works more)\nn
\[ \iff \text{“work-sharing” among the } n \text{ employees.} \]

(11)/(12) ⇒ (C)

\[
C' = \frac{n \cdot F'}{-n \cdot F' \cdot U_{CC}} \cdot U_{CL} \cdot L'
\]

Total wage payments are rising in \( s \) as \( U_{CL} > 0 \) falling in \( s \) as \( U_{CL} < 0 \)

However \( U_{CL} \leq 0 \) should be expected.

Why? Leisure is presumably a normal good.

For leisure to be a normal good, it can be shown that the following condition should be fulfilled:

\[ U_{LC} = -U_{CC} \quad U_L < 0 \quad \text{where } L = \text{Labour supply} \]

A sufficient condition is therefore \( U_{LC} \leq 0 \).

Then (C) implies

\[ \text{sign } (C') = -\text{sign } (L') < 0 \quad \text{if } U_{CL} < 0 \]

Curious?? When the state is improving, at the same time the employees work more (condition (B)) and their consumption level shrinks.

If you remember that \( C(s) \equiv w(s).L(s) \),

\[ C'(s) = w'(s).L(s) + w(s).L'(s) \]

⇒ a (sufficiently) decreasing relationship between the wage rate \( (w) \) and the state \( (s) \) is needed.

OTHER IMPLICATION:

The optimal ex-post utility level \( U(s) = U(C^*(s), L^*(s)) \) is such that

\[
U' = U_C C' + U_L L' = \ldots = \left[ -\frac{U_C}{U_{CC}} \right] \left[ U_{CL} - \frac{U_L}{U_C} \cdot U_{CC} \right] \cdot L'
\]

⇒ If leisure is a normal good, then \( U'(s) < 0 \): A complete insurance contract makes a worker facing an adverse draw better of ex post than a worker who draws a more favorable value of \( s \).
Examples with only one worker

Example 1

\[ U(C, L) = U(C - K(L)) \quad U' > 0, U'' < 0, K' > 0, K'' \geq 0 \]

Risk Neutral firm

\[ Y = sF(L) \quad F' > 0, F'' < 0 \]

\[ s = 1, \ldots, S \]

\[ \max_{s=1,\ldots,S} \mathbb{E} \left[ sF(L(s)) - C(s) \right] \quad \text{s.to. } \mathbb{E} \left( C(s) - K(L(s)) \right) \geq u \]

- With this utility function \( U_{CL} = 0 \) (i.e. no income effects on labour supply)

  \[ \Rightarrow \text{utility is constant } \Rightarrow C(s) - K(L(s)) = \text{constant } \forall s \]

- From the first order conditions, efficiency in production comes from

\[ K'(L(s)) = sF'(L(s)) \quad \forall s. \]

These properties imply that if \( s' > s \), then

\[ C(s') > C(s) \quad \text{and} \quad L(s') > L(s) \]

Example 2

\[ U(C, L) = V(C) - K(L), V' > 0, V'' < 0, K' > 0, K'' \geq 0 \]

i.e. preferences are strongly separable \((U_{CL} = 0)\).

From the previous pages (can be checked by the first order conditions):

\[ C(s) = \text{constant } \forall s. \]

If \( s' > s \), \( L(s') > L(s) \)

EXTENSIONS (under symmetric information)

- employers are risk averse or some states correspond to risks that are uninsurable (e.g. aggregate shocks) (summarized by Malcomson (1999) p. 2302-2303)

- contracts with layoffs. A good Reference is Section III of Rosen (1985).
3 Enforcement problems

3.1 When the state $s$ is not public. Information that can be verified in court

Suppose that the state is observed only by the firm. Will an optimal contract under symmetric information be incentive compatible, i.e. such that the profit in state $s$ if the firm announces $s \geq$ the profit in state $s$ if the firm lies and announces $s'$ ($\forall s' \neq s$)?

Not necessarily.

$\rightarrow$ Yes in example 1 above
$\rightarrow$ No in example 2 above


Intuitively, in example 2, the firm has always an incentive to claim that the state is “good” since $C(s)$ is constant and $L(s)$ increases with $s$.

Here it can be shown that the optimal incentive-compatible contract will have to associate higher hours with higher earnings. Moreover it can be shown that this contract leads to “overemployment”: intuitively, to force the firm not to claim that the state is “good” (when it is not), the contract forces the firm to employ a (too-) high level of employment (hours) if it announces a “good” state.

However with other utility functions and for instance risk averse firms, it can be shown that an incentive compatible contract can generate “underemployment” (i.e. less employment in “bad” states than would be the case under symmetric information.

So the main messages under asymmetric information about the realized state $s$ are:

a) generally a trade-off between risk-sharing benefits and the provision of incentives

b) often larger fluctuations in employment than under symmetric information (ex post production inefficiencies).
3.2 Enforcement problems due to alternative matches

(A more detailed summary can be found in Malcomson (1999), p 2305-10)

If in addition to contracts offering insurance, there is a “spot labor market” that opens ex post when the state $s$ is known, then

- workers have an incentive to quit in good states (high wages on the spot market)
- firms have an incentive to renego the contract in bad states (low wages on the spot market).

A major analysis here is Thomas and Worrall (1988). This paper is based on the following main assumptions:

- symmetric information : ex post, both the firm and the employee costlessly observe the state but this information is not verifiable in court $\Rightarrow$ no precommitment to a wage contract.
- infinitely lived agents (common discount factor)
- risk-averse workers and risk-neutral firms
- hours of work are fixed (extended in Malcomson (1999))
- output is not random but the spot market wage is random.

Since workers are risk averse, they would like to trade off a lower expected wage for greater stability. Thomas and Warrall assume that once an agent has reneged the insurance contract he must trade at the spot market wage from then onwards. This defines employee’s and firm’s outside options.

Basic characteristics of the self-enforcing contract
(without proof).

If the state remains such that it is possible to satisfy both outside option constraints (otherwise the firm and the worker separates)
the firm provides insurance to the employee in the form of a constant REAL wage until such time as the wage is

1. either too low to prevent the employee quitting
   (then the wage is increased by just enough to ensure the employee stays)

2. or it is too high to prevent the firm laying off the employee
   (then the wage is renegotiated downwards).
→ Fig 2 in Malcomson (1999).

→ Empirical evidence ?

Yes (Malcomson, 1999, p 2308-9)

But:

“Contracts to provide insurance are necessarily concerned with insurance of REAL earnings, whereas the data discussed in Section 2.2 provide at least some indication of NOMINAL contract effects” . . .

“to fully understand wage behavior, we need to consider contracts that arise for reasons other than insurance of employee earnings” (p 2311).

⇒

See topics 6 and 7 of these lectures.
4 Risk-sharing on the labour market and second-best wage rigidities

The main message up to now can be summarized as follows: Because employers are more able to diversify risks and the employment relationship can reduce (not eliminate!!) the extent of informational asymmetries, there is room for employment contracts which to some extent at least reconcile productive efficiency and risk-sharing efficiency. If information asymmetries are important, risk sharing will be more limited and the allocation of resources less efficient.

However, the lack of an efficient private insurance mechanism remains for new entrants on the labour market and, more generally, for job-seekers not covered by such “insurance labor contracts”. In the absence of private contracts, Drèze (1993) and Drèze and Gollier (1993) show that risk-sharing can be organized socially (or if you prefer, collectively). Reconciling competitive wages and risk-sharing efficiency is possible if lump-sum state-dependent taxes and allowances can be implemented (by the State and/or the Social Security System). Let us first consider this ‘first-best’ world (Subsection 1) and then the ‘second-best’ (Subsection 2).

4.1 Efficient labor contracts

Consider the following simple static setting where it is possible to replicate the allocation of resources when a complete set of insurance markets is available. Assume that a single aggregate consumption good is produced with homogeneous labour $L$. The technology is state-dependent. Let $f_s(L_s)$ be the production function in state $s$ ($s = 1, \ldots, S$). States are exogenous and their interpretation should be macroeconomic shocks instead of (diversifiable) firm-specific shocks. Assume that the distribution of production possibilities is common knowledge (symmetric information!). $N$ identical workers are endowed with the utility function $u(y - ml)$, where $y$ denotes income and $l$ working time ($0 \leq l \leq 1$). The parameter $m$ captures the opportunity cost of employment (including commuting costs and expenses needed to substitute market-produced goods for home production). Workers and firms are risk averse. Therefore, the competitive wage $w_s^*$ is characterized by

$$w_s^* = \max[f'_s(N), m] \text{ and } L_s = \min \left[ N, f'_{s}^{-1}(m) \right].$$

Assuming that the state-dependent lump-sum transfers/taxes accruing to employers, to employees and to unemployed people must balance in each state, the ex ante Pareto-efficient allocation maximizes the expected utility of a representative worker (assuming that if there is unemployment, it is allocated randomly), subject to an expected utility level for firm-owners. The ex ante Pareto-efficient solution features the following properties:
The wage rate in state $s$ is the competitive one $w^*_s$. Labour is therefore *ex post* efficiently allocated ($f'_s(L_s) = w^*_s$). Ex-post efficiency means an allocation of resources that maximize an ‘adjusted national product’ defined as $f_s(L_s) - mL_s$, subject to $L_s \leq N$.

The marginal rate of substitution between any pair of state contingent payoffs should be equal for employers, employees and, if any, unemployed people (the Borsh condition).

From these two properties and assumption about the utility function, one derives the following properties. When full employment occurs, workers pay a lump-sum tax. When $L_s < N$, employed and unemployed workers receive the same lump-sum transfer, say $b_s$. Hence, the utility of the unemployed is equal to $u(b_s)$. For the employed, since $L_s < N$, the utility level is equal to $u(b_s + w^*_s - m) = u(b_s)$.

If $L_s = N$ (respectively, $L_s < N$), the tax (resp., the transfer) is associated with an offsetting transfer to (resp., tax paid by) employers.

These first-best institutions would reconcile competitive wages with risk sharing. Being independent of job status, the allowance $b_s$ avoids to raise the reservation wage of jobless workers through unemployment benefits paid only to the unemployed.\(^4\)

To implement such a Pareto-efficient allocation, we need:

- First instrument: a competitive labour market or institutions that generates competitive wages.
- Second instrument: (on top of whatever taxes exist independently), a social security contribution or benefit combined with an offsetting adjustment in the (positive or negative) taxes borne by capital.
- Information problem 1: with heterogeneous workers (i.e. different values of $m$), the social security contribution or benefit should be $m$-specific.
- Information problem 2: how can the Borsch-condition be applied in ‘real world conditions’? Considering the particular case of iso-elastic utilities and assuming (as a first approximation) that (i) risk aversion is the same among workers and employers (ii) $m$ can be neglected, Drèze and Gollier (1993) present the following guiding rule: the relative share of net labour incomes (including the allowances) should be constant whatever the state of the world. If the competitive wages fail to guarantee this property, tax and transfers between capital and labour should be adjusted accordingly.

\(^4\)Notice that the proposed solution is not a basic income as it usually understood. For (i) the optimal mechanism organizes transfer between workers and employers and *vice versa* (in case of full employment, workers pay a state-dependent lump-sum tax accruing to firm owners) (ii) the level of the lump-sum allowance is state-dependent and (iii) if workers have heterogeneous preferences, the lump-sum allowance/tax would ideally depend on the individual value of $m$. 
Notice that the problems of incentives (asymmetric information) and tax evasion are ignored.

4.2 The second best

The optimal arrangement introduced in the previous subsection would involve massive transfers. For instance in bad states of the world, all workers (employed or not) would receive a transfer (and presumably a substantial one). This approach is further studied in Drèze (1993). Drèze and Gollier (1993) characterize second-best allocations by simply adding that the allowable instruments consist of labour taxes. Since lump-sum transfers to employed workers are ruled out, transfers can only be issued to the unemployed and, hence, are now interpreted as unemployment benefits.

Therefore, second-best optimal allocations will typically involve some inefficient unemployment.

The characterization of second-best ex ante Pareto-efficient allocations is quite complicate. You should read and capture only the intuition in Section 3 of Drèze and Gollier (1993) (Section 4 is too technical and can be skipped).

The basic idea is that when positive transfers to workers are impossible, raising $w_s$ above the competitive wage $w^*_s$ is the only way of increasing income in ‘bad’ state. Under plausible conditions, from an ex ante viewpoint, a wage increase in these states compensated by taxes in ‘better’ states dominates the competitive solution. This is achieved by issuing unemployment benefits that implement a downwards wage rigidity. This second-best optimal wage rigidity is a compromise between productive efficiency and risk-sharing efficiency.

4.3 Conclusion

To sum up, we cannot rely on a competitive market to offer acceptable levels of economic security. Risk-sharing of diversifiable risks can be implemented at the firm level (with problems of enforceability). However, some (macroeconomic) risks are not diversifiable. Moreover, workers not (yet) covered by long-term contracts are not protected (put another way, insurance markets are incomplete). So, to guarantee a level of economic security to every one, in a second-best world, it is often justified to implement a downward wage rigidity through unemployment benefits. Actual unemployment insurance systems (and minimum wages) are not necessarily reaching the second-best optimum but they can be understood as an endeavor to share risks for all workers (not only those employed in long-term contracts). Yet, the price to be paid is a loss of efficiency and in particular inefficient unemployment.

---

5This work provides a basis for recommendations for our current economies: Drèze and Malinvaud *et al.* (1994) and Drèze and Sneessens (1997) advocate to exempt minimum wages from employers social insurance contributions (ESIC).
4.4 Recent developments: Wage rigidities, coordination failures and supply-constrained equilibria

I here sketch a recent and difficult literature that can be seen as a come-back to the original question in the ‘implicit contract’ literature: Can workers’ risk aversion lead to wage rigidities that explain (inefficient) unemployment? The initial literature (covered in Section 2) concluded negatively under symmetric information and led to mixed results under asymmetric information (Section 3). Here, the combination of downward wage rigidities and coordination failure lead to a very different answer. There is no compulsory reading for this subsection.

The ingredients are:

**wage rigidities**: These are not necessarily motivated by uncertainty and incomplete (insurance) markets (unionization, imperfect information,... can also be invoked here). Yet, the previous subsection clearly offered an interesting rationale for downward wage rigidity.

**coordination failures**: When equilibria are multiple (for various reasons) and some are in a sense better than others, the possibility of a coordination failure arises. In some of them, there is underutilisation of resources that may persist “because it would take coordinated adjustments of quantities and/or expectations to restore equilibrium with fuller use of resources. But such coordination lies beyond the capabilities and motivations of individual agents” (Drèze, 1997, p. 1739). Put differently, “a inferior equilibrium may obtain, that could only give way to a superior one through a coordinated modification of the plans, or expectations, of some or all agents.” (see Drèze, 1999, p. 9)

**supply-constrained equilibrium**: Loosely speaking, this equilibrium is characterized by an allocation of goods, a vector of prices (and wages) and a vector of supply constraints (supply of labour, supply of goods) such that (i) consumers maximize their preferences subject to a budget set (with possibly supply constraints), (ii) firms maximize profits subject to a production set (with possibly supply constraints) and (iii) quantity constraints are binding when prices (wages) are downward rigid. (see Drèze, 1997, p. 1740).

Drèze (1997) shows that supply-constrained equilibria can be pervasive even if the level of prices and wages are compatible with competitive equilibrium! To show this one needs downward rigidity of some (relative) prices and coordination failures. To intuitively see the respective importance of these two ingredients, wage rigidity sustain (not cause) pervasive underutilisation of resources caused by coordination failures. On pages 1754-5, Drèze (1997) explicitly mentions risk-sharing and writes that the relevance of the second-best results in Drèze and Gollier (1993) is “magnified by the possibility of coordination failures”. To conclude, this recent research should be seen as seminal. At this moment, we still have to discover the implications for
the analysis of the labour market, for macroeconomics and in particular for the role and the interpretation of wage rigidity and flexibility.

References


