Lectures on Fertility, Education, Growth, and Sustainability

3b. Production vs Reproduction, and Pollution Control

David de la Croix

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Starting point

Concern about climate change

Policies aimed at reducing greenhouse gas emissions

Kyoto has set up a cap-allocate-and-trade regime

In fully certain environment, this system is the same as a (Pigovian) tax-subsidy scheme

Our question: long-term demographic impact of those?

Idea

Two ways to achieve low pollution when technology is given

- low production per capita
- low population size

Cap and trade systems: taxing production
→ lead agents to substitute tax free activities such as reproduction and leisure to production.

But reproduction generates pollution tomorrow... more taxes will be needed in the future

Capping emissions gradually impoverishes the successive generations

Two key assumptions - 1

1. “Autonomous technology”

Even if technology responds to price and tax changes, it will not be enough to keep pollution under control

The scale of environment-saving improvements required to stabilize pollution is daunting

Here we neglect technological adjustment, to focus on the demographic impact the pollution control
As if we look at a side effect of the policy

Two key assumptions - 2

2. "Production-reproduction substitution"

Raising children: opportunity cost because of the time it takes
$\rightarrow$ fertility is inversely related to (mothers') income
both across time and in any cross section

Is this substitution effect strong enough?

The impact of substitution effect is cumulative

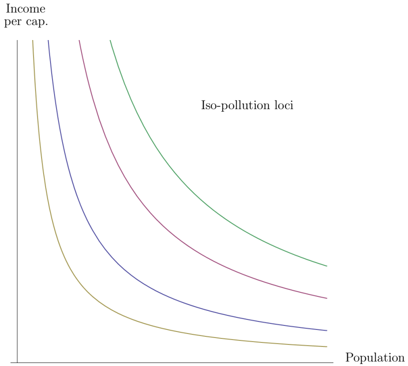
Pollution

Emissions: $E_t = a_t Y_t$
a_t: pollution coefficient,
Y_t: output

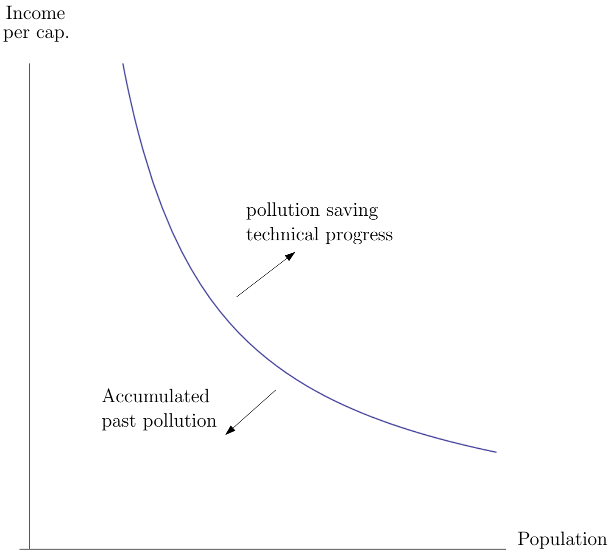
$$E_t = a_t N_t y_t$$

N_t: adult population size,
y_t: output per person
Stock of pollution S_t:

$$S_t = (1 - \delta(S_{t-1}))S_{t-1} + E_t.$$



Shifts over Time of one Iso-pollution Curve



Preferences

One generation is active during one period

Utility from consumption, leisure, fertility, and assets left for the future generation

$$u(c_t, \ell_t, n_t, k_{t+1}) = \ln c_t + \delta \ln \ell_t + \gamma \ln(n_t k_{t+1}),$$

ad-hoc altruism

logarithmic preferences (no secular trend in leisure)

Technology

Bequest b_t are invested in a productive asset k_{t+1}

$$k_{t+1} = G(k_t, b_t) = \tau k_t^{\nu} b_t^{\eta}$$

Producing x children requires time and space (land per household), with the following technology:

$$x = A\left(\frac{L}{N_t}\right)^{\alpha} T$$

Hence, to produce n_t children one needs

$$\phi \, N_t^{\alpha} n_t$$

units of time

Budget constraint without pollution cap

Households are self-employed and produce using assets k_t and hours of work $(1 - \ell_t - \phi N_t^\alpha n_t)$. The production function is

$$y_t = (1 - \ell_t - \phi N_t^\alpha n_t) k_t$$

Budget constraint:

$$y_t = c_t + n_t b_t$$

Maximization problem

$$\max_{\ell_t, n_t, b_t} \ln((1 - \ell_t - \phi n_t)k_t - n_t b_t) + \varphi \ln \ell_t$$
$$+\gamma(\ln(n_t) + \nu \ln(k_t) + \eta \ln(b_t) + \ln(\tau))$$

FOC:

$$\begin{aligned} c_t &= \frac{k_t}{1+\varphi+\gamma} \\ \ell_t &= \frac{\varphi}{1+\varphi+\gamma} \\ n_t &= \frac{\gamma(1-\eta)}{(1+\varphi+\gamma)\phi N_t^{\alpha}} \\ b_t &= \frac{\eta\phi N_t^{\alpha} k_t}{1-\eta} \\ y_t &= \frac{\gamma\eta}{1+\varphi+\gamma} k_t \end{aligned}$$

Dynamics in the benchmark case

Population dynamics

$$N_{t+1} = N_t n_t$$

$$\begin{aligned} k_{t+1} &= \tau \left(\frac{\eta\phi}{1-\eta} \right)^{\eta} N_t^{\eta\alpha} k_t^{\nu+\eta} \\ N_{t+1} &= \frac{\gamma(1-\eta)}{(1+\delta+\gamma)\phi} N_t^{1-\alpha} \end{aligned}$$

Globally stable steady state $\{\bar{k}, \bar{N}\}$

Pollution

$S_t^\star$ can be achieved by imposing an emission target $E_t^\star$

Exogenous objective of total emissions $\{E_t^\star\}_{t\geq 0}$

Kyoto-like systems

- impose each household to buy pollution rights at price p_t in proportion to their output
- provide some endowment of pollution rights q_t

$$y_t = c_t + n_t b_t + p_t(a_t y_t - q_t)$$

Lagrangian to solve the household problem

$$\max_{\ell_t, n_t, b_t} \mathcal{L}_t = \ln((1 - a_t p_t)(1 - \ell_t - \phi n_t)k_t - n_t b_t + p_t q_t) \\ + \varphi \ln \ell_t + \gamma(\ln(n_t) + \nu \ln(k_t) + \eta \ln(b_t) + \ln(\tau))$$

Partial equilibrium results

Non market activities, leisure and procreation, increase with the price of the pollution permit, as their opportunity cost is decreased by this implicit tax:

$$\frac{\partial \ell_t}{\partial p_t} > 0, \quad \frac{\partial n_t}{\partial p_t} > 0$$

Leisure and procreation increase with the endowment of pollution permits. This is because they are both normal goods:

$$\frac{\partial \ell_t}{\partial q_t} > 0, \quad \frac{\partial n_t}{\partial q_t} > 0$$

Dynamics with pollution permits

The equilibrium on the market for tradable pollution rights:

$$p_t(N_t a_t y_t - N_t q_t) = 0$$

Assuming binding pollution caps, the dynamic system is:

$$k_{t+1} = \tau k_t^{\nu} \left(\frac{\eta \phi N_t^{\alpha} k_t}{1-\eta} \left(1 - \frac{k_t(1+\gamma\eta) - q_t(1+\varphi+\gamma)}{(k_t - q_t)(1+\gamma\eta)} \right) \right)^{\eta}$$

$$N_{t+1} = N_t \left(\frac{\gamma(1-\eta)}{(1+\varphi+\gamma)\phi N_t^{\alpha}} \left(1 + \frac{\frac{k_t(1+\gamma\eta)-q_t(1+\varphi+\gamma)}{(k_t-q_t)(1+\gamma\eta)}}{1 - \frac{k_t(1+\gamma\eta)-q_t(1+\varphi+\gamma)}{(k_t-q_t)(1+\gamma\eta)}} \frac{q_t}{k_t} \right) \right)$$

Main result

Proposition

For stringent enough $E^\star$, there is a locally stable steady state population N, decreasing in $E^\star$.

Population is increasing with the restrictiveness of the pollution cap.

Overview

Each period lasts 25 years. 1983: $t = 0$. 2008: $t = 1$. 2033: $t = 2$

Initial conditions: $N_0 = 4.68$ (billions) and $y_0 = 4.541$ (dollars per capita per year).

We identify γ, φ, η, ν and α with five restrictions at steady state (next slide)

Productivity parameters, τ and ϕ, determine the size of population and income per capita.

To obtain $N_1 = 6.67$ and $y_1 = 7.614$ in 2008, we need to have $\phi = 0.0164$ and $\tau = 24.0417$.

Five restrictions

1. The share of consumption in GDP is 80%

$$\frac{c_t}{y_t} = \frac{1}{1+\gamma\eta} = 0.8$$

2. the time spent on leisure (ℓ_t) and procreation ($\phi\bar{N}^\alpha$) amounts to 2/3 of total available time:

$$\ell_t + \phi\bar{N}^\alpha = \frac{\varphi}{1+\varphi+\gamma} + \frac{\gamma(1-\eta)}{1+\varphi+\gamma} = \frac{\varphi+\gamma(1-\eta)}{1+\varphi+\gamma} = \frac{2}{3}$$

3. At steady state, time spent rearing children equals 15%:

$$\frac{\phi N^\alpha}{1-\ell_t} = \frac{\gamma(1-\eta)}{1+\gamma} = 0.15$$

Five restrictions

1. Convergence speed of income per capita is 2% per year:

$$\frac{k_{t+1}}{k_t} = \left(\frac{k_t}{k_{t-1}}\right)^{\nu+\eta} \left(\frac{N_t}{N_{t-1}}\right)^{\alpha\eta} \Rightarrow \nu + \eta = 0.98^{25}$$

2. Dynamics of population match 2007 IIASA World Population Projection:

$$\frac{N_{t+1}}{N_t} = \left(\frac{N_t}{N_{t-1}}\right)^{1-\alpha} \Rightarrow \frac{8.88}{8.18} = \left(\frac{8.18}{6.67}\right)^{1-\alpha}$$

Benchmark Simulation - World Economy 1983-2208

t	N_t	c_t	n_t	ℓ_t	y_t	Y_t
1983	4.68	4.27	1.425	0.608	5.342	25.0
2008	6.67	7.81	1.153	0.608	9.769	65.2
2033	7.69	12.59	1.059	0.608	15.737	121.0
2058	8.15	17.56	1.023	0.608	21.954	178.8
2083	8.34	21.87	1.009	0.608	27.334	227.9
2108	8.41	25.14	1.004	0.608	31.428	264.4
2133	8.44	27.43	1.002	0.608	34.290	289.6
2158	8.46	28.95	1.001	0.608	36.185	306.1
2183	8.46	29.92	1.000	0.608	37.397	316.5
2208	8.46	30.52	1.000	0.608	38.155	323.0

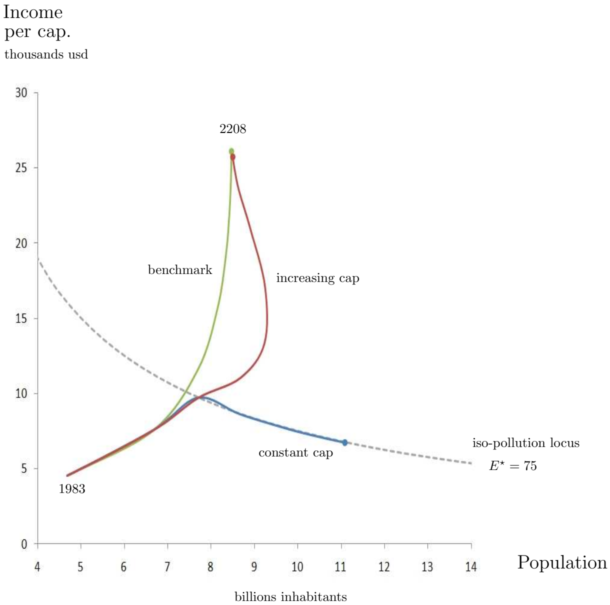
Simulation with a Constant Pollution Cap - 1983-2208

t	q_t	N_t	p_t	c_t	n_t	ℓ_t	y_t	Y_t
1983	21.37	4.680	0.00	4.27	1.425	0.608	5.342	25.0
2008	14.99	6.670	0.00	7.81	1.153	0.608	9.769	65.2
2033	13.00	7.692	0.24	10.40	1.151	0.661	13.000	100.0
2058	11.29	8.855	0.51	9.03	1.171	0.731	11.294	100.0
2083	9.65	10.366	0.56	7.72	1.087	0.746	9.647	100.0
2108	8.87	11.272	0.57	7.10	1.042	0.752	8.871	100.0
2133	8.51	11.746	0.58	6.81	1.020	0.754	8.513	100.0
2158	8.35	11.982	0.59	6.68	1.010	0.755	8.346	100.0
2183	8.27	12.097	0.59	6.61	1.005	0.756	8.266	100.0
2208	8.23	12.153	0.59	6.58	1.002	0.756	8.229	100.0

Simulation with technical progress - 1983-2208

t	q_t	N_t	p_t	c_t	n_t	ℓ_t	y_t	Y_t
1983	12.99	4.680	0.00	4.27	1.425	0.608	5.342	25.0
2008	11.69	6.670	0.00	7.81	1.153	0.608	9.769	65.2
2033	13.00	7.692	0.24	10.40	1.151	0.661	13.000	100.0
2058	14.48	8.855	0.32	11.59	1.089	0.680	14.483	128.2
2083	17.06	9.641	0.26	13.65	1.012	0.665	17.058	164.5
2108	21.62	9.755	0.14	17.30	0.964	0.638	21.622	210.9
2133	28.77	9.402	0.00	22.53	0.939	0.608	28.167	264.8
2158	39.28	8.831	0.00	26.60	0.975	0.608	33.249	293.6
2183	51.66	8.611	0.00	28.82	0.990	0.608	36.025	310.2
2208	66.92	8.525	0.00	30.01	0.996	0.608	37.511	319.8

Picture


Income
per cap.
thousands usd
30
25
20
15
10
5
0
2208
benchmark
increasing cap
constant cap
iso-pollution locus
$E^\star = 75$
1983
4
5
6
7
8
9
10
11
12
13
14
Population
billions inhabitants

Conclusion

Pollution control should not be at the cost of those in the future.

The natalist bias we identified is worrying in the latter respect.

Is our story quantitatively significant ? e.g. if we impose such a rule in India, by how much would we delay the demographic transition ?

Population could be capped directly through a separate scheme, be it in the absence of or as a complement of the pollution capping scheme.