Strategic Fertility, Education Choices, and Conflicts in Deeply Divided Societies
Online Appendix

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A Proofs of Propositions

A.1 Existence of the Nash equilibrium

A Nash equilibrium can also be defined in terms of the best response functions as a strategy profile $(n_t^{a\star}, n_t^{b\star}, e_t^{a\star}, e_t^{b\star}) = (n^a(x_t), n^b(x_t), e^a(x_t), e^b(x_t))$ such that $\forall x_t \in [0,1]$

$$\begin{cases} \mathbf{B}_{x_t}(n_t^{b\star}, e_t^{b\star}) = \arg\max_{n_t^a, e_t^a} V_t(n_t^a, n_t^{b\star}, e_t^a, e_t^{b\star}, x_t), \\ \mathbf{B}_{1-x_t}(n_t^{a\star}, e_t^{a\star}) = \arg\max_{n_t^b, e_t^b} V_t(n_t^{a\star}, n_t^b, e_t^{a\star}, e_t^b, 1-x_t). \end{cases}$$

In what follows, we denote $\mathbf{s}^i = (n^i, e^i)$. Define the set of best response functions $\mathbf{B} : \mathcal{X}^2 \to \mathcal{X}^2$ given by $\mathbf{B}(\mathbf{s}^a, \mathbf{s}^b) = \{\mathbf{B}_{x_t}(\mathbf{s}^b), \mathbf{B}_{1-x_t}(\mathbf{s}^a)\}$.

Lemma 1 *Suppose that $\alpha \leq 1/2$. There exist $\hat{\mu} \in (1, +\infty]$, $\bar{\mu} \in (1, +\infty]$, $\bar{\lambda} \in \mathbb{R}_+$, $\bar{\gamma} \in \mathbb{R}_+$ such that $\forall \mu \leq \min\{\hat{\mu}, \bar{\mu}\}$, $\forall \lambda < \bar{\lambda}$ and $\forall \gamma < \bar{\gamma}$, $\boldsymbol{B}$ is a non-empty convex-valued function.*

Proof.

Existence of the function B.

First, because the function V_t is continuous and defined on a compact set, by Weirstrass's theorem we deduce that there exists a function $\mathbf{B}_{x_t} : [0, \bar{n}] \times [0, \bar{e}] \to [0, \bar{n}] \times [0, \bar{e}]$ and a function $\mathbf{B}_{1-x_t} : [0, \bar{n}] \times [0, \bar{e}] \to [0, \bar{n}] \times [0, \bar{e}]$ given by

$$\mathbf{B}_{x_t}(n_t^b, e_t^b) = \arg\max_{n_t^a, e_t^a} V_t(n_t^a, n_t^b, e_t^{ia}, e_t^b, x_t),$$
$$\mathbf{B}_{1-x_t}(n_t^a, e_t^a) = \arg\max_{n_t^b, e_t^b} V_t(n_t^b, n_t^a, e_t^b, e_t^a, 1-x_t).$$

Hence there exists a function $\mathbf{B} : [0, \bar{n}]^2 \times [0, \bar{e}]^2 \to [0, \bar{n}]^2 \times [0, \bar{e}]^2$ given by $\mathbf{B}(\mathbf{s}^a, \mathbf{s}^b) = \{\mathbf{B}_{x_t}(\mathbf{s}^b), \mathbf{B}_{1-x_t}(\mathbf{s}^a)\}$.

Convexity of the set $\{(\mathbf{s}^a, \mathbf{s}^b), \mathbf{B}(\mathbf{s}^a, \mathbf{s}^b)\}$.

Let us skip time indexation.

Define the following functions:

$f : \mathbb{R}^+ \to \mathbb{R}^+$ given by $f(e) = e^\rho$, $\rho \in [0,1]$

$g : \mathbb{R}^+ \to \mathbb{R}^+$ given by $g(e) = e^{\rho\mu}$, $\rho \in [0,1]$, $\mu \in [1, +\infty)$.

$F : \mathbb{R}^{+2} \to \mathbb{R}^+$ given by

$$F(X,Y) = X(1-\alpha)(x^i X + B)^{-\alpha} + \frac{Y}{x^i Y + B'} \alpha (x^i X + B)^{1-\alpha}$$

with $B = f(e^{-i})n^{-i}$, $B' = g(e^{-i})n^{-i}$.

$\tilde{F} : \mathbb{R}^{+2} \to \mathbb{R}^{+}$ given by $\tilde{F}(n^i, e^i) = F(f(e^i)n^i, g(e^i)n^i)$.

$\hat{F} : \mathbb{R}^{+4} \to \mathbb{R}^{+}$ given by $\hat{F}(n^i, e^i, \lambda, \gamma) = \tilde{F}(n^i, e^i) - \frac{\lambda}{2}(n^i)^2 - \gamma n^i e^i$.

Let us express the first and second derivatives of F with respect to X and Y.

$$
\begin{aligned}
F_X &\equiv \frac{\partial F}{\partial X} = (1-\alpha)(x^i X + B)^{-\alpha}\left(1 - \alpha\frac{x^i X}{x^i X + B} + \alpha\frac{x^i Y}{x^i Y + B'}\right), \\
F_Y &\equiv \frac{\partial F}{\partial Y} = \alpha(x^i X + B)^{1-\alpha}\frac{B'}{(x^i Y + B')^2}, \\
F_{XX} &\equiv \frac{\partial^2 F}{\partial X^2} = x^i\alpha(1-\alpha)(x^i X + B)^{-\alpha-1}\left(-2 + (1+\alpha)\frac{x^i X}{x^i X + B} - \alpha\frac{x^i Y}{x^i Y + B'}\right), \\
F_{YY} &\equiv \frac{\partial^2 F}{\partial Y^2} = -2\alpha(x^i X + B)^{1-\alpha}\frac{x^i B'}{(x^i Y + B')^3}, \\
F_{XY} &\equiv \frac{\partial^2 F}{\partial X \partial Y} = x^i(1-\alpha)\alpha(x^i X + B)^{-\alpha}\frac{B'}{(x^i Y + B')^2}.
\end{aligned}
$$

We have $F_Y > 0$, $F_{YY} < 0$, $F_{XY} > 0$. Using $x^i X/(x^i X + B) < 1$ and $x^i Y/(x^i Y + B') < 1$, we easily find $F_X > 0$ $F_{XX} < 0$.

We have $\hat{F}(n^i, e^i, \lambda, \gamma) = V(n^i, n^{-i}, e^i, e^{-i}, x^i)$. Then, V is quasi-concave in (n^i, e^i), if and only if $\hat{F}$ is quasi-concave in (n^i, e^i). To determine whether $\hat{F}$ is quasi-concave, let us express the bordered Hessian matrix of $\hat{F}$ which we denote by $H(n^i, e^i)$.

$$
H(n^i, e^i) = \begin{pmatrix} 0 & \hat{F}_n & \hat{F}_e \\ \hat{F}_n & \hat{F}_{nn} & \hat{F}_{ne} \\ \hat{F}_e & \hat{F}_{ne} & \hat{F}_{ee} \end{pmatrix}
$$

where

$$
\begin{aligned}
\hat{F}_n &= \tilde{F}_n - \lambda n^i - \gamma e^i, \\
\hat{F}_e &= \tilde{F}_e - \gamma n^i, \\
\hat{F}_{nn} &= \tilde{F}_{nn} - \lambda, \\
\hat{F}_{ne} &= \tilde{F}_{ne} - \gamma, \\
\hat{F}_{ee} &= \tilde{F}_{ee},
\end{aligned}
$$

and

$$
\begin{aligned}
\tilde{F}_n &= F_X f(e^i) + F_Y g(e^i), \\
\tilde{F}_e &= F_X f'(e^i)n^i + F_Y g'(e^i)n^i, \\
\tilde{F}_{nn} &= F_{XX}(f(e^i))^2 + F_{YY}(g(e^i))^2 + 2F_{XY}f(e^i)g(e^i), \\
\tilde{F}_{ee} &= F_X f''(e^i)n^i + F_Y g''(e^i)n^i + F_{XX}(f'(e^i)n^i)^2 + F_{YY}(g'(e^i)n^i)^2 + 2F_{XY}f'(e^i)g'(e^i)(n^i)^2, \\
\tilde{F}_{en} &= F_X f'(e^i) + F_Y g'(e^i) + F_{XX}f'(e^i)f(e^i)n^i + F_{YY}g'(e^i)g(e^i)n^i + \\
&\qquad F_{XY}f'(e^i)g(e^i)n^i + F_{XY}g'(e^i)f(e^i)n^i.
\end{aligned}
$$

The function $\hat{F}$ is quasi-concave if the determinant of H is positive. The determinant of H is given by

$$\begin{aligned}
Det(H) =& -\left(\hat{F}_n\right)^2 \hat{F}_{ee} + 2\hat{F}_n\hat{F}_e\hat{F}_{en} - \left(\hat{F}_e\right)^2 \hat{F}_{nn} \\
=& -\left(\tilde{F}_n - \lambda n^i - \gamma e^i\right)^2 \tilde{F}_{ee} + 2\left(\tilde{F}_n - \lambda n^i - \gamma e^i\right)\left(\tilde{F}_e - \gamma n^i\right)\left(\tilde{F}_{en} - \gamma\right) \\
& - \left(\tilde{F}_e - \gamma n^i\right)^2 \left(\tilde{F}_{nn} - \gamma\right)
\end{aligned}$$

Define $D : \mathbb{R}^{+2} \to \mathbb{R}^+$ given by $D(\lambda, \gamma) = Det(H)$.

We will first interest in $D(0,0)$. Note that we have $\tilde{F}_n > 0$ and $\tilde{F}_e \geq 0$ so that sufficient conditions to have $D(0,0) > 0$ are

$$\tilde{F}_{nn} < 0, \text{ and } \tilde{F}_{ee} < 0, \text{ and } \tilde{F}_{en} > 0.$$

Now, we will show there exist $\hat{\mu} \in (1, +\infty]$, $\bar{\mu} \in (1, +\infty]$, such that $\forall \mu \leq \min\{\hat{\mu}, \bar{\mu}\}$ and $\alpha < 1/2$, then those conditions are satisfied.

1. We start with $\tilde{F}_{nn}$ for which we give two expressions.

$$\begin{aligned}
\tilde{F}_{nn} =& \frac{f(e^i)}{n^i} x^i \alpha(1-\alpha)(x^iX + B)^{-\alpha} \left(2\frac{B'Y}{(x^iY + B')^2}\right. \\
& \left. + \frac{x^iX}{x^iX + B}\left(-2 + (1+\alpha)\frac{x^iX}{x^iX+B} - \alpha\frac{x^iY}{x^iY + B'}\right)\right) + F_{YY}(g(e^i))^2,
\end{aligned}$$

or

$$\begin{aligned}
\tilde{F}_{nn} =& 2g(e^i)x^i\alpha\frac{(x^iX + B)^{-\alpha}B'}{(x^iY + B')^2}\left((1-\alpha) - \frac{(x^iX + B)/x^iX}{(x^iY + B')/x^iY}\right). \\
& + F_{XX}(f(e^i))^2,
\end{aligned}$$

where $X \equiv f(e^i)n^i$ and $Y \equiv g(e^i)n^i$.

Since $F_{XX} < 0$ and $F_{YY} < 0$. A sufficient condition to have $\tilde{F}_{nn} < 0$ is

$$2\frac{x^iB'Y}{(x^iY + B')^2} + \frac{x^iX}{x^iX + B}\left(-2 + (1+\alpha)\frac{x^iX}{x^iX + B} - \alpha\frac{x^iY}{x^iY + B'}\right) < 0,$$

or

$$(1-\alpha) - \frac{(x^iX + B)/x^iX}{(x^iY + B')/x^iY} < 0.$$

Let us introduce new notations. We set $z \equiv (n^ix^i)/(n^{-i}x^{-i})$ and $c \equiv (e^i/e^{-i})^\rho$. The above conditions rewrites as

$$2\frac{1}{(1 + c^{-\mu}z^{-1})(1 + c^\mu z)} + \frac{1}{(1 + c^{-1}z^{-1})}\left(-2 + (1+\alpha)\frac{1}{(1 + c^{-1}z^{-1})} - \alpha\frac{1}{(1 + c^{-\mu}z^{-1})}\right) < 0,$$

or

$$(1-\alpha) - \frac{(1 + c^{-1}z^{-1})}{(1 + c^{-\mu}z^{-1})} < 0.$$

We will consider several cases : (i) $c > 1$, (ii)-a $c < 1$ and $cz < 1$, (ii)-b $c < 1$ and $cz > 1$.

(i) The second inequality holds whenever $(1 + c^{-1}z^{-1})/(1 + c^{-\mu}z^{-1}) > 1$ which is equivalent to $c > 1$. Hence when $c > 1$ we have $\tilde{F}_{nn} < 0$.

Consider the case $c < 1$ and look at the first inequality.

Define the functions $Z : [1, +\infty) \to \mathbb{R}^+$, $L : \mathbb{R}^+ \to \mathbb{R}^+$ and $G : \mathbb{R}^+ \to \mathbb{R}^+$, $\tilde{G} : [1, +\infty) \to \mathbb{R}^+$ respectively given by

$$
\begin{aligned}
Z(\mu) &= c^\mu z \\
L(\tilde{z}) &= \frac{1}{(1 + \tilde{z}^{-1})} \left(-2 + (1 + \alpha) \frac{1}{(1 + \tilde{z}^{-1})} \right), \\
\\
G(Z) &= 2 \frac{1}{(1 + Z)(1 + Z^{-1})} + L(\tilde{z}), \\
\tilde{G}(\mu) &= G(Z(\mu)).
\end{aligned}
$$

(ii)- a A sufficient condition for the first inequality to hold is $\tilde{G}(\mu) < 0$. Suppose that $c < 1$. We show that provided that $cz < 1$, $\forall \mu \in [1, +\infty)$, $\tilde{G}(\mu) < 0$.

To do so first we can compute $G(1)$. We find

$$
\begin{aligned}
\tilde{G}(1) &= \frac{1}{(1 + c^{-1}z^{-1})} \left(\frac{2}{(1 + cz)} - 2 + \frac{1}{(1 + c^{-1}z^{-1})} \right) \\
&= -\frac{1}{(1 + c^{-1}z^{-1})(1 + c^{-1}z^{-1})(1 + cz)} < 0.
\end{aligned}
$$

Second, let us compute the derivative of $\tilde{G}$ with respect to μ. We obtain

$$
\tilde{G}'(\mu) = G'(Z(\mu))Z'(\mu).
$$

We easily show that $G'(Z(\mu)) > 0$ if and only if $Z(\mu) < 1$ and $Z'(\mu) > 0$ if and only if $c > 1$. By assumption $c < 1$ so that $Z'(\mu) < 0$. Suppose, in addition that $cz < 1$. It implies $c^\mu z < 1$ equivalent to $G'(Z(\mu)) > 0$ so that we deduce $\tilde{G}'(\mu) < 0$ which, with the continuity of G, allows us to deduce that $\tilde{G}(\mu) < 0$ $\forall \mu \in [1, +\infty)$.

(ii)- b Finally consider the case $c < 1$ and $cz > 1$. Note that $G(Z) < 1/2 + L(\tilde{z})$ $\forall Z \in \mathbb{R}^+$. A sufficient condition to have $G(Z) < 0$ (which in turn implies that the first inequality holds) is $L(\tilde{z}) < -1/2$. The function L is convex, decreasing for all $\tilde{z} \in [0, 1/\alpha]$ and increasing for all $\tilde{z} > 1/\alpha$. We have $L(1) = -1/4(3 + \alpha) < -1/2$ and $\lim_{\tilde{z} \to \infty} L = -(1 - \alpha)$ which is lower than one half whenever $\alpha \leq 1/2$. Hence we deduce that $L(\tilde{z}) < -1/2$ which implies that the second inequality holds.

Finally we can deduce that $\tilde{F}_{nn} < 0$.

2. Consider the case $\tilde{F}_{en}$.

$$
\begin{aligned}
\tilde{F}_{en} =& f^{'}(e^i)(1-\alpha)(x^iX+B)^{-\alpha}\left[1-\alpha\frac{x^iX}{x^iX+B}+\alpha\frac{x^iY}{x^iY+B'}\cdot\right.\\
&\left.+\alpha\frac{x^iX}{x^iX+B}\left(-2+(1+\alpha)\frac{x^iX}{x^iX+B}-\alpha\frac{x^iY}{x^iY+B'}\right)+\alpha\frac{x^iYB'}{(x^iY+B')^2}\right]\\
&+\alpha g^{'}(e^i)\frac{(x^iX+B)^{1-\alpha}B'}{(x^iY+B')^2}\left(1-2\frac{x^iY}{x^iY+B'}+(1-\alpha)\frac{x^iX}{x^iX+B}\right).
\end{aligned}
$$

Use the notations $z\equiv(n^ix^i)/(n^{-i}x^{-i})$ and $c\equiv(e^i/e^{-i})^\rho$ and define the function $M:[1,+\infty)\to\mathbb{R}$ given by

$$
\begin{aligned}
M(\mu) =& \rho(1-\alpha)\frac{1}{1+c^{-1}z^{-1}}\left[1-\alpha\frac{1}{1+c^{-1}z^{-1}}+\alpha\frac{1}{1+c^{-\mu}z^{-1}}\cdot\right.\\
&\left.+\alpha\frac{1}{1+c^{-1}z^{-1}}\left(-2+(1+\alpha)\frac{1}{1+c^{-1}z^{-1}}-\alpha\frac{1}{1+c^{-\mu}z^{-1}}\right)+\alpha\frac{1}{1+c^{-\mu}z^{-1}}\frac{1}{1+c^{\mu}z}\right]\\
&+\alpha\rho\mu\frac{1}{1+c^{-\mu}z^{-1}}\frac{1}{1+c^{\mu}z}\left(1-2\frac{1}{1+c^{-\mu}z^{-1}}+(1-\alpha)\frac{1}{1+c^{-1}z^{-1}}\right).
\end{aligned}
$$

The inequality $\tilde{F}_{en}>0$ is equivalent to $M(\mu)>0$.

Let us compute $M(1)$. We obtain

$$
\begin{aligned}
M(1) =& \rho(1-\alpha)\frac{1}{1+c^{-1}z^{-1}}\left[1-2\alpha\frac{1}{1+c^{-1}z^{-1}}+\alpha\left(\frac{1}{1+c^{-1}z^{-1}}\right)^2\right]\\
&+\alpha\rho\frac{1}{1+c^{-1}z^{-1}}\frac{1}{1+cz}\left(2-\alpha-(1+\alpha)\frac{1}{1+c^{-1}z^{-1}}\right).
\end{aligned}
$$

Since $\frac{1}{1+c^{-1}z^{-1}}<1$ we deduce that

$$
\begin{aligned}
M(1) >& \rho(1-\alpha)\frac{1}{1+c^{-1}z^{-1}}\left[1-2\alpha+\alpha\left(\frac{1}{1+c^{-1}z^{-1}}\right)^2\right]\\
&+\alpha\rho\frac{1}{1+c^{-1}z^{-1}}\frac{1}{1+cz}\left(1-2\alpha\right).
\end{aligned}
$$

Suppose that $\alpha\leq 1/2$. Then we the RHS is positive so that $M(1)>0$. Given that the function M is continuous on $[1,+\infty)$, we deduce that there exists $\hat{\mu}\in]1,+\infty]$ such that $\forall\mu\leq\hat{\mu}$ $M(\mu)\geq 0$ which is equivalent to $\tilde{F}_{en}\geq 0$.

3. Consider the case $\tilde{F}_{ee}$.

$$
\begin{aligned}
&\tilde{F}_{ee}=F_Xf^{''}(e^i)n^i+F_{XX}(f^{'}(e^i)n^i)^2+\\
&\rho\mu\alpha\frac{B^{'}Y}{(xY+B')^2}\left(2\rho(1-\alpha)\frac{xX}{(xX+B)}-2\rho\mu\frac{xY}{(xY+B')}+\rho\mu-1\right)
\end{aligned}
$$

The terms of the first line are all negative. Hence, $\tilde{F}_{ee}$ is negative if the term of the second line is negative. Again, we set $z \equiv (n^i x^i)/(n^{-i} x^{-i})$ and $c \equiv (e^i/e^{-i})^\rho$ and we define the function $N : [1, +\infty) \to \mathbb{R}$ given by

$$N(\mu) = \rho\mu\alpha \frac{1}{x^i} \frac{1}{(1 + c^{-\mu} z^{-1})(1 + c^{\mu} z)} \left(2\rho(1-\alpha) \frac{1}{(1 + c^{-1} z^{-1})} - 2\rho\mu \frac{1}{(1 + c^{-\mu} z^{-1})} + \rho\mu - 1 \right)$$

We have $N(1) < 0$. Given that the function N is continuous on $[1, +\infty)$, we deduce that there exists $\bar{\mu} \in]1, +\infty]$ such that $\forall \mu \leq \bar{\mu}$ $N(\mu) < 0$ which implies $\tilde{F}_{ee} < 0$.

Last step.

It is easy to check that D is continuous in $(0, 0)$ that is, if $(\lambda^n, \gamma^n) \to (0, 0)$ then $D(\lambda^n, \gamma^n) \to D(0, 0)$. Using that fact and $D(0, 0) > 0$ we can deduce that there exist two thresholds $\bar{\lambda}$, $\bar{\gamma}$ such that when $\lambda < \bar{\lambda}$ and $\gamma < \bar{\gamma}$ then $D(\lambda, \gamma) > 0$. It is equivalent to say that for $\lambda < \bar{\lambda}$ and $\gamma < \bar{\gamma}$, $\forall x^i \in [0, 1]$, the function V_t is quasi-concave in $\mathbf{s}^i$.

■

Proposition 1 *Suppose that $\alpha \leq 1/2$, $\mu \leq \min\{\hat{\mu}, \bar{\mu}\}$, $\lambda < \bar{\lambda}$ and $\gamma < \bar{\gamma}$, for any $x_t \in [0, 1]$, (i) there exists a pure-strategy Nash equilibrium, and (ii) the pure-strategy Nash equilibrium is the symmetric strategy profile $(n^a(x_t), n^a(1 - x_t), e^a(x_t), e^a(1 - x_t))$.*

Note that in this definition, the term symmetry is used to refer to a symmetry of strategic choices with respect to $x = 1/2$ (and not to an equality of choices at equilibrium).

Proof. Because the set of strategy is non-empty, convex and compact, using Lemma 1, we can apply Kakutani's fixed point theorem to deduce item (i) of Proposition 1.

Let us prove item (ii).

Define $\mathbf{f_s^a}$ and $\mathbf{f_s^b}$ the functions going from $[0, 1]$ into $[0, \bar{n}] \times [0, \bar{e}]$ which for each value of x gives the vector of strategy of one group at the Nash equilibrium. That is ,

$$\mathbf{f_s^a}(x) \equiv (n^a(x_t), e^a(x_t)),$$
$$\mathbf{f_s^b}(x) \equiv (n^b(x_t), e^b(x_t)).$$

By definition,

$$\mathbf{f_s^a}(x) = \mathbf{B}_x(\mathbf{f_s^b}(x)),$$
$$\mathbf{f_s^b}(x) = \mathbf{B}_{1-x}(\mathbf{f_s^a}(x)).$$

Therefore, the functions $\mathbf{f_s^a}$ and $\mathbf{f_s^b}$ are implicitly given by

$$\mathbf{f_s^a}(x) - \mathbf{B}_x(\mathbf{B}_{1-x}(\mathbf{f_s^a}(x))) = 0,$$
$$\mathbf{f_s^b}(x) - \mathbf{B}_{1-x}(\mathbf{B}_x(\mathbf{f_s^b}(x))) = 0.$$

Hence, we have

$$\mathbf{f_s^a}(1-x) - \mathbf{B}_{1-x}(\mathbf{B}_x(\mathbf{f_s^a}(1-x))) = 0,$$

so that we deduce $\mathbf{f_s^a}(1-x) = \mathbf{f_s^b}(x)$.

■

A.2 Proof of Proposition 2

Suppose that $\mu = 1$. The first order conditions can be rewritten as

$$\frac{x_t(e_t^a)^\rho}{n_t^a x_t(e_t^a)^\rho + n_t^b(1-x_t)(e_t^b)^\rho}\Pi_{t+1}^b N_t^{-\alpha}\frac{(n_t^a x_t(e_t^a)^\rho + n_t^b(1-x_t)(e_t^b)^\rho)^{(1-\alpha)}}{x_t}$$
$$+\Pi_{t+1}^a N_t^{-\alpha}(1-\alpha)\frac{(n_t^a x_t(e_t^a)^\rho + n_t^b(1-x_t)(e_t^b)^\rho)^{-\alpha}}{x_t}(e_t^a)^\rho x_t = \gamma e_t^a + \lambda n_t^a$$

$$\frac{\rho\,(e_t^a)^{\rho-1}\, n_t^a x_t}{n_t^a x_t(e_t^a)^\rho + n_t^b(1-x_t)(e_t^b)^\rho}\Pi_{t+1}^b N_t^{-\alpha}\frac{(n_t^a x_t(e_t^a)^\rho + n_t^b(1-x_t)(e_t^b)^\rho)^{(1-\alpha)}}{x_t}$$
$$+\rho\Pi_{t+1}^a(1-\alpha)N_t^{1-\alpha}\frac{(n_t^a x_t(e_t^a)^\rho + n_t^b(1-x_t)(e_t^b)^\rho)^{-\alpha}}{x_t}n_t^a e_t^{a\rho-1}x_t = \gamma n_t^a$$

Equalizing, the second FOC for group 1 and 2, we obtain

$$\left(\frac{e_t^a}{e_t^b}\right)^{-(1-\rho)} = \frac{1-\alpha\Pi_{t+1}^b}{1-\alpha\Pi_{t+1}^a}$$

Also using the two FOC together (for group 1) we deduce that

$$\frac{x_t(e_t^a)^\rho}{n_t^a x_t(e_t^a)^\rho + n_t^b(1-x_t)(e_t^b)^\rho}\Pi_{t+1}^b N_t^{-\alpha}\frac{n_t^a x_t(e_t^a)^\rho + n_t^b(1-x_t)(e_t^b)^\rho)^{(1-\alpha)}}{x_t}$$
$$+\Pi_{t+1}^a N_t^{-\alpha}(1-\alpha)\frac{(n_t^a x_t(e_t^a)^\rho + n_t^b(1-x_t)(e_t^b)^\rho)^{-\alpha}}{x_t}(e_t^a)^\rho x_t = \frac{\lambda}{1-p}n_t^a$$

Equalizing, this equation for for group 1 and 2, we obtain

$$\left(\frac{e_t^a}{e_t^b}\right)^{\rho} = \frac{1-\alpha\Pi_{t+1}^b}{1-\alpha\Pi_{t+1}^a}\frac{n_t^a}{n_t^b}.$$

We deduce

$$\frac{e_t^a}{e_t^b} = \frac{n_t^a}{n_t^b}.$$

Hence, $\frac{n_t^{1*}}{n_t^{2*}}$ is implicitely given by

$$\left(\frac{n_t^{1*}}{n_t^{2*}}\right)^{-(1-\rho)} - \frac{1-\alpha\Pi_{t+1}^{2*}}{1-\alpha\Pi_{t+1}^{1*}} \equiv k(\frac{n_t^{1*}}{n_t^{2*}}) = 0,$$

where

$$\Pi_{t+1}^{1*} = \frac{1}{1+\frac{n_t^a}{n_t^b}^{-1-\rho}(1-x_t)},$$
$$\Pi_{t+1}^{2*} = \frac{1}{1+\frac{n_t^a}{n_t^b}^{1+\rho}x_t}$$

One has

$$\frac{d\frac{n_t^{1*}}{n_t^{2*}}}{dx_t} = -\frac{\frac{\partial k}{\partial x_t}}{\frac{\partial k}{\partial \frac{n_t^{1*}}{n_t^{2*}}}}.$$

One easily finds that

$$\frac{\partial k}{\partial x_t} < 0,$$
$$\frac{\partial k}{\partial \frac{n_t^{1*}}{n_t^{2*}}} < 0,$$

so that we deduce $\frac{d\frac{n_t^{1*}}{n_t^{2*}}}{dx_t} = \frac{d\frac{e_t^{1*}}{e_t^{2*}}}{dx_t} < 0$.

Finally, using Proposition 1, we deduce that $de_t^a/dx_t < 0$, $de_t^b/dx_t > 0$, $dn_t^a/dx_{<}0$, $dn_t^b/dx_t > 0$.

A.3 Proof of Proposition 3

Let us drop time indexation and define

$$H \equiv \left(n^a(e^a)^\rho x + n^b(e^b)^\rho(1-x)\right).$$

Remind that $\Pi^a = \Pi^a(N^a, N^b, h^a, h^b)$ with

$$\Pi^a(N^a,N^b,h^a,h^b) = \begin{cases} \frac{(h^a)^\mu\, n^a x}{(h^a)^\mu\, n^a x + (h^b)^\mu\, n^b(1-x)}, & \text{if } h^i \neq 0 \text{ and } n^i \neq 0 \quad \forall i \in \{a,b\}, \\ \frac{n^a x}{n^a x + n^b(1-x)}, & \text{if } h^i = 0 \text{ and } n^i \neq 0 \quad \forall i \in \{a,b\}, \\ \frac{(h^a)^\mu}{(h^a)^\mu + (h^b)^\mu}, & \text{if } h^i \neq 0 \text{ and } n_t^i = 0 \quad \forall i \in \{a,b\}, \\ \frac{1}{2}, & \text{if } h^i = 0 \text{ and } n_t^i = 0 \quad \forall i \in \{a,b\}, \end{cases}$$

Each household in group a solves the following program

$$\max_{n^a,e^a} \beta\tau n^a N^{-\alpha}\left((1-\alpha)H^{-\alpha}(e^a)^\rho + \Pi^a\frac{\alpha H^{1-\alpha}}{xn^a}\right) - \gamma\, n^a e^a - \frac{\lambda}{2}\,(n^a)^2.$$

The first order conditions associated to this program are

$$\beta\tau N^{-\alpha}\left((1-\alpha)H^{-\alpha}(e^a)^\rho+\Pi^a\frac{\alpha H^{1-\alpha}}{xn^a}\right)$$
$$-\beta\tau n^a N^{-\alpha}\alpha(1-\alpha)H^{-\alpha-1}(e^a)^{2\rho}x+\beta\tau n^a N^{-\alpha}\alpha(1-\alpha)\Pi^a\frac{H^{-\alpha}}{xn^a}(e^a)^\rho x$$
$$-\beta\tau n^a N^{-\alpha}\alpha H^{1-\alpha}\frac{(h^a)^{2\mu}\,x}{\left((h^a)^\mu\,n^a x+(h^b)^\mu\,n^b(1-x)\right)^2}-\gamma\,e^a-\lambda n^a=0.$$

and

$$\beta\tau n^a N^{-\alpha}\rho(1-\alpha)H^{-\alpha}(e^a)^{\rho-1}$$
$$-\beta\tau n^a N^{-\alpha}\alpha(1-\alpha)H^{-\alpha-1}(e^a)^\rho xn^a\rho(e^a)^{\rho-1}+\beta\tau n^a N^{-\alpha}\alpha(1-\alpha)\Pi^a\frac{H^{-\alpha}}{xn^a}n^a x\rho(e^a)^{\rho-1}$$
$$+\beta\tau n^a N^{-\alpha}\alpha H^{1-\alpha}\frac{\rho\mu(e^a)^{\rho\mu-1}\left(h^b\right)^\mu n^b(1-x)}{\left((h^a)^\mu\,n^a x+(h^b)^\mu\,n^b(1-x)\right)^2}-\gamma\,n^a=0.$$

Set $x=1$ and denote (n^{a1},e^{a1}) the vector which solves the above system of equation. We find

$$\beta\tau N^{-\alpha}\left(\left(n^{a1}\right)^{-\alpha}\left(e^{a1}\right)^{(1-\alpha)\rho}\right)(1-\alpha)-\gamma\,e^{a1}-\lambda n^{a1}=0,$$

and

$$\rho\beta\tau N^{-\alpha}\left(\left(n^{a1}\right)^{-\alpha+1}\left(e^{a1}\right)^{(1-\alpha)\rho-1}\right)(1-\alpha)-\gamma\,n^{a1}=0.$$

We obtain

$$(e^{a1})^{1-\rho(1-\alpha)}=\frac{\rho\beta\tau N^{-\alpha}(1-\alpha)}{\gamma}\left(n^{a1}\right)^{-\alpha},$$

$$(n^{a1})^{1+\alpha}=\frac{\beta\tau N^{-\alpha}}{\lambda}(e^{a1})^{\rho(1-\alpha)}(1-\alpha)(1-\rho).$$

Set $x=0$ and denote (n^{a0},e^{a0}) the vector which solves the above system of equation. In that case, we obtain

$$n^{a0}=\frac{\beta\tau N^{-\alpha}}{\lambda}(e^{a0})^\rho(n^{a1})^{-\alpha}(e^{a1})^{-\alpha\rho}\left((1-\alpha)(1-\rho)-\alpha\frac{(e^{a0})^{\rho(\mu-1)}}{(e^{a1})^{\rho(\mu-1)}}(\rho\mu-1)\right)$$

and

$$(e^{a0})^{1-\rho}=\frac{\rho\beta\tau N^{-\alpha}}{\gamma}(n^{a1})^{-\alpha}(e^{a1})^{-\alpha\rho}\left((1-\alpha)+\alpha\mu\frac{(e^{a0})^{\rho(\mu-1)}}{(e^{a1})^{\rho(\mu-1)}}\right).$$

Set $x=1/2$ and denote $(n^{a1/2},e^{a1/2})$ the solution to the above system of equation. We find

$$(e^{a1/2})^{1-\rho(1-\alpha)}=\frac{\rho\beta\tau N^{-\alpha}}{\gamma}\left(n^{a1/2}\right)^{-\alpha}\left(1-\alpha+\frac{\alpha\mu}{2}\right),$$

$$(n^{a1/2})^{1+\alpha} = \frac{\beta\tau N^{-\alpha}}{\lambda}(e^{a1/2})^{\rho(1-\alpha)}\left((1-\frac{\alpha}{2}) - \rho(1-\alpha) - \frac{\rho\alpha\mu}{2}\right)$$

Now we will show that provided that $\mu \in (\mu^*, \tilde{\mu})$, we have

$$e^{a0} > e^{a1/2} > e^{a1} \quad \text{and} \quad n^{a1/2} > n^{a1} > n^{a0}.$$

Step 1. We start by comparing choices at $x = 1/2$ and $x = 1$.

Using the FOC at $x = 1$ and $x = 1/2$, we can perform the following quantities

$$e^{a1} = (\beta\tau)^{\frac{1}{(1+\alpha-\rho(1-\alpha))}} \left(\frac{\rho(1-\alpha)}{\gamma}\right)^{\frac{(1+\alpha)}{(1+\alpha-\rho(1-\alpha))}} \left(\frac{(1-\alpha)(1-\rho)}{\lambda}\right)^{\frac{-\alpha}{(1+\alpha-\rho(1-\alpha))}},$$

$$n^{a1} = (\beta\tau)^{\frac{1}{(1+\alpha-\rho(1-\alpha))}} \left(\frac{\rho(1-\alpha)}{\gamma}\right)^{\frac{\rho(1-\alpha)}{(1+\alpha-\rho(1-\alpha))}} \left(\frac{(1-\alpha)(1-\rho)}{\lambda}\right)^{\frac{(1-\rho(1-\alpha))}{(1+\alpha-\rho(1-\alpha))}},$$

$$e^{a1/2} = (\beta\tau)^{\frac{1}{(1+\alpha-\rho(1-\alpha))}} \left(\frac{\rho(1-\alpha+\alpha\mu/2)}{\gamma}\right)^{\frac{(1+\alpha)}{(1+\alpha-\rho(1-\alpha))}} \times \left(\frac{(1-\alpha/2-\rho(1-\alpha+\alpha\mu/2)}{\lambda}\right)^{\frac{-\alpha}{(1+\alpha-\rho(1-\alpha))}},$$

and

$$n^{a1/2} = (\beta\tau)^{\frac{1}{(1+\alpha-\rho(1-\alpha))}} \left(\frac{\rho(1-\alpha+\alpha\mu/2)}{\gamma}\right)^{\frac{\rho(1-\alpha)}{(1+\alpha-\rho(1-\alpha))}} \times \left(\frac{(1-\alpha/2-\rho(1-\alpha+\alpha\mu/2)}{\lambda}\right)^{\frac{(1-\rho(1-\alpha))}{(1+\alpha-\rho(1-\alpha))}}.$$

Note that at $\mu = 1$ we have $n^{a1/2} > n^{a1}$ and $e^{a1/2} > e^{a1}$. One easily shows that $\partial e^{a1/2}/\partial\mu > 0$ $\forall \mu \in \mathbb{R}^+$. Let us perform the derivative of $n^{a1/2}$ with respect to μ. We find

$$\frac{\partial n^{a1/2}}{\partial \mu} = \frac{\rho\alpha}{2\lambda(1+\alpha-\rho(1-\alpha))} \left(\frac{\rho(1-\alpha+\alpha\mu/2)}{\gamma}\right)^{\frac{\rho(1-\alpha)}{(1+\alpha-\rho(1-\alpha))}} \times \left(\frac{(1-\alpha/2-\rho(1-\alpha+\alpha\mu/2)}{\lambda}\right)^{\frac{1-\rho(1-\alpha)}{(1+\alpha-\rho(1-\alpha))}-1} \times \left[(1-\alpha)\left(\frac{(1-\alpha/2-\rho(1-\alpha)+\alpha\mu/2)}{(1-\alpha+\alpha\mu/2)}\right) - (1-\rho(1-\alpha))\right]$$

which has the same sign as

$$(1-\alpha)\left(\frac{(1-\alpha/2-\rho(1-\alpha+\alpha\mu/2)}{(1-\alpha+\alpha\mu/2)}\right)-(1-\rho(1-\alpha)).$$

Note that

$$\begin{aligned}&(1-\alpha)\left(\frac{(1-\alpha/2-\rho(1-\alpha)+\alpha\mu/2)}{(1-\alpha+\alpha\mu/2)}\right)-(1-\rho(1-\alpha))\\&<1-\alpha/2-\rho(1-\alpha)-\alpha\mu/2-1+\rho(1-\alpha)=-\alpha/2-\alpha\mu/2<0.\end{aligned}$$

Hence, $\partial n^{a1/2}/\partial\mu<0\ \forall\mu\in\mathbb{R}^+$.

When μ is such that $1-\alpha/2-\rho(1-\alpha)-\alpha\mu/2=0$, then $n^{a1/2}=0<n^{a1}$. We can deduce that there exists a unique $\tilde{\mu}$ implicitly given by

$$\left(\frac{1-\alpha+\alpha\tilde{\mu}/2}{1-\alpha}\right)^{\rho(1-\alpha)}\left(\frac{(1-\alpha/2-\rho(1-\alpha+\alpha\tilde{\mu}/2)}{(1-\alpha)(1-\rho)}\right)^{(1-\rho(1-\alpha))}-1=0,$$

such that $\mu<\tilde{\mu}\Leftrightarrow n^{a1/2}>n^{a1}$.

Hence for any $\mu\in[1,\tilde{\mu})$ we have $n^{a1/2}>n^{a1}$ and $e^{a1/2}>e^{a1}$.

Step 2. Now we show that $n^{a1}>n^{a0}$.

First, we easily find that $e^{a0}>e^{a1}$. We have

$$\begin{aligned}&n^{a0}<n^{a1}\\\Leftrightarrow&\frac{(e^{a0})^\rho}{(e^{a1})^\rho}\left((1-\alpha)(1-\rho)+\alpha\frac{(e^{a0})^{\rho(\mu-1)}}{(e^{a1})^{\rho(\mu-1)}}(1-\rho\mu)\right)-(1-\alpha)(1-\rho)<0.\end{aligned}$$

Define the function $\Gamma:\mathbb{R}^+\to\mathbb{R}$ given by

$$\Gamma(x)=x^\rho\left((1-\alpha)(1-\rho)-\alpha x^{\rho(\mu-1)}(\rho\mu-1)\right)-(1-\alpha)(1-\rho).$$

One can compute

$$\Gamma'(x)=\rho x^{\rho-1}\left((1-\alpha)(1-\rho)-\alpha x^{\rho(\mu-1)}(\mu\rho-1)\right)-\alpha x^\rho\rho(\mu-1)x^{\rho(\mu-1)-1}(\rho\mu-1).$$

Suppose that $\rho\mu>1$. Then, the function Γ reaches a maximum at $x_m=\left(\frac{(1-\alpha)(1-\rho)}{\alpha(\rho\mu-1)\mu}\right)^{\frac{1}{\rho(\mu-1)}}$. A sufficient condition for $\Gamma(e^{a0}/e^{a1})<0$ is $\Gamma(x_m)<0$ which is equivalent to $\Lambda(\mu)<0$ where $\Lambda:[1,+\infty)\to\mathbb{R}$ is given by

$$\Lambda(\mu)=(1-\alpha)(1-\rho)\left(\left(\frac{(1-\alpha)(1-\rho)}{\alpha(\rho\mu-1)\mu}\right)^{\frac{1}{(\mu-1)}}(1-\frac{1}{\mu})-1.\right)$$

First, suppose that $(1-\alpha)(1-\rho)/\left(\alpha(\rho\mu-1)\mu\right)<1$ which is equivalent to $\mu>\mu_h$ where μ_h is such that

$$\frac{(1-\alpha)(1-\rho)}{\alpha(\rho\mu_h-1)\mu_h}=1.$$

In that case, one finds $\Gamma(x_m) < 0$. Second, suppose that $1/\rho < \mu < \mu_h$. Let us perform $\Lambda'(\mu)$. We find

$$\Lambda'(\mu) = (1-\alpha)(1-\rho)\left(\frac{(1-\alpha)(1-\rho)}{\alpha(\rho\mu-1)\mu}\right)^{\frac{1}{(\mu-1)}}\frac{1}{\mu}$$
$$\times\left(-\frac{1}{(\mu-1)}\ln\left(\frac{(1-\alpha)(1-\rho)}{\alpha(\rho\mu-1)\mu}\right)-\frac{2(\rho\mu-1)}{\mu}\right) < 0.$$

Since $\Lambda(1/\rho) = +\infty$, $\Lambda(\mu_h) < 0$ and Λ is continuous, we deduce that there exists a unique $\mu^* \in [1/\rho, \mu_h]$ such that $\mu \geq \mu^*$ is equivalent to $\Lambda(\mu) \leq 0$ which implies $n^a < n^b$.

Step 3. Finally, we show that $e^{a0} > e^{a1/2}$.

$$\begin{aligned}
&e^{a0} > e^{a1/2},\\
\Leftrightarrow &(n^{a1})^{-\alpha}(e^{a1})^{-\alpha\rho}\left((1-\alpha)+\alpha\mu\frac{(e^{a0})^{\rho(\mu-1)}}{(e^{a1})^{\rho(\mu-1)}}\right)\\
&> \left(n^{a1/2}\right)^{-\alpha}(e^{a1/2})^{-\alpha\rho}\left(1-\alpha+\frac{\alpha\mu}{2}\right),
\end{aligned}$$

In Step 1 we showed that $n^{a1/2} > n^{a1}$ and $e^{a1/2} > e^{a1}$ which implies $\left(n^{a1/2}\right)^{-\alpha}(e^{a1/2})^{-\alpha\rho} < (n^{a1})^{-\alpha}(e^{a1})^{-\alpha\rho}$. A sufficient condition for the above inequality to hold is

$$\alpha\mu\frac{(e^{a0})^{\rho(\mu-1)}}{(e^{a1})^{\rho(\mu-1)}} > \frac{\alpha\mu}{2},$$

which is true since we showed in Step 2. that $e^{a0}/e^{a1} > 1$. Hence we deduce that $e^{a0} > e^{a1/2}$.

B Strategic decisions on whether to establish norms

Let us define $d^i \in \{0,1\}$, where d^i equal to one when group i chooses to build the cultural institution. Remind that d^i is chosen at date $t = 1$ but let us skip time indexation to simplify notation.

Definition 1 (Stackelberg-Nash equilibrium)
A Stackelberg-Nash equilibrium is a strategy profile $(d^{a\star}, d^{b\star}, n^{a\star}, n^{b\star}, e^{a\star}, e^{b\star}) = (d^a(x), d^b(x), n^a(x), n^b(x), e^a(x), e^b(x))$ *with* $d^i : [0,1] \to \{0,1\}$ $n^i : [0,1] \to [0,\bar{n}]$ *and* $e^i : [0,1] \to [0,\bar{e}]$ *such that for all* $i \in \{a,b\}$,

$$d^{i\star} \in \underset{d^i\in\{0,1\}}{\operatorname{argmax}} V(\hat{n}^i(d^i,d^{-i}), \hat{n}^{-i}(d^{-i},d^i), \hat{e}^i(d^i,d^{-i}), \hat{e}^{-i}(d^{-i},d^i), x^i) - \kappa d^i$$

where $\hat{n}^i : \{0,1\}\times\{0,1\} \to [0,\bar{n}]$, $\hat{e}^i : \{0,1\}\times\{0,1\} \to [0,\bar{e}]$ *are given by*

$$\left(\hat{n}^i(1,d^{-i}), \hat{e}^i(1,d^{-i})\right) \in \underset{(n^i,e^i)\in\mathcal{X}}{\operatorname{argmax}} V(n^i, \hat{n}^{-i}(d^{-i},1), e^i, \hat{e}^{-i}(d^{-i},1), x^i) \quad \forall x^i \in [0,1]$$

$$\left(\hat{n}^i(0,d^{-i}), \hat{e}^i(0,d^{-i})\right) \in \underset{(n^{ij},e^{ji})\in\mathcal{X}}{\operatorname{argmax}}\, W(n^{ji}, \hat{n}^i(0,d^{-i}), \hat{n}^{-i}(d^{-i},0), e^{ji}, \hat{e}^i(0,d^{-i}), \hat{e}^{-i}(d^{-i},0), x^i)$$
$$\forall x^i \in [0,1] \quad \forall j \in [0, Nx^i].$$

The functions $\hat{n}^i(d^i, d^{-i})$ and $\hat{e}^i(d^i, d^{-i})$ are the optimal choices of fertility and education of household j of group i for any choices of the leaders of group, d^i and d^{-i}.

When $d^i = 1$, the leader chooses to build a cultural institution. Each household j in group i acts as a representative individual of group i and chooses the level of education and fertility - n^i, e^i - which maximizes the welfare of the group, taking the choices of members of the other group - n^{-i}, e^{-i} equal to $\hat{n}^{-i}(d^{-i}, 1), \hat{e}^{-i}(d^{-i}, 1)$ at equilibrium - as given.

When $d^i = 0$, the leader chooses not to build a cultural institution. In group i, each household j chooses its level of fertility and education - n^{ji}, e^{ji} - taking the choices of all other members of group i - n^i, e^i equal to $\hat{n}^i(0, d^{-i}), \hat{e}^i(0, d^{-i})$ at equilibrium - and the choices of members of the other group - n^{-i}, e^{-i} equal to $\hat{n}^{-i}(d^{-i}, 0), \hat{e}^{-i}(d^{-i}, 0)$ at equilibrium - as given.

In what follows, we focus on the game between small groups ($x \approx 0$) and large groups ($x \approx 1$).

Proposition 2 *Suppose that $x_t^a = 0$. There exist $\tilde{\kappa}_1$, $\tilde{\kappa}_2$, $\tilde{\kappa}_3$ such that if $\tilde{\kappa}_2 < \min\{\tilde{\kappa}_1, \tilde{\kappa}_3\}$, $\tilde{\kappa}_1 \neq \tilde{\kappa}_3$, there exists a unique Stackelberg-Nash equilibrium given by*

$$(d^{a\star}, d^{b\star}, n^{a\star}, n^{b\star}, e^{a\star}, e^{b\star}) = (1, 1, \hat{n}^a(1,1), \hat{n}^b(1,1), \hat{e}^a(1,1), \hat{e}^b(1,1)) \;\forall \kappa < \tilde{\kappa}_2,$$

$$(d^{a\star}, d^{b\star}, n^{a\star}, n^{b\star}, e^{a\star}, e^{b\star}) = (1, 0, \hat{n}^a(1,0), \hat{n}^b(0,1), \hat{e}^a(1,0), \hat{e}^b(0,1)) \;\forall \kappa \in (\tilde{\kappa}_2, \tilde{\kappa}_3),$$

$$(d^{a\star}, d^{b\star}, n^{a\star}, n^{b\star}, e^{a\star}, e^{b\star}) = (0, 0, \hat{n}^a(0,0), \hat{n}^b(0,0), \hat{e}^a(0,0), \hat{e}^b(0,0)) \;\forall \kappa > \tilde{\kappa}_3.$$

The proof is developed in the following subsection.

This proposition reveals that in a game between small and large groups, there exists a unique Stackelberg-Nash equilibrium which depends on the cost of building a cultural institution. First, uniqueness holds because, the actions chosen by the small group do not influence total outcome (and so the payoff of the large group). This implies that the choices of large groups are independent of the actions chosen by small groups (i.e., the reaction function of the large group is constant). For each set of actions taken by the large group, there is a unique optimal set of actions for the small group so that the equilibrium is unique.

When the cost of building a cultural institution is small we are back to the benchmark model where the existence of norms on fertility and education are given. This is because the benefit from coordination is strictly positive since it allows to internalize intra-group externalities. Reversely, when the cost of creating a cultural institution is very high, we are back to the case where norms on education and fertility are absent. In that case, whatever the size of the group, the benefit from coordination is too low compared to the cost. Interestingly, when the fixed cost is intermediate, an asymmetric type of equilibrium emerges in which small groups create a cultural institution while large groups do not. This result requires $\tilde{\kappa}_2 < \tilde{\kappa}_3$ which means that the payoff to coordination of the small group (group a), when the large group (group b) does not coordinate, is higher than the payoff to coordination of the large group.[1] We show that this condition holds when μ, the impact of human capital on appropriation, is not too low.

[1] The condition $\tilde{\kappa}_2 < \tilde{\kappa}_1$ is imposed to simplify the exposition.

B.1 Proof of Proposition 2

This proof is divided in two steps.

In step 1, we compare (i) $\Omega(1, d^{-i}, x^i)$ and $\Omega(0, d^{-i}, x^i)$, (ii) $\Omega(1, 1, 0)$ and $\Omega(1, 1, 1)$, (iii) $\Omega(1, 0, 1)$ and $\Omega(1, 1, 1)$, (iv) $\Omega(1, 1, 1)$ and $\Omega(1, 0, 0)$.

In step 2, we show that the Stackelberg-Nash equilibrium is unique and we determine the values of d_0 and d_1 at the equilibrium. We deduce the value of education and fertility at the equilibrium.

Step 1:

We start by defining the function $\Omega^i : \{0, 1\} \times \{0, 1\} \times [0, 1] \to \mathbb{R}^+$ which is given by $\forall i \in \{a, b\}$

$$\Omega(d^i, d^{-i}, x^i) = V(\hat{n}^i(d^i, d^{-i}), \hat{n}^{-i}(d^{-i}, d^i), \hat{e}^i(d^i, d^{-i}), \hat{e}^{-i}(d^{-i}, d^i), x^i) \quad \forall x^i \in [0, 1].$$

where the functions $\hat{n}^i$ and $\hat{e}^i$ are as in Definition 1.

(i) Comparison between $\Omega(1, d^{-i}, x^i)$ and $\Omega(0, d^{-i}, x^i)$. By definition, we have

$$\begin{aligned}\Omega(1, d^{-i}, x^i) &= \max_{(n^i, e^i) \in [0, \bar{n}] \times [0, \bar{e}]} V(n^i, \hat{n}^{-i}(d^{-i}, 1), e^i, \hat{e}^{-i}(d^{-i}, 1), x^i) \\ &= V(\hat{n}^i(1, d^{-i}), \hat{n}^{-i}(d^{-i}, 1), \hat{e}^i(1, d^{-i}), \hat{e}^{-i}(d^{-i}, 1), x^i),\end{aligned}$$

and

$$\begin{aligned}\Omega(0, d^{-i}, x^i) &= \max_{(n^{ij}, e^{ji}) \in [0, \bar{n}] \times [0, \bar{e}]} W(n^{ji}, \hat{n}^i(0, d^{-i}), \hat{n}^{-i}(d^{-i}, 0), e^{ji}, \hat{e}^i(0, d^{-i}), \hat{e}^{-i}(d^{-i}, 0), x^i) \\ &= W(\hat{n}^i(0, d^{-i}), \hat{n}^i(0, d^{-i}), \hat{n}^{-i}(d^{-i}, 0), \hat{e}^i(0, d^{-i}), \hat{e}^i(0, d^{-i}), \hat{e}^{-i}(d^{-i}, 0), x^i)\end{aligned}$$

Also, by definition, we have

$$\begin{aligned}&W(\hat{n}^i(0, d^{-i}), \hat{n}^i(0, d^{-i}), \hat{n}^{-i}(d^{-i}, 0), \hat{e}^i(0, d^{-i}), \hat{e}^i(0, d^{-i}), \hat{e}^{-i}(d^{-i}, 0), x^i) \\ &= V(\hat{n}^i(0, d^{-i}), \hat{n}^{-i}(d^{-i}, 0), \hat{e}^i(0, d^{-i}), \hat{e}^{-i}(d^{-i}, 0), x^i) \\ &< V(\hat{n}^i(1, d^{-i}), \hat{n}^{-i}(d^{-i}, 1), \hat{e}^i(1, d^{-i}), \hat{e}^{-i}(d^{-i}, 1), x^i)\end{aligned}$$

which is equivalent to

$$\Omega(1, d^{-i}, x^i) > \Omega(0, d^{-i}, x^i) \quad \forall d^{-i} \in \{0, 1\} \quad \forall x^i \in [0, 1].$$

(ii) Comparison between $\Omega(1, 1, 0)$ and $\Omega(1, 1, 1)$. Remind that we set $x^a = 0$.

Define the function $\Psi : [0, \bar{n}] \times [0, \bar{e}] \to \mathbb{R}$ given by

$$\Psi(n, e) = \beta\tau n \left((1-\alpha) H^{-\alpha} (e)^\rho + \frac{(e)^{\rho\mu}}{D} \alpha H^{1-\alpha} \right) - \gamma \, ne - \frac{\lambda}{2} (n)^2 , \tag{1}$$

where

$$H = \hat{n}^b(1,1)(\hat{e}^b(1,1))^\rho$$
$$D = \hat{n}^b(1,1)(\hat{e}^b(1,1))^{\rho\mu}$$

We have

$$\Omega(1,1,0) = \Psi(\hat{n}^a(1,1), \hat{e}^a(1,1))$$
$$\Omega(1,1,1) = \Psi(\hat{n}^b(1,1), \hat{e}^b(1,1)).$$

Note that $\Psi(n^i, e^i) = V(n^i, \hat{n}^b(1,1), e^i, \hat{e}^b(1,1), 0)$ so that

$$\begin{aligned}\max_{(n^i,e^i)\in[0,\bar{n}]\times[0,\bar{e}]} V(n^i, \hat{n}^b(1,1), e^i, \hat{e}^b(1,1), 0) &= \max_{(n^i,e^i)\in[0,\bar{n}]\times[0,\bar{e}]} \Psi(\hat{n}^i, \hat{e}^i) \\ &= V(\hat{n}^a(1,1), \hat{n}^b(1,1), \hat{e}^a(1,1), \hat{e}^b(1,1), 0) \\ &= \Psi(\hat{n}^a(1,1), \hat{e}^a(1,1)).\end{aligned}$$

We deduce that

$$\begin{aligned}&\Psi(\hat{n}^a(1,1), \hat{n}^a(1,1)) > \Psi(\hat{n}^b(1,1), \hat{e}^b(1,1)), \\ \Leftrightarrow &\Omega(1,1,0) > \Omega(1,1,1).\end{aligned}$$

Using a similar reasoning we can also deduce

$$\Omega(0,1,0) > \Omega(0,1,1).$$

(iii) Comparison between $\Omega(1,1,1)$ and $\Omega(1,0,1)$.

We have

$$\Omega(1,1,1) = \Psi(\hat{n}^b(1,1), \hat{e}^b(1,1))$$
$$\Omega(1,0,1) = \Psi(\hat{n}^b(1,1), \hat{e}^b(1,1)).$$

We deduce $\Omega(1,1,1) = \Omega(1,0,1)$.

(iv) Comparison between $\Omega(1,0,0)$ and $\Omega(1,1,1)$. We have

$$\Omega(1,0,0) = \beta\tau\hat{n}^a(1,0)\left((1-\alpha)H^{-\alpha}(\hat{e}^a(1,0))^\rho + \frac{(\hat{e}^a(1,0))^{\rho\mu}}{D}\alpha H^{1-\alpha}\right)$$
$$-\gamma\,\hat{n}^a(1,0)\hat{e}^a(1,0) - \frac{\lambda}{2}\left(\hat{n}^a(1,0)\right)^2,$$

where

$$H = \hat{n}^b(0,1)(\hat{e}^b(0,1))^\rho$$
$$D = \hat{n}^b(0,1)(\hat{e}^b(0,1))^{\rho\mu}$$

$$\Omega(1,1,1) = \beta\tau\hat{n}^b(1,1)\left((1-\alpha)H^{-\alpha}(\hat{e}^b(1,1))^\rho + \frac{(\hat{e}^b(1,1))^{\rho\mu}}{D}\alpha H^{1-\alpha}\right)$$
$$-\gamma\, \hat{n}^b(1,1)\hat{e}^b(1,1) - \frac{\lambda}{2}\left(\hat{n}^b(1,1)\right)^2,$$

where

$$H = \hat{n}^b(1,1)(\hat{e}^b(1,1))^\rho$$
$$D = \hat{n}^b(1,1)(\hat{e}^b(1,1))^{\rho\mu}$$

Note that $\forall d^b \in \{0,1\}$, the FOC for education at $x^b = 1$ gives

$$(1-\alpha)H^{-\alpha} = \frac{\gamma N^\alpha}{\rho\beta\tau}\,(\hat{e}^b(d^b,1))^{1-\rho}$$

Using that in the above equations, we obtain

$$\Omega(1,0,0) = \hat{n}^a(1,0)\frac{\gamma N^\alpha}{\rho(1-\alpha)}\left((1-\alpha)\ \hat{e}^b(0,1))^{(1-\rho)}(\hat{e}^a(1,0))^\rho + \alpha(\hat{e}^a(1,0))^{\rho\mu}\ \hat{e}^b(0,1))^{(1-\rho\mu)}\right)$$
$$-\gamma\, \hat{n}^a(1,0)\hat{e}^a(1,0) - \frac{\lambda}{2}\left(\hat{n}^a(1,0)\right)^2,$$

and

$$\Omega(1,1,1) = \hat{n}^b(1,1)\frac{\gamma N^\alpha}{\rho(1-\alpha)}\left((1-\alpha)\hat{e}^b(1,1))^{(1-\rho)}(\hat{e}^b(1,1))^\rho + \alpha(\hat{e}^b(1,1))^{\rho\mu}\hat{e}^b(1,1))^{(1-\rho\mu)}\right)$$
$$-\gamma\, \hat{n}^b(1,1)\hat{e}^b(1,1) - \frac{\lambda}{2}\left(\hat{n}^b(1,1)\right)^2,$$

Define the function $\Phi : [0,\bar{n}] \times [0,\bar{e}]^2 \to \mathbb{R}$ given by

$$\Phi(n,e,y) = n\frac{\gamma N^\alpha}{\rho(1-\alpha)}\left((1-\alpha)\ (y)^{(1-\rho)}(e)^\rho + \alpha(e)^{\rho\mu}\ (y)^{(1-\rho\mu)}\right) - \gamma\, ne - \frac{\lambda}{2}\,(n)^2.$$

Then we have

$$\Phi(\hat{n}^b(1,1), \hat{e}^b(1,1), \hat{e}^b(1,1)) = \Omega(1,1,1),$$

and

$$\Phi(\hat{n}^a(1,0), \hat{e}^a(1,0), \hat{e}^b(0,1)) = \Omega(1,0,0).$$

Note that when $\rho\mu \leq 1$, the sign of $\partial\Phi/\partial y$ is positive. When $\rho\mu > 1$, the sign of $\partial\Phi/\partial y$ is ambiguous. Let us consider the two cases.

First, suppose that $\partial\Phi/\partial y < 0$. Remind that $\hat{e}^b(1,1) > \hat{e}^b(0,1)$. Together these two conditions imply that $\forall (n,e) \in [0,\bar{n}] \times [0,\bar{e}]$

$$\Phi(n, e, \hat{e}^b(1,1)) < \Phi(n, e, \hat{e}^b(0,1)).$$

In particular, we have

$$\Phi(\hat{n}^b(1,1), \hat{e}^b(1,1), \hat{e}^b(1,1)) < \Phi(\hat{n}^b(1,1), \hat{e}^b(1,1), \hat{e}^b(0,1)).$$

But, we know that

$$\Phi(\hat{n}^a(1,0), \hat{e}^a(1,0), \hat{e}^b(0,1)) = \max_{(n,e)\in[0,\bar{n}]\times[0,\bar{e}]} \Phi(n, e, \hat{e}^b(0,1)) > \Phi(\hat{n}^b(1,1), \hat{e}^b(1,1), \hat{e}^b(0,1)).$$

We deduce that $\Omega(1,0,0) > \Omega(1,1,1)$.

Second, suppose that $\partial\Phi/\partial y > 0$. Using this condition and $\hat{e}^b(1,1) > \hat{e}^b(0,1)$ we obtain

$$\Phi(n, e, \hat{e}^b(1,1)) > \Phi(n, e, \hat{e}^b(0,1)).$$

Then, on the one hand, we have

$$\Phi(\hat{n}^b(1,1), \hat{e}^b(1,1), \hat{e}^b(1,1)) > \Phi(\hat{n}^b(1,1), \hat{e}^b(1,1), \hat{e}^b(0,1)).$$

But, on the other hand, we know that

$$\Phi(\hat{n}^a(1,0), \hat{e}^a(1,0), \hat{e}^b(0,1)) = \max_{(n,e)\in[0,\bar{n}]\times[0,\bar{e}]} \Phi(n, e, \hat{e}^b(0,1)) > \Phi(\hat{n}^b(1,1), \hat{e}^b(1,1), \hat{e}^b(0,1)).$$

In this case, the sign of $\Omega(1,0,0) - \Omega(1,1,1)$ is ambiguous.

(v) Comparison between $\Omega(1,0,0)$ and $\Omega(1,1,0)$. Using similar arguments (to the ones used in case iv) we can deduce that

$$\begin{aligned}
\frac{\partial\Phi}{\partial y} < 0 \quad &\Leftrightarrow \quad \Omega(1,0,0) > \Omega(1,1,0) \\
\frac{\partial\Phi}{\partial y} > 0 \quad &\Leftrightarrow \quad \Omega(1,0,0) < \Omega(1,1,0).
\end{aligned}$$

(vi) Comparison between $\Omega(0,0,0)$ and $\Omega(0,1,0)$. As well we can deduce the following.

$$\frac{\partial \Phi}{\partial y} < 0 \quad \Leftrightarrow \quad \Omega(0,0,0) > \Omega(0,1,0)$$
$$\frac{\partial \Phi}{\partial y} > 0 \quad \Leftrightarrow \quad \Omega(0,0,0) < \Omega(0,1,0).$$

Step 2. The different outcomes of the game can be represented in the payoff matrix below.

		Individual a	
		$d^a = 0$	$d^a = 1$
Individual b	$d^b = 0$	$\Omega(0,0,1) > 0, \Omega(0,0,0) > 0$	$\Omega(0,1,1) > 0, \Omega(1,0,0) - \kappa$
	$d^b = 1$	$\Omega(1,0,1) - \kappa, \Omega(0,1,0) > 0$	$\Omega(1,1,1) - \kappa, \Omega(1,1,0) - \kappa$

Let us look at the values of these payoffs. To do so, we define

$$\tilde{\kappa}_1 \equiv \Omega(1,1,0) - \Omega(0,1,0),$$
$$\tilde{\kappa}_2 \equiv \Omega(1,1,1) - \Omega(0,1,1) = \Omega(1,1,1) - \Omega(0,0,1) = \Omega(1,0,1) - \Omega(0,0,1),$$
$$\tilde{\kappa}_3 \equiv \Omega(1,0,0) - \Omega(0,0,0).$$

From (i) we deduce that $\tilde{\kappa}_1 > 0$, $\tilde{\kappa}_2 > 0$, $\tilde{\kappa}_3 > 0$. From (ii) we deduce that the sign of $\tilde{\kappa}_1 - \tilde{\kappa}_2$ is ambiguous. From (iii) and (iv) we deduce that the sign of $\tilde{\kappa}_2 - \tilde{\kappa}_3$ is ambiguous. Finally, from (v) and (vi) we deduce that the sign of $\tilde{\kappa}_1 - \tilde{\kappa}_3$ is ambiguous. Hence, depending on the model's parameters, there are six possibilities for the ranking of the thresholds. Let us focus on the case $\tilde{\kappa}_2 < \min\{\tilde{\kappa}_3, \tilde{\kappa}_1\}$.

a. Suppose that $\kappa > \max\{\tilde{\kappa}_3, \tilde{\kappa}_1\}$. Then $\Omega(1,0,0) - \kappa < \Omega(0,0,0)$, $\Omega(1,1,0) - \kappa < \Omega(0,1,0)$, $\Omega(1,0,1) - \kappa < \Omega(0,0,1)$ and $\Omega(1,1,1) - \kappa < \Omega(0,1,1)$. We deduce that there exists only one equilibrium of this game: $(d_t^{a\star}, d_t^{b\star}, n_t^{a\star}, n_t^{b\star}, e_t^{a\star}, e_t^{b\star}) = (0, 0, \hat{n}^a(0,0), \hat{n}^b(0,0), \hat{e}^a(0,0), \hat{e}^b(0,0))$.

b. Suppose that $\kappa < \tilde{\kappa}_2$. Then $\Omega(1,0,0) - \kappa > \Omega(0,0,0)$, $\Omega(1,1,0) - \kappa > \Omega(0,1,0)$, $\Omega(1,0,1) - \kappa > \Omega(0,0,1)$ and $\Omega(1,1,1) - \kappa > \Omega(0,1,1)$. We deduce that there exists only one equilibrium of this game: $(d_t^{a\star}, d_t^{b\star}, n_t^{a\star}, n_t^{b\star}, e_t^{a\star}, e_t^{b\star}) = (1, 1, \hat{n}^a(1,1), \hat{n}^b(1,1), \hat{e}^a(1,1), \hat{e}^b(1,1))$.

c. Suppose that $\tilde{\kappa}_3 > \kappa > \tilde{\kappa}_2$. We know that $\Omega(1,0,1) - \kappa < \Omega(0,0,1)$ and $\Omega(1,1,1) - \kappa < \Omega(0,1,1)$ so that $d_t^{b\star} = 0$. Furthermore, we know that $\Omega(1,0,0) - \kappa > \Omega(0,0,0)$ so that $d_t^{a\star} = 1$.

C Numerical example

Assume $\alpha = 0.25$, $\lambda = 0.2$, $\rho = 0.5$, $\gamma = 0.3$. Figure 1 shows how fertility and education of group a change when the share of the group varies, in the case $\mu = 1$. This illustrates the results of Proposition 2. Figure 2 shows the same variables in the case $\mu = 3$. This illustrates the results of Proposition 3.

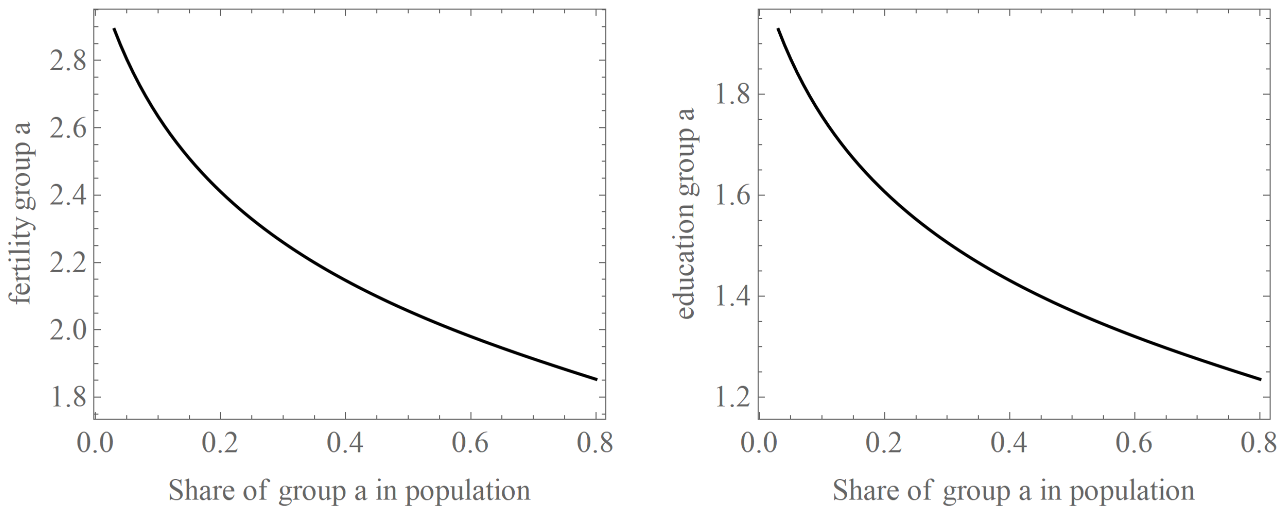


Figure 1: Numerical example: Case $\mu = 1$

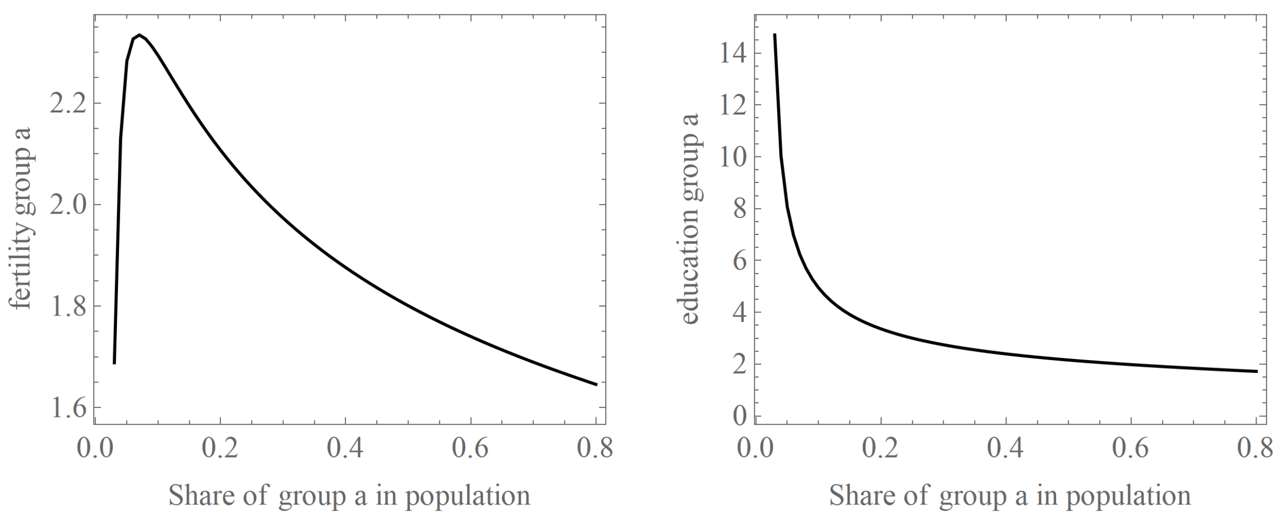
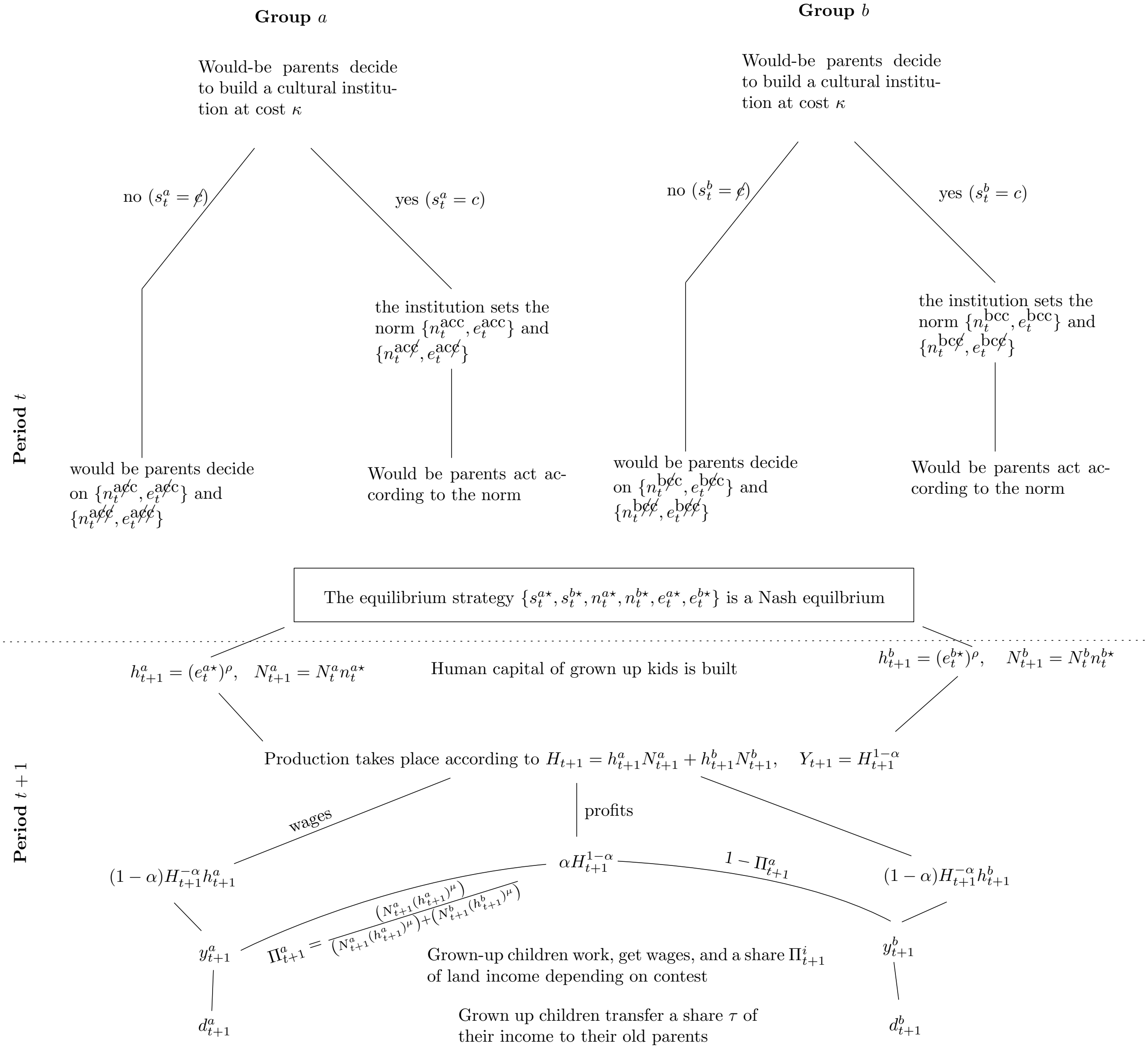


Figure 2: Numerical example: Case $\mu = 3 (> 1/\rho)$

D The full game



E The Indonesian Administrative System

Indonesia's administrative system is composed of four main levels: 34 provinces, about 500 regencies or cities, over 6,000 districts and 75,000 villages. It has been a rather centralized system since the Dutch era and after independence until the early 2000, with the central power in Jakarta appointing governors at the province level, based on their loyalty rather than their knowledge of the local context. The village administration level and its elected village chief (*lurah* or *kepala desa*) in contrast has probably the most direct influence on a citizen's daily life. However, residential segregation is very strong in Indonesia. As noted by Bazzi, Koehler-Derrick, and Marx (2020), the ethnic fractionalization index (the probability that any two residents belong to different ethnicities) is of around 0.81 nationally, while it drops to a median of 0.04 at the village level. For this reason, we believe that the village is not the most relevant level at which to measure religious divisions, as it is not where the conflict about resource appropriation by different groups happens.

Regencies (*kabupaten*) and cities (*kota*), led by regents (*bupati*) and mayors (*walikota*) respectively, have become chief administrative units since the early 2000, responsible for providing most government services, such as provision of public schools and public health facilities. Indeed, following the implementation of drastic regional autonomy measures in 2001, regencies have received the largest part of the decentralized competencies while the regents and the representative council members have become elected officials for 5-year terms. This makes it the ideal level of disaggregation to examine conflict over resource appropriation.

The decentralization movement has been accompanied by a massive wave of creation of new regencies, stemming from older regencies splitting up, known as the *pemekaran* or blossoming, taking the total number of regencies from about 300 to just over 500 in the interval of a few years. We acknowledge that this movement may have been influenced by existing power struggles across religious groups. It may also have inflected the dynamics of those struggles in return by creating more homogenous regencies. We however rely on regency boundaries from before the decentralization happened and consider that the division movement is too recent to have substantially influenced cultural norms of different groups.

Nonetheless, to exclude this possibility altogether, we ran a robustness check using only the data from the 1971, 1980 and 1990 Censuses, for which the religious shares correspond closely to the regencies in use at the time. Results present very similar patterns to those in the benchmark sample, though at a higher level of fertility and a lower level of educational attainment, consistent with the general time trend in these variables. The lower fertility of the first ventile is less sharp that in the overall sample, especially in terms of number of surviving children, suggesting that the phenomenon has grown stronger in recent years with more access to fertility control and lower rates of infant mortality. Figures are available upon request.

F Additional descriptive statistics

Table 1 shows summary statistics on the size of religious groups at the regency level and decomposes its variance into a between observations and a within observation (across time) standard deviation. The overall shares are somewhat different from those at the population level because they are taken unweighted at the regency level.[2] The distribution of religion shares has not drastically changed over time. Most of the variation occurs across observations, while that within variation is limited to between 2 and 6 percentage points. In contrast, Figure 3 shows strong trends in educational attainments, with a doubling of the number of years of schooling over the period, and in child mortality, which has been divided by four.[3]

Religion	Mean size	Std. Dev.	Min	Max	Between obs. Std. Dev.	Within obs. Std. Dev.
Buddhist	0.01	0.032	0.00	0.45	0.024	0.021
Hindu	0.03	0.157	0.00	1.00	0.156	0.018
Muslim	0.82	0.293	0.00	1.00	0.291	0.058
Catholic	0.05	0.150	0.00	1.00	0.157	0.026
Protestant	0.08	0.180	0.00	0.99	0.175	0.042
Confucian	0.01	0.054	0.00	0.94	0.037	0.039
# of obs.	1336					
# of regencies	269					

Table 1: Summary statistics on size of religious group

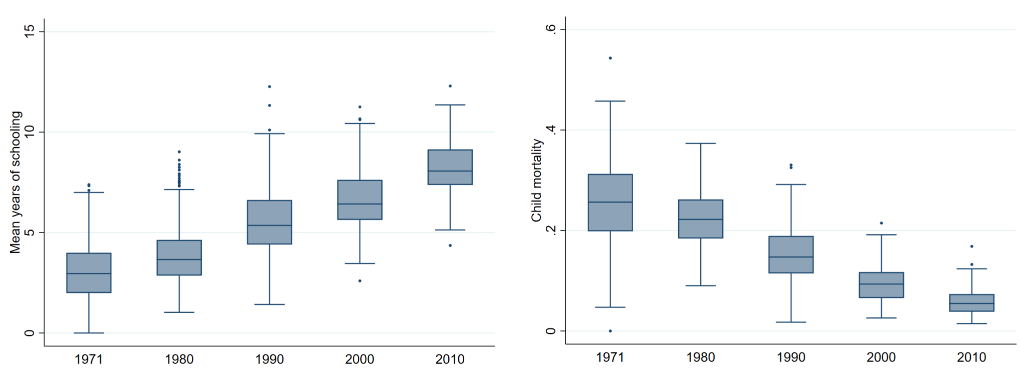


Figure 3: Distribution of regency-level controls

[2]Regencies with large shares of Protestants tend to be smaller in size, and conversely for Muslims, which explains the discrepancies between regency-level and population means.

[3]The child mortality rate is computed following Baudin, de la Croix, and Gobbi (2018) as one minus the share of surviving children among those ever born.

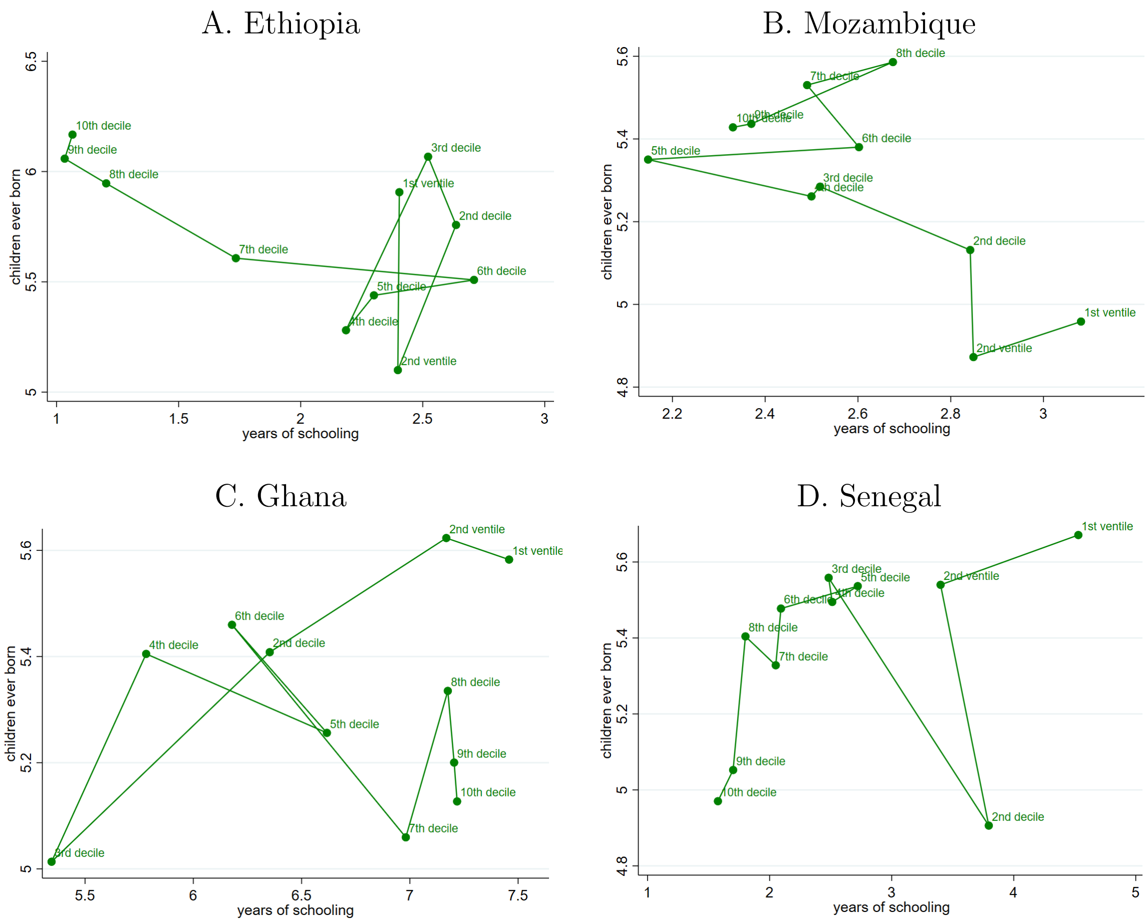


Figure 4: Fertility and education choices by group size

Source: Ethiopian Census, waves 1994-2007, Mozambican Census, wave 2007, Ghanaian Census, waves 2000-2010, Senegalese Census, waves 1988-2013

Notes: Fertility corresponds to the count of children ever born predicted by the model estimated using Poisson regression. Education is the number of years of schooling predicted by the model estimated using OLS. Corresponding tables of coefficients are available upon request.

G Robustness analysis

G.1 Analysis in terms of ethnic groups

Interreligious conflict is not necessarily the most salient in Indonesia and is definitely not the dimension along which there is the largest amount of diversity. However, information on ethnicity is available in ipums only in the 2010 census, which limits drastically the scope of the analysis. We anyway performed a robustness check on the 2010 Census, using ethnicity rather than religion to define the groups. To this end, we apply to the raw data on ethnicity the new classification suggested by Ananta et al. (2015) to go from 1331 down to 119 groups. We obtain deciles with a much more balanced variation in group size than for religion, as shown in Figure 5.

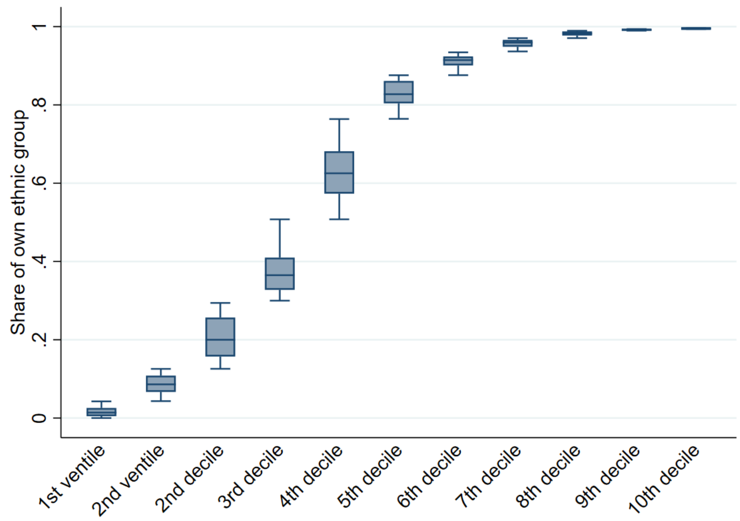
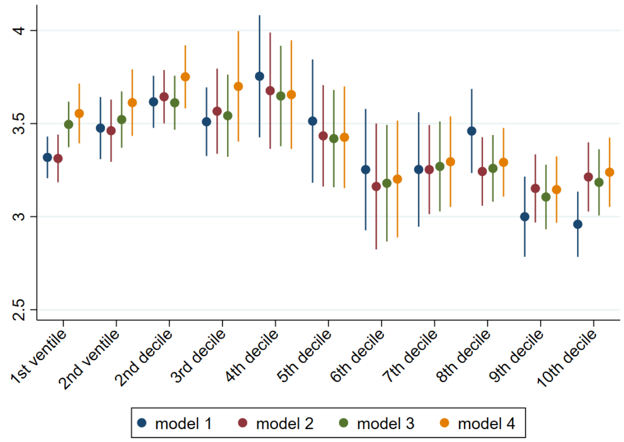
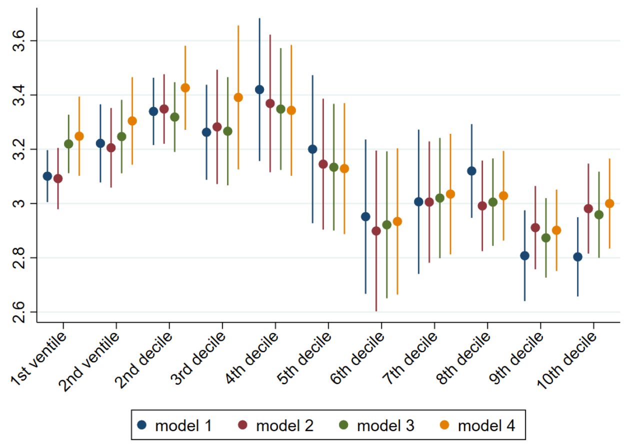


Figure 5: Distribution of size of ethnic group by deciles

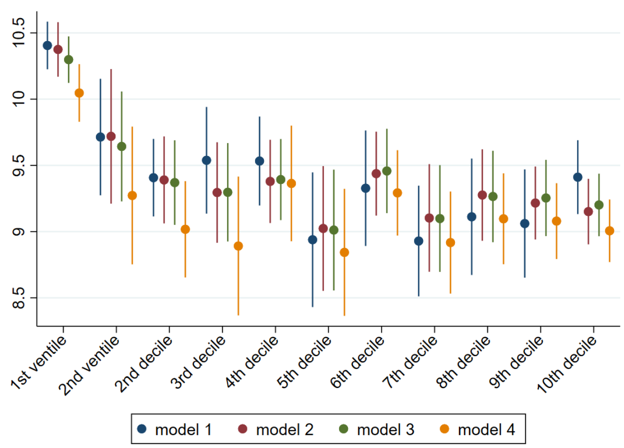
We then run the exact same regressions as in our benchmark and obtain Figure 6. We confirm that fertility, whether measured by children ever born or surviving children, is substantially higher for smaller groups (up to 4th/5th decile). Additionally, we re-establish that the first ventile attains about 10 years of schooling, while all other categories are significantly below that. The rest of the picture in terms of years of schooling is less sharp. While the point estimates of the 2nd ventile and deciles 2, 3 and 4 suggest about 9.5 years of education, deciles 5, 7, 8 and 9 hover around 9 years only. Confidence intervals are large however, so we cannot establish with certainty that education choices for intermediate groups is higher than those of large majorities. Additionally, the point estimates in model 4 are all much closer to 9 years, suggesting that the population that migrates is on average more educated. One issue may be that we are using the delimitations of regencies from the 1990s while they have changed drastically during the 2000s. We believe that this exercise suggests that we would find a similar picture with ethnicity as we do with religion, if we had data on ethnicity in all censes waves.



(a) Predicted count of children ever born by group size deciles



(b) Predicted count of surviving children by group size deciles



(c) Predicted years of education by group size deciles

Source: Indonesian Census 2010
Notes: Panels (a) and (b) are based on the fertility equation estimated using Poisson regression. Panel (c) is based on the education equation estimated by OLS. Confidence intervals are all computed based on standard errors clustered at the regency level. Corresponding tables of coefficients are available upon request.

Figure 6: Fertility and Education by Ethnic Group Size

G.2 Exclusion of the last census from the sample

The decentralization movement of the early 21st century has been accompanied by a massive wave of creation of new regencies, stemming from older regencies splitting up, known as the *pemekaran* or blossoming, taking the total number of regencies from about 300 to just over 500 in the interval of a few years. In the main analysis, we rely on regency boundaries from before the decentralization happened and consider that the division movement is too recent to have substantially influenced cultural norms of different groups.

Nonetheless, to exclude the possibility of an effect of this multiplications of the number of regencies, we run a robustness check using only the data from the 1971, 1980 and 1990 Censuses, for which the religious shares correspond closely to the regencies in use at the time. Results are given in Figure 7. They present very similar patterns to those in the benchmark sample, though at a higher level of fertility and a lower level of educational attainment, consistent with the general time trend in these variables. The lower fertility of the first ventile is less sharp that in the overall sample, especially in terms of number of surviving children, suggesting that the phenomenon has grown stronger in recent years with more access to fertility control and lower rates of infant mortality.

G.3 Intensive vs extensive margins of fertility

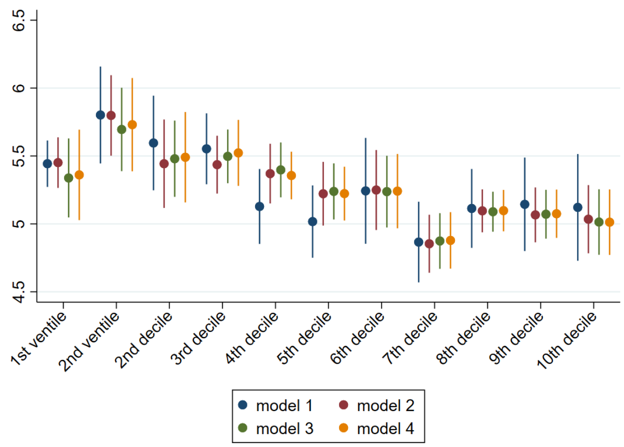
In this appendix, we analyse separately the behavior of the extensive and intensive margins of fertility.

We run two separate regressions: one with a dummy equal to one when a woman is childless as the dependent variable and a second one with the count of children ever born, but restricting to the sample of mothers of at least one child. Results are shown in Figure 8. Panel 8a shows a pattern very similar to the one found for fertility overall, suggesting a limited role for childlessness. Panel 8b confirms that childlessness varies little after the second ventile, hovering between 5.5 and 7%. Interestingly, over this range, childlessness seems to be positively correlated to completed fertility of mothers, while they generally move in opposite directions.

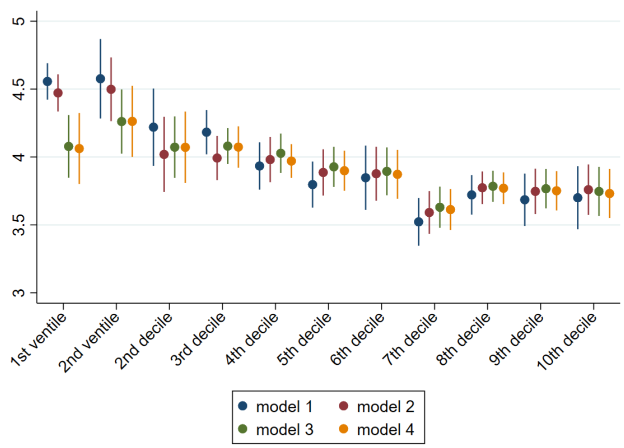
The first ventile is apart however, with a childlessness rate of 7 to 8% depending on the specifications. It suggests that the limitation of fertility in order to invest in education for that group comes at the cost of increasing accidental childlessness, which may arise from postponing marriage and conception attempts. One may have thought that childlessness should be particularly low for that group, as one needs at least one child to be able to reap the benefits of high investment in education. However, this is not what we observe. Childlessness might not be as detrimental in reality as in our model if there is some sort of cooperative breeding, whereby childless women help mothers in their extended family raise and educate their children.

G.4 Rural vs urban

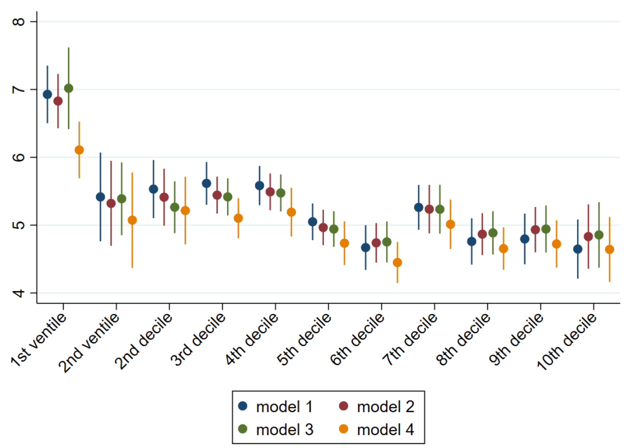
Finally, we split the sample according to urban versus rural status and run the analysis again. Results are shown in Figure 9. The higher fertility of smaller groups seems to be especially marked in rural areas, while the very low fertility of very small groups is a rather urban



(a) Predicted count of children ever born by group size deciles



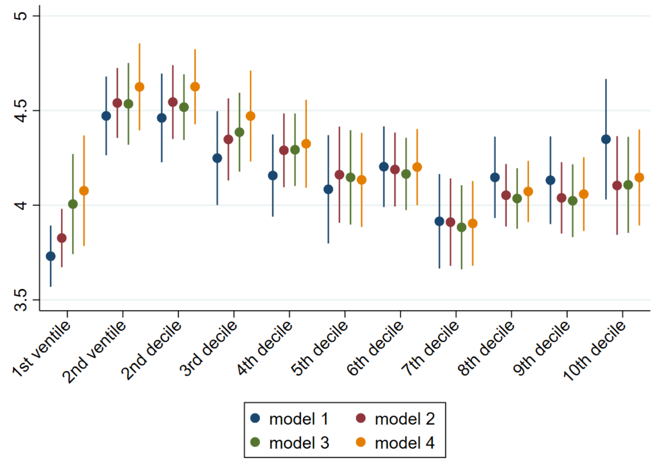
(b) Predicted count of surviving children by group size deciles



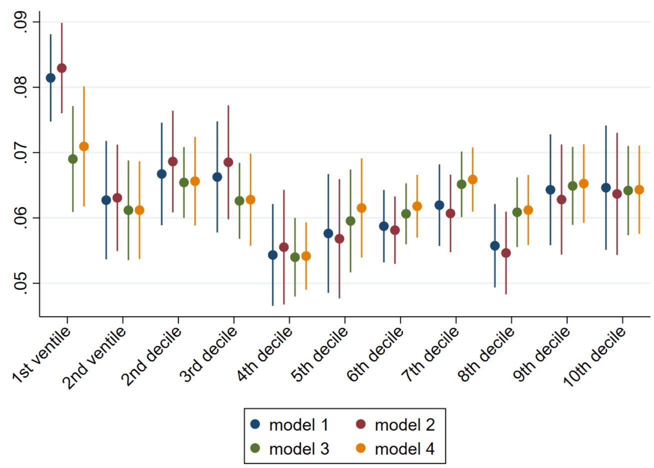
(c) Predicted years of education by group size deciles

Source: Indonesian Census waves 1971, 1980 and 1990
Notes: Panels (a) and (b) are based on the fertility equation estimated using Poisson regression. Panel (c) is based on the education equation estimated by OLS. Confidence intervals are all computed based on standard errors clustered at the regency level. Corresponding tables of coefficients are available upon request.

Figure 7: Fertility and Education by Religious Group Size, excluding 2000 and 2010



(a) Predicted count of children ever born by group size deciles, sample of mothers



(b) Predicted probability to remain childless by group size deciles

Source: Indonesian Census, waves 1971-2010

Notes: Panel (a) is based on the fertility equation estimated using Poisson regression. Panel (b) has the same controls as the fertility equation equation but is estimated by OLS as the dependent variable is a dummy equal to 1 if a woman is childless and 0 otherwise. Confidence intervals are all computed based on standard errors clustered at the regency level. Corresponding tables of coefficients are available upon request.

Figure 8: Extensive and Intensive Margins of Fertility by Ethnic Group Size

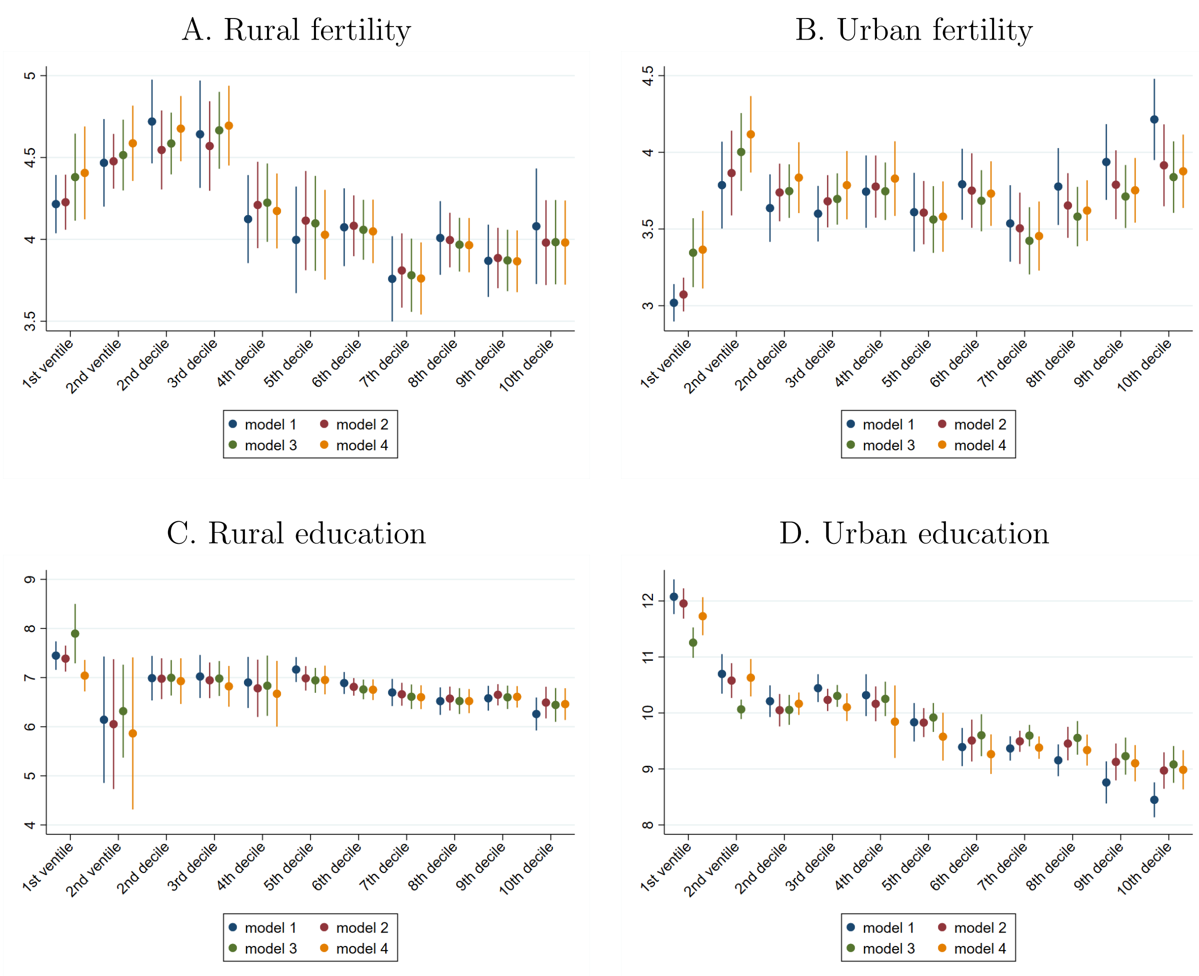


Figure 9: Fertility and education choices by group size and urban status

Source: Indonesian Census, waves 1971-2010

Notes: Fertility corresponds to the count of children ever born predicted by the model estimated using Poisson regression. Education is the number of years of schooling predicted by the model estimated using OLS. Corresponding tables of coefficients are available upon request.

phenomenon. In terms of education, we observe a negative relation with group size in both urban and rural contexts, but the gradient is much steeper in urban areas.

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