Lectures on Fertility, Education, Growth, and Sustainability

1b. Growth and Inequality

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Question

We extent the benchmark model to address a precise quantitative question:

Inequality bad for growth

Many channels are invoked: political economy, sociopolitical unrest, borrowing constraints...

One neglected channel: differential fertility

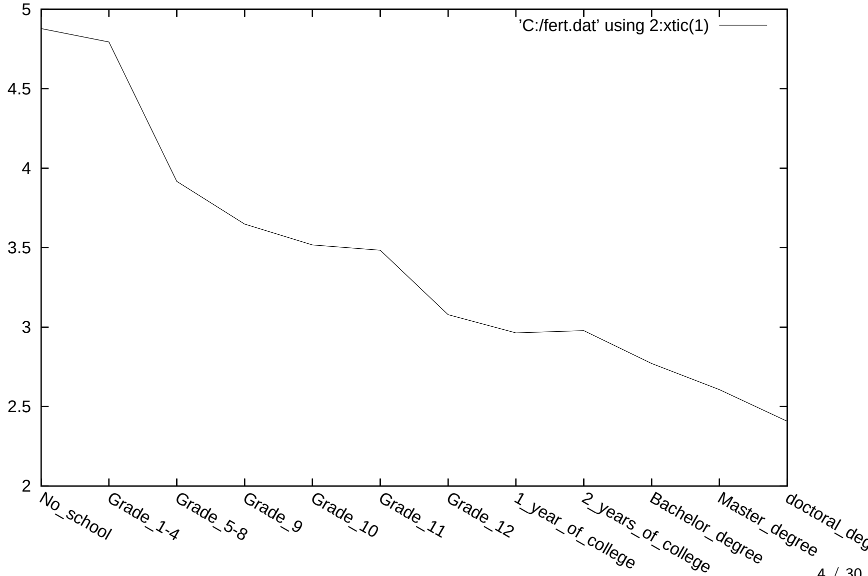
We study its importance
– Theoretical model
– Calibration

Differential fertility: Fertility Rates by Education

		Total Fertility Rate		
Survey	# of Countries	<Elementary	Elementary	Secondary+
WFS, 1975-1979	13 EUR/US	2.40	2.17	1.79
WFS, 1974-1982	30 DC	6.5	5.5	4.0
DHS, 1985-1989	26 DC	5.7	4.9	3.6
DHS, 1990-1994	27 DC	5.29	4.72	3.29

Source: WFS: World Fertility Survey. DHS: Demographic and Health Survey. "Secondary+" is the average of low secondary, high secondary, and post-secondary, where appropriate.

Differential fertility: US census 1990



The Model

- People live for three periods: childhood, adulthood, old age
- Individual state: Human capital h_t
- All decisions taken by adults: savings s_t, number of children n_t and education e_t

Compared to benchmark:

- physical capital is included,
- distribution of human capital is continuous,
- technical progress is introduced,
- marginal productivity of labor is no longer constant.

Decision problem of an adult

$$\max\left\{\ln(c_t) + \beta\ln(d_{t+1}) + \gamma\ln(n_t h_{t+1})\right\}$$

subject to:

$$\begin{aligned} c_t + s_t + e_t n_t w_t \bar{h}_t &= w_t h_t(1-\phi n_t), && (1)\\ d_{t+1} &= R_{t+1}s_t, && (2)\\ h_{t+1} &= B_t(\theta + e_t)^{\eta}(h_t)^{\tau}(\bar{h}_t)^{\kappa}. && (3) \end{aligned}$$

$\gamma > 0$: altruism factor
$\beta > 0$: psychological discount factor
$\phi \in (0,1)$: rearing time cost
$\theta > 0$, $\eta \in (0,1)$: education technology
$\tau \in [0,1]$: intergenerational transmission of human capital within family
$\kappa \in [0, 1-\tau]$: externality at the community or society level.

Technology

Technical progress in human capital production:

$$B_t = B\left(1+\rho\right)^{(1-\tau-\kappa)t}. \tag{4}$$

compatible with endogenous growth for $\kappa = 1 - \tau$, and with exogenous growth otherwise.

Final good production:

$$Y_t = AK_t^{\alpha}L_t^{1-\alpha}$$

Aggregate state:

- Capital K_t
- Human capital distribution $F_t(h_t)$
- Population P_t

Equilibrium conditions

Total population P_t evolves over time according to:

$$P_{t+1} = P_t \int_0^\infty n_t \, \mathrm{d}F_t(h_t), \tag{5}$$

Distribution function of human capital evolves according to:

$$F_{t+1}(h) = \frac{P_t}{P_{t+1}} \int_0^\infty n_t \; I(h_{t+1} \leq h) \, \mathrm{d}F_t(h_t). \tag{6}$$

Average human capital $\overline{h}_t$ is given by:

$$\overline{h}_t = \int_0^\infty h_t \, \mathrm{d}F_t(h_t). \tag{7}$$

The market-clearing conditions for capital and labor are:

$$K_{t+1} = P_t \int_0^\infty s_t \, \mathrm{d}F_t(h_t), \tag{8}$$

$$L_t = P_t \left[\int_0^\infty h_t(1 - \phi n_t) \, \mathrm{d}F_t(h_t) - \int_0^\infty e_t n_t \overline{h}_t \, \mathrm{d}F_t(h_t) \right]. \tag{9}$$

Equilibrium definition

Definition (Intertemporal Equilibrium with Heterogeneity)

Given an initial distribution of human capital $F_0(h_0)$, an initial stock of physical capital K_0, and an initial population size P_0, an equilibrium consists of sequences of prices $\{w_t,\ R_t\}$, aggregate quantities $\{L_t,\ K_{t+1},\ \bar{h}_t,\ P_{t+1}\}$, distributions $F_{t+1}(h_{t+1})$, and decision rules $\{c_t,\ d_{t+1},\ s_t,\ n_t,\ e_t,\ h_{t+1}\}$ such that:

1. households' decision rules c_t d_{t+1}, s_t, n_t, e_t, h_{t+1} maximize utility subject to the constraints (1), (2), and (3);
2. firm's choices L_t and K_t maximize profits;
3. prices w_t and R_t are such that markets clear, i.e., (8) and (9) hold;
4. the distribution of human capital evolves according to (6);
5. aggregate variables P_t and $\bar{h}_t$ are given by (4), (5) and (7).

Solution of the Adult's Problem(1)

We denote the relative human capital of a household as: $x_t \equiv \frac{h_t}{\bar{h}_t}$.
For a household that has enough human capital, s.t. $x_t > \frac{\theta}{\phi\eta}$:

$$s_t = \frac{\beta}{1+\beta+\gamma} w_t h_t, \tag{10}$$

$$e_t = \frac{\eta\phi x_t - \theta}{1-\eta}, \tag{11}$$

$$n_t = \frac{(1-\eta)\gamma x_t}{(\phi x_t - \theta)(1+\beta+\gamma)}. \tag{12}$$

For poorer households s.t. $x_t \leq \frac{\theta}{\phi\eta}$, equation (10) and:

$$e_t = 0, \tag{13}$$

$$n_t = \frac{\gamma}{\phi(1+\beta+\gamma)}. \tag{14}$$

Equilibrium with stationary variables (1)

Define stationary variables:

$$k_t \equiv \frac{K_t}{L_t}, \qquad g_t \equiv \frac{\bar{h}_{t+1}}{\bar{h}_t}, \qquad N_t \equiv \frac{P_{t+1}}{P_t}, \qquad \hat{h}_t \equiv \frac{\bar{h}_t}{(1+\rho)^t}.$$

Define the distribution of the relative human capital levels:

$$G_t(x_t) \equiv F_t(x_t \bar{h}_t).$$

Rewrite equations (4), (5), (6) and (7):

$$\hat{h}_{t+1} = \frac{g_t}{1+\rho}\hat{h}_t \tag{15}$$

$$N_t = \int_0^\infty n_t \, \mathrm{d}g_t(x_t), \tag{16}$$

$$G_{t+1}(x) = \frac{1}{N_t}\int_0^\infty n_t \; I(x_{t+1} \leq x) \; \mathrm{d}g_t(x_t), \tag{17}$$

$$1 = \int_0^\infty x_t \; \mathrm{d}g_t(x_t). \tag{18}$$

Equilibrium with stationary variables (2)

Prices follow from the competitive behavior of firms:

$$w_t = A(1-\alpha)k_t^{\alpha}, \tag{19}$$

$$R_t = A\alpha k_t^{\alpha-1}.$$

The number of children for an adult with relative human capital x_t is thus given by:

$$n_t = \min\left[\frac{(1-\eta)\gamma x_t}{(\phi x_t - \theta)(1+\beta+\gamma)}, \frac{\gamma}{\phi(1+\beta+\gamma)}\right]. \tag{20}$$

From equation (3), the children's human capital is given by:

$$x_{t+1} = \frac{B x_t^{\tau}}{g_t}\left(\theta + \max\left[0, \frac{\eta\phi x_t - \theta}{1-\eta}\right]\right)^{\eta}(\hat{h}_t)^{\tau+\kappa-1}. \tag{21}$$

Equilibrium with stationary variables (3)

From equation (9), labor input satisfies:

$$\frac{L_t}{P_t \bar{h}_t} = \int_0^{\frac{\theta}{\eta\phi}} \frac{(1+\beta)x_t}{1+\beta+\gamma} \, \mathrm{d}g_t(x_t) + \int_{\frac{\theta}{\eta\phi}}^{\infty} \left(1 - \gamma \frac{\phi(1-\eta)x_t + (\eta\phi x_t - \theta)}{(\phi x_t - \theta)(1+\beta+\gamma)}\right) x_t \, \mathrm{d}g_t(x_t).$$

which leads to:

$$\frac{L_t}{P_t \bar{h}_t} = \frac{1+\beta}{1+\beta+\gamma}. \tag{22}$$

Using (8), (10), (19) and (22), the capital stock evolves according to the following law of motion:

$$k_{t+1} = \frac{\beta}{1+\beta} \, \frac{1}{g_t N_t} \, A(1-\alpha) k_t^{\alpha}. \tag{23}$$

Equilibrium with stationary variables (4)

Given initial conditions k_0, $\hat{h}_0$ and $G_0(x_0)$, an equilibrium can be characterized by sequences $\{\hat{h}_{t+1},\ g_t,\ n_t,\ G_{t+1}(x),\ N_t,\ x_t,\ k_{t+1}\}$ such that (15), (16), (17), (18), (20), (21), and (23) hold at all dates.

Proposition (Existence and Uniqueness of the Equilibrium)

Given any initial conditions, an equilibrium exists and is unique.

Balanced Growth Path

Proposition (Balanced Growth Path & Limiting Distribution)

If $\eta\phi > \theta$, there is a balanced growth path characterized by $dG(1) = 1$ (i.e. the limiting distribution is degenerate). The growth factor of output and human capital is:

$$g^\star = \begin{cases} B\left(\frac{\eta(\phi-\theta)}{1-\eta}\right)^{\eta} & \text{if} \quad \kappa = 1-\tau \text{ (endogenous growth)}, \\ 1+\rho & \text{otherwise (exogenous growth)}, \end{cases}$$

and the growth factor of population is:

$$N^\star = \frac{(1-\eta)\gamma}{(\phi-\theta)(1+\beta+\gamma)}.$$

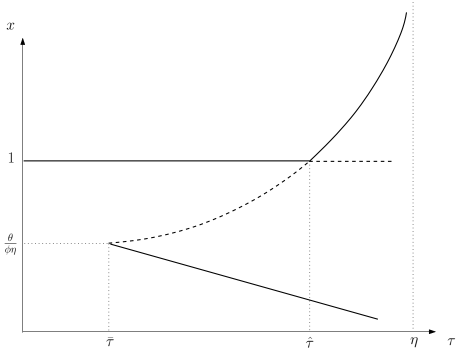
Dynamics of Individual Human Capital

Examine $x_{t+1} - x_t = \Psi(x_t; \tau)$ for $g = g^\star$, $h = h^\star$:

$$\Psi(x;\tau) = \frac{Bx^\tau}{g^\star}\left(\theta + \max\left[0, \frac{\eta\phi x - \theta}{1-\eta}\right]\right)^\eta (\hat{h}^\star)^{\kappa+\tau-1} - x.$$

$$\Psi(x;\tau) = \left(\frac{1-\eta}{\eta(\phi-\theta)}\right)^\eta x^\tau \left(\theta + \max\left[0, \frac{\eta\phi x - \theta}{1-\eta}\right]\right)^\eta - x. \quad (24)$$

Steady State Human Capital as a Function of τ



$$\hat{\tau} = 1 - \frac{\eta\phi}{\phi - \theta}$$

The transcritical bifurcation

Proposition (Dynamics around the Transcritical Bifurcation)

At the point:

$$\hat{\tau} = 1 - \frac{\eta\phi}{\phi - \theta}$$

the dynamics of individual capital described by $x_{t+1} - x_t = \Psi(x_t; \tau)$ *undergo a transcritical bifurcation. There are two steady-state equilibria, 1 and* $\bar{x}$*, near* $(1, \hat{\tau})$ *for each value of* τ *smaller or larger than* $\hat{\tau}$*. The equilibrium 1 (resp.* $\bar{x}$*) is stable (resp. unstable) for* $\tau < \hat{\tau}$ *and unstable (resp. stable) for* $\tau > \hat{\tau}$*.*

Calibration

one period (or generation) has a length of thirty years

- $\beta = .99^{120}$,
  $\alpha = 1/3$.
- $\rho = 1.02^{30} \rightarrow$ exo growth of 2% per year.
- $B = 1$ (only a scale parameter)
  $\gamma = .271 \rightarrow N = 0\%$ per year.
- $\eta = .637$: fertility differential in Brazil
  $\phi = .075$: rearing cost= 15% of time endowment
  $\theta = .0119$: share of educ along the bgp =7.3 of GDP%
- $\kappa = .1$: externalities are small (social return only slightly larger than the private return)
  τ: does not affect the bgp. $\hat{\tau} = .246$. Sensitivity analysis

The implied interest rate per year is 4.7 percent

Calibration: ϕ

Time-cost ϕ determines the overall opportunity cost of children.

Evidence in Haveman and Wolfe (1995) suggests that the opportunity cost of a child is about 15 percent of the parents' time endowment.

This cost only accrues as long as the child is living with the parents.

Assume that children live with parents for 15 years.

Accordingly, we choose $\phi = 0.075$.

The parameter ϕ also sets an upper limit on the number of children a person can have.

Calibration: the return to education

Combined choices for η and θ imply elasticity of human capital with respect to education of 0.6 in the bgp.

This number is within the range of estimates of the elasticity of earnings with respect to schooling. (Mincer equations)

Surveys: return to schooling in developed countries 8-10 percent, with higher estimates for developing countries.

Assume that an additional year of schooling raises education expenditure by 20 percent.

10% return of schooling translates into 0.5 earnings elasticity to spending.

Initial conditions

The initial distribution of human capital follows a log-normal distribution $F(\mu, \sigma^2)$.

μ is set such that $\hat{h}_t$ is at its balanced-growth level.

We provide simulations for different variances of the distribution in order to examine the effects of inequality.

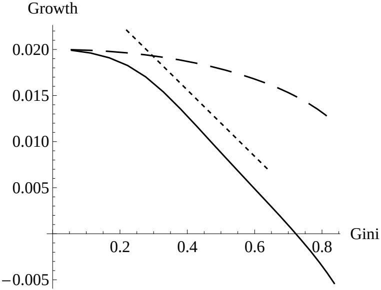
K_0 such that the ratio of physical to human capital is equal to its value in the balanced growth path.

Initial effect of inequality (1)

	Endogenous Fertility				Exogenous Fertility			
σ^2	g_0	N_0	I_0	D_0	g_0	N_0	I_0	D_0
0.10	2.00%	0.00%	0.056	0.09	2.00%	0%	0.056	0
0.75	1.26%	0.66%	0.404	1.95	1.87%	0%	0.400	0
1.00	0.80%	1.08%	0.520	2.76	1.78%	0%	0.513	0
1.50	0.01%	1.71%	0.707	2.77	1.53%	0%	0.700	0

I_0: initial Gini on earnings. D_0: initial fertility differential

Initial effect of inequality (2)



The Relationship of Inequality and Growth with Endogenous Fertility (Solid), Exogenous Fertility (Dashed), and in Barro's Regression (Dotted)

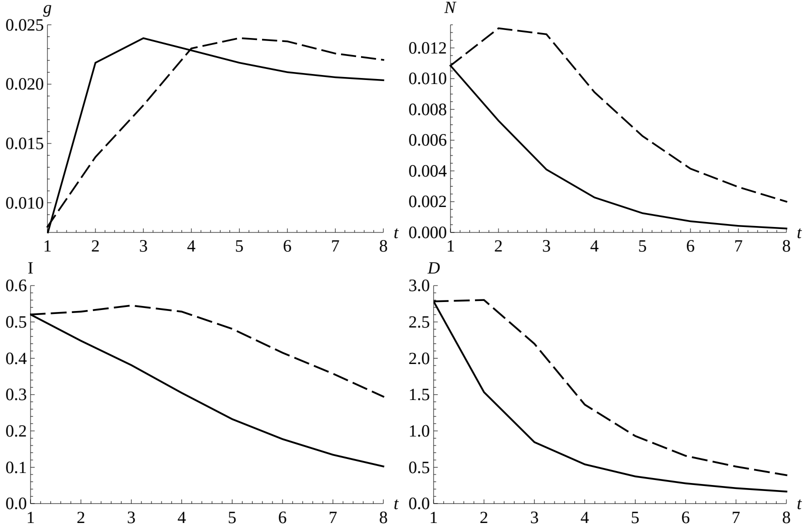
Dynamics

Horizon of eight periods, corresponding to 240 years.

Initial inequality $\sigma^2 = 1$, corresponding to an initial Gini coefficient of 0.5.

Growth, Fertility, Inequality, and Differential Fertility for $\tau = 0.05$ and $\tau = 0.2$

Dynamics: results



Dynamics: results

The non-monotone behavior of inequality and fertility related to the corner solution for education.

A moderate degree of intergenerational persistence in human capital is essential.

The main disparity between the model and the data is that in the data growth rates were slowly increasing throughout the nineteenth century

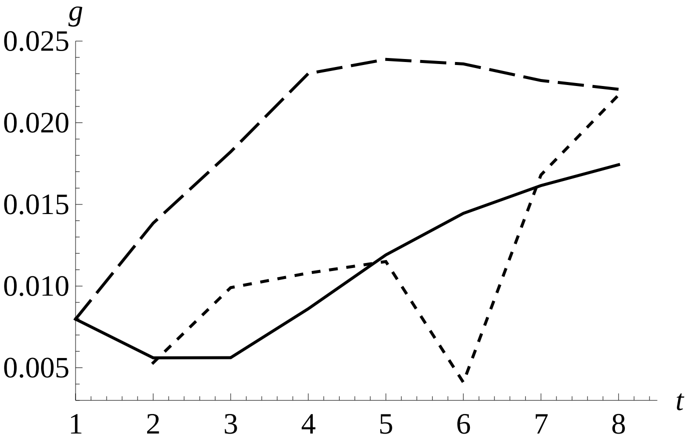
Exogenous vs Endogenous growth

We increase the human capital externality to $\kappa = 1 - \tau = 0.8$

The overall productivity B in the production function for human capital now governs the growth rate of output per capita. We pick $B = 0.367$ to get the 2% in the bgp

Individual decisions and the evolution of inequality, fertility, and differential fertility are independent of the assumption on κ

Exogenous vs Endogenous growth



The pattern under the endogenous growth assumption is closer to the observed pattern in Western Europe.

Conclusion

Families with less human capital decide to have more children and invest less in education.

When income inequality is high, large fertility differentials lower the growth rate of average human capital by a substantial amount.

Dynamic implications for the evolution of fertility and inequality in industrializing countries in the last 200 years are matched.

If allow for endogenous growth, implications for income are matched too.

A natural direction for further research concerns the policy implications of our model. Tomorrow.