

# Lectures on Fertility, Education, Growth, and Sustainability

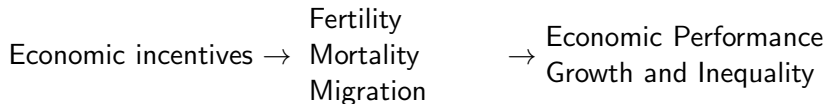
## 1a. Introduction and Benchmark model

David de la Croix<sup>1</sup>

IRES and CORE, Univ. cath. Louvain (UCL)

March 2012

# Population Economics



Population changes slowly over time ...  
but has large effects in the long-run

In these lectures, focus on **net** fertility

## methodology: OLG

### Preferred toolkit: Overlapping Generations Model (OLG)

- Households have a finite lifespan
- Discrete time can be used if we are not interested in length of things (such as birth spacing, age at first birth etc...)
- Age and stages of life are modeled explicitly  
example of life: childhood - adulthood, adulthood - old age,  
junior worker - senior worker - retired.
- Perfect foresight but limited horizon (unless rational altruism is assumed)

## Methodology: Quantitative theory

Quantitative theory uses simple, abstract economic models together with a small amount of economic data to highlight major economic mechanisms

Models are combined with economic data to display successes and failures of particular theoretical mechanisms

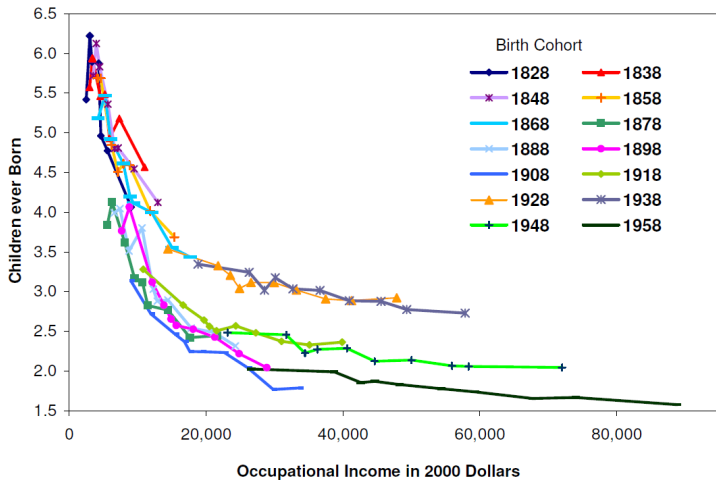
No statistical inference

Approach that may complement others. E.g. in population history, complement the historical approach, to lead to further insights.

## Facts about Fertility

- Fact 1: In all species, when resources are more abundant, reproduction increases. This is true for plants, animals, and humans before the industrial revolution.
- Fact 2: Before the industrial revolution, the rich had more surviving children than the poor.
- Fact 3: The transition from stagnation to economic growth is accompanied by a demographic transition from high to low fertility.
- Fact 4: Now, both within and across countries, rich and educated mothers have less children than poor and unskilled ones.
- Fact 5: Most of the literature finds that the income of the father positively affects fertility, while the income of the mother negatively affects fertility.

# The decline in fertility



from Jones and Tertilt (2007)

## Reasons for the decline in fertility (1)

Demographers would stress:

**Contraception:** Better contraception technology. But how large is the gap between desired and effective fertility ?

**Mortality:** Lower child mortality may imply lower birth, to get the same number of surviving children (child-replacement hypothesis)

**Culture:** Change in cultural norms (drops in fertility across Europe often followed linguistic and religious contours)

## Reasons for the decline in fertility (2)

Economists would stress:

**Mortality:** Same mechanism as demographers

**Old-age support:** Children as a way to save resources for the future and to obtain some support when old. Declines with pension systems.

**Ban on child labor:** Reduces the return from children

**Parents' income:** Opportunity cost of child-rearing time is high for high income/education mothers

**Return to education:** Industrial revolution accompanied by a rise in the skill premium (Galor).



## Quality - Quantity Tradeoff Model

Most economic models are based on the QQ model

In the budget constraint:

Total cost of children = number  $\times$  spending on quality  
(education+health)

When number of children (quantity) becomes too costly, or if quality becomes more profitable parents may want to invest more in the quality of a small number of children. Becker.

## Differential fertility

QQ account for fertility over time in the demographic transition, but also for fertility rates in the cross-section of a given country.

Since for **EDUCATED WOMEN** the opportunity cost of child-rearing time is high, they prefer to invest in the “quality” of a small number of children.

For **LESS EDUCATED WOMEN**, the opportunity cost of raising children is low, while providing education is expensive relative to their income. They would therefore prefer to have many children, but invest little in the education of each child.

# 1. Surveys

Do people really choose their number of children ?? four arguments

## 1. Surveys (Pritchett 1994)

Ninety percent of the differences across countries in total fertility rates are accounted for solely by differences in women's reported desired fertility.

But one can certainly find the opposite conclusion in another survey

And, anyway, economists do not believe in what people say, they prefer to look at what people do

## 2. Common sense

Having one more child is a huge investment

Similar in cost to buying a small house (Cigno)

Cumulative hours of child care that the wife devotes:

1 kid family: 9.274

2 kids family: 12.946

3 kids family: 18.389

+ Husbands time

Mean hours per child: 1 kid family: 9.274 2 kids family: 6.473 3  
kids family: 6.130

## 3. Historical data

Look at forerunners in fertility decline

Fertility started to decline in some European cities as early as in the 18th century

Because some incentives changed ? return to education ?

## Historical data - Rouen (Bardet)

	notables	merchants	craftsmen	workmen
<i>Literacy rate</i>				
1670-99	91.99	76.00	60.50	29.50
1700-29	97.42	79.00	71.50	34.00
1730-59	96.11	83.50	77.00	42.50
1760-92	95.70	90.50	82.50	47.50
<i>Fertility per women</i>				
1670-99	6.23	6.53	7.19	7.21
1700-29	4.87	5.51	6.29	6.06
1730-59	4.84	4.81	5.48	5.67
1760-92	3.77	3.28	4.84	4.84
<i>Survival probability</i>				
15 → 30	0.865	0.875	0.875	0.857
0 → 15	0.521	0.474	0.474	0.408

## Data for Geneva (Perrenoud)

	notables	craftmen	workmen
<i>Literacy rate</i>			
1700-4	86.00	54.50	12.50
1741-5	96.50	83.50	33.50
1770-4	98.50	87.50	58.00
<i>Fertility per women</i>			
1675-96	6.70	7.10	6.20
1700-4	6.70	7.30	5.50
1741-5	4.70	5.70	4.20
1770-4	2.80	5.20	4.70
<i>Survival probability</i>			
15 → 30	0.89	0.84	0.80
0 → 15	0.61	0.452	0.338

## 4. Fathers vs mothers

Many studies find that fertility increases with father's wage but decreases with mother's wage

- Mother's wage: income effect + substitution effect (opportunity cost story)
- Father's wage: income effect only

If those studies are right, strong evidence in favor of the economic approach

against the idea that wage reflect “culture” or “norms” only



## Map of the lectures (1)

### Lecture 1 - Differential fertility and the dynamics of inequality

- Benchmark model
- Growth and inequality
- Forerunners in fertility decline

De la Croix D. and M. Doepke, Inequality and growth: why differential fertility matters, American Economic Review, 2003

## Map of the lectures (1)

### Lecture 2 - Education policy

- Private vs public schools
- Education Politics
- Empirical Evidence

De la Croix D. and M. Doepke, Private versus public education when differential fertility matters, *Journal of Development Economics*, 2004

De la Croix D. and M. Doepke, To Segregate or to Integrate: Education Politics and Democracy, *Review of Economic Studies*, 2009

## Map of the lectures (3)

### Lecture 3 - Sustainability

- Environmental collapse
- Production vs Reproduction, and Pollution Control
- Population policy

De la Croix D. and D. Dottori, Easter Island Collapse: a Tale of Population Race, *Journal of Economic Growth*, 2008

De la Croix D. and A. Gosseries, Population Policy through Tradable Procreation Entitlements, *International Economic Review*, 2009

## Basics

A simple model linking together fertility, inequality and growth

Time is discrete and runs from 0 to  $\infty$ .

People live for two periods, childhood, and adulthood.

Unitary representation of the household.

Two types of agents, unskilled (group  $i = A$ ) and skilled (group  $i = B$ ).

The size of each group is denoted  $N_t^i$ .

Agents represent households within a country, but we can also interpret them as countries within the global economy.

## Preferences

Parents choose consumption  $c_t^i$ , fertility  $n_t^i$  and education  $e_t^i$

$$\max \ln[c_t^i] + \gamma \ln[n_t^i \pi(e_t^i)]. \quad (1)$$

Probability for a child to become skilled:

$$\pi^i(e) = \tau^i (\theta + e)^\eta, \quad \eta \in (0, 1).$$

Budget constraint

$$c_t^i = [w_t^i(1 - \phi n_t^i) - n_t^i e_t^i]. \quad (2)$$

Maximum fertility:  $1/\phi$

## Firms and markets

Output:

$$Y_t = \omega^A L_t^A + \omega^B L_t^B.$$

The equilibrium condition on both labor markets

$$N_t^i(1 - \phi n_t^i) = L_t^i$$

Implies that wages are equal to marginal productivity:

$$w_t^i = \omega^i.$$

# Equilibrium

## Definition (Benchmark Inter-temporal Equilibrium)

Given initial population sizes  $N_0^A$  and  $N_0^B$ , an equilibrium is a sequence of individual quantities  $(\hat{c}_t^i, \hat{e}_t^i, \hat{n}_t^i)_{i=A,B,t \geq 0}$  and group sizes  $(\hat{N}_t^i)_{i=A,B,t \geq 0}$  such that

- Consumption, education and fertility maximize households' utility (1) subject to the budget constraint (2);
- Group sizes evolve according to:

$$\begin{bmatrix} \hat{N}_{t+1}^A \\ \hat{N}_{t+1}^B \end{bmatrix} = \begin{bmatrix} \hat{n}_t^A(1 - \pi^A(\hat{e}_t^A)) & \hat{n}_t^B(1 - \pi^B(\hat{e}_t^B)) \\ \hat{n}_t^A \pi^A(\hat{e}_t^A) & \hat{n}_t^B \pi^B(\hat{e}_t^B) \end{bmatrix} \begin{bmatrix} \hat{N}_t^A \\ \hat{N}_t^B \end{bmatrix} \quad (3)$$

- Labor market clears, i.e.

$$\hat{N}_t^i(1 - \phi \hat{n}_t^i) = L_t^i \quad \forall i. \quad (4)$$

## Solution to the household problem

If  $w_t^i > \theta/(\eta\phi)$  [interior regime],

$$\hat{e}_t^i = \frac{\eta\phi w_t^i - \theta}{1 - \eta}, \quad \text{and:} \quad (5)$$

$$\hat{n}_t^i = \frac{(1 - \eta)\gamma w_t^i}{(\phi w_t^i - \theta)(1 + \gamma)}. \quad (6)$$

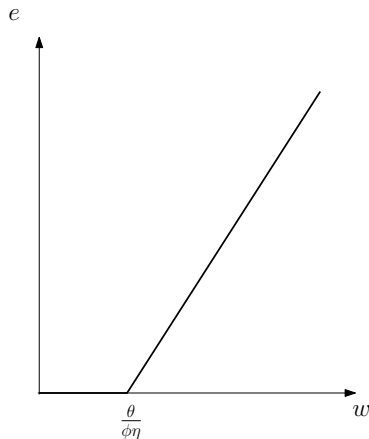
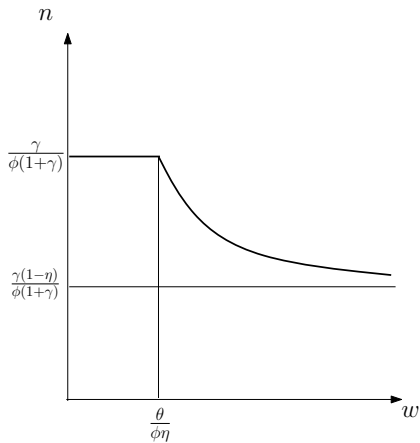
otherwise,

$$\hat{e}_t^i = 0, \quad \text{and:} \quad (7)$$

$$\hat{n}_t^i = \frac{\gamma}{\phi(1 + \gamma)} \quad (8)$$



# Fertility as a Function of Parents' Human Capital



## Inequality

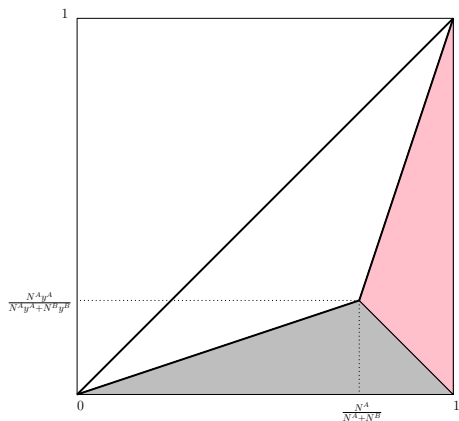
Upper bound on the fertility differential:

$$\frac{\lim_{x_t \rightarrow 0} n_t}{\lim_{x_t \rightarrow \infty} n_t} = \frac{1}{1 - \eta}.$$

Denote individual income by  $y^i = \omega^i(1 - \phi \hat{n}^i)$ .

$$\text{Gini} = \frac{N^B}{N^A y^A / y^B + N^B} - \frac{N^B}{N^A + N^B}.$$

## Construction of the Gini Coefficient with two Groups



Gini =  $1/2$  minus the surface of the pink triangle  
 (=  $(1/2)N^B / (N^A + N^B)$ ) minus the surface of the grey triangle  
 (=  $(1/2)N^A y^A / (N^A y^A + N^B y^B)$ )

## Dynamics (1)

Population ratio:

$$z_t = \frac{N_t^A}{N_t^B}.$$

The dynamic system (3) can be reduced to:

$$z_{t+1} = \frac{n^A(1 - \pi^A)z_t + n^B(1 - \pi^B)}{n^A\pi^Az_t + n^B\pi^B} \equiv f(z_t).$$

The function  $f$  has the following properties:

$$f(0) = \frac{1 - \pi^B}{\pi^B} > 0$$

$$f'(z) = \frac{n^An^B(\tau^B - \tau^A)}{(n^A\pi^Az + n^B\pi^B)^2} > 0$$

$$f''(z) = \frac{2(n^A)^2\pi^An^B(\tau^A - \tau^B)}{(n^A\pi^Az + n^B\pi^B)^3} < 0$$

The last two results are guaranteed by the fact that  $\tau^B > \tau^A$ .

## Dynamics (2)

### Proposition (Dynamics of the Composition of Population)

The dynamics of  $z_t$  given by  $z_{t+1} = f(z_t)$  admit a single positive steady state:

$$\bar{z} = \frac{n^A(1 - \pi^A) - n^B\pi^B \sqrt{(n^B\pi^B - n^A(1 - \pi^A))^2 + 4n^A\pi^An^B(1 - \pi^B)}}{2n^A\pi^A},$$

which is globally stable.

## Income and inequality

Average income per capita is a negative function of  $z_t$ .

Income inequality as measured by the Gini coefficient is

$$\begin{aligned} \text{Gini}_t &= \frac{N_t^B y^B}{N_t^A y^A + N_t^B y^B} - \frac{N_t^B}{N_t^A + N_t^B} \\ &= \frac{\omega^B (1 - \phi n^B)}{z_t \omega^A (1 - \phi n^A) + \omega^B (1 - \phi n^B)} - \frac{1}{z_t + 1} \end{aligned}$$

It is equal to 0 when  $z_t = 0$  or when  $z_t \rightarrow \infty$ .

Maximum inequality at

$$\hat{z} = \sqrt{\frac{\omega^B (1 - \phi n^B)}{\omega^A (1 - \phi n^A)}}$$

# Calibration

Assume one period in the model lasts 25 years.

Calibrate  $\eta$ ,  $\phi$ ;  $\theta$ , and  $\gamma$  on a cross-section of countries.

Data from the World Development Indicators, averaging those available for the years 1998-2002.

## Data

$n$ : “Fertility rate, total (births per woman)” divided by two to obtain a fertility rate per person, and multiplied by  $(1 - \text{“Mortality rate, infant (per 1,000 live births)”} / 1000)$  to measure net fertility per capita.

Total education  $\theta + e$ : “Adjusted savings: education expenditure (% of GNI)”  $\times$  “GNI per capita, PPP (current international \$)”

Population size  $N$ : population aged 15-64.

Productivity per person  $\omega$  obtained from GNI per capita  $y$  as follows:

$$y = \omega(1 - \phi n / 25). \quad (9)$$



## Maximum likelihood estimation

$$e_i = \begin{cases} 0 & \text{if } \omega_i \leq \frac{\theta}{\eta\phi} \\ \frac{\eta\phi\omega_i - \theta}{1-\eta} & \text{if } \omega_i > \frac{\theta}{\eta\phi} \end{cases} + \varepsilon_i^e \quad (10)$$

$$n_i = \begin{cases} \frac{\gamma}{\phi(1+\gamma)} & \text{if } \omega_i \leq \frac{\theta}{\eta\phi} \\ \frac{(1-\eta)\gamma\omega_i}{(\phi\omega_i - \theta)(1+\gamma)} & \text{if } \omega_i > \frac{\theta}{\eta\phi} \end{cases} + \varepsilon_i^n \quad (11)$$

$$y_i = \omega_i(1 - \phi n_i/25)$$

## Maximum likelihood results

---

---

Number of observations = 158

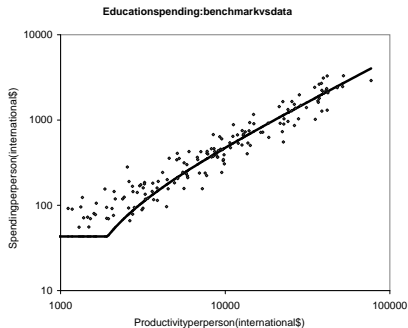
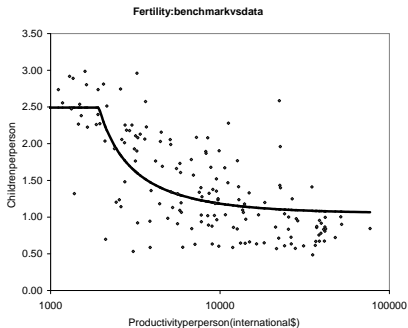
Parameter	Estimate	Standard Error
$\eta$	0.578	0.0356
$\phi$	0.039	0.0057
$\theta$	43.18	4.9365
$\gamma$	0.107	0.0127

Equation	education	fertility
R-squared	0.88	0.50

---

---

# Calibrated Fertility and Education Relationships



below a productivity of  $\theta/(\phi\eta) = 1915$  dollars per person and per year,  $e = 0$

## Conclusion

A very simple pedagogical model

Interactions over time between demography and economics

Income and population dynamics can be computed explicitly

Inequality is endogenous