

# Lectures on Fertility, Education, Growth, and Sustainability

## 2b. Education politics

David de la Croix

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# Education Funding

- Share of private funding in total education funding varies greatly across countries.
- 44.5% of total spending in Chile, 25% in the US, only 1.9% in Norway.
- **Research Question** why such big differences ?  
What are the determinants of the mix ?

# Segregation

- Important factor: whether elites participate to public schools
- If elites go to private schools, *segregation*.  
They vote for low funding levels of public schools.
- *Segregation* varies greatly across countries.  
PISA data - we compute private school attendance by social class.
  - Programme for International Student Assessment.
  - Year 2000, 15 year-old students, 30 countries.
  - Math or language test + student questionnaire + school questionnaire.

## PISA for Norway and the U.K.

Country	social status	N. obs.	subsidy rate	% in priv. schools	fertility
Norway	16-35	418	99.57	0.72	3.40
	36-53	1737	99.71	0.63	2.98
	54-70	1148	99.53	1.13	2.99
	71-90	538	99.39	1.12	2.95
United Kingdom	16-35	1858	98.24	0.65	3.44
	36-53	3166	96.50	2.46	2.99
	54-70	2276	89.99	8.92	2.82
	71-90	856	84.93	14.02	2.82

## PISA for Brazil and Korea

Country	social status	N. obs.	subsidy rate	% in priv. schools	fertility
Brazil	16-35	1699	87.93%	2.35%	3.67
	36-53	831	79.52%	10.59%	3.36
	54-70	926	66.77%	23.00%	3.07
	71-90	125	41.60%	49.60%	2.86
Korea	16-35	1554	53.63%	47.23%	2.46
	36-53	1840	48.12%	50.00%	2.25
	54-70	803	46.47%	49.69%	2.18
	71-90	96	42.19%	45.83%	2.20

# What we do

A model to understand education funding and segregation

## Key features

- Heterogenous agent models
- Agents vote for the quality of public education
- And can opt out of the public system
- Fertility is endogenous

# Objective

Obtain a mapping:

Distribution of income

Distribution of political power  $\implies$  Schooling system

Government commitment

-level of funding

-level of segregation

## Preferences

Continuum of people differentiated by income  $x$ .

Parents care about consumption  $c$ , child quantity  $n$  and quality  $h$ :

$$U = \ln(c) + \gamma [\ln(n) + \eta \ln(h)]. \quad (1)$$

$\gamma > 0$  : taste for children.  $0 < \eta < 1$ : weight attached to quality.

**Trade-off between quantity and quality**, affected by parents skills and schooling regime.

## Constraints

Two modes of education:

- public: free, of quality  $s$ , funded by a general income tax  $v$
- private: of quality  $e$ , costs  $ne$  and is tax deductible.  
( $e$ =teaching hours, teacher's wage=1)

Budget constraint:

$$c = (1 - v) [x(1 - \phi n) - ne]. \quad (2)$$

Rearing time:  $\phi$ .

Utility function for household:

$$u[x, v, n, e, s] = \ln(1-v) + \ln(x(1-\phi n) - ne) + \gamma \ln n + \gamma \eta \ln \max\{e, s\}.$$

# Technology

Aggregate production function is linear in labor.

Distribution of productivity over the interval  $[1 - \sigma, 1 + \sigma]$

$$Y = \int_0^{\infty} x L g[x] dx.$$

Uniform distribution:  $g[x] = 1/(2\sigma)$  if  $1 - \sigma \leq x \leq 1 + \sigma$ ,  
 $g[x] = 0$  otherwise.

$L$ : input of every worker, smaller than the total number of hours –  
some hours are used as teaching time.

## Timing of decisions

Benchmark timing.

Motivation: Public spending adjusted frequently, fertility not.

Switching costs between public versus private education.

1. Parents choose fertility  $n$ , and schooling (private or public).  
If they choose private schools, they also fix the amount spent  $e$ .
2. Probabilistic voting on taxes and corresponding quality of public schools.

When choosing fertility and education households have perfect foresight about the quality of public schools, and the tax rate.

## Fertility and private education

Parents planning to send their children to public choose:

$$n^s = \arg \max_n u[x, v, n, 0, s] = \frac{\gamma}{\phi(1 + \gamma)}. \quad (3)$$

Households planning to provide private schooling choose:

$$n = \arg \max_n u[x, v, n, e, s] = \frac{x\gamma}{(1 + \gamma)(e + \phi x)},$$

$$e[x] = \arg \max_e u[x, v, n, e, s] = \frac{\eta\phi x}{1 - \eta}. \quad (4)$$

$$n^e = \frac{\gamma(1 - \eta)}{\phi(1 + \gamma)}. \quad (5)$$

Fertility is higher when parents choose public education.

Private education spending depends positively on wage  $x$ .

## Constant parental spending on children

### Lemma

*For given  $s$ ,  $v$  and  $x$ , parental spending on children does not depend on the choice of private versus public schooling and is equal to  $\frac{\gamma}{1+\gamma} x$ .*

Parents choosing private education have fewer children.

Tax base does not depend on the fraction of people participating in public schools.

## Opting out decision

### Lemma

*There exist an income threshold:*

$$\tilde{x} = \frac{1 - \eta}{\delta \phi \eta} E[s] \quad \text{with: } \delta = (1 - \eta)^{\frac{1}{\eta}}. \quad (6)$$

*such that households prefer private education if and only if  $x > \tilde{x}$ .*

Skilled households are more inclined to choose private education.

Endogenous percentage of children in public schools:

$$\psi = \begin{cases} 0 & \text{if } \tilde{x} < 1 - \sigma \\ \frac{\tilde{x} - (1 - \sigma)}{2\sigma} & \text{if } 1 - \sigma \leq \tilde{x} \leq 1 + \sigma \\ 1 & \text{if } \tilde{x} > 1 + \sigma \end{cases} \quad (7)$$

## Budget constraint

Balanced budget:

$$\int_0^{\tilde{x}} n^s s g[x] dx = \int_0^{\tilde{x}} v(x(1 - \phi n^s)) g[x] dx + \int_{\tilde{x}}^{\infty} v(x(1 - \phi n^e) - e[x]n^e) g[x] dx, \quad (8)$$

reduces to:

$$v = \Psi \frac{\gamma}{\phi} s \quad (9)$$

## Probabilistic voting

2 political parties,  $q$  and  $z$ . Proposed policy:  $s^q, s^z$ .

Probability that voter  $i$  votes for party  $q$ :  $F^i(u^i[s^q] - u^i[s^z])$   
 $F^i(\cdot)$  is a continuous cumulative distribution function.

Party  $q$  maximizes its expected vote share:  $\int_0^\infty g[x] F(\cdot) dx$

This implements the maximum of a social welfare function:

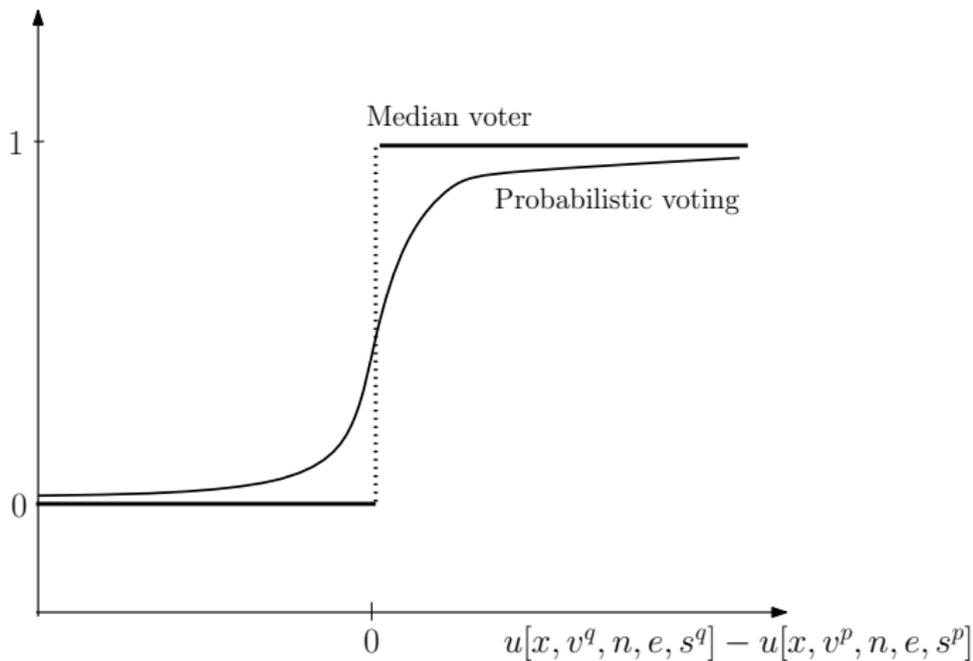
$$\int_0^\infty g[x] (F)'(0) u[s^q] dx.$$

At equilibrium,  $s = s^q = s^z$ .

Weights  $(F^i)'$ : responsiveness of voters  $\rightarrow$  “political power”.

# Probabilistic Voting vs Median Voter

Vote share of party  $q$



## Objective function

Maximize a social welfare function for given  $\tilde{x}$  :

$$\Omega[s] \equiv \int_0^{\tilde{x}} u[x, v, n^s, 0, s]g[x]dx + \int_{\tilde{x}}^{\infty} u[x, v, n^e, e[x], 0]g[x]dx. \quad (10)$$

Assumption: All have the same political power  $\rightarrow$  effective weights = population densities.

Solution:  $s$  decreases with the participation rate in public school.

$$s = \frac{\eta\phi}{1 + \gamma\eta\Psi} \equiv s[\Psi]. \quad (11)$$

$$v = \frac{\eta\gamma\Psi}{1 + \gamma\eta\Psi}, \quad (12)$$

## Definition of Equilibrium

Voting:  $\Psi$  was given. In equilibrium, it should be optimal.

### Definition

*An equilibrium consists of:*

- *an income threshold  $\tilde{x}$  satisfying (6),*
- *private choices: ( $n = n^s$ ,  $e = 0$ ) for  $x \leq \tilde{x}$  and ( $n = n^e$ ,  $e = e[x]$ ) for  $x > \tilde{x}$ ,*
- *aggregate variables ( $\Psi$ ,  $s$ ,  $v$ ) given by (7), (11) and (12),*

*such that the perfect foresight condition holds:*

$$E[s] = s. \tag{13}$$

# Existence and Uniqueness

## Proposition

*An equilibrium exists and is unique.*

Intuition:

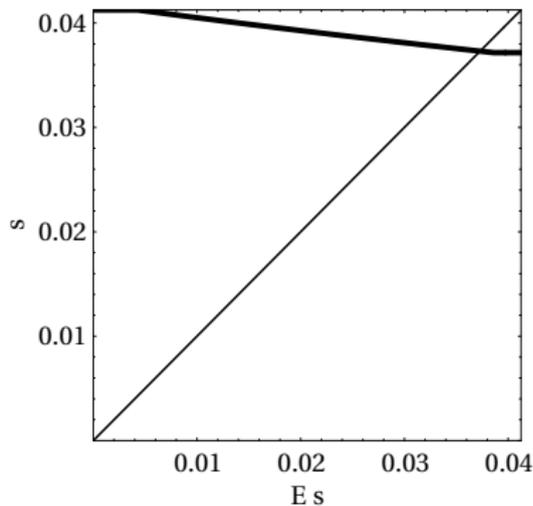
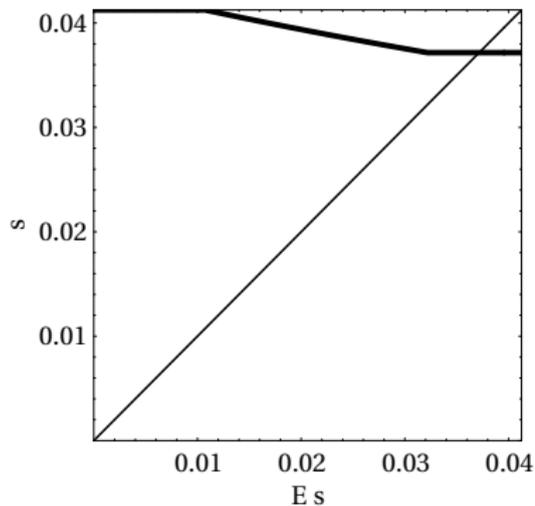
(A) participation  $\Psi$  is a continuous increasing function of  $E[s]$ .

(B)  $s$  is a continuous and decreasing function of participation.

→ continuous and decreasing mapping from  $E[s]$  to  $s$ .

This mapping has a unique fixed point.

# Example



The fixed point with  $\sigma = 0.5$  (left) and  $\sigma = 0.8$  (right)

## Role of fertility

Endogenous fertility is critical in having (B).

If fertility is exogenous and constant, Lemma 1 no longer holds.  
The tax basis increases with participation  $\Psi$ .

$s$  *increases* in participation if the “tax basis effect” dominates.

The mapping from  $E[s]$  to  $s$  is no longer guaranteed to have a unique fixed point.

→ when looking at education decision, interaction with fertility decision is important.

## Comparing the education regimes

Regime	$\Psi$
Public	1
Segregation	$\in (0, 1)$
Private	0

conditions for each regime to arise ?

## Results

### Proposition (Occurrence of education regimes)

*The private regime is not an equilibrium outcome.*

*Whether public schooling can arise in equilibrium depends on the preference parameters  $\gamma$  and  $\eta$ . Let  $\hat{\gamma} = (1 - \delta - \eta)/(\delta\eta)$ .*

*If  $\gamma > \hat{\gamma}$ , public education is not an equilibrium outcome and  $\Psi < 1/2$  for any  $\sigma$ .*

*If  $\gamma < \hat{\gamma}$ , the public regime prevails if and only if*

$$\sigma \leq \hat{\sigma} = \frac{1 - \eta}{(1 + \gamma\eta)\delta} - 1.$$

*Otherwise, we have segregation with  $\Psi > 1/2$ .*

## Intuitions

When participation is very low ( $\Psi \rightarrow 0$ ), high quality public education can be provided at very low tax levels.  $\rightarrow$  Private regime never occurs.

The public regime arises only if the income distribution is sufficiently compressed, so that the preferred education level varies little in the population.

## Assumption

*The model parameters satisfy:*

$$\gamma < \hat{\gamma} \equiv \frac{1 - \delta - \eta}{\delta\eta}.$$

(with  $\eta = 0.6$  and  $\phi = 0.075$ , requires fertility per woman  $< 15.6$ )

## Proposition (Inequality and segregation)

*Under Assumption 1, an increase in inequality leads to a lower share of public schooling, a higher quality of public schooling, and lower taxes.*

High income inequality maps into segregation.

## Introducing Political Power

Simple way: Only individuals with income  $x \geq \bar{x}$  are allowed to vote

$$\Omega[s] \equiv \int_{\bar{x}}^{\max\{\bar{x}, \tilde{x}\}} u[x, v, n^s, 0, s] g[x] dx + \int_{\max\{\bar{x}, \tilde{x}\}}^{\infty} u[x, v, n^e, e[x], 0] g[x] dx. \quad (14)$$

## Possibility of private regime

We can no longer exclude pure private education.

If voters expect to send their children to private schools ( $\tilde{x} < \bar{x}$ )  
→ the chosen school quality is zero.

Private schooling becomes attractive to all agents.

More generally: If the influence of the poor is sufficiently low,  
entirely private education systems are possible.

## Multiple Equilibria

Proposition (Multiplicity of equilibria for  $\bar{x} > 1 - \sigma$ )

If  $\bar{x}$ ,  $\gamma$ , and  $\sigma$  satisfy the conditions

$$\bar{x} > 1 - \sigma, \quad \gamma < \hat{\gamma}, \quad \text{and} \quad \sigma \leq \hat{\sigma} = \frac{1 - \eta}{(1 + \gamma\eta)\delta} - 1,$$

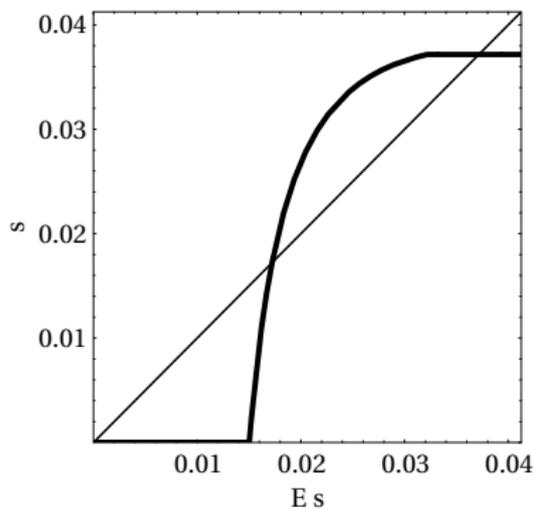
there are at least three equilibria.

Proof: Private regime always exists.

With the conditions of the proposition, Public regime also exists.

By continuity, a regime with segregation also exists.

## Example



The fixed point with multiple equilibria ( $\sigma = 0.5, \bar{x} = 0.7$ ).

## Why multiplicity ?

Strategic complementarity:

education choice of skilled people  $\longleftrightarrow$  quality of public schools.

If all skilled people switch to the public system, the quality of public schools rises since they have all the political power.

Countries with similar characteristics can choose different educational systems, provided that there is a strong concentration of political power.

## Alternative Timing

Idea: education systems are set for very long periods.

1. Government sets taxes (or total spending on public education)
2. Parents choose fertility and public versus private education
3. Public schooling per child: ratio of pre-committed total spending to the number of children in public schools.

Problem can be solved backward

## Endogenous Participation and Income Threshold

Participation in public schools

$$\Psi[s] = \begin{cases} 0 & \text{if } \tilde{x}[s] < 1 - \sigma \\ \frac{\tilde{x}[s] - (1 - \sigma)}{2\sigma} & \text{if } 1 - \sigma \leq \tilde{x}[s] \leq 1 + \sigma \\ 1 & \text{if } \tilde{x}[s] > 1 + \sigma \end{cases} \quad (15)$$

Income threshold

$$\tilde{x}[s] = \frac{1 - \eta}{\delta\phi\eta} s \quad (16)$$

## Objective Function

Same objective function but  $\tilde{x}[s]$  and  $\Psi[s]$  endogenous.

$$\Omega[s] \equiv \int_0^{\tilde{x}[s]} u[x, v, n^s, 0, s]g[x]dx + \int_{\tilde{x}[s]}^{\infty} u[x, v, n^e, e[x], 0]g[x]dx. \quad (17)$$

objective function not globally concave (kinks at the values of  $s$  corresponding to  $\tilde{x}[s] = 1 - \sigma$  and  $\tilde{x}[s] = 1 + \sigma$ )

## Equilibrium with commitment

### Proposition

*An equilibrium with commitment exists. Public school quality is lower than or equal to the level reached without commitment. The inequality is strict, if participation  $\Psi$  satisfies:  $0 < \Psi < 1$ .*

Existence: objective function is continuous on a compact set.

Multiplicity however occurs for knife-edge cases.

Lower public school quality: comparing the F.O.C.s

## More realistic timing

With regards to fertility the realistic assumption is that households move first.

1. Fertility decision
2. Government commits to education spending
3. Parental schooling decisions

## Objective Function

There is an income threshold  $\bar{x}$  below which people have large families (corresponding to the expectation of public schooling).

For  $\bar{x} < \tilde{x}[s]$ , the objective is:

$$\begin{aligned}\Omega[s] = & \int_0^{\bar{x}} u[x, v, n^s, 0, s]g[x]dx + \int_{\bar{x}}^{\tilde{x}[s]} u[x, v, n^e, 0, s]g[x]dx \\ & + \int_{\tilde{x}[s]}^{\infty} u[x, v, n^e, e[x], 0]g[x]dx,\end{aligned}$$

Similar expressions for  $\bar{x} = \tilde{x}[s]$  and  $\bar{x} > \tilde{x}[s]$ .

## Results

In equilibrium, agents have perfect foresight, and  $\bar{x} = \tilde{x}[s]$  should hold.

For  $\bar{x} = \tilde{x}[s]$ , the first-order condition is the same as in our original timing, and the outcome is the same.

Once you have chosen a large family, you have little incentives to go to private schools.

Local argument.

## Conclusion

### A political economy model of education funding:

Segregation goes along with low public funding.

High income inequality maps into a segregated education system.

Segregation does not imply low quality public schools.

Accounting for endogenous fertility is important.

Multiple equilibria arise when the rich are in charge.