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Altruism and Self-Restraint

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ABSTRACT. – Parental altruism plays a particular role when standard-of-living aspirations are transmitted from one generation to the next. The influence of altruistic parents is not limited to the bequest they could leave; they also direct the evolution of children's aspirations by restraining their own consumption standard. We show that, even if there is no bequest, altruism always increases capital accumulation and has a stabilizing effect on the economy. However, its effect on steady state welfare can be negative. Inherited standard-of-living can also generate regime shifts (bequest/no bequest) along the *equilibrium* path.

Altruisme, transmission d'habitudes et frugalité

RÉSUMÉ. – L'altruisme parental joue un rôle particulier lorsque des aspirations sociales sont transmises d'une génération à l'autre. L'influence des parents altruistes ne se limite pas aux legs qu'ils peuvent offrir; ils influencent aussi les aspirations de leurs enfants en restreignant leur propre niveau de vie. Nous montrons que, même en l'absence de legs, l'altruisme accroît toujours l'accumulation du capital et a un effet stabilisateur sur l'économie. Toutefois, son effet sur le bien-être de long terme peut être négatif. La transmission des aspirations sociales peut aussi engendrer des changements de régimes le long de la trajectoire d'équilibre.

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1 Introduction

The utility of altruistic parents depends partly on the utility of their children. One demonstration of this altruism consists in the bequests that the parents decide to leave to their offspring. However, bequests are operative if the intensity of altruism is strong enough (WEIL [1987] and ABEL [1987]). Private intergenerational transfers like bequests are able to neutralize public intergenerational transfers (debt, pensions) and Ricardian equivalence holds (BARRO [1974]). In that case, overlapping generations model have the same properties as the models with infinitely lived agents. On the other hand, if the optimal bequests are zero, the model with altruistic agents is equivalent to the overlapping generations model without altruism (DIAMOND [1965]).

Parents' influence on children is however not limited to resource transfers. Indeed, BECKER [1992] notices that: "*The habits acquired as a child or young adult generally continue to influence behavior even when the environment changes radically. For instance, Indian adults who migrate to the United States often eat the same type of cuisine they had in India, and continue to wear the same type of clothing.(...) Childhood-acquired habits then continue, even though these would not have developed if the environment when growing up had been the same as the environment faced as an adult.(...)*" Denoting $u(.)$ the instantaneous utility of an adult, the influence of parents could be introduced by assuming that u depends both on the own consumption of the agent c_t and on a state variable a_t which itself depends on his parents' past consumption $a_t = c_{t-1}$.

When $u'_a < 0$, which is the case studied in this paper, parents' consumption has a negative influence on children's utility. This is the case for instance when the young generation inherits standard-of-living aspirations from their parents. As in the psychological models of the "*goal-achievement gap*" (MICHALOS [1980]), the instantaneous satisfaction depends on the gap between the actual consumption and the aspirations, *ie*, the consumption of the previous generation. These aspirations serve as a benchmark consumption level determining a goal to reach for the new generation. As already stressed by EASTERLIN [1971], this implies that "*in a steadily growing economy successive generations are raised in increasingly affluent households and hence develop successively higher living aspirations*." If $u''_{ca} < 0$, the aspiration effect generates distaste. If $u''_{ca} > 0$, which is the interesting case, the aspiration effect induces a desire of catching-up, pushing the new generation to consume more than what their parents did. In a world without altruism, this kind of intergenerational externality can be responsible for fluctuations at the macroeconomic level (de la CROIX [1996]). Indeed, in a booming period, there is a point at which standard-of-living aspirations that are inherited from the previous generation grow more rapidly than income and have a negative impact on saving and growth. The consecutive depression ceases when aspirations reverts to lower levels.

In a previous paper (de la CROIX and MICHEL [1999]), we have studied the policy that should be implemented to internalize the "*taste*" externality and hence avoid or reduce the sub-optimal fluctuations. This policy amounts to subsidize savings, in addition to the usual public intergenerational transfer.

This allows to equalize the return on saving to its social value and therefore take into account the effect of parents' consumption on children's habits.

In this paper, we analyze the effect of bequeathed tastes when agents are altruistic. In this case, the altruistic parents will take into account the effect of their own consumption on the bequeathed habits. Quoting BECKER [1992] again, "*they would try to direct the evolution of children's preferences toward raising the utility of children. For example, parents may refrain from smoking even when that gives them much pleasure because thei smoking raises the likelihood that the children will smoke. Or they may take children to church, even when not religious, because they believe exposure to religion is good for children. Indeed, many parents stop going after their children leave home.*" Hence, the optimal behavior of the parents is to promote self-restraint.

To analyze these issues we develop, in section 2, a specific model in which the utility of a given generation is influenced by the level of consumption of the previous generation. The altruistic parents should thus modify their optimal behavior to increase the utility of their children, regardless of their degree of altruism. Altruism determines agents' optimal decisions, even when bequests are not operative.

In the presence of positive bequests (section 3), the decentralized economy is equivalent to the centralized one, and the results of de la CROIX and MICHEL [1999] still apply. This illustrates the regulatory role of altruism in the economy when it is operative in the sense of generating positive bequests. In the absence of positive bequests (section 4), altruism remains operative in the sense that the optimal consumption choice takes into account the effect of this choice on the standard-of-living aspirations of the next generation. This defines a novel situation with respect to the literature whose implications should be studied carefully.

One additional difficulty comes from the interaction between the choice of the optimal bequest in terms of resources and the choice of the optimal standard-of-living. The result of WEIL [1987] in an altruistic economy without bequeathed tastes will be modified by this interaction. The internalization by the parents of the intergenerational spill-over indeed modifies the frontier between the regime with operative bequests and the regime with constrained bequest.

The main results on the role of self-restraint on the accumulation of capital are established in section 5. All these aspects are illustrated is section 6 in the case of Cobb-Douglas preferences and technology. Finally, although a bunch of results is established analytically, the study of the global dynamics which display endogenous shifts of regime (zero vs positive bequests) can only be analyzed using numerical simulations. In section 7, we analyze whether it is possible for an economy to be subject to regime shift along the convergence path from a given initial situation to the steady state.

2 The Model

Time is discrete and goes from 0 to $+\infty$. In each period, the economy is populated by three generations, each living for three periods. The growth rate of the population is zero. When young, the representative agent inherits aspirations from his parents but does not take any decision. When adult, he works, might receive a bequest and draws utility from consuming given his aspiration level; he has to decide how much to save for future consumption. When old, he consumes and can leave a bequest to his children. At each date a single good is produced. This good can either be consumed by the middle-aged and old generation during the period or accumulated as capital for future production.

Firms

Production occurs through a constant returns to scale technology. Output per capita y_t is a function of capital intensity k_t:

$$y_t = f(k_t),$$

in which $f(.)$ is a neo-classical production function with $f' > 0$, $f'' < 0$. The optimal behavior of firms under perfect competition leads to the equalization between marginal costs and marginal revenues:

$$w_t = w(k_t) = f(k_t) - k_t f'(k_t) \tag{1}$$

$$R_t = R(k_t) = f'(k_t), \tag{2}$$

in which we assume that capital depreciates entirely after one period. R_t is the gross interest rate and w_t is the real wage.

Households

Each household lives for three periods. When young, the representative agent inherits aspirations a_t from his parents, following:

$$a_t = c_{t-1}, \quad \forall t > 0 \tag{3}$$

where c_{t-1} is the parents' consumption level. The young agent does not take any decision.[1] When adult, he draws utility from consuming the quantity c_t given his aspiration level a_t. When old, he draws utility from consuming d_{t+1}. Since agents are assumed to be altruistic, they also draw utility from the well-being of their children $\mathcal{V}_{t+1}$, discounted at the rate $0 \leqslant \beta < 1$, which is called the degree of altruism (or the inter-cohort discount factor). The utility of the typical agent is thus:

$$\mathcal{V}_t = u(c_t, a_t) + v(d_{t+1}) + \beta \mathcal{V}_{t+1}$$

Bequeathed tastes affect thus the way current consumption generates satisfaction. The forgetting rate of tastes is high so that they no longer affect the evaluation of consumption when old.

We assume that $u'_c > 0$, $u'_a < 0$, $u''_{cc}, u''_{aa} < 0$, $u''_{ca} > 0$ and $v' > 0$, $v'' < 0$. The assumption $u''_{ca} > 0$ amounts postulating that a rise in the

1. His consumption is implicitly included in his parents' consumption.

aspirations increases the marginal utility of (*ie*, the desire for) consumption. We also assume that:

$$\lim_{d\to 0} v'(d) = \infty,$$

and that u is strictly concave:

$$u''_{cc}u''_{aa} - {u''_{ca}}^2 > 0.$$

When adult, the agent sells one unit of labor inelastically at any real wage w_t, receives a bequest x_t from the previous generation, consumes the quantity c_t and saves s_t for next period consumption by holding capital:

$$c_t = w_t + x_t - s_t. \tag{4}$$

When old, the agent leaves a bequest x_{t+1} to the next generation and spends all its remaining saving plus interest accrued to consume d_{t+1}:

$$d_{t+1} + x_{t+1} = R_{t+1}s_t. \tag{5}$$

The programme of the representative individual is thus to choose $\{x_{t+1}, c_t, d_{t+1}\}$ in order to maximize:

$$\begin{aligned} \mathcal{V}_t(x_t, a_t) &= \max\{u(c_t, a_t) + v(d_{t+1}) + \beta\mathcal{V}_{t+1}(x_{t+1}, a_{t+1})\} \\ &\text{s.t.} \quad (3), (4) \text{ and } (5) \\ &\qquad c_t \geqslant 0,\ d_{t+1} \geqslant 0,\ x_{t+1} \geqslant 0 \end{aligned} \tag{6}$$

The sequence of these problems can be rewritten as an infinite horizon problem:

$$\begin{aligned} \max \sum_{t=0}^{\infty} \quad & \beta^t[u(c_t, c_{t-1}) + v(d_{t+1})], \\ \text{s.t.} \quad & x_{t+1} = R_{t+1}(w_t + x_t - c_t) - d_{t+1}, \ \forall t \geqslant 0 \\ & c_{-1}, x_0 \text{ given.} \end{aligned}$$

The constraint is the inter-temporal budget constraint of generation t obtained from equations (4) and (5). The problem (6) is the Bellman equation of the infinite horizon problem. We build the following Lagrangian:

$$\begin{aligned} \mathcal{L} = \sum_{t=0}^{\infty} \beta^t [u(c_t, c_{t-1}) + v(d_{t+1}) \\ + q_{t+1}\{R_{t+1}(w_t + x_t - c_t) - d_{t+1} - x_{t+1}\} + \lambda_{t+1}x_{t+1}], \end{aligned}$$

in which q_{t+1} is the shadow price of x_t and λ_{t+1} is the Kuhn-Tucker multiplier for the positivity constraint $x_{t+1} \geqslant 0$. The first order necessary conditions are:

$$u'_c(c_t, c_{t-1}) + \beta u'_a(c_{t+1}, c_t) - R_{t+1}q_{t+1} = 0 \tag{7}$$

$$v'(d_{t+1}) - q_{t+1} = 0 \tag{8}$$

$$(9) \qquad -q_{t+1} + \lambda_{t+1} + \beta q_{t+2} R_{t+2} = 0$$

$$(10) \qquad \lambda_{t+1} \geqslant 0$$

$$(11) \qquad \lambda_{t+1} x_{t+1} = 0.$$

Equilibrium: The clearing condition on the goods market is:

$$f(k_t) = k_{t+1} + c_t + d_t.$$

Using equations (1), (2), (4) and (5), it implies:

$$(12) \qquad k_{t+1} = s_t$$

Given the initial conditions $a_0 = c_{-1}$, x_0 and k_0, an *equilibrium* is a sequence:

$$\{w_t, R_{t+1}, a_{t+1}, c_t, s_t, d_{t+1}, q_{t+1}, \lambda_{t+1}, x_{t+1}, k_{t+1}\}$$

that solves the maximization problem of the firms and of the households, and that fulfills the clearing condition on the goods market. It thus satisfies (1)-(5) and (7)-(12) for all $t \geqslant 0$.

Using (9) and (2) to compute the steady state capital stock, we find:

$$(13) \qquad \frac{-\lambda}{q} = \beta f'(k) - 1$$

From this equation, we obtain the traditional result that when bequests are positive at steady state ($x > 0, \lambda = 0$), the steady state capital stock satisfies the modified golden rule $f'(k) = 1/\beta$. When altruists are constrained with respect to bequests ($\lambda > 0$, $x = 0$), $f'(k) < 1/\beta$ and the steady state capital stock is larger than the modified golden rule level. In the limit case ($\lambda = 0$, $x = 0$), the altruists are not constrained and bequests are zero, we have $f'(k) = 1/\beta$. Before considering these cases in turn, it is convenient to build a transformed utility function $\hat{u}(c)$ and make the following assumption.

ASSUMPTION 1: Let us define:

$$\hat{u}(c) = \int [u'_c(c,c) + \beta u'_a(c,c)] \, dc.$$

We assume that $\hat{u}(.)$ is increasing, concave and satisfies the Inada condition in 0, that is:

$$(14) \qquad \hat{u}'(c) = u'_c(c,c) + \beta u'_a(c,c) > 0$$

$$(15) \qquad \hat{u}''(c) = u''_{cc} + (1+\beta) u''_{ca} + \beta u''_{aa} \leqslant 0$$

$$(16) \qquad \lim_{c \to 0} \hat{u}'(c) = +\infty.$$

Equation (14) states that the negative effect of habits does not offset the desire for consumption. In particular, we exclude Veblenian worlds: Veblen believed that the welfare of a typical person primarily depends on his relative income position. In that case, the value of social environment causing envy exactly offset the value of own income. A rise in all incomes in a community by the same percentage would not improve anyone's welfare in VEBLEN's world (see BECKER [1974], VEBLEN [1934]). We thus exclude satiation equilibria like those exhibited by RYDER and HEAL [1973] in an infinite horizon set-up with habit formation.

Equation (15) imposes the concavity of $\hat{u}$. Notice that given the concavity of $u(c,a)$, the condition (15) is always satisfied for $\beta = 1$. In the other cases, (15) imposes a restriction on u''_{ca} conditionally on β, u''_{cc} and u''_{aa}:

$$u''_{ca} \leqslant \frac{|u''_{cc}| + \beta |u''_{aa}|}{1+\beta}.$$

This condition is satisfied for every β as long as $u''_{ca} \leqslant |u''_{cc}|$ and $u''_{ca} \leqslant |u''_{aa}|$. If not, there is a restriction on β.

Finally, equation (16) states that the negative effect of habits does not offset the infinite desire for consumption when $c = 0$.

It is possible to find the explicit functional form of this function $\hat{u}$ in the following example. Let us assume that $u(c,a) = \tilde{u}(c - \gamma a)$, with γ being a positive parameter measuring the intensity of the externality. In that case, assumption 1 amounts to assume that $\tilde{u}$ is concave and $\gamma < 1$. We obtain:

$$\hat{u}(c) = \frac{1-\beta\gamma}{1-\gamma}\, \tilde{u}((1-\gamma)c). \tag{17}$$

The Associated Standard Economy

In order to study the steady state properties of the model, it is useful to introduce an associated standard economy; it is an economy without bequeathed tastes to which it will be possible to apply the results of WEIL [1987].

Let us denote our economy with altruism and bequeathed tastes $\mathcal{E}$. We define a standard economy without bequeathed tastes (but with altruism) $\hat{\mathcal{E}}$ where preferences can be represented by the life-cycle utility function is $\hat{u} + \beta v$. For this economy $\hat{\mathcal{E}}$ the first order conditions are (8)-(11) with, instead of (7):

$$\hat{u}'(c_t) - R_{t+1} q_{t+1} = 0$$

or

$$u'_c(c_t,c_t) + \beta u'_a(c_t,c_t) - R_{t+1} q_{t+1} = 0. \tag{18}$$

The dynamics are completely different in the two economies but the steady states are the same. Indeed, with constant values of the variables, equations (7) and (18) are the same.

3 Unconstrained Altruism

In this section, we study the steady state and the local dynamics. Assuming there is an *equilibrium* such that $\lambda_t = 0$ for all large $t \geqslant T$: the economy is in a regime with unconstrained altruism from T onwards. Using (8) to eliminate q_{t+1} in (7), the *equilibrium* dynamics can be described by the following system of four first-order non-linear difference equations:

$$(19) \qquad \begin{aligned} f'(k_{t+1})v'(d_{t+1}) &= u'_c(c_t, a_t) + \beta u'_a(c_{t+1}, a_{t+1}) \\ f'(k_{t+1})v'(d_{t+1})\beta &= v'(d_t) \\ k_{t+1} &= f(k_t) - c_t - d_t \\ a_{t+1} &= c_t, \end{aligned}$$

in which a_t and k_t are predetermined, and c_{t+1} and d_{t+1} are anticipated variables. The constraint $x_t \geqslant 0$ is equivalent to:

$$c_t + k_{t+1} - w(k_t) \geqslant 0.$$

This system is equivalent to the system derived from an optimal growth problem applied to our economy that has been analyzed in de la Croix and Michel [1999]. Then we may use their results.

3.1 Steady State

A steady state $\{c^\star, a^\star, k^\star, d^\star\}$ of this economy is defined by:

$$(20) \qquad u'_c(c^\star, a^\star) = \beta^{-1} v'(d^\star) - \beta u'_a(c^\star, a^\star)$$

$$(21) \qquad f'(k^\star) = \beta^{-1}$$

$$(22) \qquad f(k^\star) = d^\star + c^\star + k^\star$$

$$(23) \qquad a^\star = c^\star$$

Equation (21) yields the modified golden rule. Hence, if bequests are positive, the introduction of standard-of-living aspirations does not modify the steady state stock of capital which remains fixed at the modified golden rule level. Equation (20) shows that the tradeoff between c and d is modified by the presence of inherited tastes. The marginal utility of the adults u'_c, all things being equal, is increased by the presence of the negative intergenerational spill-over $\beta u'_a$. This implies that adults should consume less.

PROPOSITION 1. **[Existence and uniqueness of the steady state]**:
Under assumption 1, a steady state $\{c^\star,a^\star,k^\star,d^\star\}$ with positive bequest $x^\star > 0$ exists if, and only if:

$$\hat{u}'(w(k^\star) - k^\star) - \beta^{-1}v'(\beta^{-1}k^\star) < 0, \tag{24}$$

with $k^\star$ satisfying $\beta f'(k^\star) = 1$. If this steady state exists, it is unique.

PROOF: See Appendix.

Note that, in the limit case ($x^\star = 0, \lambda = 0$), condition (24) holds with equality. Proposition 1 implicitly imposes a condition on β as a function of the other parameters. However, condition (24) depends on β in a non-trivial way (through β and indirectly through $k^\star$ and $\hat{u}'$). As we shall see in the Cobb-Douglas example (section 6) there does not necessarily exist one threshold $\tilde{\beta}$ separating the two types of long-run equilibria, positive bequest and zero bequest.

A sufficient condition for (24) to be satisfied is

$$u_c'(w(k^\star) - k^\star, w(k^\star) - k^\star) - R(k^\star)v'(R(k^\star)k^\star) < 0 \tag{25}$$

which is nothing else that (24) in the case $u_a' = 0$. We obtain (24) by adding $\beta u_a'$ to the left-hand side of (25) implying that the larger $\mid u_a' \mid$, the less restrictive is the condition (24). This point is illustrated in the Cobb-Douglas case in section 5.

3.2 Local Dynamics

For the equilibrium defined by (19) to be unique and (locally) converging to the steady state, two and only two eigenvalues of its linearization should have a modulus larger than one, since there are two anticipated variables in the system.

PROPOSITION 2. **[Local stability of the steady state]**:
Under assumption 1 and condition (24), the steady state $\{c^\star,a^\star,k^\star,d^\star\}$ is a saddle-point. Let us define, at the steady state,

$$\Delta \equiv \left[\hat{u}'' + u''_{ca}\frac{\beta^2 v'}{v''}f''\right]^2 + 4\beta u''_{ca}v' f''$$

If $\Delta > 0$, the stable eigenvalues are real and the local dynamics are monotonic. If $\Delta < 0$, the stable eigenvalues are complex and the local dynamics display damped oscillations.

PROOF: See de la CROIX and MICHEL [1999].

As a conclusion, the solution when bequests are positive is always stable in the saddle-point sense provided that the world is non-Veblenian, *ie*, that the welfare of a typical agent does not only depend on its relative position. The solution may however display oscillations, even when the degree of altruism is very high. Indeed, we have shown in de la CROIX and MICHEL [1999] that even when $\beta \to 1$, that is near the golden rule case, the condition $\Delta < 0$ can be satisfied.

4 Constrained Altruism (No Bequest)

We now consider the case of an *equilibrium* where $\lambda_t > 0$ for all large $t \geqslant T$. The equilibrium dynamics can then be described by the following system of three first-order non-linear difference equations:

$$\begin{aligned} R(k_{t+1})v'(R(k_{t+1})k_{t+1}) &= u'_c(c_t,a_t) + \beta u'_a(c_{t+1},a_{t+1}) \\ k_{t+1} &= w(k_t) - c_t \\ a_{t+1} &= c_t, \end{aligned} \tag{26}$$

in which a_t and k_t are predetermined, and c_{t+1} is an anticipated variable. Notice that when $\beta = 0$, we retrieve the dynamics of the perfect competition case without altruism analyzed in de la CROIX [1996].

4.1 Steady State

A steady state *equilibrium* $\{\bar{c},\bar{a},\bar{k}\}$ of this economy is defined by:

$$u'_c(\bar{c},\bar{a}) + \beta u'_a(\bar{c},\bar{a}) = R(\bar{k})v'(R(\bar{k})\,\bar{k}) \tag{27}$$

$$\bar{c} = w(\bar{k}) - \bar{k} \tag{28}$$

$$\bar{a} = \bar{c} \tag{29}$$

As we have seen the steady state equilibria of our economy $\mathcal{E}$ are the same as the equilibria of the economy $\hat{\mathcal{E}}$ in which agents have a utility function $\hat{u}(c) + v(d)$:

PROPOSITION 3. **[Characterization of the steady state]**:
A steady state *equilibrium* of the economy $\hat{\mathcal{E}}$ without altruism and with utility function $\hat{u}(c) + v(d)$, which verifies the condition $\beta f'(k) < 1$, is also a steady state *equilibrium* without bequest of the economy $\mathcal{E}$ with inherited tastes. The converse holds.

PROOF: See Appendix.

In the example with $u(c,a) = \tilde{u}(c - \gamma a)$, equation (27) becomes $(1 - \beta\gamma)\tilde{u}'((1 - \gamma)c) = R(k)v'(d)$ which corresponds to the Diamond equation in which the utility function is $\hat{u}(c)$ given by (17).

Proposition 3 implies that, in the regime without bequest, the introduction of inherited aspirations does not modify the properties of the basic model in terms of steady states. However, as shown in the next sub-section, the stability properties of these steady states are affected by the presence of aspirations.

4.2 Local Dynamics

For the equilibrium dynamics defined by (26) to be unique and (locally) converging to the steady state, one and only one eigenvalue of its linearization should have a modulus larger than one, since there is one anticipated variable in the system.

The dynamics underlying (26) can be described by a difference equation of the third order in k_t:

$$-u_c'(c_t,c_{t-1}) - \beta u_a'(c_{t+1},c_t) + R(k_{t+1})v'(R(k_{t+1})k_{t+1}) = 0,$$

where:

$$c_{t+i} = w(k_{t+i}) - k_{t+i+1}, \quad i = -1,0,1.$$

Let us consider a steady state $\{\bar{c},\bar{a},\bar{k}\}$ that satisfies $\beta f'(\bar{k}) < 1$ and linearize around it:

$$(30) \qquad -(u_{cc}'' + \beta u_{ca}'')\mathrm{d}c_t - u_{ca}''\mathrm{d}c_{t-1} + \beta u_{aa}''\mathrm{d}c_{t+1} + g'(\bar{k})\mathrm{d}k_{t+1} = 0$$

in which:

$$\mathrm{d}c_{t+1} = w'(k_{t+i})\mathrm{d}k_{t+i} - \mathrm{d}k_{t+i+1}, \quad i = -1,0,1$$

$$g(\bar{k}) \equiv R(\bar{k})v'(R(\bar{k})\bar{k})$$

$$g'(\bar{k}) = R'(\bar{k})v'(R(\bar{k})\bar{k}) + R(\bar{k})v''(R(\bar{k})\bar{k})(R(\bar{k}) + R'(\bar{k})\bar{k}).$$

We shall use the following assumption

ASSUMPTION 2:

$$g'(\bar{k}) < 0.$$

Assumption 2 bears on both the production function and the utility function v. However, there is a sufficient condition bearing on the production function only: since we have $R' < 0, v' > 0, v'' < 0$, it is enough to have $R(\bar{k}) + \bar{k}\,R'(\bar{k}) \geqslant 0$ for assumption 2 to be satisfied. This sufficient condition is always satisfied, *eg*, when the production function is Cobb-Douglas.

Replacing $\mathrm{d}c_{t+i}$ by $w'\mathrm{d}k_{t+i} - \mathrm{d}k_{t+i+1}$ in (30), we obtain the characteristic polynomial by substituing $\mathrm{d}k_{t+i}$ by λ^{i+1} for $i = -1,0,1,2$:

$$-(u_{cc}'' + \beta u_{ca}'')(w'\lambda - \lambda^2) - u_{ca}''(1 + \beta\lambda^2)(w' - \lambda) + g'\lambda^2 = 0,$$

with $w' = w'(\bar{k})$ and $g' = g'(\bar{k})$. After division by u''_{ca} we obtain the characteristic polynomial:

$$P(\lambda) = (\lambda - w')Q(\lambda) - n\lambda^2 \tag{31}$$

with

$$Q(\lambda) = \beta\lambda^2 - m\lambda + 1$$

$$n = \frac{-g'}{u''_{ca}} > 0$$

$$m = -\frac{u''_{cc} + \beta u''_{aa}}{u''_{ca}} > 0$$

LEMMA 1: Under assumption 1, the polynomial $Q(\lambda)$ has two positive real roots, μ_1 and μ_2 which verify:

$$0 < \mu_1 \leqslant 1 < 1/\beta \leqslant \mu_2.$$

PROOF: See Appendix.

LEMMA 2. **[Determination]**:
Under assumptions 1 and 2, the larger real eigenvalue λ_1 verifies:

$$\lambda_1 > \frac{1}{\beta} \quad \text{and} \quad \lambda_1 > w'(\bar{k}).$$

PROOF: See Appendix.

The case with all three eigenvalues with a modulus lying between -1 and 1 is thus excluded by assumption 2. An infinity of stable trajectories and the corresponding indeterminacy are thus not possible.

The two other eigenvalues λ_2 and λ_3 are real or complex conjugates. We deduce from:

$$\lambda_1\lambda_2\lambda_3 = \frac{w'}{\beta} \quad \text{and} \quad \lambda_1 + \lambda_2 + \lambda_3 = w' + \frac{m+n}{\beta}$$

that λ_2 and λ_3 are the roots of:

$$\lambda^2 - \left(w' + \frac{m+n}{\beta} - \lambda_1\right)\lambda + \frac{w'}{\beta\lambda_1} = 0 \tag{32}$$

PROPOSITION 4. **[Local stability of the steady state]**:
The steady state *equilibrium* is (locally) stable in the saddle-point sense if, and only if:

$$P(1) < 0 \quad \text{and} \quad \lambda_1 > \frac{w'}{\beta}. \tag{33}$$

PROOF: See Appendix.

Notice that the condition $P(1) < 0$, which can be written $(1 - w')(1 + \beta - m) - n < 0$, bears on the coefficients of the characteristic polynomial. It is verified if $w' \leqslant 1$ since $1 + \beta - m = Q(1)$ is negative or null and n is positive (assumption 2). We also notice that if $w' \leqslant 1$,

$$\frac{w'}{\beta\lambda_1} \leqslant \frac{1}{\beta\lambda_1} < 1.$$

Hence, a sufficient condition for stability in the saddle-point sense is:

$$w' \leqslant 1.$$

When $w' > 1$, the second condition $\lambda_1 > w'/\beta$ bears on the larger real eigenvalue λ_1 of $P(\lambda)$. This condition can be expressed in terms of the coefficients of the characteristic polynomial:

$$\lambda_1 > \frac{w'}{\beta} \Longleftrightarrow w' < \frac{1}{2}\left(m + \frac{n}{1-\beta}\right) + \frac{1}{2}\sqrt{\left(m + \frac{n}{1-\beta}\right)^2 - 4\beta}$$

Local stability allows us to analyze the effect of an increase in the degree of altruism on the steady state capital stock:

COROLLARY 1. **[Altruism and self-restraint under constrained altruism]**: A stable steady state stock of capital increases with the degree of altruism.

PROOF: See Appendix.

Even when there is no bequest, altruism has a positive influence on the steady state capital stock in the case of saddle-point stability. This is due to the self-restraint property: at a given degree of intergenerational comparisons, a higher degree of altruism urges the parents to reduce their consumption and increase their saving, which is beneficial for the accumulation of capital. Contrary to the standard model (WEIL [1987]), altruism has a positive influence on capital accumulation even in the case of no bequest.[2]

Since the stability is not guaranteed in the regime without bequest, it is useful to study the bifurcations that arise when the *equilibrium* looses its stability.

4.3 Bifurcations

Three types of bifurcations are possible.

Flip Bifurcation

When the eigenvalues λ_2 and λ_3 are real and when one of them becomes equal to minus one, there is a flip bifurcation. In that case, $P(-1) = 0$. This implies that:

2. Similarly, in a model with negative environmental externalities (pollution), JOUVET, MICHEL and VIDAL [2000] show that altruism modifies the stock of capital in the absence of bequest.

$$n = -(1 + w')(\beta + m + 1) < 0$$

and Assumption 2 is violated. Assumption 2 is thus sufficient to exclude flip bifurcation.

Saddle-Node Bifurcation

When the eigenvalues λ_2 and λ_3 are real and when one of them becomes equal to one, there is a saddle-node bifurcation. In that case we have the largest eigenvalue, say $\lambda_2 = 1$, $P(1) = 0$ and $\lambda_3 = w'/(\beta\lambda_1) < 1$. Hence, there is a saddle-node bifurcation if and only if:

$$P(1) = 0 \quad \text{and} \quad w' < \beta\lambda_1 \tag{34}$$

When there exists such a bifurcation, from Hale and Koçak [1991], on the one side of the saddle-node bifurcation (here when $P(1) < 0$) there are at least two steady-state equilibria, the one satisfying $\lambda_1 > w'/\beta$ being stable in the saddle-point sense. When $P(1) = 0$ these two fixed points collapse into a non-hyperbolic *equilibrium*. On the other side of the bifurcation point (when $P(1) > 0$) this equilibrium disappears. Since a saddle-node bifurcation is always associated with the disappearance of steady states we conclude from Proposition 3 that if there is a bifurcation of this type in our economy $\mathcal{E}$, then this bifurcation should also exist in the economy $\hat{\mathcal{E}}$. It is thus not directly linked to the dynamics of bequeathed tastes.

Neimark-Sacker Bifurcation

When the eigenvalues λ_2 and λ_3 are complex conjugates, a Neimark-Sacker bifurcation takes place if, and only if, their product is equal to 1: $w' = \beta\lambda_1$, which is then equivalent to $P(w'/\beta) = P(\lambda_1) = 0$. Furthermore λ_2 and λ_3 are the roots of equation (32) with $\lambda_1 = w'/\beta$:

$$\lambda^2 - \left(\frac{m + n - (1 - \beta)w'}{\beta}\right)\lambda + 1 = 0.$$

These roots are complex if, and only if, the discriminant is negative, *ie*,

$$| m + n - (1 - \beta)w' | < 2\beta.$$

Hence, there is a Neimark-Sacker bifurcation if, and only if,

$$P\left(\frac{w'}{\beta}\right) = 0 \quad \text{and} \quad | m + n - (1 - \beta)w' | < 2\beta \tag{35}$$

This bifurcation is often called "*Hopf bifurcation for maps*". The condition (35) suggests that a Neimark-Sacker bifurcation arises when the effect of the taste externality on marginal utility is strong enough. Indeed, in that case, $| u''_{ca} |$ is large and $m + n$ is small. Contrary to the saddle-node bifurcation, the Neimark-Sacker bifurcation does not exist in simple Diamond economies since it requires a dynamics of order 2 or more. Here, the presence of bequeathed tastes is crucial for the appearance of such a bifurcation. The Neimark-Sacker theorem is a powerful tool for the detection of limit cycles in discrete maps. Indeed a limit cycle appears either on the low or on the high side of the critical parameter value in a neighborhood of the bifurcation point.

The interpretations of cycles in the same as in de la CROIX [1996] and de la CROIX [2001]: during an expansion, there is a point where aspirations grow faster than resources; this reduces savings and start a depression period. A new turning point is reached when aspirations have reverted to lower levels.

5 Altruism, Accumulation and Consumption

5.1 Accumulation of Capital

In order to make a global analysis of altruism on the accumulation of capital in both regimes, we assume the existence of a unique stable steady state. More precisely we make the assumption of WEIL [1987], *ie*, the existence and uniqueness of the steady state *equilibrium* in the corresponding Diamond economy without altruism. But, in our case, this assumption is made for the simple standard economy $\hat{\mathcal{E}}$.

ASSUMPTION 3: For any $\beta < 1$, in the simple Diamond model with life-cycle utility $\hat{u}(c) + v(d)$, there exists a unique steady state $k^D(\beta) > 0$ which is globally stable.

With this assumption we apply the result of WEIL [1987][3] to our economy $\hat{\mathcal{E}}$, for a fixed β:[4]

- if $f'^{-1}(1/\beta) > k^D(\beta)$, bequests are operative and the unique steady state in $\hat{\mathcal{E}}$ is $k^\star = f'^{-1}(1/\beta)$.
- if $f'^{-1}(1/\beta) < k^D(\beta)$, bequests are zero and the unique steady state in $\hat{\mathcal{E}}$ is $k^D(\beta)$.
- in the limit case the unique steady state is $k^\star = f'^{-1}(1/\beta) = k^D(\beta)$.

Thus, in all cases, there exists a unique steady state *equilibrium* in $\hat{\mathcal{E}}$ which is

$$\hat{k}(\beta) = \max\left[f'^{-1}(1/\beta), k^D(\beta)\right]. \tag{36}$$

3. See the proposition 2 of WEIL [1987]. Notice that this proposition was generalized by ABEL [1987] to allow for non-separable utility functions and for local stability of steady state (instead of global stability).
4. Remember that $\hat{u}$ depends on β.

This $\hat{k}(\beta)$ is also the unique steady state in the economy $\mathcal{E}$. We now obtain the following result:

PROPOSITION 5. **[Capital accumulation and the degree of altruism]**: Under assumption 3, the unique steady state capital stock of $\mathcal{E}$ given by (36) increases with the degree of altruism.

PROOF: See Appendix.

The result of Proposition 5 is not a consequence of the local analysis (Corollary 4) when $\hat{k} = k^D(\beta) = \bar{k}$ is not stable in our economy $\mathcal{E}$.[5] Indeed the stability in $\hat{\mathcal{E}}$ does not imply the stability in $\mathcal{E}$, as it is shown in the Cobb-Douglas case of section 5.

From Proposition 5, we can easily generalize the result of WEIL [1987] concerning the link between over-accumulation of capital and inoperative bequests. Indeed, if there is no over-accumulation of capital for $\beta = 1$, there is no over-accumulation for any $\beta < 1$. Moreover, if for $\beta = 1$, $k^D(\beta) > f'^{-1}(1/\beta)$, we have over-accumulation.

5.2 Consumption

An interesting question concerns the effect of altruism on consumption when young at steady state. Intuitively, rising the degree of altruism has two effects: self-restraint reduces consumption against that the positive effect on capital (proposition 5) increases wages and, hence, consumption.

The effect of altruism on total consumption is easy to determine: in general, $c + d = f(k) - k$ increases with β when there is under-accumulation ($f'(k) > 1$), and decreases with β when there is over-accumulation.

The sign of the effect of altruism on consumption when young should be analyzed in the two different regimes.

In the case of zero bequest, $\hat{c} = w(\hat{k}) - \hat{k}$ increases with β when $w' > 1$ at k^D, and decreases when $w' < 1$.[6]

In the case of positive bequests ($\hat{k} = f'^{-1}(1/\beta)$), $\hat{c}$ is given by $\zeta(\hat{c},\beta) = 0$ with:

$$\zeta(c,\beta) = \hat{u}'(c) - \beta^{-1} v'(f(\hat{k}) - \hat{k} - c).$$

We notice that:

$$\zeta'_c = \hat{u}'' + \beta^{-1} v'' < 0$$

$$\zeta'_\beta = u'_a + \beta^{-2} v' - b^{-1} v''(\beta^{-1} - 1)\hat{k}'(\beta).$$

5. In fact, $k^D(\beta)$ increasing corresponds to the condition $P(1) < 0$ which is not sufficient to guarantee the stability of $\bar{k}$ (Proposition 4).

6. With a Cobb-Douglas production function $y = k^\alpha$, $w' = (1-\alpha)R$. When the steady state is not too far from the Golden Rule, the case $w' < 1$ prevails.

All terms of ζ'_β are positive except u'_a. If $\mid u'_a \mid$ is not too large, *ie*, if self-restraint is not too strong, $\hat{c}'(\beta) = -\zeta'_\beta/\zeta'_c$ is positive and consumption c increases with altruism.

The negative effect of altruism on consumption when there is over-accumulation of capital and no bequest has an important consequence for welfare. In this case, altruism causes household members to behave in ways that leaves all parties worse off. The possible negative role of altruism was already stressed in another context by BERNHEIM and STARK [1988]. In our set-up, altruism might be a counterproductive social force when it induces parents to save "*too much*" in the case they want to avoid to transmit bad habits to their children without leaving any bequest. Altruism might thus increase the loss of efficiency due to over-accumulation in the economy.

6 Analysis of the Cobb-Douglas Case

In the preceding sections, the dynamics of the two regimes have been analyzed separately. In order to look at the changes of regime as well as to discuss the properties of the model in terms of a small set of parameters, we propose in this section a global analysis of the Cobb-Douglas case, that is, we assume simple forms for preferences and technology. The inter-temporal utility of the consumer is given by:

$$\sum_{t=0}^{\infty} \beta^t \left[\ln(c_t - \gamma a_t) + \delta \ln(d_{t+1})\right]$$

The production function is given by $y_t = k_t^\alpha$. In that case, the properties of the model depend on four parameters: the psychological discount factor $\delta > 0$, the intensity of inter-generational comparisons $\gamma \in [0,1[$, the degree of altruism $\beta \in [0,1[$ and the share of capital in added-value $\alpha \in]0,1[$. The interesting characteristic of this example is to nest the Barro-Weil model (obtained when $\gamma = 0$).

6.1 Steady State

Assumption 1 is always satisfied and:

$$\hat{u}(c) = \frac{1-\beta\gamma}{1-\gamma} \ln(c).$$

Following Proposition 1, the steady state exists and is unique in the regime with positive bequests. Following Proposition 3, the steady state exists and is unique also in the regime without bequest, because the Diamond economy

FIGURE 1
The Two Regimes at Steady State in the Space $\{\beta, \gamma\}$

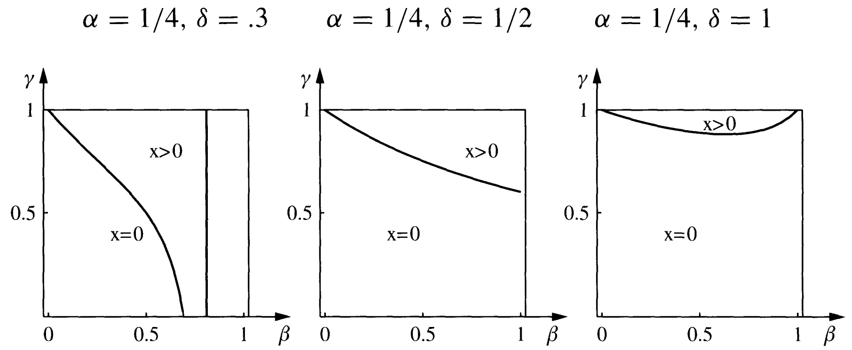


FIGURE 2
The Stability Region in the Space $\{\beta, \gamma\}$

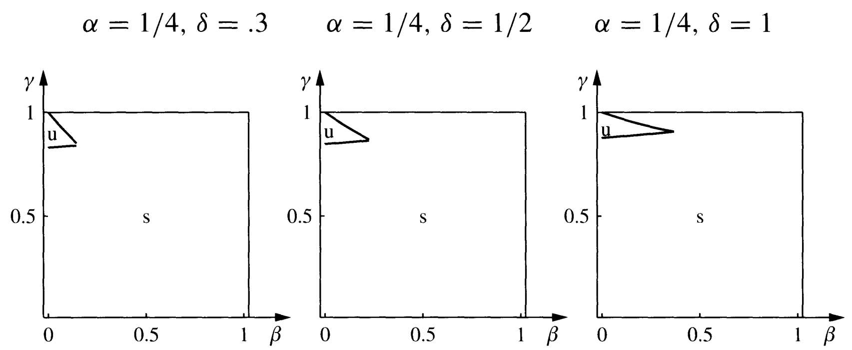
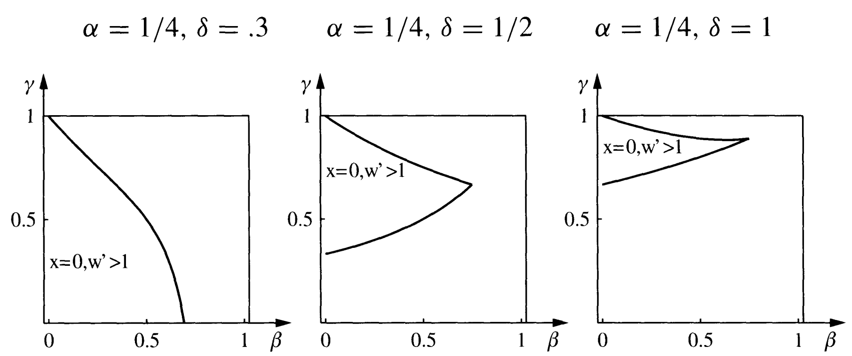


FIGURE 3
The Region in which $\mathrm{d}c/\mathrm{d}\beta > 0$ ***and*** $x = 0$ ***in the Space*** $\{\beta, \gamma\}$



with the utility $\hat{u}$ has one and only one non degenerate steady state in this regime, which is:

$$k^D = \left(\frac{(1-\alpha)(1-\gamma)\delta}{(1-\gamma)\delta + 1 - \beta\gamma} \right)^{\frac{1}{1-\alpha}}.$$

As it has been proven in the corollary of Proposition 4, since k^D is stable, the steady state capital stock is influenced positively by the degree of altruism. Intuitively, the size of this effect depends negatively on δ. If the agent puts much weight on consumption when old relative to consumption when young, the self-restraint effect has a less important role to play.

Moreover, the effect of γ on k^D is given by:

$$\frac{\partial k^D}{\partial \gamma} = \frac{-(1-\beta)\delta k^{D\alpha}}{(\beta\gamma - 1 - (1-\gamma)\delta)^2} < 0,$$

which is negative. The presence of bequeathed tastes always reduces the steady-state capital stock in the constrained case.

When bequests are positive at steady state, the capital stock satisfies the modified golden rule $\alpha k^{\alpha-1} = 1/\beta$. When there are no bequest, $\alpha k^{\alpha-1} < 1/\beta$ and the steady state capital stock is larger. The stock of capital at steady state is given by:

$$k = \max\left[k^D, (\alpha\beta)^{\frac{1}{1-\alpha}} \right].$$

The frontier that delimits the two regimes can be computed as a function linking the four parameters. Expressed in terms of δ, bequests are positive at steady state if $\delta \leqslant \tilde{\delta}$ with:

$$\tilde{\delta} = \frac{\alpha\beta(1-\gamma\beta)}{(1-\gamma)(1-\alpha-\alpha\beta)}.$$

This expression shows that a high psychological discount factor δ incites households to consume more when old and prevents them to leave bequest to their descendants.

It is interesting to plot the frontier between the two regimes in the space $\{\beta, \gamma\}$. Figure 1 shows three possible cases. The zone below the curve corresponds to the no-bequest regime. In the first case ($\delta = .3$) the frontier has a vertical asymptote, indicated in the chart: there exists always a threshold $\bar{\beta}$ such that bequests are positive for $\beta > \bar{\beta}$ for any $\gamma < 1$. Moreover, the degree of altruism necessary to yield positive bequest is a negative function of γ. Indeed, when γ is high and δ is low, the parents anticipate that their children will be very demanding about their future standard-of-living, and they thus wish to provide their children with more resources to fulfill their future aspirations.

The two last panels of Figure 1 shows that if γ is high enough bequests are positive, even when this is never the case in the economy without bequeathed tastes (which is our horizontal axis), that is when there is over-accumulation of capital in the Diamond case. Moreover, the last panel shows that there is not always a simple relationship between β and the kind of regime. In this case, the regime with positive bequest appears only if β is within a given interval; a too high β prevents the occurrence of the regime with positive bequest.

6.2 Local Dynamics

We know that if $\delta < \tilde{\delta}$ bequests are positive and, following Proposition 2, the steady state is locally stable. When $\delta > \tilde{\delta}$ it is not necessarily the case. Let us notice the following elements:

– since the production function is Cobb-Douglas, Assumption 2 is always satisfied and indeterminacy is excluded (Lemma 2).

– when $\gamma = 0$, the standard Diamond model with Cobb-Douglas preferences and technology is stable.

– we obtain:

$$m = \frac{1+\beta\gamma^2}{\gamma}, \quad n = \frac{\delta(1-\gamma)^2\bar{c}^2}{\gamma\bar{k}^2} = \frac{(1-\beta\gamma)^2}{\delta\gamma},$$

and:

$$Q(\lambda) = (\lambda - \gamma)\left(\beta\gamma - \frac{1}{\gamma}\right)$$

– the characteristic polynomial is:

$$P(\lambda) = -\frac{(1-\beta\gamma)^2}{\delta\gamma}\lambda^2 + (\lambda-\gamma)\left(\beta\gamma - \frac{1}{\gamma}\right)(\lambda - w').$$

– the product of the three eigenvalues is:

$$\frac{w'}{\beta} = \frac{\alpha(1-\beta\gamma+(1-\gamma)\delta)}{\beta(1-\gamma)\delta}.$$

The condition to have a saddle-node bifurcation, $P(1) = 0$, amounts to:

$$\delta = -\frac{1-\beta\gamma}{1-\gamma} < 0.$$

which is excluded.

– the necessary condition to have a Neimark-Sacker bifurcation $P(w'/\beta) = 0$ gives several relationships among the parameters, among which one is relevant given the admissible values of the different parameters and the requirement to have complex eigenvalues. In terms of δ it is given by:

$$\delta = \frac{\sqrt{\alpha}(1-\beta\gamma)(2\alpha^{3/2}(1-\beta)\gamma + \sqrt{\alpha}\mu + \sqrt{\nu}}{2(1-\beta)(1-\gamma)(1-\alpha\gamma)(\alpha-\beta\gamma)},$$

with:

$$\begin{aligned}\mu &= \gamma - \alpha + \beta^2\gamma^2 + \beta(1+\gamma-2\gamma^2)\\ \nu &= 4(1-\beta)\beta(1-\gamma)\gamma(1-\beta\gamma)\\ &\quad + \alpha(\gamma^2 + \beta^4\gamma^4 + 2\beta\gamma(1+\gamma)(1+\beta^2\gamma)\\ &\quad + \beta^2(1-2\gamma+9\gamma^2-2\gamma^3)).\end{aligned}$$

The region of instability is plotted in Figure 2 in the space $\{\beta,\gamma\}$ for different values of δ. "u" stands for unstable and "s" for stable.

The striking feature of this analysis is that the *equilibrium* could be unstable if, for a given β low enough, γ lies in a particular interval, let's say $[\underline{\gamma},\overline{\gamma}]$. If $\gamma < \underline{\gamma}$ there are no bequest and the *equilibrium* is stable. At the point $\underline{\gamma}$ the intergenerational spill-over is so strong that the economy experiences a Neimark-Sacker bifurcation and the steady state looses its stability. At the point $\overline{\gamma}$ there is a regime switch, bequests become positive and the *equilibrium* becomes stable again.

This figure also shows that instability occurs in the no bequest regime for very large values of γ. Moreover, we observe that altruism has the effect of reducing the scope for instability. The components of this reduction are twofold: first, an increase in altruism at low levels of β widens the region of the positive bequest regime which is always stable; second, in the no bequest regime, an increase in altruism makes the condition for stability less restrictive in terms of γ.

Let us now finally investigate the condition under which, in the no bequest regime, a rise in altruism increases the steady state consumption level c. We have seen in the previous section that when $w' > 1$, the rise in wages (income effect) dominates the self-restraint effect in the face of a rise in β, implying a positive net effect on c. Using the result that, with a Cobb-Douglas production function,

$$w' = (1-\alpha)\alpha k^{\alpha-1}$$

and the value of k^D computed above, the condition $w' > 1$ can be written:

$$\delta > \frac{\alpha}{1-\alpha}\,\frac{1-\beta\gamma}{1-\gamma}.$$

The region for which this condition is verified is plotted in figure 3.

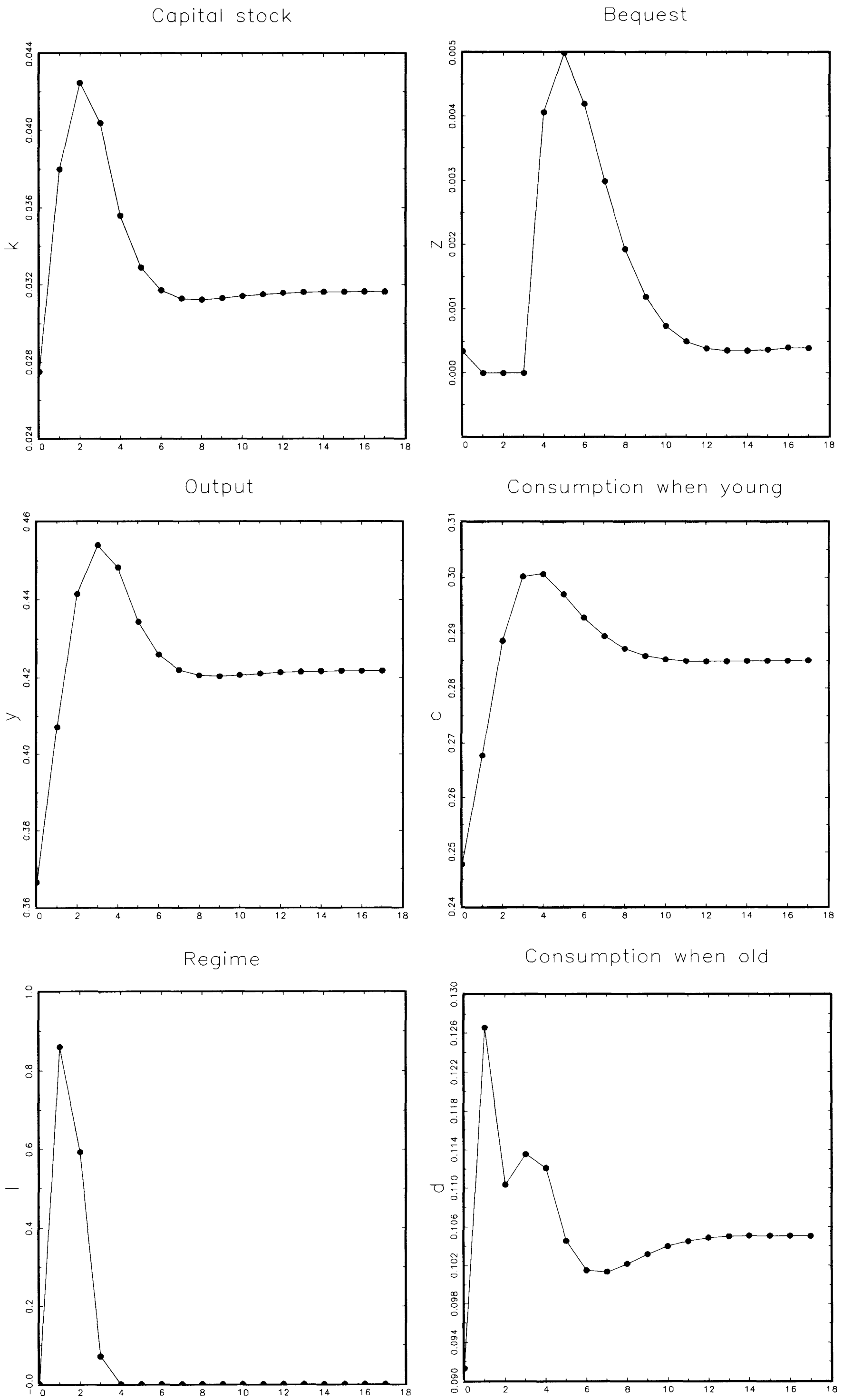
We first notice that the income effect always dominates the self-restraint effect if δ is low, that is if the capital stock is low. In that case, the interest factor is high, w' is high too, and the wage reacts much to a change in β. Secondly, the income effect dominates the self-restraint effect when the degree of altruism is low. Indeed, a low level of altruism implies a low level of capital, and we may conduct the same reasoning as above.

Finally, notice that figure 3 compared to figure 2 illustrates the fact that $w' < 1$ is a sufficient condition for stability in the no bequest regime.

7 Global Dynamics with a Numerical Example

The study of the global dynamics cannot be done analytically because endogenous shifts of regime (constrained vs positive bequests) are possible along the trajectories. We illustrate this using numerical simulations and analyze whether

FIGURE 4
Global Dynamics in an Example



it is possible for the economy to start growing in a given regime and converge to a steady state in the other regime. Our intuition is that the presence of local oscillatory trajectories (demonstrated in the previous sections) makes regime shifts likely if the parameter configuration is not too far from the configuration of the frontier separating the two regimes.

To perform this experiment, we need to use an algorithm that preserves the non-linear nature of the model (7)-(11). We thus follow the methodology proposed by BOUCEKKINE [1995] for saddle-point trajectories of non-linear deterministic models. The infinite horizon problem is approximated by a horizon of 100 periods. Increasing the simulation horizon further does not modify the results. Only the first 18 periods are displayed since the steady state is almost attained after 18 periods.

Figure 4 shows the convergence path of an economy starting at $t = 0$ with a capital stock which is lower than its long-run value. The parameters have been set such that the steady state is in the regime with positive bequests, but bequests are not too high ($\alpha = 0.25$, $\delta = 0.43$, $\beta = 0.3$, $\gamma = 0.805$). The eigenvalues of the linearized system around the steady state are $0.55 \pm 0.21i$ and $5.34 \pm 2.01i$. The initial level of aspirations and capital are set by computing a fictitious steady state with a production function $y = 0.9k^{\alpha}$.

At $t = 0$ there are slightly positive bequests and, accordingly, the variable λ_0 is equal to zero (panel "*regime*"). In the periods $t = 1,2$ there is a very rapid growth of the capital stock and of consumption. This very rapid take off makes optimal for the parents not to leave bequest, because they know that their future generations will soon benefit from higher resources. In fact, they would like to make negative bequests, transferring resources away from the future generations to their generation but that is impossible. We thus observe a regime shift at $t = 1$. Standard-of living aspirations rise very rapidly and attain a maximum at $t = 5$, *ie*, one period after the consumption when adult. These rising aspirations depress saving considerably and the capital stock starts declining. At $t = 4$ the middle-aged generation anticipates that their children will not be able to maintain their consumption level compared to the one of their parents, and it becomes optimal for them to leave positive bequests again. The regime of the economy changes, and will remain in the positive bequest case until convergence to the steady state.

This numerical analysis illustrates the complex interaction of altruism and inherited tastes and shows that the global dynamics can be characterized by regime shifts.

8 Concluding Comments

Assuming that children compare their consumption level with a benchmark derived from the standard-of-living of their parents, we have obtained a bunch of original results concerning the role of altruism in the economy.

We have first analyzed in details the dynamics in two regimes, one with and the other without bequest. The regime with (positive) bequest is similar to the analysis of the central planner performed in de la CROIX and MICHEL [1999]. The regime without bequest is new. In this new regime, altruism and bequests are dissociated, *ie*, altruism has economic effects though bequests are zero. The parents internalize the intergenerational spill-over and direct the evolution of children's aspirations towards raising their utility by restraining their own consumption standard, all things being equal. We have shown that, in that case, altruism is always beneficial to the accumulation of capital. It can thus potentially amplify the over-accumulation of capital.

Altruism has also an ambiguous effect on the consumption of the parents which results from the interaction of the self-restraint property with the increase in wage due to the higher stock of capital. The first effect dominates if wages are not too sensitive to capital, which is also a sufficient condition for stability.

To analyze the steady state of our economy, we have shown that it is possible to study a standard associated economy without inherited tastes whose steady state is the same as in our economy. However, the dynamics are completely different in these two economies. Inherited standard-of-living aspirations can be responsible for damped oscillations in the regime with positive bequests and for damped or exploding oscillations in the regime with no bequest. Altruism reduces the scope for instability in this last regime. Furthermore, in an example nesting the Barro-Weil model, we have shown that bequests can be positive even if this is never the case in the Barro-Weil economy. This is because parents wish to provide their children with more resources to fulfill their inherited aspirations.

The global analysis of the dynamics shows that the growth process of an economy can be characterized by regime shifts, some generations wishing to leave bequests, some other not. If the overlapping generations model with altruism is thought as micro-foundations for the infinite-horizon-representative-agent model, one message of this paper is that the introduction of habit formation and its corresponding oscillatory dynamics can break down the inter-generational linkages through bequests, as some generations may want not to leave bequest along the transition path.

In this paper, agents are homogenous. In particular they all have the same level of aspirations. An extension with heterogenous agents with different initial aspirations might be useful to study how migrants (with lower aspirations) can progressively be integrated in a given economy. ■

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APPENDIX

Proof of Proposition 1

Let us define:

$$\phi(c) = \hat{u}'(c) - \beta^{-1} v'(b^\star - c)$$

in which $b^\star = f(k^\star) - k^\star$. Steady state consumption, $c^\star$, is the solution of $\phi(c^\star) = 0$. Notice that:

$$\lim_{d \to 0} v'(d) = \infty \quad \Longrightarrow \quad \lim_{c \to b^\star} \phi(c) = -\infty.$$

From Assumption 1, $\phi(c)$ is decreasing and $\phi(0) = +\infty$. Thus there exists a unique $c^\star$ such that $\phi(c^\star) = 0$. Moreover, the positive bequest condition $c^\star > w(k^\star) - k^\star$ holds if and only if $\phi(w(k^\star) - k^\star) < 0$, *ie*, if (24) holds.

Proof of Proposition 3

Using $\hat{u}(c)$, equation (27) becomes $\hat{u}' = R\, v'$ which characterizes the standard Diamond economy without altruism.

Proof of Lemma 1

For $\lambda = 1$ we have

$$Q(1) = 1 + \beta - m = \frac{u''_{cc} + (1+\beta)u''_{ca} + \beta u''_{aa}}{u''_{ca}} = \frac{\hat{u}''}{u''_{ca}} \leqslant 0$$

following assumption 1 (equation (15)).

Since $Q(0) = 1 > 0$, $Q(\lambda)$ is equal to zero at $\mu_1 \in]0,1]$. Furthermore, the product of the roots is equal to $1/\beta$ implying that $\mu_2 = 1/(\beta\mu_1) \geqslant 1/\beta$.

Proof of Lemma 2

The limit of $P(\lambda)$ when $\lambda \to +\infty$ is equal to $+\infty$. When $\lambda = \mu_2$ we have $P(\mu_2) = -n\mu_2^2 < 0$. When $\lambda = w'$ we have $P(w') = -n(w')^2 < 0$. Hence, the conclusion.

Proof of Proposition 4

Necessary condition: On the one hand, if $P(1) \geqslant 0$, we deduce from $P(\mu_2) < 0$ that there exists a second real eigenvalue $\lambda_2 \geqslant 1$ and the steady state is unstable. Hence, stability implies $P(1) < 0$. On the other hand, stability implies $w'/(\beta\lambda_1) = \lambda_2\lambda_3 < 1$ and the two conditions (33) are necessary.

Sufficient condition: (a) When λ_2 and λ_3 are real, $P(1) < 0$ implies that λ_2 and λ_3 are on the same side of 1. (if only one eigenvalue is lower than 1, we have $P(1) > 0$, since $P(-\infty) = -\infty$). Since the product $\lambda_2\lambda_3 = w'/(\beta\lambda_1)$ is lower than 1, both eigenvalues are lower than 1.
(b) When λ_2 and λ_3 are complex, we have: $|\lambda_2|^2 = |\lambda_3|^2 = \lambda_2\lambda_3 = w'/(\beta\lambda_1)$ and (33) implies stability.

Proof of Corollary 1

The steady state is defined by:

$$\hat{u}'(w(\bar{k}) - \bar{k}) - g(\bar{k}) = 0.$$

Differentiation yields:

$$(\hat{u}''(w' - 1) - g')\mathrm{d}\bar{k} + \frac{\partial \hat{u}'}{\partial \beta}\mathrm{d}\beta = 0.$$

Since

$$\frac{\partial \hat{u}'}{\partial \beta} = u'_a < 0,$$

and

$$\hat{u}''(w' - 1) - g' = -u''_{ca}P(1) > 0 \quad \text{when} \quad P(1) < 0,$$

we have $\mathrm{d}\bar{k}/\mathrm{d}\beta > 0$.

Proof of Proposition 5

Since $f'^{-1}(1/\beta)$ is increasing in β, we simply have to prove that $k^D(\beta)$ is increasing in β in order to show that, according to (36), $\bar{k}(\beta)$ is also increasing in β.

For any β the saving function of the Diamond economy $s(w,R;\beta)$ is given by:

$$s(w,r;\beta) = \arg\max \hat{u}(w - s) + v(R\,s).$$

It depends on β through $\hat{u}$. Defining:

$$\psi(s,w,R;\beta) = \hat{u}'(w - s) - Rv'(R\,;s)$$

the saving function is such that $\psi(s,w,r;\beta) = 0$. Notice that $\psi'_s > 0$ and $\psi'_\beta = u'_a < 0$, which implies that $s'_\beta > 0$, the degree of altruism has a positive effect on saving at given wages and interest rate.

$k^D(\beta)$ is such that $\xi(k^D(\beta),\beta) = 0$ with

$$\xi(k,\beta) = s(w(k),R(k),\beta) - k.$$

Thanks to the stability and uniqueness of $k^D(\beta)$,

$$\xi'_k = s'_w w' + s'_R R' - 1 < 0$$

holds at $k = k^D(\beta)$ Hence, since $\xi'_\beta = s'_\beta > 0$, $k^D(\beta)$ is an increasing function of β.