Regional Equilibrium Unemployment Theory at the Age of the Internet*

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Abstract

This paper studies equilibrium unemployment in a two-region economy with matching frictions, where workers and jobs are free to move and wages are bargained over. Job-seekers choose between searching locally or searching in both regions. Search-matching externalities are amplified by the latter possibility and by the fact that some workers can simultaneously receive a job offer from each region. The rest of the framework builds upon Moretti (2011). Increasing the matching effectiveness out of the region of residence has an ambiguous impact on unemployment rates. While it reduces the probability of remaining unemployed, it also decreases labor demand because of a lower acceptance rate. We characterize the optimal allocation and conclude that the Hosios condition is not sufficient to restore efficiency. A numerical exercise indicates that the loss in net output is non negligible and rising in the matching effectiveness in the other region.

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JEL: J61, J64, R13, R23.

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1 Introduction

While an abundant literature in urban economics addresses unemployment issues within cities (see Zenou, 2009, for a detailed coverage), less effort has been devoted to analyze the causes of unemployment at the regional level. “Given the large geographical differences in the prevalence of unemployment observed in the real world, understanding spatial equilibrium when the labor market does not instantly clear would appear to be of primary importance.” (Kline and Moretti, 2013, p. 239). The main purpose of this paper is to contribute to this understanding.

The Internet allows both sides of the labor market to find more easily potential partners, even faraway, thanks to job boards and meta-search engines. Moreover, the recruitment process can now also be conducted online through virtual recruiting tools. Marinescu and Rathelot (2013) provide evidence that the distance between the job-seeker and the job vacancy exceeds 100 km (63 miles) in about 10% of the online applications on CareerBuilder.com. This suggests that a non-negligible share of US job-seekers ramp up their job search by expanding it over long distances.

While most of the literature dealing with regional unemployment assumes that people need to migrate before they can start searching locally, we relax the assumption of segmented regional labor markets. Our main contribution consists in developing a general equilibrium search-matching framework where job-seekers choose whether they search in their region of residence only or all over the country. To the best of our knowledge, this has not been done yet. In this setting, regions are strongly interdependent and several sources of inefficiency explained later are present. A numerical exercise suggests a non-negligible gap between the efficient and the equilibrium allocations.

As a secondary contribution, our analysis also sheds some light on a puzzle. Expectations that the Internet would improve the functioning of the labor market by reducing search-matching frictions were great (see e.g. Autor, 2001). A decade later, the evidence is mixed. Some microeconometric evaluations find that online job search shortens unemployment duration in the US (Kuhn and Mansour, 2014; Choi, 2011). For graduate students in Italy, Bagues and Labini (2009) conclude that the use of an online platform reduces the probability of unemployment and raises geographical mobility. However, via a difference-in-differences approach, Kroft and Pope (2014) find no evidence that the rapid expansion of a major online job board (during the years 2005-2007) has affected city-level unemployment rates in the US. So, the reasons why improvements at the individual level disappear at a more aggregate level need to be understood. This paper proposes an explanation in a spatial economy.

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1 In 2010, 25% of the interviewed Americans who use the Internet declared to do so to find a position (U.S. Census Bureau, 2012, Survey of Income and Program Participation, 2008 Panel). In Europe, in 2005, among the unemployed workers, 25% used the Internet to search for a job. This share has increased to 50% in 2013 (Eurostat, 2014, see http://ec.europa.eu/eurostat/data/database?node_code=isoc_ci_ac_i).


3 The evidence is more mixed for the UK (see Manning and Petrongolo, 2011).
We build upon the synthesis of Moretti (2011) who develops a two-region static spatial equilibrium model à la Rosen (1979)-Roback (1982). Contrary to these authors, Moretti (2011) assumes that the supply of labor is not perfectly elastic. This property is obtained by assuming that economic agents have heterogeneous idiosyncratic preferences for regions. The aim of Moretti (2011) is to analyze how local shocks propagate in the long run to the rest of the economy, with a focus on the labor market. He discusses the case where agents have different skills, while we keep labor homogeneous. However, regional unemployment disparities are not studied by Moretti. We introduce search-matching frictions and wage bargaining within this framework (Pissarides, 2000) but we abstract from the housing market.

Contrary to most of the search and matching literature, the spatial heterogeneity is explicit in our framework. In each region, imperfect information and lack of coordination among agents create frictions summarized by a constant-returns-to-scale regional-specific matching function in which the number of job-seekers is a weighted sum of the residents and of the non-residents who decide to search all over the country, both numbers being endogenous. We characterize the equilibrium. We show how regional unemployment differentials are affected by the partition of the population between the two regions and between the statuses of national and regional job-seekers. We also conclude that a rise in search effectiveness out of the region of residence has an ambiguous effect on the unemployment rates. It decreases the probability of remaining unemployed but it also reduces labor demand through a lower acceptance rate of job offers. This ambiguity echoes the main conclusion of Kroft and Pope (2014).

In the standard search-matching literature, frictions generate congestion externalities which are not internalized by decentralized agents unless the Hosios (1990) condition is met. As soon as some workers search all over the country, new sources of inefficiency arise. First, when decentralized agents decide whether to search nationally, they look at their private interest and ignore the consequences of their choices on job creation in all regions. Second, when opening vacancies in a region, firms do not internalize the changes in the matching probability and hence in net output in the rest of the economy. We show that the Hosios condition is never sufficient to decentralize the constrained efficient allocation.

We develop a numerical exercise for a very stylized US economy made of two regions that are initially symmetric and where the Hosios condition prevails. The decentralized economy appears to be far from efficient. For a very wide range of parameters, efficiency requires that nobody searches in the whole country while 10% of the workforce does it in the decentralized economy. Furthermore, the efficient unemployment rate level is lower than the decentralized one. As this exercise assumes symmetric regions, this conclusion is not in contradiction with the recent evidence that geographical mismatch is negligible in the US (see e.g. Sahin et al., 2014, Marinescu and Rathelot, 2013 and Nenov, 2014).

Although a spatial equilibrium model with genuine unemployment has for long been missing, some papers have recently partly filled the gap. Leaving aside the literature where regions are so close that commuting is an alternative to relocation, the literature about regional unemployment differentials can be divided in two groups according to the
type of search: either one needs to move before starting to seek a job in the region of residence or one can search all over the country and then move if needed.

In the first case, some papers extend the island model of Lucas and Prescott (1974) whose economy is populated by a large number of segmented perfectly competitive labor markets where only labor is mobile (workers being allowed to visit only one island per period). Lkhagvasuren (2012) adds search-matching frictions as well as match-location specific productivity shocks in an otherwise standard islands model to reproduce the volatility of unemployment rates in the United States. Focusing also on one (small) region out of many, Wrede (2014) studies the relationships between wages, rents, unemployment and the quality of life in a dynamic framework. He assumes a standard search-matching framework and analyzes how regional amenities affect unemployment and the quality of life. Beaudry et al. (2014) introduce search-matching frictions in a spatial equilibrium setting with wage bargaining, free mobility of jobs, a very stylized housing market, and amenities with congestion externalities. In their paper, with some exogenous probability, the jobless population gets the opportunity to move to another city in order to seek jobs, while we let agents choose between two strategies: regional and national search. Furthermore, Beaudry et al. (2014) do not look at efficiency while we do.

Second, some recent papers assume that workers can seek a job in the whole country. In a setting with many regions, Amior (2012) studies wages’ responses to a housing shock in the presence of skill heterogeneity. He assumes national search in a search-matching framework as well as a random migration cost. Domingues Dos Santos (2011) builds a search-matching dynamic framework with two regions that are each considered as a line. She finds that increasing search effectiveness is beneficial for unemployment rates in both regions. However, she keeps wages exogenous. Using a search-matching dynamic framework with national search and endogenous wages, Antoun (2010) assumes two types of agents who differ in their preference for a region. He finds that a positive productivity shock in one region decreases unemployment locally but raises it in the other region. We extend these models by endogenizing the choice between regional and national search under wage bargaining. Contrary to these papers, we also develop a normative analysis by looking at efficiency. However, we keep our framework static while they all assume a dynamic setting.

In the new economic geography literature, Epifani and Gancia (2005) analyze the simultaneous emergence of both agglomeration economies and unemployment rate differentials. For this purpose, they build a dynamic two-sector two-region model with transport costs and search-matching frictions. They assume a congestion effect in the utility which could reflect the housing market. They emphasize the role of migration following a productivity shock, which raises the unemployment rate in the short run but decreases it in the long run. Francis (2009) extends this framework to endogenous job

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4Molho (2001) develops a partial equilibrium job-search framework with both types of search. Manning and Petrongolo (2011) build a partial equilibrium framework where job-seekers choose their search field. See also Marinescu and Rathelot (2013). We share a common interest with these papers in a general equilibrium model with endogenous wages and vacancies.
Galenianos and Kircher (2009) build a directed-search wage-posting model where workers simultaneously apply to a $N > 1$ jobs. They show that multiple applications lead to an inefficient allocation when a vacancy remains unmatched if the job applicant refuses the offer (an assumption revisited by Kircher, 2009). We stick to the random matching assumption with wage bargaining popularized by Pissarides (2000). More importantly, by assumption in our paper, the search for an efficient allocation is constrained by the free choice of agents when they get two job offers. So, the possibility of unmatched vacancies is common to both the decentralized equilibrium and the efficient allocation.

The rest of this paper is organized as follows. Section 2 describes the model and its equilibrium. We first adopt the the standard assumption that once a job-seeker has decided to live in a region she can only search there. Next, we open the possibility of national job-search. Section 3 studies efficiency. A numerical analysis is conducted in Section 4. Section 5 concludes.

2 The model

This section develops a model with two distant regions. We first introduce the main assumptions and the matching process. Then, it is assumed that a match can only be formed if the vacant job and the unemployed worker are located in the same region. This standard assumption is relaxed in a last subsection where vacancies and job-seekers located in different regions are allowed to meet.

We consider a static model of an economy made of two large regions ($i \in \{1, 2\}$). Each region is a point in space. The distance between the two regions is such that commuting is ruled out, while inter-regional migration is allowed. Topel (1986) and Kennan and Walker (2011) among others have stressed the importance of migration costs. As will soon be clear, idiosyncratic preferences for regions will in our setup play the role of individual-specific relocation costs. The aggregate national labor force is made of an exogenous large number $N$ of homogeneous risk-neutral workers. A worker living in region $i$ supplies one unit of labor if the wage is above the value of time if she stays at home, denoted $b_i$. Firms are free to locate in the region they prefer. They use labor to produce a unique consumption good which is sold in a competitive market and taken as the numeraire.

Workers have idiosyncratic preference for regions. Agent $j$ gets utility $c_{ij}$ from living in region $i$. As Moretti (2011), we assume that the relative preference for region 1 over region 2, $c_{1j} - c_{2j}$, is uniformly distributed on a given support $[-v; v]$, $v > 0$. The presence of a distribution of relative preferences implies that the elasticity of inter-regional labor mobility is finite. A higher value of $v$ entails less intense responses to regional differences.

The indirect utility $V_{ij}^e$ of an employed individual $j$ living in region $i$ is, as in Moretti (2011), assumed to be additive and defined by:

$$V_{ij}^e = w_{ij} + a_i + c_{ij}$$ (1)
where $w_{ij}$ represents the wage earned by agent $j$ in region $i$ and $a_i$ is a measure of exogenous local consumption amenities in region $i$, such as the climate. These amenities are public goods and are not affected by the number of inhabitants in a region (no rivalry).\footnote{Contrary to what is sometimes done in the literature (see e.g. Wrede, 2014 or Brueckner and Neumark, 2014), amenities $a_i$ do not affect the production function either.} Similarly, the indirect utility $V_{ij}^u$ of an unemployed person $j$ residing in region $i$ is:

\[ V_{ij}^u = b_i + a_i + c_{ij}. \] (2)

In each regional labor market, we assume a regional-specific random matching process. Adopting a one-job-one-firm setting, firms decide in which region they open at most one vacancy. The cost $\kappa_i$ of opening a vacancy is constant, exogenous and region-specific.\footnote{Capital is assumed to move freely across regions through vacancy creation. Ignoring credit market imperfections, entrepreneurs have no problem financing their vacancy cost $\kappa_i$.} Throughout the paper, we assume constant returns in the production of the consumption good.\footnote{Although very standard in the search-matching literature, this assumption does not account for an empirical regularity according to which firms are more productive in larger cities. The elasticity is quite small, especially when controlling for characteristics such as education, but differences in population sizes can be substantial. In the US however, Beaudry et al. (2014) find no significant evidence of agglomeration effects on productivity (over 10-year periods). So, we think that our assumption is not too strong a simplification, at least in the US context.} If the vacancy is filled, a firm produces $y_i > b_i$ units of the consumption good. So, depending on the origin of the worker, a firm makes a profit $J_{ij} = y_i - w_{ij}$ on a filled position.

### 2.1 The timing of decisions

At the beginning of the unique period, everybody is unemployed, chooses in which region to reside, and decides to search for a job either regionally or nationally (i.e. either one only searches for a job in the region where one lives or one searches in both regions at the same time). The reason why some workers would only search in their region of residence rather than nationally is intuitive. If a worker has a sufficiently strong relative idiosyncratic preference for her region of residence, she will not accept to migrate to take a position. Since, following Decreuse (2008), we assume a small cost of refusing a job offer, this individual will then not take part in the matching process in the other region.

In a second step, firms open vacancies and possibly meet a worker. This process occurs simultaneously in both regions. If a vacancy meets a job-seeker, this worker then accepts or not the job offer. When a match is formed with a job-seeker who does not live in the firm’s region, this worker relocates. Allowing unemployed workers to relocate at this stage would complicate the exposition without yielding more insights. After the relocation step, employed workers and firms bargain over wages. Fourth, production takes place and good markets clear.

The moment at which wages are negotiated matters when a relocation of the worker is involved. If this moment occurs before the decision to migrate is taken, through Nash bargaining, the worker will get a partial compensation for the difference in the
regional non-wage components of utility $a_i + c_{ij}$. To implement this timing, one has to assume that the employer is aware of the idiosyncratic preferences of the worker for both regions. One can doubt that this information is available.\footnote{Notice that if the framework was dynamic this timing would raise another issue. Under the standard assumption of automatic renegotiation (Pissarides 2000, p. 15), the wage would be revised after the relocation step and would be chosen exactly as proposed in the timing of events we privilege.} A survey conducted by CareerBuilder.com at the end 2011 indicates that less than a third of employers are ready to pay for relocation costs of their new employees.\footnote{See http://www.careerbuilder.com/share/aboutus/pressreleasesdetail.aspx?id=pr677&sd=1/18/2012&ed=1/18/2099&siteid=cbpr&sc_cmp1=cb_pr677.} This is casual evidence in favor of the timing indicated above: The bargained wage will then not compensate the worker for the difference in $a_i + c_{ij}$. We will return to the timing of the wage bargain in Section 3.

Some additional notations have to be introduced. Before the matching process, $N_i$ agents choose to reside in region $i$ ($N_i$ is called the \textit{ex-ante} population in region $i$, with $N_1 + N_2 = N$). Population in region $i$ is composed of $N_i^N$ agents who search nationally and $N_i^R$ individuals who only search in their region of residence ($N_i = N_i^N + N_i^R$). For agents located in region $i$, the notation $-i$ will designate the other region.

### 2.2 The matching process

We allow for distant search, meaning that search in a region can be conducted while living in the other one. The search-matching effectiveness of those living in the region where vacancies are open is normalized to one. For residents of the other region, this effectiveness takes an exogenous value $\alpha$ with $0 \leq \alpha \leq 1$. The number of hirings in each region is given by a regional-specific matching function $M_i(\cdot, \cdot)$ with:

$$M_i(V_i, N_i + \alpha N_{-i}) < \min\{V_i, N_i + \alpha N_{-i}\}, \quad i \in \{1, 2\},$$

where $V_i$ represents the endogenous number of vacancies opened in region $i$ and $N_i + \alpha N_{-i}$ is the endogenous number of job-seekers measured in efficiency units. As Molho (2001) and Manning and Petrongolo (2011) do in a partial equilibrium framework, we endogenize search effort by letting job-seekers choose their search field. Following Pissarides (2000) and a large empirical literature, the matching function has constant returns to scale\footnote{Manning and Petrongolo (2011) provide recent evidence at the local level for the UK.} and is increasing and concave in both arguments. Defining tightness in region $i$ as

$$\theta_i = \frac{V_i}{N_i + \alpha N_{-i}},$$

$m_i(\theta_i)$ designates the probability $M_i/V_i$ that a vacancy in region $i$ meets a worker, with $0 < m_i(\theta_i) < 1$ by the inequality in (3) and $m_i'(\theta_i) < 0$ because of search-matching congestion externalities. So, unfilled jobs find a partner more easily in a region able to attract more job-seekers. The probability that an unemployed worker living in $i$ meets a firm located in region $i$ is $p_i(\theta_i) = \theta_i m_i(\theta_i)$, with $0 < p_i(\theta_i) < 1$. Job-seekers find a
job more easily in a thicker local labor market: $[p_i(\theta_i)]' > 0$. The probability that an unemployed worker searching nationally and living in $-i$ meets a firm settled in region $i$ is $\alpha p_i(\theta_i)$. In case of national search, for someone living in $i$, the probability of getting an offer in $i$ and no offer from the other region is $p_i(\theta_i)(1-\alpha p_{-i}(\theta_{-i}))$. The probability of the opposite event is $\alpha p_{-i}(\theta_{-i})(1-p_i(\theta_i))$. The probability of getting an offer from each region is $\alpha p_i(\theta_i)p_{-i}(\theta_{-i})$. In this case, the worker accepts the position that offers the highest indirect utility level. Finally, this worker living in $i$ faces a probability $(1-p_i(\theta_i))(1-\alpha p_{-i}(\theta_{-i}))$ of remaining unemployed.

2.3 A model with regional search only

Before discussing the general case of Subsection 2.2, let us briefly consider the standard assumption, $\alpha = 0$, according to which an unemployed can only search in the region where she lives.

Individual Nash bargaining takes place ex-post, once the cost of opening a vacancy is sunk. So, when a vacancy and a job-seeker have met, the wage solves the following maximization:

$$\max_{w_{ij}} (V_i^{u_j} - V_i^{u_i})^{\beta_i} (J_{ij} - V_i)^{1-\beta_i}$$

where $V_i$ is the value of an unfilled vacancy and $\beta_i \in [0,1)$ denotes the bargaining power of a worker in region $i$. The first-order condition can be rewritten as:

$$w_{ij} = \beta_i y_i + (1-\beta_i)b_i - \beta_i V_i.$$  \hspace{1cm} (5)

Hence, the wage is independent of the location of the unemployed and can therefore be denoted by $w_i < y_i$. As $w_i > b_i$, under free-entry, workers always take the position.

The expected value of a vacant position $V_i$ is equal to $-\kappa_i + m_i(\theta_i)(y_i - w_i)$. Firms open vacancies freely until this value $V_i$ is nil in both regions. Anticipating correctly the outcome of the wage bargain, the free-entry condition becomes:

$$\frac{\kappa_i}{(1-\beta_i)m_i(\theta_i)} = y_i - b_i, \quad \forall i \in \{1, 2\}.$$ \hspace{1cm} (6)

The probability of filling a vacancy $m_i(\theta_i)$ increases with the (ex-post) surplus of a match $y_i - b_i$ and decreases with the cost of opening a vacancy $\kappa_i$ and workers’ bargaining power $\beta_i$.

Agents decide in which region to locate in order to maximize their expected utility. They thus compare the expected utility of living in region 1, $p_1(\theta_1)V_1^{e_1} + (1-p_1(\theta_1))V_1^{e_n}$, with the expected utility of living in region 2, $p_2(\theta_2)V_2^{e_2} + (1-p_2(\theta_2))V_2^{u_2}$. The worker whose relative preference for region 1 over region 2 is above $b_2 - b_1 + a_2 - a_1 + p_2(\theta_2)(w_2 - b_2) - p_1(\theta_1)(w_1 - b_1)$ chooses to live in region 1, while if their relative preference is below this threshold, workers reside in region 2. Let us define

$$\Delta = b_2 - b_1 + a_2 - a_1$$ \hspace{1cm} (7)

\textsuperscript{11}As is standard, we assume Inada conditions: $\lim_{\theta \to 0} m(\theta) = 1; \lim_{\theta \to a_0} p(\theta) = 0; \lim_{\theta \to a_0} m(\theta) = 0; \lim_{\theta \to +\infty} p(\theta) = 1.
We get the following lemma:

**Lemma 1.** Given that agents perfectly anticipate the wage \( w_i \) defined in (5), there is a threshold

\[
x = \Delta + p_2(\theta_2)\beta_2(y_2 - b_2) - p_1(\theta_1)\beta_1(y_1 - b_1),
\]

assumed to be in \((-v, v)\), such that a job-seeker decides to live in region 2 if \( c_{1j} - c_{2j} < x \). Otherwise, she resides in region 1.

When unemployed workers need to move before starting to search for a job, a higher (respectively, lower) time value of being unemployed in region 2 (resp., 1) or higher relative levels of amenities \( a_2 - a_1 \) induce a higher threshold \( x \), meaning that more workers locate in 2. A higher probability of getting a job in region \( i \), i.e. a higher \( \theta_i \), induces more people to locate in this region.

Let \( u_i \) designate the unemployment rate in region \( i \) at the end of the matching process.

**Definition 1.** When \( \alpha = 0 \), an interior equilibrium is a vector \( \{w_i, \theta_i, u_i, N_i\}_{i \in \{1, 2\}} \) and a scalar \( x \). \( w_i \) is given by (5), in which under free-entry \( V_i = 0 \), \( \theta_i \) is fixed by (6), \( u_i = 1 - p_i(\theta_i) \), \( N_1 = \frac{v - x}{2v} N \), \( N_2 = \frac{v + x}{2v} N \), and \( x \) solves equation (8) and is assumed to be in \((-v, v)\).

The equilibrium is determined recursively and is unique. Once tightness is fixed in each region by the free-entry condition, the equilibrium value of \( x \) is known and population sizes are determined as well. The equilibrium unemployment rate in a region is only affected by the determinants of tightness in this region. By looking at (6), these determinants are regional-specific. So, a change in say the marginal product of labor in a region has no spillover effect on the equilibrium unemployment rate in the other region.

**Condition 1.** A necessary and sufficient condition for an interior equilibrium is \( v > |\Delta + \beta_2 p_2(\theta_2)(y_2 - b_2) - \beta_1 p_1(\theta_1)(y_1 - b_1)| \) where, by (6), \( \theta_i = m^{-1} (\kappa_i / [(1 - \beta_i)(y_i - b_i)]) \).

### 2.4 Regional and national search

To capture some of the possibilities created by the Internet, this section lets workers search simultaneously in both regions \( 0 < \alpha \leq 1 \) if they prefer to do so. Then, if they meet a vacancy and accept a job offer in the other region, they migrate at no cost. At the time of individual bargaining in any region \( i \), a worker migrating from \(-i\) has already moved in region \( i \) and thus has the same fall-back position as a worker settled in region \( i \) from the start. The generalized Nash bargaining process is therefore (4). The wage is still given by (5) and denoted \( w_i \).

#### 2.4.1 Acceptance of a job offer

A worker searching locally always accepts a job offer, as \( V^e_{ij} > V^u_{ij} \) in a free-entry equilibrium with Nash bargaining. Similarly, a worker searching nationally who only gets
a job offer from a firm located in the region where she lives always takes the position. In case this worker only receives a job offer from a firm settled in the other region, she always accepts the job, as she decided to search for a job there (as shown in Appendix A). However, if a worker searching nationally gets two offers, one from each region, she rejects one of them (incurring an arbitrary small cost $\varepsilon$) and accepts the other one. To take this decision, the unemployed worker compares $V_{1j}^e$ with $V_{2j}^e$. The agent whose relative preference $c_{1j} - c_{2j}$ is above the threshold $w_2 - w_1 + a_2 - a_1$ chooses to work in region 1 rather than in region 2. So,

**Lemma 2.** When a job-seeker searching nationally has one job offer from each region, there is a threshold

$$\hat{x} = \Delta + \beta_2(y_2 - b_2) - \beta_1(y_1 - b_1),$$

assumed to be in $(-v, v)$, such that she accepts the job offer in region 2 if $c_{1j} - c_{2j} < \hat{x}$. Otherwise, she accepts the job offer in region 1.

### 2.4.2 Vacancy creation

Firms open vacancies in region $i$ until the expected gain $V_i$ is nil ($i \in \{1, 2\}$). This condition is now $m_i(\theta_i)p_i(y_i - w_i) = \kappa_i$, where $\pi_i$ is new and designates the conditional probability that the meeting leads to a filled vacancy (see Section 2.4.4 for more details). Combining (5) and the free-entry condition yields

$$\frac{\kappa_i}{(1 - \beta_i)m_i(\theta_i)} = \pi_i(y_i - b_i), \quad \forall i \in \{1, 2\}. \tag{10}$$

The rate at which vacancies are on average filled, $\pi_i m_i(\theta_i)$, varies with the parameters exactly as when $\alpha = 0$.

### 2.4.3 Search decision and location choice

Appendix A shows that taking search and location decisions simultaneously or choosing first the location and then the searching area is equivalent. Therefore, to ease the exposition, the presentation below opts for the second timing.

**Search decision**

Let $p_i$ be a short notation for $p_i(\theta_i)$. An individual $j$ living in region 2 decides to search regionally or nationally by comparing the expected utility in both cases. The expected utility if the agent located in 2 searches nationally is

$$p_2(1 - \alpha p_1)V_{2j}^e + \alpha p_1(1 - p_2)V_{1j}^e + \alpha p_1 p_2(\max\{V_{1j}^u; V_{2j}^u\}) + (1 - p_2)(1 - \alpha p_1)V_{2j}^u. \tag{11}$$

The expected utility of a job-seeker living in region 2 and searching for a job in this region only is $p_2V_{2j}^e + (1 - p_2)V_{2j}^u$.

When the small cost of refusing a job offer $\varepsilon$ tends to zero, someone searches nationally if her relative preference for region 1 over region 2, $c_{1j} - c_{2j}$, is above a threshold
\[ z_1 = b_2 - w_1 + a_2 - a_1. \] Otherwise, she decides to look for a job in region 2 only. This is shown in Appendix A (the comparison of cases e and f).

A similar development is conducted for a worker settled in region 1. A job-seeker living in region 1 whose relative preference for region 1 over region 2 is higher than a threshold \( z_2 = w_2 - b_1 + a_2 - a_1 \) searches in region 1 only. Below this threshold, the worker looks for a job in the whole country (see the comparison of cases a and c in Appendix A). Under perfect anticipation of bargained wages,

**Lemma 3.** When \( \alpha > 0 \) and the cost of refusing an offer \( \varepsilon \to 0 \), let

\[ z_1 = \beta_1(b_1 - y_1) + \Delta \quad \text{and} \quad z_2 = \beta_2(y_2 - b_2) + \Delta \]

with \( z_1 < z_2 \). Assuming that both \( z_i \)’s lie in \((-v, v)\),

- If \( c_{1j} - c_{2j} < z_1 \), agent \( j \) searches in region 2 only;
- If \( z_1 \leq c_{1j} - c_{2j} \leq z_2 \), agent \( j \) searches nationally;
- If \( c_{1j} - c_{2j} > z_2 \), agent \( j \) searches in region 1 only.

The shares of these three groups in the total population are respectively \( \frac{v + z_1}{2v} \), \( \frac{z_2 - z_1}{2v} \) and \( \frac{v - z_2}{2v} \). Remembering (9), it is easily seen that \( z_1 \leq \hat{x} \leq z_2 \).

By comparing their expected utility in case of regional and national search, unemployed workers turn out to compare the utility levels when they are actually employed in the other region and when they remain unemployed in their region of residence. These utility levels are not in expected terms and so search decisions are independent of probabilities to get a job offer.\(^{12}\) Therefore, the number of workers who search nationally is independent of search effectiveness \( \alpha > 0 \).\(^{13}\) A rise in \( \Delta \) shifts \( z_1 \) and \( z_2 \) upwards, while keeping \( z_2 - z_1 \) unchanged. Hence, more unemployed workers search in region 2 only and less do so in 1 only, while the share of the population searching nationally remains constant. If any gap \( y_i - b_i \) rises, the corresponding wage \( w_i \) increases and so less residents of region \( -i \) search only locally.

**Location choice**

As an unemployed worker who decides to look for a job regionally only locates in her region of search, we have to compare the expected utility of an agent \( j \) who searches nationally while being located in region 1 or in region 2. These expected utility levels are respectively

\[ p_1(1 - \alpha p_2)V_{1j}^e + \alpha p_2(1 - p_1)V_{2j}^e + \alpha p_1 p_2 (\max \{V_{1j}^e; V_{2j}^e\} - \varepsilon) + (1 - p_1)(1 - \alpha p_2)V_{1j}^n \]

and (11), as shown in Appendix A.

\(^{12}\)If \( \varepsilon \) was non negligible, these probabilities would however play a role in the definition of the \( z_i \)’s.

\(^{13}\)This would still be true if \( \varepsilon \) was non negligible. When \( \alpha \) is nil, searching all over the country cannot be ruled out but the probability of finding a job in the other region is zero. So, every job-seeker searches locally.
Lemma 4. If relative idiosyncratic preferences $c_{1j} - c_{2j} < x$, agent $j$ chooses to reside in region 2, where

\[ x = \Delta + \frac{1 - \alpha}{1 - \alpha p_1 - \alpha p_2 + \alpha p_1 p_2} \left( p_2 \beta_2(y_2 - b_2) - p_1 \beta_1(y_1 - b_1) \right), \tag{14} \]

with $0 < \frac{1 - \alpha}{1 - \alpha p_1 - \alpha p_2 + \alpha p_1 p_2} < 1$ and, by (12), $z_1 \leq x \leq z_2$. Otherwise agent $j$ settles in region 1. The share of the population living ex-ante in region 2 (respectively, 1) is then $\frac{v + x}{2v}$ (respectively $\frac{v - x}{2v}$).

Compared to (8), which is the limit case of (14) when $\alpha$ tends to 0, increasing the differential in expected rents $p_2 \beta_2(y_2 - b_2) - p_1 \beta_1(y_1 - b_1)$ has now a less positive effect on the number of people choosing to locate in region 2 since there is the opportunity of searching nationwide wherever one lives. At the limit, if search is equally efficient wherever one looks for a job ($\alpha = 1$), this differential does not affect the location choice any more. When $\alpha > 0$, an increase in relative amenities in region 2, $a_2 - a_1$, as well as a rise (respectively, a drop) in the value of home production in region 2 (resp., 1) still induce more workers to locate in 2 ex-ante. A rise in $\alpha$ attracts more workers in region 1 if $p_2 \beta_2(y_2 - b_2) - p_1 \beta_1(y_1 - b_1) > 0$ and conversely. An increase in tightness in region 1 has several effects if $0 < \alpha < 1$. First, if one lives in region 1, the increase in the probability of being employed in this region equals the decrease in the probability of being unemployed. As the individual stays in the same region, the net gain is proportional to $w_1 - b_1$. Second, if one lives in region 2, the increase in the probability of being employed in region 1 equals the decrease in the probability of staying unemployed in region 2. This effect is proportional to $V_{1j} - V_{2j}$. Third, the decline in the probability of being employed in 2 is the same wherever one lives. So, this effect cancels out. The first and the second effects push the difference in idiosyncratic preference of the indifferent agent, $x$, in opposite directions so that the net effect is ambiguous. This conclusion also holds if $\theta_2$ increases. So, a first major difference with the case where $\alpha = 0$ is that a rise in the number of vacancies in a region has no clear-cut impact on the location choice any more. However, the first effect described just above dominates if $\alpha$ is sufficiently small.

Proposition 1. When $0 < \alpha < 1$, a tighter labor market in region $i$ induces more people to reside there under the following sufficient condition:

\[ \alpha < 1 - \frac{(\beta_1(y_1 - b_1) - \beta_2(y_2 - b_2))^2}{(\beta_1(y_1 - b_1) + \beta_2(y_2 - b_2))^2}. \tag{15} \]

When $\alpha = 1$, the levels of tightness do not influence the choice of residence any more.

The proof is provided in Appendix B. Recalling (14), Condition (15) is easy to interpret: The higher the inter-regional difference in workers’ shares in the surpluses,$^{16}$...
the lower $\alpha$ should be in order to get the intuitive relations between the levels of tightness and $x$.

### 2.4.4 Summary of the acceptance, search and location decisions

Combining Lemmas 3 and 4, the total labor force is made of four groups with distinct behaviors:

**Proposition 2. Partition of the population**

- If $c_{1j} - c_{2j} < z_1$, agent $j$ locates in region 2 and searches there only;
- If $z_1 \leq c_{1j} - c_{2j} < x$, agent $j$ settles in region 2 and searches in the whole country;
- If $x \leq c_{1j} - c_{2j} \leq z_2$, agent $j$ locates in region 1 and looks for a job nationally;
- If $c_{1j} - c_{2j} > z_2$, agent $j$ settles in region 1 and looks for a job in region 1 only.

Figure 1 illustrates this partition of the total population if $-v < z_1, z_2 < v$.

![Figure 1: The partition of the population in case of an interior solution.](image)

In general, one cannot rank threshold values $x$ and $\hat{x}$ since $x$ varies with the levels of tightness. When regions are symmetric however, these two thresholds are equal to zero. When $\alpha = 1$ and regions are asymmetric, the comparison of the thresholds is obvious since $x = \Delta$ then: $\hat{x} \leq x \iff \beta_2(y_2 - b_2) - \beta_1(y_1 - b_1) \leq 0$.

### 2.4.5 Acceptance probability and vacancy creation

A detailed explanation is provided in Appendix C. Consider a vacant position in region 1. The mass of job-seekers searching for a job in 1 is $[v - x + \alpha(x - z_1)][N/2v]$ in efficiency units. Conditional on meeting one of these unemployed workers, all those whose relative
preference $c_{1j} - c_{2j}$ lies above $\hat{x}$ accept for sure an offer from region 1. For those between $z_1$ and $\hat{x}$, this is only the case if they get no offer from region 2. So, conditional on a contact between a vacancy in region 1 and a job-seeker, the acceptance probability $\pi_1$ is (with a corresponding expression for $\pi_2$):

\[
\pi_1 = 1 - \frac{\alpha p_2(\hat{x} - z_1)}{v - x + \alpha(x - z_1)} \\
\pi_2 = 1 - \frac{\alpha p_1(z_2 - \hat{x})}{v + x + \alpha(z_2 - x)}
\]

It is easily checked that the higher $\hat{x}$, the more workers accept job offers in region 2 and so the lower the acceptance rate in 1. The same is true for $x$. The higher the number of workers searching in region 2 only (an increasing function of $z_1$), the higher the acceptance rate in 1. Finally, an increase in the probability of getting a job offer in region 2 decreases the acceptance rate in region 1. Similarly, $\pi_2$ increases with $\hat{x}$ and $x$ and decreases with $z_2$ and $p_1$. The impact of search-matching effectiveness $\alpha$ should be stressed: A higher $\alpha$ leads to a lower conditional acceptance probability, as the probability of getting two job offers increases. Conversely, $\lim_{\alpha \to 0} \pi_i = 1$.

Combining (10) with (16)-(17) leads to the following free-entry conditions:

\[
\kappa_1 = \frac{1}{(1 - \beta_1)m_1(\theta_1)} = \frac{v - x + \alpha(x - z_1) - \alpha p_2(\theta_2)(\hat{x} - z_1)}{v - x + \alpha(x - z_1)}(y_1 - b_1) \\
\kappa_2 = \frac{1}{(1 - \beta_2)m_2(\theta_2)} = \frac{v + x + \alpha(z_2 - x) - \alpha p_1(\theta_1)(z_2 - \hat{x})}{v + x + \alpha(z_2 - x)}(y_2 - b_2)
\]

Through the endogenous acceptance rate, vacancy creation in any region is now affected by the level of tightness and the value of parameters in the other region. This is a second major difference with the case where people cannot search nationally ($\alpha = 0$).

### 2.4.6 Populations and unemployment rates

Since $z_1 \leq x \leq z_2$, and assuming an interior solution, the sizes $N_i^N$ of the workforce searching nationally and $N_i^R$ of those searching in their region of residence only are given by:

\[
N_1^R = \frac{(v - z_2)}{2v}N \\
N_2^R = \frac{(z_1 + v)}{2v}N \\
N_1^N = \frac{(z_2 - x)}{2v}N \\
N_2^N = \frac{(x - z_1)}{2v}N
\]

so that the total ex-ante population sizes are $N_1 = N_1^N + N_1^R = \frac{v - x}{2v}N$ and $N_2 = N_2^N + N_2^R = \frac{z_2 - x}{2v}N$, with $N_1 + N_2 = N$.

The unemployment rates which are the ratio of the number of (ex-post) unemployed
workers over the (ex-post) population, are:
\[
\begin{align*}
  u_1 &= \frac{(1 - p_1)(v - x - \alpha p_2(z_2 - x))}{v - x - \alpha p_2(1 - p_1)(z_2 - x) + \alpha p_1(1 - p_2)(x - z_1) + \alpha p_1 p_2(x - \hat{x})} \\
  u_2 &= \frac{(1 - p_2)(v + x - \alpha p_1(x - z_1))}{v + x + \alpha p_2(1 - p_1)(z_2 - x) - \alpha p_1(1 - p_2)(x - z_1) - \alpha p_1 p_2(x - \hat{x})}
\end{align*}
\]

The number of (ex-post) unemployed workers in, say, region 1 is composed of the agents living ex-ante in 1 who did not get a job offer in region 1, \((1 - p_1)^{x - \hat{x}} N\), to which we subtract the workers who did not get an offer from region 1 but well from region 2 \((\alpha p_2(1 - p_1)^{x - \hat{x}} N)\). The denominator corresponds to the population living ex-post in the region (up to a \(N/2v\) term). Ex-post, the number of inhabitants in, say, region 1 is the sum of 4 terms. The first term represents the population living ex-ante in region 1. The second term corresponds to the workers who were living ex-ante in 1 and who leave region 1 as they only get a position in region 2. The third term is composed of the agents who lived ex-ante in region 2 and who move as they only get an offer from region 1. Finally, the fourth term represents the number of workers who get two offers. This term is positive whenever \(x > \hat{x}\), meaning that some more workers living in 2 ex-ante accept a position in region 1.

The following lemma provides the signs of the partial derivatives of (22) and (23) with respect to the other endogenous variables and \(\alpha\).

**Lemma 5.** When \(\alpha > 0\), the unemployment rate \(u_i\) decreases with the level of tightness \(\theta_i\) in the same region (as was the case when \(\alpha = 0\)). The following properties only hold when \(\alpha > 0\): (i) The sign of the cross-derivatives \(\partial u_i/\partial \theta_{-i}\) varies with the sign of \(p_2 \beta_2(y_2 - b_2) - p_1 \beta_1(y_1 - b_1)\) (more on this below). (ii) A rise in the threshold \(x\) (i.e. a bigger workforce \(N_2\) living ex-ante in region 2) decreases the unemployment rate in region 1 while it increases it in region 2. (iii) An increase in the number of regional job-seekers increases the unemployment rate in both regions. (iv) The unemployment rate in region 1 increases with \(\hat{x}\), while the opposite holds for the unemployment rate in region 2. (v) A rise in search effectiveness \(\alpha\) lowers regional unemployment rates.

An increase in region \(i\)'s labor market tightness \(\theta_i\) boosts the probability that a worker living in region \(i\) finds a job there and it rises the probability that a worker located in the other region gets a position in region \(i\) (which increases the labor force living in region \(i\)). Consequently, the unemployment rate in region \(i\) goes down. Turning to property (i) of Lemma 5, augmenting \(\theta_{-i}\) rises the probability of leaving region \(i\) and this reduces both the number of unemployed workers and the size of the labor force in region \(i\). The net effect on the unemployment rate depends on the inter-regional difference in the gains \(p_i \beta_i(y_i - b_i)\). More precisely, Appendix D shows that

\[
\begin{align*}
  p_2 \beta_2(y_2 - b_2) < p_1 \beta_1(y_1 - b_1) &\Rightarrow \frac{\partial u_1}{\partial \theta_2} < 0, \quad \lim_{\alpha \to 0^+} \frac{\partial u_2}{\partial \theta_1} > 0 \quad \text{and} \quad \lim_{\alpha \to 1} \frac{\partial u_2}{\partial \theta_1} < 0, \\
  p_2 \beta_2(y_2 - b_2) > p_1 \beta_1(y_1 - b_1) &\Rightarrow \frac{\partial u_2}{\partial \theta_1} < 0, \quad \lim_{\alpha \to 0^+} \frac{\partial u_1}{\partial \theta_2} > 0 \quad \text{and} \quad \lim_{\alpha \to 1} \frac{\partial u_1}{\partial \theta_2} < 0.
\end{align*}
\]

\(^{17}\text{The proof, which is available upon request, consists in reorganizing these partial derivatives.}\)
Consequently, the cross-partial derivatives are both negative when $\alpha$ is big enough. When $\alpha = 0$, the unemployment rate in a region does not depend on tightness in the other. However, for sufficiently small positive values of $\alpha$, one cross-derivative $\partial u_i/\partial \theta_{-i}$ is positive.

As $x$ goes up (property (ii)), the number $N_1$ of agents living *ex-ante* in region 1 shrinks while $N_2$ increases. These population sizes are (up to a $N/2v$ term) present in the numerators and the denominators of (22) and (23). In addition, a rise in $x$ affects the numbers of national job-seekers who depletes the regional workforces if they are recruited in the other region. All in all, a rise in $x$ reduces (resp., increases) the unemployment rate in region 1 (resp., 2). A corollary of property (iii) is that more workers searching all over the country reduces the unemployment rates in both regions. These properties as well as the favorable role of $\alpha$ on the unemployment rates ($v$) are conditional on the other endogenous variables.

In a standard Mortensen-Pissarides setting (where geographical heterogeneities are concealed in an aggregate matching function), the size of the labor force does not affect the equilibrium unemployment rate (as eventually the number of vacancies rises proportionately, leaving the equilibrium level of tightness unaffected). This equilibrium property is not different here ($N$ plays no role in (22)-(23)). However,

**Proposition 3.** If $\alpha > 0$, the equilibrium unemployment rates are affected by the partition of the population between the two regions and between the two statuses of national versus regional job-seekers.

### 2.4.7 Equilibrium

**Definition 2.** When $0 < \alpha \leq 1$, an interior equilibrium is a vector $\{x, \hat{x}, z_1, z_2\}$ assumed to be in $(-v,v)$ and a vector $\{w_i, \theta_i, u_i, N_i^N, N_i^R\}_{i \in \{1,2\}}$, solving (5) under free-entry $V_i = 0$, (9), (10), (12), (14), (16), (17), (20), (21), (22) and (23).

As we already know that $z_1 \leq \hat{x}, x \leq z_2$, an equilibrium is interior if $-v < z_1$ and $z_2 < v$. From (7) and Lemma 3, these conditions become:

**Condition 2.** Necessary and sufficient conditions for an interior solution are

$$v > \beta_1(y_1 - b_1) - (b_2 - b_1 + a_2 - a_1) \quad \text{and} \quad v > \beta_2(y_2 - b_2) + (b_2 - b_1 + a_2 - a_1)$$

When $\alpha \neq 0$, the system of equations characterizing an equilibrium is block recursive. The thresholds $z_i$’s and $\hat{x}$ being explicit functions of the parameters only, the central endogenous variables are $\{\theta_1, \theta_2, x\}$. They are determined by the system (14)-(18)-(19) of three non-linear equations. Once this system is solved, all other endogenous variables get unique values. Under Condition (15), Definition (14) is an implicit relationship which is decreasing in $\theta_1$ and increasing in $\theta_2$ (unless $\alpha = 1$ in which case $x$ is constant).
Substituting this relationship into the free-entry conditions (18)-(19) yields:

\[
\frac{\kappa_1}{(1 - \beta_1)m_1(\theta_1)} - \pi_1(\theta_1, \theta_2)(y_1 - b_1) = 0 \quad (27)
\]

\[
\frac{\kappa_2}{(1 - \beta_2)m_2(\theta_2)} - \pi_2(\theta_1, \theta_2)(y_2 - b_2) = 0, \quad (28)
\]

where the inequality signs under the \(\theta_i\)'s designate those of the partial derivatives. The free-entry condition in region 1, (27), can be seen as an implicit reaction function \(\theta_1 = \Theta_1(\theta_2)\). Similarly, (28) defines an implicit relationship \(\theta_2 = \Theta_2(\theta_1)\). Figure 2 draws these functions for the limit cases, namely \(\alpha = 0\) (the two relationships being then orthogonal) and \(\alpha = 1\) (the dotted curves). By looking at (16) and (17), it is easily seen that the values of \(\Theta_i(0)\) are independent of the magnitude of \(\alpha\).

The net effect of a change in a regional gap \(y_j - b_j\) on the equilibrium levels of tightness is hard to sign because it affects almost all thresholds present in the acceptance rates \(\pi_i, i \in \{1, 2\}\). A rise in \(\alpha\) has a clear effect on one of the acceptance probabilities but an ambiguous one on the other. So, the impact on both equilibrium levels of tightness is hard to sign except in the symmetric case discussed in the next subsection. The cost of opening a vacancy is a determinant of unemployment differentials emphasized in the urban search-matching literature (see e.g. Coulson et al., 2001). A rise in the cost of opening a vacancy in, say, region 1 only shifts the \(\Theta_1(\theta_2)\) function to the left, leading to a lower equilibrium value of \(\theta_1\) and, more interestingly, to a rise in \(\theta_2\) (see the red curve

---

If \(\alpha = 1\), \(\pi_i\) is a function of tightness in the other region only. \(\theta_1 = \Theta_1(\theta_2)\) and \(\theta_2 = \Theta_2(\theta_1)\) are negatively sloped. It can easily be checked that \(+\infty > \lim_{\theta_2 \to 0} \Theta_1(\theta_2) > \lim_{\theta_2 \to +\infty} \Theta_1(\theta_2) > 0\) and \(+\infty > \lim_{\theta_1 \to 0} \Theta_2(\theta_1) > \lim_{\theta_1 \to +\infty} \Theta_2(\theta_1) > 0\). Furthermore, the \(\Theta_i(\theta_{-i})\) functions are convex if the matching function is a Cobb-Douglas. With more general matching functions, the same conclusions hold if the matching rates \(m_i(\theta_i)\) are sufficiently convex.
and compare equilibrium $E$ and $E'$ in Figure 2). According to Lemma 5, the direct implications are a rise (resp., a decline) in the unemployment rate in region 1 (resp., 2). There are however also induced effects in various directions. Given Proposition 1, more people decide to reside in region 2 ($x$ rises). Then, by Result (ii) of Lemma 5, the unemployment rate in region 1 (resp., 2) shrinks (resp., increases). In addition the cross-effects $\partial u_i/\partial \theta_{-i}$ can reinforce or not the direct effects (see (24) and (25)). The net effect on the unemployment rates is therefore ambiguous. In a symmetric case however, to which we turn now, the net effect is negative.

### 2.4.8 The symmetric equilibrium

When regions are symmetric, as explained in Appendix E.1, the unique symmetric equilibrium tightness and unemployment rate are characterized by the following system:

$$
\pi(\theta, \alpha) m(\theta) = \frac{\kappa}{(1-\beta)(y-b)}, \quad \text{where} \quad \pi(\theta, \alpha) = \frac{v + \alpha(1-p(\theta))\beta(y-b)}{v + \alpha\beta(y-b)} \tag{29}
$$

$$
u(\theta, \alpha) = \frac{(1-p(\theta))(v-\alpha p(\theta)z_2)}{v} \tag{30}
$$

together with $x = \hat{x} = 0$, $z_2 = \beta(y-b) = -z_1 > 0$. In the upper part of Figure 3 we draw the left-hand side of (29) when $\alpha = 0$ (in black) and when $0 < \alpha \leq 1$ (in red). A rise in $\alpha$ induces a leftward shift of the curve. The equilibrium level of tightness therefore declines because the acceptance rate $\pi$ shrinks with $\alpha$. The lower part of the figure draws (30) and illustrates the end of Lemma 5, namely the favorable partial effect of a rise in $\alpha$ on the unemployment rate conditional on tightness. Depending on the importance of the shifts of the two curves the equilibrium unemployment rate can vary in both directions.

If $\alpha = 0$ (resp., $\alpha > 0$) can be interpreted as a world without (resp., with) the modern communication technologies using the Internet, Figure 3 illustrates that the introduction of these technologies can have ambiguous effects on the regional equilibrium unemployment rates. This is a way of rationalizing the results of Kroft and Pope (2014).

### 2.4.9 Extensions

Our main mechanisms should be robust to the introduction of more than two regions. With three regions for instance, the matching functions takes the form: $M_i(V_i, N_i + \alpha N_{-i}^N)$ where now $N_{-i}^N$ stands for the population of the other two regions. Everything else equal, the importance of national job-seekers in the unemployed population (in efficiency units) is growing, which lowers the probability of accepting a job offer.

Lutgen and Van der Linden (2013) extend the theoretical framework by including regional housing markets. As in Moretti (2011), housing demand is totally inelastic (one unit of dwelling per resident) and housing supply is increasing in the population size. The introduction of an endogenous housing market complicates the model a lot. Many more effects become ambiguous since they hinge upon the slope of the housing supply. For example, more valuable amenities in a region has now an ambiguous impact on the
Figure 3: The symmetric equilibrium: $\alpha = 0$ ($\pi = 1$; the black curves) versus $\alpha > 0$ ($\pi < 1$; the red curves).

search and location decisions. At given rents, the size of the population increases in the better endowed region. When rents are endogenous, their rise in this region is a disincentive to search and locate there. If the housing supply is very inelastic, one could get that the population shrinks in the better endowed region.

Consider next a dynamic extension with infinitely-lived agents. As in the static framework, for national job-seekers, the partial effect of a rise in $\alpha$ is an increase in the probability of finding a job. In a continuous time setting, the probability of receiving two offers during a very small interval of time tends to zero. A rise in $\alpha$ now negatively affects equilibrium tightness through wages. When $\alpha$ rises, national job-seekers have an improved outside option and hence their bargained wages rise. Then, through the free entry of vacancies, labor demand shrinks. So, although the mechanism at work differs, the ambiguity remains.

3 Efficiency

This section studies the efficiency of the *laissez-faire*\(^{19}\) decentralized equilibria introduced in the previous section. We first derive the optimal allocation when workers can only search in the region where they live ($\alpha = 0$) and compare it with the decentralized equilibrium derived in Section 2.3. It will turn out that the decentralized equilibrium

\(^{19}\)This expression is added since there is no public intervention in Section 2.
is efficient when the Hosios condition is satisfied. In a second stage, we derive the optimal allocation when $\alpha > 0$ and analyze the differences between this allocation and the decentralized equilibrium characterized in Section 2.4. The Hosios condition is then not sufficient to guarantee efficiency of the decentralized equilibrium.

### 3.1 The case where $\alpha = 0$

The constrained central planner’s chooses the levels of tightness and the threshold $x$ to maximize net output subject to the same matching frictions as decentralized agents. Net output is the sum of the indirect utility levels of the $N_i$ agents living ex-post in each region $i$ and of firms’ profits. To maximize net output, the central planner decides how many vacancies to open and how many job-seekers will take part in the regional matching processes. For a given number of participants in the matching functions, it is obvious that the planner will allocate those with the highest relative preference for region 1 over region 2, $c_{1j} - c_{2j}$, in region 1 and conversely. Put differently, the planner also chooses a threshold $x$. Therefore, we define the planner’s problem as:

$$
\max_{\theta_1, \theta_2, x} y_1 L_1 + y_2 L_2 + b_1 (N_1^P - L_1) + b_2 (N_2^P - L_2) + a_1 N_1^P + a_2 N_2^P - \kappa_1 V_1
$$

$$
-\kappa_2 V_2 + \frac{N}{2v} \left[ \int_{-v}^{x} c_{2j} dj \right] + \frac{N}{2v} \left[ \int_{x}^{v} c_{1j} dj \right]
$$

(31)

where

$$
V_1 = \theta_1 \frac{N}{2v} (v - x) \quad L_1 = \frac{N}{2v} p_1 (v - x) \quad N_1^P = \frac{N}{2v} (v - x),
$$

$$
V_2 = \theta_2 \frac{N}{2v} (v + x) \quad L_2 = \frac{N}{2v} p_2 (v + x) \quad N_2^P = \frac{N}{2v} (v + x),
$$

and $-v \leq x \leq v$. The first-order conditions write:

$$
\frac{\kappa_i}{(1 - \eta_i)m_i(\theta_i)} = y_i - b_i, \ i \in \{1, 2\}
$$

(32)

$$
x = \Delta + p_2(y_2 - b_2 - \frac{\kappa_2}{m_2(\theta_2)}) - p_1(y_1 - b_1 - \frac{\kappa_1}{m_1(\theta_1)})
$$

(33)

where $\eta_i = -\frac{\theta_i m_i'(\theta_i)}{m_i(\theta_i)}$ is the elasticity of the probability at which vacancies are filled in region 1 with respect to $\theta_1$ in absolute value.

Imagine that the Hosios condition is satisfied (i.e. $\beta_i = \eta_i$). Then, as in a standard Mortensen-Pissarides setting, the equilibrium levels of tightness, and hence the unemployment rates, are chosen optimally (compare equations (6) and (32)). Furthermore, the partition of the population is optimal. By (32), the optimality condition (33) can be rewritten as:

$$
x = \Delta + p_2 \eta_2(y_2 - b_2) - p_1 \eta_1(y_1 - b_1)
$$

(33)

In expressions $\int_{-v}^{x} c_{2j} dj$ and $\int_{x}^{v} c_{1j} dj$, there is a slight abuse of notation since $v$ and $x$ are values for the difference $c_{1j} - c_{2j}$. This notation is equivalent to assuming a bijective relationship between the identifier of workers, $j$, and their relative preference for region 1, $c_{1j} - c_{2j}$.
which is equivalent to the corresponding condition in the decentralized equilibrium (8) when the Hosios condition is satisfied. In sum,

**Proposition 4.** If $\alpha = 0$, in the laissez-faire economy, the sizes of the workforce and the decentralized equilibrium unemployment rates are efficient if the Hosios condition is met in both regions.

### 3.2 The case where $\alpha > 0$

We start with the general case and consider symmetric regions later on. In a directed search framework with multiple applications, wage posting with commitment and no recall of applications, Galenianos and Kircher (2009) conclude that the decentralized equilibrium is inefficient because firms cannot influence the probability of retaining applications when they get more than one. By allowing firms to recall all the applicants they receive, Kircher (2009) shows that efficiency is restored. In our framework, firms cannot receive more than one application while some job-seekers can get two job opportunities. Consequently, in the decentralized equilibrium, the probability that a vacancy is rejected is not nil. This feature, which is also present in Galenianos and Kircher (2009), is as such a source of efficiency that could be avoided in a dynamic model in continuous time (see Subsection 2.4.9). **Here, we show that there are other sources of inefficiency that are specific to our regional model.**

In our static framework, we will first look at the case where the constrained social planner only chooses the levels of tightness. All thresholds are determined as in the decentralized economy. In this environment, the probability of rejecting an offer will be exactly the same in the decentralized and the efficient economies if the corresponding tightness levels are equal too. Next, we will let the planner choose in addition all the threshold values but we will impose that the arrival of two offers cannot be avoided by the planner if it is efficient that some workers search all over the country.

First, the planner solves\(^\text{21}\):

\[
\max_{\theta_1, \theta_2} y_1 L_1 + y_2 L_2 + b_1 (N_1^P - L_1) + b_2 (N_2^P - L_2) + a_1 N_1^P + a_2 N_2^P \\
+ \frac{N}{2v} \left[ \int_{-v}^{x} c_2 j \, dj \right] + \frac{N}{2v} \left[ \int_{v}^{x} c_1 j \, dj \right] + \frac{N}{2v} \alpha p_1 p_2 \left[ \int_{-v}^{x} (c_1 j - c_2 j) \, dj \right] \\
+ \frac{N}{2v} \alpha p_1 (1 - p_2) \left[ \int_{z_1}^{x} (c_1 j - c_2 j) \, dj \right] - \frac{N}{2v} \alpha p_2 (1 - p_1) \left[ \int_{z_2}^{x} (c_1 j - c_2 j) \, dj \right] \\
- \kappa_1 V_1 - \kappa_2 V_2 \tag{34}
\]

subject to (9), (12), (14) and to the following definitions:

\(^{21}\)With the same slight abuse of notations as in the previous subsection.
\[
\begin{align*}
\mathcal{V}_1 &= \theta_1 \frac{N}{2v} [v - x + \alpha(x - z_1)] \\
\mathcal{V}_2 &= \theta_2 \frac{N}{2v} [v + x + \alpha(z_2 - x)] , \\
L_1 &= \frac{N}{2v} [p_1(v - x) + \alpha p_1(1 - p_2)(x - z_1) + \alpha p_2(x - \hat{x})] , \\
L_2 &= \frac{N}{2v} [p_2(v + x) + \alpha p_2(1 - p_1)(z_2 - x) - \alpha p_1 p_2(x - \hat{x})] , \\
N_1^P &= \frac{N}{2v} [v - x - \alpha p_2(1 - p_1)(z_2 - x) + \alpha p_1(1 - p_2)(x - z_1) + \alpha p_2(x - \hat{x})] , \\
N_2^P &= \frac{N}{2v} [v + x + \alpha p_2(1 - p_1)(z_2 - x) - \alpha p_1(1 - p_2)(x - z_1) - \alpha p_2(x - \hat{x})] .
\end{align*}
\]

Given the decentralized values of \(z_1, z_2, \hat{x}\) and the decentralized relationship between \(x\) and tightness levels, an efficient allocation is a vector \(\{\theta_1, \theta_2\}\) solving the following first-order conditions:

\[
\begin{align*}
\frac{\kappa_1}{(1 - \eta_1)m_1(\theta_1)} &= \pi_1(y_1 - b_1) - \frac{\alpha p_2(2 - \hat{x})}{v - x + \alpha(x - z_1)} (y_2 - b_2) \\
&- \frac{\alpha(1 - p_2)(x - z_1)}{v - x + \alpha(x - z_1)} \left( \Delta - \frac{x}{2} + z_1 \right) - \frac{\alpha p_2(z_2 - x)}{v - x + \alpha(x - z_1)} \left( \Delta - \frac{z_2 + x}{2} \right) \\
\frac{\kappa_2}{(1 - \eta_2)m_2(\theta_2)} &= \pi_2(y_2 - b_2) - \frac{\alpha p_1(x - \hat{x})}{v + x + \alpha(z_2 - x)} (y_1 - b_1) \\
&+ \frac{\alpha(1 - p_1)(z_2 - x)}{v + x + \alpha(z_2 - x)} \left( \Delta - \frac{z_2 + x}{2} \right) + \frac{\alpha p_1(x - z_1)}{v + x + \alpha(z_2 - x)} \left( \Delta - \frac{x + z_1}{2} \right) \\
&+ \frac{\alpha p_1(\hat{x} - x)}{v + x + \alpha(z_2 - x)} \left( \Delta - \frac{\hat{x} + x}{2} \right)
\end{align*}
\]

in which \(\pi_1\) (resp., \(\pi_2\)) verifies the same expression as in the decentralized economy, namely (16) (resp., (17)). If the planner only looked at the influence of \(\theta_i\) on output and the cost of vacancy creation in region \(i\), the above first order conditions would write \(\kappa_i/[1 - \eta_i]m_i(\theta_i)] = \pi_i(y_i - b)\). This condition and (10) in the decentralized economy would be reconciled thanks to the Hosios condition. For then the efficient and decentralized values of \(\pi_i\) would be equal. However, the planner internalizes various other effects to which we turn now.

The planner recognizes that an additional vacancy in region \(i\) reduces the chances of a match between residents of region \(i\) and vacancies in region \(-i\), while firms in region \(i\) have no reason to share the same concern. By subtracting the nonnegative term \([\alpha p_2(z_2 - \hat{x})(y_2 - b_2)]/[v - x + \alpha(x - z_1)]\) from \(\pi_1(y_1 - b_1)\) in (35), the planner internalizes an induced effect on expected output in region 2 and this pushes optimal tightness downwards in region 1. A corresponding mechanism applies in (36) as well.

Furthermore, when choosing the number of vacancies in each region, the planner takes into account the impacts of a rise in the number of vacancies on the value of
leisure, the amenities and the idiosyncratic preference when a worker has to migrate to take a job offer (see the second and third lines of (35) and (36)). These compensations are not present in the decentralized equilibrium. Appendix F verifies that shifting the wage bargain before the migration decision, a case which we henceforth call ex-ante bargaining, does not entirely eliminate these sources of inefficiencies. For, as Appendix F.6 explains, the decentralized number of vacancies created in a region does not take into account the induced effects (in terms of leisure, amenities and idiosyncratic preferences) on workers who live in this region but are recruited in the other one.

Next, we turn to the case where the planner also chooses the threshold values. For given numbers of participants to the matching process, the planner will divide the population in two groups as far as the choice of residence is concerned (as in the previous subsection). In addition, to offer them the highest idiosyncratic level of preference, the planner will put the regional job-seekers at the extremes of the \([-v, v]\) segment and the national job-seekers, if any, in the middle. The national job-seekers located in region 1 are more likely to stay there than national job-seekers located ex-ante in region 2 are likely to live in region 1 (if \(\alpha < 1\)). So, it is efficient to put them to the right of the national job-seekers living in region 2. All this implies that the planner will select unique thresholds \(x, z_1, z_2\) under the constraints \(-v \leq z_1 \leq x, \hat{x} \leq z_2 \leq v\). Therefore, the planner’s problem consists here in maximizing Objective (34) with respect to \(\{\theta_1, \theta_2, z_1, z_2, x, \hat{x}\}\).

An efficient interior allocation solves (35) and (36) and:

\[
\begin{align*}
\hat{x} &= \Delta + y_2 - b_2 - (y_1 - b_1), \quad (37) \\
z_1 &= b_1 - y_1 + \Delta + \frac{\kappa_1}{m_1(\theta_1)} - \frac{1}{1 - p_2}, \quad (38) \\
z_2 &= y_2 - b_2 + \Delta - \frac{\kappa_2}{m_2(\theta_2)} - \frac{1}{1 - p_1}.
\end{align*}
\]

\[
x = \Delta + (1 - \alpha) \left[ p_2(y_2 - b_2 - \frac{\kappa_2}{m_2(\theta_2)}) - p_1(y_1 - b_1 - \frac{\kappa_1}{m_1(\theta_1)}) \right] - \frac{1}{1 - \alpha p_1 - \alpha p_2 + \alpha p_1 p_2}. \quad (40)
\]

Additional sources of inefficiency arise. Considering first the partition of regional workforces between the two statuses of job search, we need to compare (38) or (39) and (12).

---

\(22\) Three terms are distinguished in both equations. We now interpret them after multiplication by \((1 - \eta_i) m_i(\theta_i)\). The first part of the second line measures the marginal impact of vacancy creation in region \(i\) on workers who seek a job in region \(i\), live in the other region and get a position in region \(i\). This first part is the product of the change in the share of these workers when an additional vacancy is created in \(i\) and an expression between parentheses that recognizes that these workers do not enjoy leisure, amenities and their idiosyncratic preference in region \(-i\) since they move to region \(i\). The second part of the second line is also the product of two terms. The first one quantifies the change in the share of workers who live in region \(i\), only get an offer in region \(-i\) and migrate there when an additional vacancy is created in \(i\). The second term captures that these workers do not enjoy leisure, amenities and their idiosyncratic preference in region \(i\) since they move to region \(-i\). Finally, the sign of the third line depends on the sign of \(x - \hat{x}\). The first term of the line corresponds to the change in the share of workers who get two job offers when an additional vacancy is created in \(i\). These workers are located ex-ante in region 2 (resp. 1) whenever \(x \geq \hat{x}\) (resp. \(x < \hat{x}\)). The second term takes into account that these workers do not (resp. do) enjoy leisure, amenities and their idiosyncratic preference in region 2. All these compensations cannot appear if \(\alpha = 0\) since workers never migrate to take a position.
Two effects are at work. First, because the planner considers the net gain in output, \( y_i - b_i \), while workers only consider the share \( \beta_i(y_i - b_i) \) that accrues to them, the decentralized values of \( z_i \) are lower than the efficient ones. As Appendix F.6 explains, this source of inefficiency disappears under *ex-ante* bargaining. The second effect is instead present whenever the wages are set. A marginal rise in, say, \( z_1 \) reduces the size of the workforce living in region 2 and searching also in region 1. This lowers expected net output by an amount \( \alpha p_1(1 - p_2)(y_1 - b_1) \) but reduces the cost of vacancy creation by \( \alpha \kappa_1 \theta_1 \). After division by \( \alpha p_1(1 - p_2) \), the loss in expected output becomes \( y_1 - b_1 \) while the gain becomes the product of the expected cost of opening a vacancy in region 1, \( \kappa_1/m_1(\theta_1) \), and of \( 1/(1 - p_2) \). This gain is not taken into account by decentralized decisions (see (12)) but well by the planner (see the expressions \( \kappa_i/[(1 - p_i)m_i(\theta_i)] \) in (38) and (39)). So, through this second effect, too many workers search for a job nationally.

Turning to the choice of residence, a comparison of (40) and (14) indicates one additional source of inefficiency. When fixing \( x \), the planner compares the expected increase in net output in each region, \( p_i(y_i - b_i - \kappa_i/m_i(\theta_i)) \), while job-seekers compare the expected net increase in income \( p_i\beta_i(y_i - b_i) \). Contrary to the case where \( \alpha = 0 \), the Hosios condition does not reconcile the two perspectives since in any case (35) and (36) do not yield an equality between the expected cost of opening a vacancy and \( (1 - \eta_1)(y_i - b_i) \) when \( \alpha > 0 \). This remains true under *ex-ante* bargaining (see Appendix F.6). It should be mentioned that this source of inefficiency in the choice of \( x \) would disappear in the limit case where job-seekers are equally effective in the matching process wherever they search (\( \alpha = 1 \)). The decentralized threshold \( x \) would in this case be efficient and equal to \( \Delta \). In sum,

**Proposition 5.** If \( \alpha > 0 \), even if the Hosios condition is met in both regions, the decentralized laissez-faire equilibrium unemployment rates, regional partition of the workforce and numbers of job-seekers searching in the whole country are inefficient. One cannot rank the optimum and the equilibrium and hence one cannot determine the direction of changes in the workers’s bargaining power that could be welfare improving. This proposition is robust to changes in the timing of the wage bargain.

If we assume two symmetric regions, the efficient allocation is symmetric as well. Hence, the optimal \( x \) and \( \bar{x} \) are set to zero. We can denote the thresholds \( z_2 = -z_1 = z \in [0,v] \) and the probability of being recruited \( p_1 = p_2 = p \). The planner’s objective function becomes

\[
2( (y - b)L - \kappa V - \frac{\kappa}{2E} \alpha p(\theta)(1 - p(\theta)) \left[ \int_0^x (c_{ij} - c_{2j}) dj \right] ) = \frac{\kappa}{v} (y - b)p(\theta)(v + \alpha z(1 - p(\theta))) - \kappa \theta (v + \alpha z) - \alpha p(\theta)(1 - p(\theta))(z^2/2) 
\]

plus a constant term. Tightness and \( z \) are determined by a simplified version of (35)-(36) and (38)-(39), namely:

\[
\frac{\kappa}{(1 - \eta)m(\theta)} = (y - b) \left( \frac{v + \alpha z(1 - 2p(\theta))}{v + \alpha z} \right) - \frac{\alpha(1 - 2p(\theta))}{v + \alpha z} 
\]

\[
z = y - b - \frac{\kappa}{m(\theta)(1 - p(\theta))} \frac{1}{(1 - \eta)m(\theta)} 
\]

24
if the constraints \(0 = x < z < v\) hold. In a decentralized equilibrium, tightness solves (29) and \(z = \beta(y - b)\) (or \(y - b\) under \textit{ex-ante} bargaining). The only source of inefficiency that disappears concerns the choice of residence. As regions are symmetric, the location threshold stands in the middle of the \([-v, v]\) segment both in the decentralized and the optimal cases. The other sources of inefficiency remain. First, firms do not take into account their impact on the opening of vacancies in the other region. As a result, the ratio \(- (y - b)p(\theta)\alpha z / (v + \alpha z)\), corresponding to the last term in the first lines of (35) and (36), is present on the RHS of (42) as well.\(^{23}\) Furthermore, firms do not internalize the change in workers’ utility induced by getting (or not) a job offer in region \(i\). This is represented by the term in \(z^2\) on the RHS (42). Second, as a comparison between (43) and the decentralized value of \(z\) shows, workers do not take their search decision optimally since they do not internalize the impact of their decision on the cost of opening a vacancy in the other region (see the last term in (43)).

4 Numerical exercise

In this section, we consider a stylized symmetric economy. We highlight that nobody searches nationally in the efficient allocation. This conclusion resists to many changes in the parameters. We also look at the gap between the efficient and the decentralized allocation when parameter \(\alpha\) rises.

4.1 Calibration

We consider two symmetric regions in a stylized US economy. We normalize the total size of the population \(N\) to 1. We assume Cobb-Douglas matching functions \(M_i = \bar{h}_i V_i^{0.5} (N_i + \alpha N)^{0.5}\) with \(\bar{h} = 0.7\).\(^{24}\) Following common practice, we assume that the Hosios condition holds (workers’ bargaining powers \(\beta_i = 0.5\)). In the dynamic framework of Pissarides (2009), the expected cost of opening a vacancy amounts to 43% of \textit{monthly} output. Normalizing regional productivity levels \(y_i\) to 1 in our static framework can be interpreted as equating monthly output divided by the sum of the discount rate and the separation rate to 1. So, following Pissarides (2009), monthly output equals the sum of the interest rate (0.004) and the separation rate (0.036). Therefore, in our static setting, the expected cost of opening a vacancy, \((\kappa_i / [\pi_i m_i(\theta_i)]\)), is \(0.43 \times (0.004 + 0.036) = 0.0172\), i.e. 1.72% of intertemporal output. From the free-entry conditions (10), we then derive the value of home production: 0.9656. By (5), the wage equals then 0.98. The difference between the wage and home production may look small. However, as Pissarides (2009) explains, the permanent income of employed workers is only marginally above the permanent income of unemployed workers, even if the difference in a dynamic framework between current wage and current home production is quite large. Plugging

\(^{23}\)Merged with the probability of rejecting an offer, it leads to the term \(1 - 2p(\theta)\) in (42) while it is only \(1 - p(\theta)\) in (29).

\(^{24}\)In a static setting, the Cobb-Douglas specification does not guarantee that the hiring rate tends to 1 when \(\theta_i\) becomes sufficiently big. In the simulations we take care of this difficulty.
the above-mentioned values for $\beta, y$ and $b$ into (12) yields $z_2 = 0.0172$. On the basis of the evidence provided by Marinescu and Rathelot (2013) for the US, we assume that 10% of the population is looking for a job nationally. This information is then introduced in the definition of $N^R_0$ (20)-(21) and yields $v = 0.172$. In the absence of evidence about $\alpha$, we arbitrarily set this parameter to 0.12, which means that workers searching out of their region of residence are 8 times less efficient than in their own region. However, we develop a sensitivity analysis with respect to $\alpha$. Finally, we calibrate the cost of opening a vacancy $\kappa_i$ to match an average unemployment rate in the US in the period 2005-2011, namely 7%. So, $\kappa_i = 0.0090$. In the calibrated economy, $p(\theta) = 0.93$, a value well above 0.5.

4.2 The efficient allocation and the efficiency gap

Let us first look at the efficient allocation. The central planner chooses higher levels of tightness than in the decentralized equilibrium, so that the efficient unemployment rate is lower than in the decentralized economy (6% versus 7%). As regions are symmetric, this difference is not an indicator of regional mismatch (as measured by e.g. Sahin et al., 2014) but the impact on unemployment of the sources of inefficiencies explained after Equality (43). While 10% of the population searches nationally in the decentralized economy, it is optimal that everyone searches regionally only ($z = 0$). A high value of tightness implies that $m(\theta)(1 - p(\theta))$ is low in the RHS of (43) and this explains why the optimal $z$ is zero. Recall that $-\kappa / [m(\theta)(1 - p(\theta))]$ captures the consequences of search decisions on the cost of opening vacancies in the region where the job-seeker does not live.

The conclusion that workers should only search regionally appears to be very robust to changes in the parameters. Varying search effectiveness in the other region, $\alpha$, from 0.01 to 1 does not modify this conclusion. Let superscript $c$ designate the calibrated values. With $\alpha^c = 0.12$, $z = 0$ is still optimal when we successively consider the following changes in the other parameters: $\kappa \in [0.95 * \kappa^c, 1.85 * \kappa^c]$, $v \in [0.1 * v^c, 30 * v^c]$, $y \in [y^c, 1.0025 * y^c]$ and $\bar{h} \in [0.5 * \bar{h}^c, 1 * \bar{h}^c]$.

Net output levels at the social optimum and at the decentralized equilibrium differ only by the net gain of firms’ production minus some idiosyncratic preferences. We compute an “efficiency gap” as this difference in net output divided by the optimal value of net output. Figure 4 draws the evolution of this efficiency gap with $\alpha$. As already mentioned, the efficiency gap is positive whenever $\alpha$ is positive. The gap is increasing with $\alpha$ and amounts to about 7% when $\alpha \rightarrow 1$.

---

25To compute the optimal values of $-z_1 = z_2 = z \geq 0$ and of $p_1 = p_2 = p \geq 0$ (from which the corresponding value of $\theta$ is deducted), we discretize $z$ in $[0; 0.172]$ and $p$ in $[0, 1]$ (allowing each to take 9000 values), then we evaluate the social objective of the planner for each of the 9000 × 9000 values. Finally, we select the global optimum.

26The value of $-\kappa / [m(\theta)(1 - p(\theta))]$ is -0.28 at the optimum, which is 8 times bigger than the difference $y - b$. This implies that the optimal $z$ is bounded below by $x = 0$.

27The efficient unemployment rate is already nil when $y = 1.0025$. 

---
Conclusion

This paper studies equilibrium unemployment in a two-region static economy where wages are endogenous and homogeneous workers and jobs are free to move. We develop a tractable search-matching equilibrium in which job-seekers can decide to search for a job in another region without first migrating there. Current communication technologies motivate this assumption. Since individuals have idiosyncratic and heterogeneous preferences for regions, part of the population chooses to seek a job all over the country while the rest only searches in the region where they live. These decisions affect the regional unemployment rates. Compared to the case where job-seekers can only search in their region of residence, search-matching externalities are amplified by the opportunity of searching in a region where one does not live and by the fact that some workers can simultaneously receive a job offer from each region. Hence, some vacant positions remain unfilled, which leads to a waste of resources. By this effect, if it is now much easier to search for a job all over the country thanks to the Internet, firms are less inclined to open vacancies. The shrinking labor demand and the higher probability of finding a job conditional on tightness are two opposite forces that hold true in a dynamic framework. They can explain the disagreement between Kroft and Pope (2014) at the city level and Kuhn and Mansour (2014) and Choi (2011) at the individual level.

We characterize the constrained efficient allocation, where the central planner is subject to the same matching frictions as the decentralized agents. In standard non-spatial search-matching models with wage bargaining, the laissez-faire decentralized economy is efficient if the Hosios condition is verified. This is also true when search effort is endogenous. In our model where job-seekers decide over their search field, the Hosios condition is not sufficient to guarantee efficiency when workers can search nationally. Workers and firms take decisions without internalizing the effect of their choices on net output in both regions. Simulations show that the optimal allocation is a corner solution where no one searches all over the country (this result is very robust to
changes in parameters) and the efficient regional unemployment rates are one percentage point lower than in the decentralized equilibrium.

This paper does not claim to have evaluated the general equilibrium impact of the Internet on the matching process. It has only focused on the implications of searching before possibly moving to another region under the standard assumptions of constant returns to scale in the matching process and in production. Beaudry et al. (2014) find no significant effects of agglomeration forces on productivity in the US. So, we feel confident that the latter assumption is not too strong a simplification. However, the presence of non negligible agglomeration forces would affect our conclusions.

References


Appendices

A Search and location decisions are taken simultaneously

The following table summarizes the different cases an agent faces when $\alpha > 0$:

<table>
<thead>
<tr>
<th>Where to live</th>
<th>Where to search</th>
</tr>
</thead>
<tbody>
<tr>
<td>Region 1</td>
<td>Region 1</td>
</tr>
<tr>
<td></td>
<td>case a</td>
</tr>
<tr>
<td>Region 2</td>
<td>Region 2</td>
</tr>
<tr>
<td></td>
<td>case d</td>
</tr>
<tr>
<td></td>
<td>Region 1 and region 2</td>
</tr>
<tr>
<td></td>
<td>case c</td>
</tr>
<tr>
<td></td>
<td>case e</td>
</tr>
<tr>
<td></td>
<td>case f</td>
</tr>
</tbody>
</table>

We will proceed by first computing the expected utility of an individual in each case. By doing so, we will be able to drop cases b and d. In a second step, we will define 3 thresholds out of the 6 that could be computed from the 4 remaining cases. By ranking these thresholds and comparing the expected utility levels, we will be able to rank the expected utility levels and show that these thresholds are those we get if we assume that agents take the location and search decisions sequentially.
1. Expected utility in each case

Case a: The utility if the individual lives in 1 and searches in 1 only
\[ p_1 V_{1j}^e + (1 - p_1) V_{1j}^a \]

Case b: The utility if the individual lives in 1 and searches in 2 only
\[ \alpha p_2 V_{2j}^e + (1 - \alpha p_2) V_{1j}^a \]

Case c: The utility if the individual lives in 1 and searches in both regions
\[ p_1 (1 - \alpha p_2) V_{1j}^e + \alpha p_2 (1 - p_1) V_{2j}^e + (1 - p_1)(1 - \alpha p_2)V_{1j}^a + \alpha p_1 p_2 \left[ \max \left\{ V_{1j}^e; V_{2j}^e \right\} - \varepsilon \right] \]

Case d: The utility if the individual lives in 2 and searches in 1 only
\[ \alpha p_1 V_{1j}^e + (1 - \alpha p_1) V_{2j}^a \]

Case e: The utility if the individual lives in 2 and searches in 2 only
\[ p_2 V_{2j}^e + (1 - p_2) V_{2j}^a \]

Case f: The utility if the individual lives in 2 and searches in both regions
\[ p_2 (1 - \alpha p_1) V_{1j}^e + \alpha p_1 (1 - p_2) V_{1j}^e + (1 - p_2)(1 - \alpha p_1)V_{2j}^a + \alpha p_1 p_2 \left[ \max \left\{ V_{1j}^e; V_{2j}^e \right\} - \varepsilon \right] \]

We assume that the cost \( \varepsilon \) of refusing a job offer tends to zero.

2. Case b is dominated by case c if
\[ \alpha p_2 V_{2j}^e + (1 - \alpha p_2) V_{1j}^a < p_1 (1 - \alpha p_2) V_{1j}^e + \alpha p_2 (1 - p_1) V_{2j}^e + (1 - p_1)(1 - \alpha p_2)V_{1j}^a + \alpha p_1 p_2 \left[ \max \left\{ V_{1j}^e; V_{2j}^e \right\} - \varepsilon \right] \]
\[ \Leftrightarrow 0 < (1 - \alpha p_1 p_2) V_{1j}^a + p_1 (V_{1j}^e - V_{1j}^a) + \alpha p_1 p_2 \left[ \max \left\{ V_{1j}^e; V_{2j}^e \right\} - V_{1j}^e - V_{2j}^a + V_{1j}^a \right] \]

Two sub-cases should be considered:

- If \( \max \left\{ V_{1j}^e; V_{2j}^e \right\} = V_{2j}^e \), then the comparison becomes:
  \[ 0 < (1 - \alpha p_1 p_2) V_{1j}^a + p_1 (V_{1j}^e - V_{1j}^a) + \alpha p_1 p_2 (V_{1j}^e - V_{1j}^a) \]
  \[ 0 < (1 - \alpha p_1 p_2) V_{1j}^a + (1 - \alpha p_2) p_1 (V_{1j}^e - V_{1j}^a) \]

  This always holds. Similarly,

- If \( \max \left\{ V_{1j}^e; V_{2j}^e \right\} = V_{1j}^e \), then the comparison becomes:
  \[ 0 < (1 - \alpha p_1 p_2) V_{1j}^a + p_1 (V_{1j}^e - V_{1j}^a) + \alpha p_1 p_2 (V_{1j}^e - V_{1j}^a) \]
  \[ 0 < (1 - \alpha p_1 p_2) V_{1j}^a + (1 - \alpha p_2) p_1 (V_{1j}^e - V_{1j}^a) + \alpha p_1 p_2 (V_{1j}^e - V_{2j}^e) \]

  This always holds since \( V_{1j}^e \geq V_{2j}^e \). As a result, case b will never be optimal for agent \( j \).
3. **Case d** is dominated by **case f** if

\[ \alpha p_1 V_{ij}^c + (1 - \alpha p_1) V_{ij}^u < p_2(1 - \alpha p_1) V_{ij}^c + \alpha p_1(1 - p_2) V_{ij}^u + (1 - p_2)(1 - \alpha p_1) V_{ij}^u + \alpha p_1 p_2 \max \{ V_{ij}^c; V_{ij}^u \} \]

\[ \Leftrightarrow 0 < (1 - \alpha p_1 p_2) V_{ij}^u + p_2(V_{ij}^c - V_{ij}^u) + \alpha p_1 p_2 \max \{ V_{ij}^c; V_{ij}^u \} - V_{ij}^c - V_{ij}^u \]

Two sub-cases should be considered:

- If \( \max \{ V_{ij}^c; V_{ij}^u \} = V_{ij}^c \), then the comparison becomes:

  \[ 0 < (1 - \alpha p_1 p_2) V_{ij}^u + p_2(V_{ij}^c - V_{ij}^u) + \alpha p_1 p_2(V_{ij}^u - V_{ij}^c) \]

  \[ 0 < (1 - \alpha p_1 p_2) V_{ij}^u + (1 - \alpha p_1) p_2(V_{ij}^u - V_{ij}^c) \]

  This always holds. Similarly,

- If \( \max \{ V_{ij}^c; V_{ij}^u \} = V_{ij}^u \), then the comparison becomes:

  \[ 0 < (1 - \alpha p_1 p_2) V_{ij}^u + p_2(V_{ij}^c - V_{ij}^u) + \alpha p_1 p_2(V_{ij}^u - V_{ij}^c) \]

  \[ 0 < (1 - \alpha p_1 p_2) V_{ij}^u + (1 - \alpha p_1) p_2(V_{ij}^u - V_{ij}^c) + \alpha p_1 p_2(V_{ij}^u - V_{ij}^c) \]

  This always holds since \( V_{ij}^c \geq V_{ij}^u \). As a result, **case d** will never be optimal for agent \( j \).

4. **Defining the thresholds**

With the 4 remaining cases, we define 3 threshold values and show that this is sufficient to get a dominant case for each value of \( c_{1j} - c_{2j} \):

**Definition of the threshold between case c and case f**

\[ p_1(1 - \alpha p_2) V_{ij}^c + \alpha p_2(1 - p_1) V_{ij}^u + (1 - p_1)(1 - \alpha p_2) V_{ij}^u + \alpha p_1 p_2 \max \{ V_{ij}^c; V_{ij}^u \} \]

\[ = p_2(1 - \alpha p_1) V_{ij}^u + \alpha p_1(1 - p_2) V_{ij}^c + (1 - p_2)(1 - \alpha p_1) V_{ij}^u + \alpha p_1 p_2 \max \{ V_{ij}^c; V_{ij}^u \} \]

\[ \Leftrightarrow (1 - \alpha) p_1 V_{ij}^c - (1 - \alpha) p_2 V_{ij}^u + (1 - p_1)(1 - \alpha p_2) V_{ij}^u - (1 - p_2)(1 - \alpha p_1) V_{ij}^u = 0 \]

\[ \Leftrightarrow (1 - \alpha) p_2(V_{ij}^u - V_{ij}^c) - (1 - \alpha) p_1(V_{ij}^c - V_{ij}^u) = 0 \]

\[ (1 - \alpha) p_1 - \alpha p_2 + \alpha p_1 p_2(V_{ij}^u - V_{ij}^c) = 0 \]

Using the definitions of utilities, we get equation (14):

\[ x = b_2 - b_1 + a_2 - a_1 + (1 - \alpha) \frac{p_2(w_2 - b_2) - p_1(w_1 - b_1)}{1 - \alpha p_1 - \alpha p_2 + \alpha p_1 p_2} \]

**Definition of the threshold between case c and case f**

\[ p_2 V_{ij}^c + (1 - p_2) V_{ij}^u = p_2(1 - \alpha p_1) V_{ij}^u + \alpha p_1(1 - p_2) V_{ij}^c \]

\[ + (1 - p_2)(1 - \alpha p_1) V_{ij}^u + \alpha p_1 p_2 \max \{ V_{ij}^c; V_{ij}^u \} \]

\[ \Leftrightarrow \alpha p_1(V_{ij}^c - V_{ij}^u) + \alpha p_1 p_2 \max \{ V_{ij}^c; V_{ij}^u \} - V_{ij}^c + V_{ij}^u = 0 \]

Two sub-cases should be considered:
Two sub-cases should be considered:

- If \( \max \left\{ V_{ij}^{e}; V_{2j}^{e} \right\} = V_{1j}^{e} \),
  \[
  \alpha p_1(1 - p_2)(V_{ij}^{e} - V_{2j}^{u}) = 0 \\
  \Leftrightarrow V_{ij}^{e} = V_{2j}^{u} \text{ as } 0 < p_i < 1 \text{ and } \alpha > 0 \\
  \Leftrightarrow z_1 = b_1 - w_1 + b_2 - b_1 + a_2 - a_1
  \]

- If \( \max \left\{ V_{ij}^{e}; V_{2j}^{e} \right\} = V_{2j}^{e} \),
  \[
  \alpha p_1(1 - p_2)(V_{ij}^{e} - V_{2j}^{u}) + \alpha p_1 p_2 (V_{ij}^{e} - V_{2j}^{e}) = 0 \\
  \Leftrightarrow \tilde{z}_1 = b_1 - w_1 + b_2 - b_1 + a_2 - a_1 + p_2(w_2 - b_2) \text{ as } 0 < p_i < 1 \text{ and } \alpha > 0
  \]

However, the assumption of the sub-cases, \( V_{ij}^{e} \geq V_{2j}^{e} \), implies that in this case relative preference are such that:

\[
c_{1j} - c_{2j} \geq b_1 - w_1 + b_2 - b_1 + a_2 - a_1 + w_2 - b_2 > \tilde{z}_1
\]

which leads to a contradiction as we assume in the definition of \( \tilde{z}_1 \) that \( V_{ij}^{e} \geq V_{2j}^{e} \).

Therefore, the only possible threshold value between case e and case f is \( z_1 \).

**Definition of the threshold between case a and case c**

\[
p_1 V_{ij}^{e} + (1 - p_1)V_{ij}^{u} = p_1(1 - \alpha p_2)V_{ij}^{e} + \alpha p_2(1 - p_1)V_{2j}^{e} \\
+ (1 - p_1)(1 - \alpha p_2)V_{ij}^{u} + \alpha p_1 p_2 \max \{ V_{ij}^{e}; V_{2j}^{e} \} \\
\Leftrightarrow \alpha p_2(V_{2j}^{e} - V_{1j}^{u}) + \alpha p_1 p_2 \max \{ V_{1j}^{e}; V_{2j}^{e} \} - V_{1j}^{e} - V_{2j}^{e} + V_{ij}^{u} = 0
\]

Two sub-cases should be considered:

- If \( \max \left\{ V_{ij}^{e}; V_{2j}^{e} \right\} = V_{1j}^{e} \),
  \[
  \alpha p_2(1 - p_1)(V_{2j}^{e} - V_{1j}^{u}) = 0 \\
  \Leftrightarrow V_{2j}^{e} = V_{1j}^{u} \text{ as } 0 < p_i < 1 \text{ and } \alpha > 0 \\
  \Leftrightarrow z_2 = w_2 - b_2 + b_2 - b_1 + a_2 - a_1
  \]

- If \( \max \left\{ V_{ij}^{e}; V_{2j}^{e} \right\} = V_{2j}^{e} \),
  \[
  \alpha p_2(1 - p_1)(V_{2j}^{e} - V_{1j}^{u}) + \alpha p_1 p_2 (V_{2j}^{e} - V_{1j}^{e}) = 0 \\
  \Leftrightarrow \tilde{z}_2 = w_2 - b_2 + b_2 - b_1 + a_2 - a_1 - p_1(w_1 - b_1) \text{ as } 0 < p_i < 1 \text{ and } \alpha > 0
  \]

However, the assumption of the sub-case, \( V_{2j}^{e} \geq V_{1j}^{e} \), implies that in this case relative preference are such that:

\[
c_{1j} - c_{2j} \leq w_2 - b_2 + b_2 - b_1 + a_2 - a_1 + w_1 < \tilde{z}_1
\]

which leads to a contradiction as we assume in the definition of \( \tilde{z}_2 \) that \( V_{2j}^{e} \geq V_{1j}^{e} \).

Therefore, the only possible threshold value between case a and case c is \( z_2 \).
5. Ranking the thresholds

It is easily seen that \( z_1 \leq x \leq z_2 \).

6. Dominant strategies

<table>
<thead>
<tr>
<th></th>
<th>( z_1 )</th>
<th>( x )</th>
<th>( z_2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( f ) dominates c</td>
<td>( f ) dominates c</td>
<td>( c ) dominates f</td>
<td>( c ) dominates f</td>
</tr>
<tr>
<td>( e ) dominates f</td>
<td>( f ) dominates e</td>
<td>( f ) dominates e</td>
<td>( f ) dominates e</td>
</tr>
<tr>
<td>( c ) dominates a</td>
<td>( c ) dominates a</td>
<td>( c ) dominates a</td>
<td>( a ) dominates c</td>
</tr>
<tr>
<td>( e &gt; f &gt; c &gt; a )</td>
<td>( f &gt; c &gt; a ) and ( f &gt; e )</td>
<td>( c &gt; f &gt; e ) and ( c &gt; a )</td>
<td>( a &gt; c &gt; f &gt; e )</td>
</tr>
</tbody>
</table>

- the individual whose relative preference is \( x \) is indifferent between living in 1 and searching in both regions (\( \text{case c} \)) and living in 2 and searching in both regions (\( \text{case f} \));
- the individual whose relative preference is \( z_1 \) is indifferent between living in 2 and searching in 2 only (\( \text{case e} \)) and living in 2 and searching in both regions (\( \text{case f} \));
- the individual whose relative preference is \( z_2 \) is indifferent between living in 1 and searching in 1 only (\( \text{case a} \)) and living in 1 and searching in both (\( \text{case c} \));

We conclude from Figure 5 that the three threshold values we chose at first are sufficient to get a dominant strategy for each value of the relative preference \( c_{1j} - c_{2j} \). These values are equivalent to those obtained when location and search decisions are taken sequentially.

B Proof of Proposition 1

When \( \alpha = 1 \), Proposition 1 is obvious.

When \( 0 < \alpha < 1 \), we first provide conditions guaranteeing that \( \frac{\partial x}{\partial p_2} > 0 \) and \( \frac{\partial x}{\partial p_1} < 0 \). These conditions depend on endogenous variables. We finally derive a unique sufficient condition on \( \alpha \).

\[
\frac{\partial x}{\partial p_2} = \frac{1 - \alpha}{(1 - \alpha p_1 - \alpha p_2 + \alpha p_1 p_2)^2} \left( (1 - \alpha p_1) \beta_2 (y_2 - b_2) - \alpha p_1 (1 - p_1) \beta_1 (y_1 - b_1) \right)
\]
which has a positive sign if:

\[
\alpha < \frac{\beta_2(y_2 - b_2)}{p_1(\beta_2(y_2 - b_2) + (1 - p_1)\beta_1(y_1 - b_1))}
\]  

(44)

To obtain a sufficient condition depending on parameters only, we should now minimize the RHS of (44) by maximizing the denominator with respect to \( p_1 \in (0, 1) \). The unique maximum is

\[
p_1^* = \min \left( \frac{\beta_1(y_1 - b_1) + \beta_2(y_2 - b_2)}{2\beta_1(y_1 - b_1)}, 1 \right)
\]

If \( \beta_2(y_2 - b_2) < \beta_1(y_1 - b_1) \), \( p_1^* < 1 \) and a sufficient condition for (44) is

\[
\alpha < \frac{4\beta_1(y_1 - b_1)\beta_2(y_2 - b_2)}{(\beta_1(y_1 - b_1) + \beta_2(y_2 - b_2))^2}
\]  

(45)

Otherwise, a sufficient condition is \( \alpha < 1 \).

Similarly, \( \partial x/\partial p_1 < 0 \) if

\[
\alpha < \frac{\beta_1(y_1 - b_1)}{p_2(\beta_1(y_1 - b_1) + (1 - p_2)\beta_2(y_2 - b_2))}
\]  

(46)

Minimizing this ratio with respect to \( p_2 \in (0, 1) \) yields

\[
p_2^* = \min \left( \frac{\beta_1(y_1 - b_1) + \beta_2(y_2 - b_2)}{2\beta_2(y_2 - b_2)}, 1 \right)
\]

If \( \beta_2(y_2 - b_2) > \beta_1(y_1 - b_1) \), \( p_2^* < 1 \) and a sufficient condition for (46) is (45) Otherwise, a sufficient condition is \( \alpha < 1 \). To guarantee that both \( \partial x/\partial p_2 > 0 \) and \( \partial x/\partial p_1 < 0 \), we conclude that (45), i.e. (15) in Proposition 1, is the unique sufficient condition, its RHS being in any case lower than one.

C Conditional acceptance rates

In this appendix, we show formulas (16) and (17). In the first part, we focus on the conditional acceptance rate when the vacancy is located in region 1. We then turn to the opposite case.

C.1 Conditional acceptance rate in region 1

A vacant position located in region 1 faces \( \frac{\hat{x} - x + \alpha(x-z_1)}{2v} N \) possible workers in efficiency units. These workers always accept a job offer from the firm if their relative preference for region 1 over region 2, \( c_{1j} - c_{2j} \), is higher than \( \hat{x} \). If their relative preference is below \( \hat{x} \), job-seekers only accept the offer if they have not received one from a firm located in region 2.

We thus compute the conditional acceptance rate as:

\[
\frac{2v}{v - x + \alpha(x - z_1)} \left\{ P(c_{1j} - c_{2j} \geq \hat{x})1 + P(c_{1j} - c_{2j} < \hat{x})P(\text{no offer from 2}) \right\}
\]
There are two sub-cases: Whether $x$ is lower or greater than $\hat{x}$.

Whenever $x < \hat{x}$,

<table>
<thead>
<tr>
<th>Relative preference</th>
<th>Proba to have this preference</th>
<th>Proba to accept a position in 1</th>
</tr>
</thead>
<tbody>
<tr>
<td>$v &gt; c_{1j} - c_{2j} &gt; \hat{x}$</td>
<td>$\frac{v - \hat{x}}{2v}$</td>
<td>1</td>
</tr>
<tr>
<td>$\hat{x} &gt; c_{1j} - c_{2j} &gt; x$</td>
<td>$\frac{\hat{x} - x}{2v}$</td>
<td>$1 - \alpha p_2$</td>
</tr>
<tr>
<td>$x &gt; c_{1j} - c_{2j} &gt; z_1$</td>
<td>$\frac{\alpha(x - z_1)}{2v}$</td>
<td>$1 - p_2$</td>
</tr>
</tbody>
</table>

The conditional acceptance rate is thus:

$$\frac{2v}{v - x + \alpha(x - z_1)} \left\{ \frac{v - \hat{x}}{2v} + \frac{\hat{x} - x}{2v} (1 - \alpha p_2) + \frac{x - z_1}{2v} (1 - p_2) \right\}$$

which leads to equation (16).

Whenever $x > \hat{x}$,

<table>
<thead>
<tr>
<th>Relative preference</th>
<th>Proba to have this preference</th>
<th>Proba to accept a position in 1</th>
</tr>
</thead>
<tbody>
<tr>
<td>$v &gt; c_{1j} - c_{2j} &gt; x$</td>
<td>$\frac{v - x}{2v}$</td>
<td>1</td>
</tr>
<tr>
<td>$x &gt; c_{1j} - c_{2j} &gt; \hat{x}$</td>
<td>$\frac{\alpha(x - \hat{x})}{2v}$</td>
<td>1</td>
</tr>
<tr>
<td>$\hat{x} &gt; c_{1j} - c_{2j} &gt; z_1$</td>
<td>$\frac{\alpha(\hat{x} - z_1)}{2v}$</td>
<td>$1 - p_2$</td>
</tr>
</tbody>
</table>

The conditional acceptance rate is thus:

$$\frac{2v}{v - x + \alpha(x - z_1)} \left\{ \frac{v - x}{2v} + \frac{\alpha(x - \hat{x})}{2v} + \frac{\alpha \hat{x} - z_1}{2v} (1 - p_2) \right\}$$

which leads to equation (16) as well.

C.2 Conditional acceptance rate in region 2

A vacant position located in region 2 faces $\frac{v + x + \alpha(z_2 - x)}{2v} N$ possible workers in efficiency units. These workers always accept a job offer from the firm if their relative preference for region 1 over region 2, $c_{1j} - c_{2j}$, is lower than $\hat{x}$. If their relative preference is higher $\hat{x}$, job-seekers only accept the offer if they have not received one from a firm located in region 1.

We thus compute the conditional acceptance rate as:

$$\frac{2v}{v + x + \alpha(z_2 - x)} \{P(c_{1j} - c_{2j} < \hat{x})1 + P(c_{1j} - c_{2j} \geq \hat{x}) P(\text{no offer from 1})\}$$

Here again there are two sub-cases: Whether $x$ is lower or greater than $\hat{x}$.

Whenever $x < \hat{x}$,

<table>
<thead>
<tr>
<th>Relative preference</th>
<th>Proba to have this preference</th>
<th>Proba to accept a position in 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>$-v &lt; c_{1j} - c_{2j} &lt; x$</td>
<td>$\frac{v + x}{2v}$</td>
<td>1</td>
</tr>
<tr>
<td>$x &lt; c_{1j} - c_{2j} &lt; \hat{x}$</td>
<td>$\frac{\alpha(x - \hat{x})}{2v}$</td>
<td>1</td>
</tr>
<tr>
<td>$\hat{x} &lt; c_{1j} - c_{2j} &lt; z_2$</td>
<td>$\frac{\alpha(z_2 - \hat{x})}{2v}$</td>
<td>$1 - p_1$</td>
</tr>
</tbody>
</table>

36
The condition acceptance rate is thus:
\[
\frac{2v}{v + x + \alpha(z_2 - x)} \left\{ \frac{v + x + \alpha(\hat{x} - x)}{2v} + \frac{\alpha(z_2 - x)}{2v} \left( 1 + \frac{z_2 - x}{2v} (1 - p_1) \right) \right\}
\]
which leads to equation (17).

Whenever \( x > \hat{x} \),

<table>
<thead>
<tr>
<th>Preference</th>
<th>Proba to have this preference</th>
<th>Proba to accept a position in 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>(-v &lt; c_{1j} - c_{2j} &lt; \hat{x} )</td>
<td>( \frac{v + x}{2v} )</td>
<td>1</td>
</tr>
<tr>
<td>( \hat{x} &lt; c_{1j} - c_{2j} &lt; x )</td>
<td>( \frac{x - \hat{x}}{2v} )</td>
<td>( 1 - \alpha p_1 )</td>
</tr>
<tr>
<td>( x &lt; c_{1j} - c_{2j} &lt; z_2 )</td>
<td>( \frac{\alpha(z_2 - x)}{2v} )</td>
<td>( 1 - p_1 )</td>
</tr>
</tbody>
</table>

The condition acceptance rate is thus:
\[
\frac{2v}{v + x + \alpha(z_2 - x)} \left\{ \frac{v + \hat{x} + x - \hat{x}}{2v} \left( 1 - \alpha p_1 \right) + \frac{z_2 - x}{2v} \left( 1 - p_1 \right) \right\}
\]
which leads again to equation (17).

### D Cross partial derivative of unemployment rates

The cross-partial derivatives of the unemployment rates with respect to tightness in the other region are respectively:

\[
\text{sign} \left\{ \frac{\partial u_1}{\partial \theta_2} \right\} = \text{sign} \{ \alpha p_1 \} \left\{ (v - x)(x - \Delta) - \alpha(z_2 - x)(x - z_1) \right\}
\]

\[
\text{sign} \left\{ \frac{\partial u_2}{\partial \theta_1} \right\} = \text{sign} \{ \alpha p_2 \} \left\{ (v + x)(\Delta - x) - \alpha(z_2 - x)(x - z_1) \right\}
\]

We directly notice that one of these two derivatives is always negative:

\[
x - \Delta < 0 \quad \text{i.e.} \quad p_2 \beta_2(y_2 - b_2) < p_1 \beta_1(y_1 - b_1) \Rightarrow \frac{\partial u_2}{\partial \theta_1} \geq 0 \quad \text{and} \quad \frac{\partial u_1}{\partial \theta_2} < 0,
\]
\[
x - \Delta > 0 \quad \text{i.e.} \quad p_2 \beta_2(y_2 - b_2) > p_1 \beta_1(y_1 - b_1) \Rightarrow \frac{\partial u_1}{\partial \theta_1} \geq 0 \quad \text{and} \quad \frac{\partial u_2}{\partial \theta_2} < 0.
\]

To limit the extent of these ambiguities, we first compute the limits when \( \alpha \) tends to 0 and to 1. We then check how the functions could behave between these two limits.

From (47), (48) and (14), the following limit values are obvious:

\[
\lim_{\alpha \to 0} \frac{\partial u_1}{\partial \theta_2} = \lim_{\alpha \to 0} \frac{\partial u_2}{\partial \theta_1} = 0;
\]

\[
\lim_{\alpha \to 1} \text{sign} \left\{ \frac{\partial u_1}{\partial \theta_2} \right\} = \lim_{\alpha \to 1} \text{sign} \left\{ \frac{\partial u_2}{\partial \theta_1} \right\} = \text{sign} \{ - (z_2 - x)(x - z_1) \} < 0.
\]
It can furthermore be shown\(^{28}\) that:

\[
\lim_{\alpha \to 0^+} \operatorname{sign} \left\{ \frac{\partial u_1}{\partial \theta_2} \right\} = \operatorname{sign} \left\{ (x - \Delta)(v - x) \right\},
\]

\[
\lim_{\alpha \to 0^+} \operatorname{sign} \left\{ \frac{\partial u_2}{\partial \theta_1} \right\} = \operatorname{sign} \left\{ (\Delta - x)(v + x) \right\}.
\]

Between the two extremes, we cannot go further in the analysis. Because of the limit when \(\alpha \to 1\), we however can conclude that:

- When \(x - \Delta > 0\), \(\frac{\partial u_2}{\partial \theta_1} < 0 \forall \alpha > 0\) and \(\frac{\partial u_1}{\partial \theta_2} < 0\) if \(\alpha\) is large enough;
- When \(x - \Delta < 0\), \(\frac{\partial u_1}{\partial \theta_2} < 0 \forall \alpha > 0\) and \(\frac{\partial u_2}{\partial \theta_1} < 0\) if \(\alpha\) is large enough;
- When \(x - \Delta = 0\), both partial derivatives are always negative.

## E Existence of an equilibrium

### E.1 Existence of a symmetric equilibrium when regions are symmetric

When both regions have the same exogenous characteristics, a free-entry symmetric equilibrium is characterized by the following conditions:

\[
\dot{x} = x = 0
\]
\[
z_2 = \beta(y - b) = -z_1
\]
\[
\pi m(\theta) = \frac{\kappa}{(1 - \beta)(y - b)}
\]
\[
\pi = 1 - \frac{\alpha p(\theta) z_2}{v + \alpha z_2}
\]
\[
u = \frac{(1 - p(\theta))(v - \alpha p(\theta) z_2)}{v + \alpha z_2}
\]
\[
N_1^P = N_2^P = 1/2
\]

and (20) and (21). This system of equations can be solved recursively. First, the \(z_1\) and \(z_2\) thresholds are functions of parameters only. Second, combining equations (49) and (50) leads to the following implicit relationship in equilibrium tightness:

\[
\frac{v + \alpha(1 - p(\theta)) \beta(y - b)}{v + \alpha \beta(y - b)} = \frac{1}{m(\theta)} \frac{\kappa}{(1 - \beta)(y - b)}
\]

\(^{28}\)A proof is available upon request. It can actually be shown that the partial derivatives (47) and (48) have the same sign as a polynomial of degree four in \(\alpha\), whose constant term is respectively \((v - x)(x - \Delta)\) and \((v + x)(\Delta - x)\).
The left-hand side is a negative function of tightness, while the right-hand side depends positively on tightness. So, there is at most one equilibrium. To show the existence of the equilibrium, consider the limit of each side of the last equality when \( \theta \) tends to 0:

\[
\lim_{\theta \to 0} \frac{v + \alpha(1 - p(\theta))\beta(y - b)}{v + \alpha(1 - p(\theta))\beta(y - b)} = 1
\]

\[
\lim_{\theta \to 0} \frac{\kappa}{m(\theta)(1 - \beta)(y - b)} = 0, \text{ by the Inada conditions.}
\]

So, a unique symmetric equilibrium tightness exists. The other endogenous variables are then determined uniquely as well.

**F Ex-ante bargaining**

The aim of this appendix is to check whether the *ex-ante* bargaining process leads to an efficient allocation under the Hosios condition. In the five following subsections, we derive and briefly explain the key equations that are modified compared to the model described in the paper. In the last subsection, we compare the decentralized equilibrium free-entry conditions with the efficient allocation, and show that the Hosios condition is never sufficient to restore efficiency.

When bargaining *ex-ante*, the timing is defined as follows\(^{29}\):

1. Workers decide where to locate and where to search.
2. Firms open vacancies and the matching process takes place.
3. In case a worker gets two offers, she chooses one of them.
4. Workers bargain with the firm.
5. Workers relocate if the accepted position is in the other region.
6. Production takes place and both the housing and the good markets clear.

**F.1 Wage bargaining**

Since there are, for each worker, two potential fall-back positions when bargaining (being unemployed in region 1 or in region 2), there will be four wages in the economy, which may depend on the worker’s relative idiosyncratic preference. Let \( w_{ikj} \) denote the wage of individual \( j \) living in region \( k \) *ex-ante* and who works for a firm located in \( i \) and \( V_{ikj}^e \) the utility she gets in this case. The wage \( w_{ikj} \) verifies:

\[
\max_{w_{ikj}} (V_{ikj}^e - V_{ikj}^u) \beta_i (J_{ij})^{1 - \beta_i}
\]

\(^{29}\)An alternative timing would be that workers who get two job offers bargain with the firms before refusing one of the job offers. This would lead to Bertrand competition between the two firms for this worker. It has however been checked that this lead to further sources of inefficiencies.
where \[ V_{ikj}^e - V_{kj}^u = w_{ikj} - b_k + a_i - a_k + c_{ij} - c_{kj} \]

This leads to the following wages:

\[ w_{ikj} = \beta_i y_i + (1 - \beta_i) b_i - (1 - \beta_i) (b_i - b_k + a_i - a_k + (c_{ij} - c_{kj})) \] (51)

or

\[ w_{11j} = \beta_1 y_1 + (1 - \beta_1) b_1 \]
\[ w_{12j} = \beta_1 y_1 + (1 - \beta_1) b_1 + (1 - \beta_1) (\Delta - (c_{1j} - c_{2j})) \] (52)
\[ w_{22j} = \beta_2 y_2 + (1 - \beta_2) b_2 \]
\[ w_{21j} = \beta_2 y_2 + (1 - \beta_2) b_2 - (1 - \beta_2) (\Delta - (c_{1j} - c_{2j})) \] (53)

It is worth mentioning that *ex-ante* bargaining allows workers coming from the other region to be compensated for the difference in leisure, amenities and idiosyncratic preferences (see the term \( \Delta - (c_{1j} - c_{2j}) \) in (52) and (53)).

### F.2 Acceptance decisions

Acceptance decisions are conditional on the region of residence, as wages in the other region depend on it. Workers located in region 1 decide whether they prefer working in region 1 by comparing \( V^e_{11j} \) and \( V^u_{21j} \). This leads to the following threshold:

\[ \hat{x}_2 = \Delta + y_2 - b_2 - \beta_1 \beta_2 (y_1 - b_1) \]

Considering now workers located in region 2 *ex-ante*, one gets:

\[ \hat{x}_1 = \Delta + \beta_2 \beta_1 (y_2 - b_2) - (y_1 - b_1) \]

When regions are asymmetric, these two thresholds are equal if and only if \( \beta_1 = \beta_2 \). As we aim at checking whether an *ex-ante* bargaining process leads to an efficient allocation under Hosios, we make the assumption that \( \beta_1 = \beta_2 \), so that we get a unique threshold \( \hat{x} \). This assumption is made as the central planner would always choose a unique \( \hat{x} \) threshold. For, assume that \( \beta_1 \neq \beta_2 \), so that there exist two threshold values \( \hat{x}_1 \) and \( \hat{x}_2 \). The only possibility that these two thresholds are simultaneously meaningful is when we have \( \hat{x}_1 < x < \hat{x}_2 \).\(^{30}\) This case is represented by Figure 6 and is never optimal. For, take a first worker whose relative preference for region 1 over region 2 lies between \( \hat{x}_1 \) and \( x \). This worker would choose to work in region 1. Take then a second worker whose relative preference lies between \( x \) and \( \hat{x}_2 \). She chooses to work in region 2. As assumed in the paper, workers are homogenous in productivity in a given region. Thus, interchanging these 2 workers (the first worker would then work in region

\(^{30}\)This situation assumes that the threshold \( x \) is unique. This unicity is verified with the assumption we make regarding the \( \beta \)'s.
2 and the second one in region 1) does not modify the total levels of production nor the ex-post population sizes. However, this will change the levels of workers’ idiosyncratic preferences. In this regard, interchanging the first worker and the second worker would lead to a higher level of preferences (as workers having a relative preference between $x$ and $\hat{x}_2$ value more region 1 relative to region 2 than workers with a relative preference between $\hat{x}_1$ and $x$), without modifying the level of production nor the population sizes. The central planner thus prefers this situation than the initial one. This induces that having two distinct thresholds $\hat{x}$ is never optimal. Therefore, for the rest of this appendix, we assume that $\beta_1 = \beta_2 = \beta$, so that we do not face a multi-thresholds equilibrium for the acceptance decision.

![Diagram](image)

Figure 6: Case of two different $\hat{x}$ thresholds

### F.3 Opening of vacancies

We assume free-entry of firms. Firms open vacancies until the expected profit is nil:

$$\pi_i m_i(\theta_i)(y_i - w^e_i) - \kappa_i = 0,$$

where $w^e_i$ is the wage a firm located in region $i$ is expected to pay. This can be rewritten as:

$$\frac{\kappa_i}{m_i(\theta_i)} = \pi_i(y_i - w^e_i)$$

### F.4 Location and search decisions

The location and search decisions are taken simultaneously. With the assumption on the $\beta$’s, it is easily shown that taking the decision simultaneously of choosing first where to locate and then where to search is equivalent (one needs to proceed as in Appendix A). To ease the exposition, we focus on the second procedure.
F.4.1 Search decision

A worker located in region 2 searches regionally when:

\[0 > -\alpha p_1 p_2 V_{22j}^e + \alpha p_1 p_2 \left[ \max \{ V_{22j}^e; V_{12j}^e \} - \epsilon \right] + \alpha p_1 (1 - p_2) (V_{12j}^e - V_{2j}^e)\]

Assuming that \(\epsilon\) tends to 0, one gets the following threshold value whenever \(V_{22j}^e \geq V_{12j}^e\):

\[z_1 = \Delta - (y_1 - b_1)\]  \hspace{1cm} (54)

When \(V_{22j}^e < V_{12j}^e\), one gets:

\[\tilde{z}_1 = \Delta - (y_1 - b_1) + p_2(y_2 - b_2)\]

However, \(V_{22j}^e < V_{12j}^e\) implies that:

\[c_{1j} - c_{2j} = \Delta - (y_1 - b_1) + y_2 - b_2 > \tilde{z}_1\]

which leads to a contradiction, as we assume in the definition of \(\tilde{z}_1\) that \(V_{22j}^e < V_{12j}^e\). We thus get a unique threshold value, \(z_1\), for the search decision of agents located in region 2.

Similarly, we get the following threshold for workers located in region 1:

\[z_2 = \Delta + y_2 - b_2\]  \hspace{1cm} (55)

F.4.2 Location choice

The marginal worker that decides where to locate is a national job-seeker. One thus needs to compare the expected utility of a national job-seeker in both regions. One gets as threshold value:

\[x = \Delta_2 + \frac{(1 - \alpha)(p_2 \beta(y_2 - b_2) - p_1 \beta(y_1 - b_1))}{1 - \alpha \beta p_1 - \alpha \beta p_2 + \alpha \beta p_1 p_2}\]  \hspace{1cm} (56)

F.5 Population, acceptance rates and expected wages

Populations can be described as in the paper, using the new threshold definitions. Acceptance rates are given, as in the ex-post bargaining case, by equations (16)-(17). Knowing how workers locate and search, one can compute the expected wage of a firm. In region 1, the wage bill can be rewritten as:

\[\frac{N}{2v} (\beta y_1 + (1 - \beta) b_1) [p_1 (v - x) + \alpha p_1 (1 - p_2) (x - z_1) + \alpha p_1 p_2 (x - \hat{x})] + \frac{N}{2v} (1 - \beta) \alpha p_1 (1 - p_2) \int_{z_1}^{x} (\Delta - (c_{1j} - c_{2j})) dj + \max \left\{ \frac{x - \hat{x}; 0} {x - \hat{x}} \right\} \frac{N}{2v} (1 - \beta) \alpha p_1 p_2 \int_{\hat{x}}^{x} (\Delta - (c_{1j} - c_{2j})) dj\]
As the number of employed workers in region 1 is given by

\[ L_1 = \frac{N}{2v} [p_1(v - x) + \alpha p_1 (1 - p_2)(x - z_1) + \alpha p_1 p_2 (x - \hat{x})], \]

we can write the expected wage in region 1 as:

\[ w_1^e = \beta y_1 + (1 - \beta) b_1 + \frac{(1 - \beta)(1 - p_2)\alpha (x - z_1)(\Delta - \frac{x + z_1}{2})}{v - x + \alpha (x - z_1) - \alpha p_2(\hat{x} - x)} \]

\[ + \frac{(1 - \beta)\alpha p_2 \max \{x - \hat{x}; 0\} (\Delta - \frac{x + \hat{x}}{2})}{v - x + \alpha (x - z_1) - \alpha p_2(\hat{x} - z_1)} \]

So, plugging this equation in the free-entry condition yields:

\[ \frac{\kappa_1}{(1 - \beta)m_1(\theta_1)} = \frac{\pi_1(1 - b_1) - \frac{\alpha (1 - p_2)(x - z_1)}{v - x + \alpha (x - z_1)} \left( \Delta - \frac{x + z_1}{2} \right)}{v - x + \alpha (x - z_1) - \alpha p_2(\hat{x} - x)} \]

\[ - \frac{\alpha p_2 \max \{x - \hat{x}; 0\} (\Delta - \frac{x + \hat{x}}{2})}{v - x + \alpha (x - z_1) - \alpha p_2(\hat{x} - z_1)} \] (57)

Similarly, one gets for the expected wage in region 2:

\[ w_2^e = \beta y_2 + (1 - \beta) b_2 - \frac{(1 - \beta)(1 - p_1)\alpha (z_2 - x)(\Delta - \frac{z_2 + x}{2})}{v + x + \alpha (z_2 - x) + \alpha p_1(\hat{x} - x)} \]

\[ - \frac{(1 - \beta)\alpha p_1 \max \{\hat{x} - x; 0\} (\Delta - \frac{z_2 + \hat{x}}{2})}{v + x + \alpha (z_2 - x) - \alpha p_1(z_2 - \hat{x})} \]

so that the free-entry condition in region 2 writes:

\[ \frac{\kappa_2}{(1 - \beta)m_2(\theta_2)} = \frac{\pi_2(y_2 - b_2) + \frac{\alpha (1 - p_1)(z_2 - x)}{v + x + \alpha (z_2 - x)} \left( \Delta - \frac{z_2 + x}{2} \right)}{v + x + \alpha (z_2 - x) + \alpha p_1(\hat{x} - x)} \]

\[ + \frac{\alpha p_1 \max \{\hat{x} - x; 0\} (\Delta - \frac{z_2 + \hat{x}}{2})}{v + x + \alpha (z_2 - x) - \alpha p_1(z_2 - \hat{x})} \] (58)

### F.6 Efficiency

Comparing (57)-(58) with (35)-(36), one directly sees that the ex-ante bargaining helps restoring efficiency, but is not sufficient.

First, regarding the search thresholds definitions (equations (38)-(39) and (54)-(55)), we notice that it partially corrects the search decisions, by taking the whole surplus formed rather than a share \( \beta \). Workers however still do not take into account the impact of their search decision on the opening of vacancies in the other region.

Second, even if thresholds were set optimally, firms do not open the optimal number of vacancies. This is due to the fact that workers get compensated for leisure, amenities, rents and idiosyncratic preferences by a firm if they are currently working in it, while the central planner compensate all workers. For example, for a firm settled in region 1, the third term on the RHS of (35) is equivalent to the second term on the RHS of (57).
Furthermore, if $x > \dot{x}$, then the fifth term in (35) corresponds to the third term of (57). As workers, when getting two job offers and being located in region 2, prefer to work in region 1, firms located in region 1 compensate them for not living in region 2 anymore. However, when deciding how many vacancies they open in region 1, firms do not take into account the implications of their decisions on the workers who live in region 1 but accept a job offer in region 2. Such an event is of course influenced among other things by the number of vacancies created in region 1.

Furthermore, *ex-ante* bargaining does not allow to internalize the loss in output in the other region (in (57)-(58) there is no expression corresponding to the second term on the RHS of (35)-(36) because when they bargain over wages agents have no reason to internalize the induced effect of the wage *via* vacancy creation on the net output created in the other region).