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Externalities in Wage Formation and Structural Unemployment

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ABSTRACT. – A sectorial general equilibrium model in which externalities among sectors arise through wage envy is presented. Without externalities, equilibrium unemployment is only a function of the product market power of the firm and of demand uncertainty. With externalities, unemployment is higher. It is increasing with union power even though bargaining is efficient. Aggregate demand shock do not modify the magnitude of unemployment. However, when externalities are present, sectorial demand shocks modify the allocation of output across sectors ; this reallocation may increase or decrease unemployment depending on the initial situation of the economy.

Externalités dans la formation des salaires et chômage structurel

RÉSUMÉ. – On présente un modèle d'équilibre général sectoriel dans lequel des externalités inter-sectorielles existent par la présence d'envie dans la formation des salaires. Sans externalités le chômage d'équilibre est seulement fonction du pouvoir de marché des firmes et de l'incertitude sur la demande. Avec externalités, le chômage est plus important. Il augmente avec le pouvoir syndical bien que la négociation soit efficace. Des chocs de demandes agrégés n'affectent pas le chômage. Toutefois, des chocs de demandes sectoriels modifient l'allocation de la production entre les secteurs ; cette réallocation peut accroître ou diminuer le chômage en fonction de la situation de l'économie.

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1 Introduction

The idea that sector-specific unions are influenced by the wages obtained by other unions in other sectors has been used mainly to support the empirical observation that wages are highly correlated among industries (see e.g. MITCHELL [1980]). Furthermore, the presence of externalities and of strategic complementarity among unions leads to sub-optimal equilibria (COOPER and JOHN [1988]) which may generate non-desired inflation [1] and/or unemployment (GYLFASON and LINDBECK [1984]). This leads to the recognition that part of current unemployment can be eliminated by improving the coordination between social actors (JACOBS and JANSSEN [1990]).

In this paper, we apply a simple decentralised wage bargaining framework to a world where firms set their prices in a monopolistic competition environment. The general idea is to introduce externalities among sectors through decentralised wage bargaining structures and to analyse their effect on the equilibrium. In particular, we are interested in answering the following key question:

• What is the impact of externalities on the equilibrium unemployment rate?

The concept of equilibrium unemployment will be defined in the spirit of LAYARD and NICKELL [1986] and SNEESSENS and DRÈZE [1986]. In order to lead to a more precise "anatomy" of unemployment, we combine structural factors such as demand-supply mismatch [2] with market imperfections [3] such as union power, firm market power and externalities. Compared to DIXON's [1988] model, which presents a two-sector economy with externalities in wage formation, our framework is more specific (e.g. the union utility is directly derived from households utility), includes quantity rationing features and shows the influence of various parameters such as union power and firm market power on the equilibrium unemployment rate.

Three related questions are also treated in the paper; they pertain mainly to the interaction between externalities and other characteristics of the model:

• What is the role played by externalities when a sector faces a "real" demand shock (e.g. a change in the propensity to consume)?

• What is the link between union power and the intensity of externalities?

• Which are the sectorial shocks that can be transmitted to other sectors through wage formation (contagion effect)? In particular, do increases in union power or in labour productivity affect the other sectors (as they do in the Scandinavian model of AUKRUST [1977])?

1. As in AKERLOF [1969] and the well-known Taylor and Fisher models.
2. Which have been stressed by quantity rationing models as in LAMBERT [1988] and SNEESSENS [1987].
3. As in MC DONALD and SOLOW [1981], BLANCHARD and KIYOTAKI [1987] and COOPER and JOHN [1988].

2 The Model

The economy under consideration is divided into sectors in which there are segmented labour markets. In each sector, a large number of consumption goods are produced, the goods being imperfect substitutes. There is a large number of households deriving utility from consumption of all goods and from the services of real money balances and also providing at some cost labour on markets which are firm-specific in each sector. Each sector has a large number of firms all under constant return to scale. The only difference between the firms of a sector is about how they are affected by the realisation of some random shock on the demand to the sector. There is for each firm in each sector a union whose objective function is derived by summing up the indirect utilities of the households who provide labour to that firm in that sector. An equilibrium for a sector is defined, given other sectors' variables, by efficient contracts obtained by unions and firms in the sector. An equilibrium for the entire economy is made of equilibria for each sector either assuming the prices and wages are formed independently in the different sectors (the no externality case) or assuming that the sectors' efficient contracts are obtained with respect to a reference wage derived from the average labour earning in the economy (the externality case).

The model has the following properties: at the *sectorial level*, the confrontation of firms' and unions' claims leads to the determination of an equilibrium level for the sectorial unemployment rate (i.e., taking the actions of the other sectors as given). This mechanism is similar to the one that leads to the equilibrium unemployment rate in LAYARD and NICKELL [1986] except that it results here from a cooperative agreement between each firm-union pair (i.e., there is efficient bargaining). At the *aggregate level* and in the absence of externalities, unemployment is a function of firm market power and demand uncertainty. With externalities, the interaction between sectors through wage formation raises the unemployment rate and makes it sensitive to union power and relative demand in addition to firm market power and demand uncertainty [4].

2.1. Households

The economy is divided into K sectors. Each sector k is composed of a large number n^k of firms i, each producing a single consumption good. The goods are imperfectly substitutable. The utility function of the representative household j is defined over the consumption goods c_{ij}^k. The elasticity of substitution between the different goods of the same sector is

4. To help the comparison with DIXON's [1988] model, an interpretation can also be given in terms of natural range of unemployment. We show that this range is determined by union power, firm market power, demand uncertainty and by the intensity of wage externalities. The observed rate of unemployment, which lies within the natural range, is pegged by relative demand.

a constant $-\varepsilon^k$, with $\varepsilon^k > 1$. The elasticity of substitution between the goods baskets of different sectors is -1. Therefore, the utility function is a Cobb-Douglas of different baskets of goods, each basket being a CES of different goods. The utility function is also defined over real money holdings m_j/p. Money holdings enter the utility function in order to take into account future consumption. p is the aggregate price level. The households are risk neutral [5].

$$\max_{c_{ij}^k, m_j, l_j} \mathrm{U}_j = \left[\frac{m_j/p}{\alpha^m}\right]^{\alpha^m}$$

$$\times \prod_{k=1}^{\mathrm{K}} \left[\frac{1}{\alpha^k}\left(\sum_{i=1}^{n^k}\left(\frac{u_i^k}{n^k}\right)^{1/\varepsilon^k} c_{ij}^{k\left(\varepsilon^k-1\right)/\varepsilon^k}\right)^{\varepsilon^k/\left(\varepsilon^k-1\right)}\right]^{\alpha^k} - \frac{r}{p} l_{ij}^k$$

$$\text{s.t.} \quad m^0 + \mathrm{I}_j = \sum_{k=1}^{\mathrm{K}} \sum_{i=1}^{n^k} p_i^k c_{ij}^k + m_j$$

with $\sum_k \alpha^k + \alpha^m = 1$, $\sum_k u_i^k = 1$ for all k.

Note the presence of the u_i^k in the CES functions. The distribution function of the u_i^k is the same for all households and determines the allocation of a given sectorial demand across the various firms *of this sector*. We assume that this distribution is of mean 1. These good-specific weights, which are due to LICANDRO [1991], make the utility function more general than the one presented in DIXIT and STIGLITZ [1977] or BLANCHARD and KIYOTAKI [1987] and will be used to model firm-level uncertainty in the spirit of LAMBERT [1988], SNEESSENS and DRÈZE [1986] and SNEESSENS [1987].

The utility function is separable in consumption and leisure. l_{ij}^k is the amount of work done by j in firm i of sector k. The marginal disutility of work, which is equal to the real reservation wage, is r/p. Moreover, as in SNEESSENS [1987], we assume that labour supply is firm-specific: if the offered wage is larger than the reservation wage, ls_i^k workers supply inelastically one unit of labour to firm i of sector k. This assumption, which is common in quantity rationing models, is the simplest way to allow for the coexistence of vacancies and unemployment [6].

The budget constraint of household j includes initial money holdings m^0 (the same for all households) and income I_j. The first order conditions for utility maximisation yield notional consumption functions and demand for money as linear functions of wealth and of the vector of prices:

$$\begin{cases} c_{ij}^k = \left(\dfrac{p_i^k}{p^k}\right)^{-\varepsilon^k} \dfrac{\alpha^k}{n^k}\left[\dfrac{m^0+\mathrm{I}_j}{p^k}\right] u_i^k \\ m_j = \alpha^m \left[m^0 + \mathrm{I}_j\right] \end{cases}$$

5. The risk neutrality assumption does not imply any change in demand functions. It implies that the indirect utility is a linear function of income.
6. Together with firm-level demand uncertainty, labour market segmentation implies sectorial unemployment, even when total sectorial demand equals total sectorial supply, since demand and supply are not uniformly distributed among firms.

$$\text{with } p^k = \frac{1}{n^k}\left(\sum_{i=1}^{n^k} u_i^k\, p_i^{k\left(1-\varepsilon^k\right)}\right)^{1/(1-\varepsilon)} \qquad \text{and } p = \prod_{k=1}^{K} p^{k\left(\alpha^k / \sum_{z=1}^{K} \alpha^z\right)}$$

In the next sections, the effect on unemployment of the four following parameters will be analysed: the average propensity to save $\alpha^m = 1 - \sum_{k=1}^{K} \alpha^k$; the relative propensity to consume α^x/α^y, $x \neq y$, $x, y = 1, \ldots, K$; the elasticity of substitution between goods of a given sector ε^k; the distribution function of the u_i^k.

2.2. Note on Consumer Rationing

In deriving the consumer's demands, we have, as in SNEESSENS [1987] and all papers related to his model, implicitly assumed that households are never rationed on any good, which is not the case. The alternative is to assume a rational expectation property with respect to the rationing scheme and to derive effective demands, as we do in ARNSPERGER-DE LA CROIX [1992] following BENASSY [1975] and LICANDRO [1991]. Our conclusion in this other paper is that the incorporation of effective demand in this one-sector model has no implication for employment, output and real wages and affects only the level of prices. Therefore, to avoid further substantial complications which do not change the nature of the model with respect to real magnitudes, we prefer to keep our shortcut: effective demand is equal to notional demand while effective consumption will be equal to output (and will be smaller than notional demand). The households will keep a quantity of money which is larger than the desired (and expected) one.

2.3. Unions

The utility of the firm-specific union V_i^k is obtained by computing the sum of the indirect utilities of the members [7]. The indirect utility of each member (obtained by replacing (1) in the utility function) is equal to its real income $(m^0 + I_j)/p$ since households are risk neutral. If every worker supplying its work to the firm is a union member, the total membership is ls_i^k. The utility of the union is:

$$\sum_{j \in ls_i^k} U_j = V_i^k = \frac{1}{p}\left(ls_i^k\, m^0 + (w_i^k - r)\, l_i^k + \sum_{j \in ls_i^k} \sum_{k=1}^{K} \sum_{h=1}^{n^k} \theta_{hj}^k\, F_h^k\right)$$

The labour income net of the reservation wage is $(w_i^k - r)\, l_i^k$. The capital income is proportional to the shares θ_{hj}^k of firm h of sector k being in

7. Note that, usually, the utility of the union is not derived from households utility. For instance, in DIXON [1988], the households have a Cobb-Douglas utility while unions have a Stone-Geary utility function.

possession of household j. F_h^k is the nominal profit of firm h. It is equal to output y_i^k times the difference between the output price p_i^k and the unit labour cost w_i^k/a^k, a^k being the (constant) mean labour productivity:

$$F_i^k = y_i^k \left(p_i^k - \frac{w_i^k}{a^k} \right)$$

The fall-back utility $\tilde{V}_i^k$, which is the status quo point in the bargaining process, is the income attanable in case of breakdown in the negotiation. In this situation, there is no production, no labour income and no employment. Each household will simply enjoy its initial money holdings and its capital income (the fact that the profit of its own firm is zero affects its capital income in a negligible way):

$$\tilde{V}_i^k = \frac{1}{p} \left(ls_i^k \, m^0 + \sum_{j \in ls_i^k} \sum_{k=1}^{K} \sum_{h=1}^{n^k} \theta_{hj}^k \, F_h^k \right)$$

The corresponding net utility of the union is:

$$V_i^k - \tilde{V}_i^k = l_i^k \, \frac{w_i^k - r}{p}$$

Let us assume that the reservation wage is a function of the mean wage in the economy, $\tilde{w}$, which can also be called the reference wage: $r = \phi \, \tilde{w}$. This simply says that, when households evaluate their gain from working, they compare the wage they would earn with a reference wage which is the average labour earning in the economy: at a given wage, if the other workers gain more, the household would more and more prefer to work elsewhere (but it is not possible) or to stay home [8]. Consequently, the net union utility is defined over employment and over the difference between the negociated wage and a portion of the average wage in the economy [9].

$$V_i^k - \tilde{V}_i^k = l_i^k \, \frac{w_i^k - \phi \, \tilde{w}}{p}$$

The parameter ϕ measures the intensity of the externality between unions [10]. It will allow us to study the impact of the intensity of the externality on the equilibrium, including the special case where $\phi = 0$ (no externalities).

Note that, since unemployed workers receive the same income (here, nothing) than the strikers, the *net* utility of the union does not incorporate the

8. This has something to do with the usual 'rivalry' or 'jealously' effect: the workers look at the other workers to evaluate their gain of reaching an agreement during the bargaining. This interpretation is often used in the literature, from KEYNES [1936] to BHASKAR [1990].
9. The use of an arithmetic mean to model the reference wage is arbitrary. However, using a geometric mean (or any other devices increasing in all sectoral wages) would not change the qualitative nature of the results.
10. There is another attractive way of modelling externalities in this context: If the disutility of work were evaluated as a function of the value of domestic work, the reservation wage would be indexed on the general price level p. Since this price will turn out to be a function of the wages in the economy, we would retrieve the same qualitative relation in a different way. An unemployment insurance scheme could also provide some kind of interdependence.

utility of unemployed, as it is the case in insiders models. The unemployed workers are a third party, all the burden of externalities is on the unemployed.

2.4. Firms

The firm's supply is determined as in SNEESSENS [1987]. In a given sector k, the only difference between the firms is that they are affected by a different realisation of the demand shock u_i^k. The production function uses labour with constant returns with a^k as the mean productivity of labour (subscript i is omitted since the firms in sector k have the same labour productivity).

(2) $$y_i^k = a^k \, l_i^k$$

The firm is limited in its production by the availability of labour. The total supply of labour addressed to the firm is $ls_i^k = ls^k / n^k$ and is assumed to be the same in all the firms of sector k [11]. In this case, the productive capacity ys_i^k is:

(3) $$ys_i^k = a^k \, ls_i^k$$

The notional demand yd_i^k addressed to the firm is obtained by aggregating (summing) over households the consumption functions (1):

(4) $$yd_i^k = \sum_{j=1}^{J} c_{ij}^k = \left(\frac{p_i^k}{p^k}\right)^{-\varepsilon^k} \frac{\alpha^k}{n^k} \left[\frac{J\, m^0 + \sum_{z=1}^{K} p^z\, y^z}{p^k}\right] u_i^k$$

According to equation (4), the demand addressed to firm i is a share of total income depending on the relative price of the firm with respect to the price of its sector and on the weight u_i^k of good i in the utility function. The role of these weights is to introduce demand uncertainty in the model: firms and unions know only the probability distribution of these weights at the time of their decision about prices and wages.

The timing of the decisions is the following: 1) Unions and firms bargain at the firm level over prices and wages, knowing the distribution of the u_i^k. 2) Firm-specific shocks u_i^k become known. 3) Firms determine output and employment. Since output is determined after the realisation of the shock, it is equal to the minimum of the two constraints:

(5) $$y_i^k = \min\left(ys_i^k, \, yd_i^k\right)$$

Let us assume that the shock u_i^k is lognormally distributed among firms. We then apply Lambert's (1988) theorem and approximate expected output as a CES function of the two expected constraints:

(6) $$\mathrm{E}\left(y_i^k\right) = \left[\left(ys_i^k\right)^{-\rho^k} + \mathrm{E}\left(yd_i^k\right)^{-\rho^k}\right]^{-1/\rho^k}$$

11. All firms have the same exogenous labour market share; of course, if the offered wage is lower than the reservation wage, labour supply is 0.

The parameter ρ^k is a function of the variance of the shock u_i^k. In particular, if this variance goes to zero, ρ^k goes to infinity, it can be shown that the CES would tend to the minimum function (5). In this case, there would be no more uncertainty and the model would be equivalent to a regime-switching (unemployment or not) framework where all firms are in the same situation [12].

A crucial variable at the firm level is the probability of facing a demand constraint. This probability is also equal to the elasticity of firm output with respect to demand. It is defined by LAMBERT (1988) as:

$$\text{(7)} \qquad \Pr\left[y_i^k \leqslant yd_i^k\right] \equiv \pi_{\mathrm{D}i}^k = \left(\frac{\mathrm{E}\left(y_i^k\right)}{\mathrm{E}\left(yd_i^k\right)}\right)^{\rho^k}$$

2.5. Efficient Bargaining Outcome

In each firm of the K sectors, the union and the firm negotiate an efficient outcome (MAC DONALD and SOLOW, [1981]), bargaining jointly to determine the nominal wage and the output price. We have chosen this cooperative solution at the firm level in order to limit the loss of efficiency to the aggregate level at which a non-cooperative framework between firm-union pairs will be introduced. Using the asymmetric Nash bargaining solution, with β^k being the parameter which weights the two objective functions in the Nash product and which is called "union power", and assuming a zero fall-back profit, the maximisation problem is [13]

$$\max_{w_i^k,\, p_i^k} \left[\frac{\mathrm{E}\left(y_i^k\right)}{a^k}\, \frac{w_i^k - \phi\, \tilde{w}}{p}\right]^{\beta^k} \left[\mathrm{E}\left(y_i^k\right)\, \frac{p_i^k - w_i^k/a^k}{p}\right]^{\left(1-\beta^k\right)}$$

$$\text{s.t.} \quad (2), (3), (4) \quad \text{and} \quad (6)$$

The agents take $\tilde{w}, p, p^z$ and $y^z \; \forall\, z$ as exogenous macroeconomic variables. The first-order conditions are:

$$\text{(8.1)} \qquad p_i^k = \left[1 - \frac{1-\beta^k}{\varepsilon^k\, \pi_{\mathrm{D}i}^k}\right]^{-1} \frac{w_i^k}{a^k}$$

$$\text{(8.2)} \qquad w_i^k = \left(1-\beta^k\right)\phi\tilde{w} + \beta^k\, a^k\, p_i^k$$

The first-order conditions determine price and wage equations (which are the variables that are decided before the realisation of the shock). Concerning the price equation, the firm's price is a markup on marginal variable cost, with the markup rate depending on goods' substitutability, union power and the probability of a demand constraint. The second order

12. We would then always have to distinguish the two cases as in MALINVAUD [1977], BENASSY [1982] or, more recently, in JACOBSEN and SCHULTZ (1990].
13. Note that p is the aggregate price level, which is exogenous a the firm level. The fact that we deflate the firm's profit by p can be rationalised by saying that the firm is interested in the overall purchasing power of the dividends.

condition requires $\pi^k_{Di} \geq (1-\beta^k)/\varepsilon^k$. As stated before, the households refuse to work if $w^k_i > \phi\tilde{w}$ is not verified. As in SNEESSENS [1987], the introduction of firm-specific uncertainty allows to express the mark-up rate as a function of the probability of a demand constraint. In addition, at given π^k_{Di}, firm's mark-up is a decreasing function of union power. The intuition behind this is relatively straightforward: in the standard efficient bargaining model, the union bargains over employment and thus indirectly influences the firm's price; in our model, the efficient contract between the firm and the union contains an implicit clause about expected employment which forces the firm to reduce its output price in order to increase the demand for its good. The Lerner index $(1-\beta^k)/(\varepsilon^k\,\pi^k_{Di})$ is negatively affected by union power: a "powerful" union extracts some part of the pure monopoly profits, which amounts to lowering the firm's effective monopoly power.

In equation (8.2), the wage is an average of the worker's reference wage and of the firm's labour productivity in value; the weigths are a function of union power. The inclusion of the reference wage reflects what has been called the "rivalry effect": the presence of the reference wage in the bargaining function introduces a negative externality between unions: since the resulting wage is a positive function of the reference wage, we also have strategic complementarity between unions. The presence of both externalities and strategic complementarities leads to sub-optimal equilibria which are treated in the next section. This formulation shows why the rational behaviour of unions derived from household preferences does not force the union to require full compensation for inflation unless it assumes other unions to be fully compensated. This model corresponds closely to the wage determination process described by Keynes where each labour group is restrained by the at least temporary fixity of the wages of the other labour group. Stated in real terms, the wage equation can be rewritten

$$w^k_i/p = (1-\beta^k)\,\phi\,\tilde{w}/p + \beta^k\,a^k\,p^k_i/p$$

which shows that workers will ask for full compensation of aggregate inflation as long as $\tilde{w}/p$ and p^k_i/p remains constant. What is important here, comparing our rivalry model with the one of GYLFASON and LINDBECK [1984], is that the weights of the two elements in the wage equation are a function of union power: if union power is high, the workers capture a higher part of firms added-value. If it is low, the wage is determined to a larger extent by the wages of the other sectors.

3 The Equilibrium

We compute three kinds of equilibria: (a) The sectorial equilibrium is conditional to the variables of the other sectors. (b) The macroeconomic equilibrium without externalities is a juxtaposition of sectorial equilibria, when there are no externalities; it will serve as the reference case. (c) The macroeconomic equilibrium with externalities allows to detail the influence of externalities on the equilibrium unemployment rate. (b) is in fact a special case of (c) when $\phi=0$ but it is presented first for exposition purposes.

3.1. Sectorial Equilibrium

We have assumed that sectorial demand is randomly distributed among a large number of firms. When they take their decision about prices and wages, firms do not know their position in the distribution of demand. However, total sectorial demand yd^k is known and is proportional to the expected demand perceived by the firm: $\mathrm{E}\left(yd_i^k\right) = yd^k/n^k$. For this reason, we say that uncertainty is only *firm-specific*. At the sectorial level, the model becomes deterministic.

DEFINITION 1: A sectorial equilibrium E^k is a vector $[p^k, w^k, y^k, yd^k, l^k, \pi_{\mathrm{D}}^k]$ which satisfies

$$\left.\begin{array}{lc}
(9.1) & p^k = \left[1 - \dfrac{1-\beta^k}{\varepsilon^k\,\pi_{\mathrm{D}}^k}\right]^{-1} \dfrac{w^k}{a^k} \\
(9.2) & w^k = \left(1-\beta^k\right)\phi\,\tilde{w} + \beta^k\, a^k\, p^k \\
(9.3) & y^k = \left[(a^k\, ls^k)^{-\rho^k} + (yd^k)^{-\rho^k}\right]^{-1/\rho^k} \\
(9.4) & yd^k = \alpha^k \left[\dfrac{\mathrm{J}\, m^0 + \sum_{z=1}^{\mathrm{K}} p^z\, y^z}{p^k}\right] \\
(9.5) & l^k = y^k/a^k \\
(9.6) & \pi_{\mathrm{D}}^k = \left(\dfrac{y^k}{yd^k}\right)^{\rho^k}
\end{array}\right\} \quad \mathrm{E}^k \stackrel{\mathrm{def}}{=}$$

To derive (9), we use the properties of a symmetric equilibrium. All firms in each sector are the same ex-ante (when they decide about prices and wages) but differ after the realisation of the shock (when they set output and employment). In each sector, all agents set the same price and wage (equations (9.1) and (9.2)).

After the realisation of the shocks, aggregate demand is distributed among firms following the same distribution as the probability distribution of the shock. For this reason, we can equalise the ex-ante probability distribution of the shocks with the ex-post distribution of demand across firms in each sector. Consequently, the sectorial economy is similar to the firm-level one, where the expected variables are replaced by aggregate variables. Transacted quantities follow the CES formulation (9.3). Sectorial demand (9.4) is obtained by summing the firm-level demands. What was before the ex-ante probability of being constrained by demand now becomes the ex-post proportion of firms actually constrained by demand (9.6). Note that this proportion is linked to the sectorial unemployment rate UR^k. Defining $\mathrm{UR}^k = 1 - l^k/ls^k$ and using (9) it comes:

$$\mathrm{UR}^k = 1 - (1-\pi_{\mathrm{D}}^k)^{1/\rho^k} \tag{10}$$

This sectorial unemployment rate is equal to the expected firm-level unemployment rate. However, the ex-post firm-level unemployment rates differ.

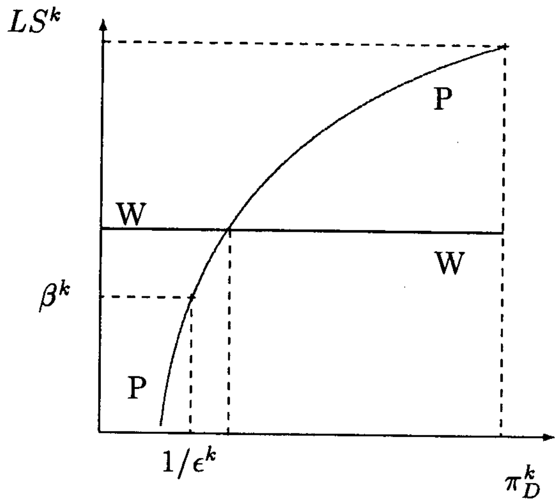

FIGURE 1

The Sectorial Equilibrium.

Denoting the labour share $\mathrm{LS}^k = w^k/(p^k\, a^k)$, the sectorial equilibrium can be graphically represented in $\{\pi_{\mathrm{D}}^k, \mathrm{LS}^k\}$ space by computing the intersection of (9.1) and (9.2). These two equations can be rewritten in terms of labour share:

$$\text{(11)} \qquad \begin{cases} \mathrm{LS}^k = 1 - \dfrac{1-\beta^k}{\varepsilon^k\, \pi_{\mathrm{D}}^k} \\ \mathrm{LS}^k = \beta^k + \phi\,(1-\beta^k)\, \dfrac{\tilde{w}}{a^k\, p^k} \end{cases}$$

The first equation of (11) defines a positive relation between LS^k and π_{D}^k which is drawn in Figure 1 (PP curve). This relation can be assimilated to the share compatible with the mark-up requirement of the firms. However, this comparison is limited, since (11) results from a *cooperative* agreement between each firm and each union. Note that the PP curve does not go beyond the point where $\pi_{\mathrm{D}}^k < (1-\beta^k)/\varepsilon^k$, in order to verify the second order condition. The second equation of (11) allows to determine a labour share which is drawn as the WW curve. Since households are risk-neutral and bargaining is efficient, the labour share is not a function of π_{D}^k along the WW line. Note that if there are no externalities ($\phi = 0$), the labour share is simply β^k. If $\phi > 0$, the labour share is larger by the amount $\phi\,(1-\beta^k)\,\tilde{w}/\,(a^k\, p^k)$.

3.2. Macroeconomic Equilibrium without Externalities

DEFINITION 2: A macroeconomic equilibrium without externalities E* is a set of K sectorial equilibria E^k with $\phi = 0$.

$$E^* \stackrel{\text{def}}{=} \begin{cases} \{E^k\}_{k=1}^{K} \\ \phi = 0 \end{cases}$$

When there are no externalities between sectors at the level of wage formation, price and wage relationships (9.1)-(9.2) alone determine the equilibrium E*:

PROPOSITION 1: A macroeconomic equilibrium in the absence of externalities E* is characterized by a vector $\{\pi_D^k\}$ which satisfy:

$$\pi_D^k = \frac{1}{\varepsilon^k} \tag{12}$$

and generates a sectorial unemployment rate which is a function only of firm market power (ε) and of the degree of uncertainty (ρ).

Proof: (9.1) and (9.2) give (12) when $\phi = 0$. Using (10) and (12) it comes:

$$UR^k = 1 - \left(1 - \frac{1}{\varepsilon^k}\right)^{1/\rho^k}$$

The risk neutrality of households is an important assumption since it makes employment not a function of union power when $\phi = 0$. In the presence of risk aversion, the above unemployment rate would also be a (negative) function of union power.

The unemployment rate generated by an equilibrium without externalities can be compared with the SURE, the structural unemployment rate which is defined as follows:

DEFINITION 3: The structural unemployment rate is the prevailing unemployment rate when sectorial goods demand yd^k equals sectorial goods supply $a^k\, ls^k$ (which implies $\pi_D^k = 1/2$):

$$\text{SURE}^k \stackrel{\text{def}}{=} 1 - \left(1 - \frac{1}{2}\right)^{1/\rho^k}$$

At this rate, the number of vacancies is equal to the number of unemployed. From Definition 3, it is clear that if $\varepsilon^k < 2$, the economy without externalities experiments an unemployment rate which is higher than the structural unemployment rate. Proposition 1 stress very well the role played by monopolistic competition and by firm-specific uncertainty. If either the goods becomes infinitely substitutable ($\varepsilon^k \to \infty$) or the variance of the shocks tends to zero ($\rho^k \to \infty$), the unemployment rate tends to zero. In order to illustrate more intuitively the implications of this model we have computed the unemployment rate at E* for different values of ρ^k and ε^k; these are presented in Table 1. We also present the corresponding value of the standard-error σ^k of the distribution of demand across firms [14].

TABLE 1

Unemployment rate at E*

		ε^k 1.1	1.5	SURE^k	3	5	10
σ^k	ρ^k						
29%	5	38%	20%	13%	8%	4%	2%
15%	10	21%	10%	7%	4%	2%	1%
10%	15	15%	7%	5%	3%	1%	1%
7%	20	11%	5%	3%	2%	1%	1%
5%	30	8%	4%	2%	1%	1%	0%
3%	50	5%	2%	1%	1%	0%	0%
1%	100	2%	1%	1%	0%	0%	0%

3.3. Macroeconomic Equilibrium with Externalities

DEFINITION 4: A macroeconomic equilibrium with externalities in wage formation E is characterised by K sectorial equilibria E^k and by a scalar $\tilde{w}$ which satisfies (13).

$$\text{(13)} \qquad E \stackrel{\text{def}}{=} \begin{cases} \{E^k\}_{k=1}^{K} \\ \tilde{w} = \sum_k \lambda^k w^k \end{cases}$$

where $\lambda^k = ls^k / \sum_z ls^z$ is the size of sector k in percentage of total labour market.

Equation (13) says that the E equilibrium is computed by assuming that the expectations about the reference wage $\tilde{w}$ are equal to the effective mean wage in the economy. Let us now solve the model for this class of equilibria.

The wage and price equations (13), (9.1) and (9.2) allow us to determine a first relation between the π_D^k of the various sectors. This relation is derived in the Appendix. The idea is that, since all price and wage equations are linear homogeneous in the other nominal variables, their combination leads to a relation between the mark-up rates and other real variables, which can be expressed as a function of the π_D^k only. This relation, called SS hypersurface, is:

$$\text{(14)} \qquad 1 = \phi \sum_{k=1}^{K} \lambda^k \left(1 + \frac{\beta^k}{\varepsilon^k \pi_D^k - 1}\right)$$

14. The link between ρ^k and σ^k is given by $\rho^k = -1 + (2/\sigma^k)\, f(-\sigma^k/2)/\mathrm{F}(-\sigma^k/2)$, where F is the standard normal distribution and f is the normal density function (Cf. LAMBERT [1988]).

This is a hypersurface in the $\{\pi_D^k\}$ space on which the economy has to stay in order to make the claims of all players compatible. This SS hypersurface defines a tradeoff between the output of the different sectors. The mechanism of this tradeoff is the following: if output in sector x falls, the price and the nominal wage in this sector also fall because of the mark-up rule. This decreases the reference wage for the other sectors, thus lowering their labour share and allowing price cuts, which increases demand and output. The endogenous mark-up rate plays a crucial role here.

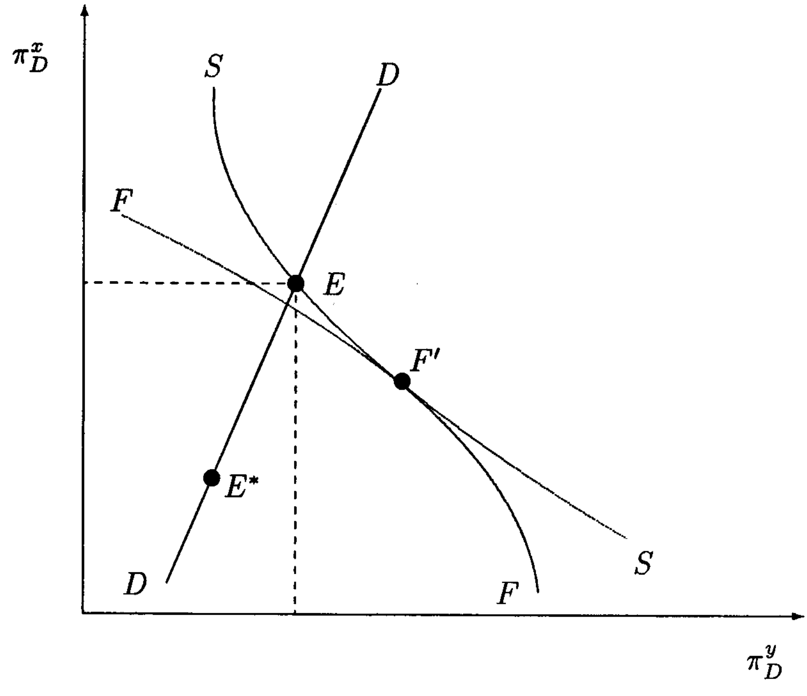

FIGURE 2

Macroeconomic Equilibria

Figure 2 displays for two sectors the different types of derived macroeconomic equilibria [15]. The SS hypersurface is drawn for the two-sector case as the SS curve in Figure 2. Note that total employment $\sum_k l^k$ varies when we move from one point of the hypersurface to another. This becomes clear when we compute a hypersurface along which employment is constant.

15. The π_D^k are on the axes. We may also consider that the axes measure sectorial unemployment since (10) defines a positive monotonic relation between π_D^k and UR^k.

DEFINITION 5: An iso-employment hypersurface in $\{\pi_D^k\}$ space is a set of vectors $\{\pi_D^k\}$ which achieves the same level of total employment $\tilde{l}$. It is defined by the following relation:

$$\tilde{l} = \sum_{k=1}^{K} ls^k \left(1 - \pi_D^k\right)^{1/\rho^k}$$

This relation is found by transforming $\sum_k l^k = \tilde{l}$ using (9.3), (9.5) and (9.6). Note that this curve is only affected by the variance of the shocks (reflected in ρ^k). Figure 2 plots the iso-employment curve (FF curve) corresponding to the highest employment level compatible with the SS curve.

If $\phi = 0$, we retrieve the preceding case where $\varepsilon^k \pi_D^k \rightarrow 1$ for all k. The labour shares in both sectors are equal to β^k and the π_D^k are equal to $1/\varepsilon^k$. In $\{\pi_D^x, \pi_D^y\}$ space this determines a unique point E*.

Demand equations determine another set of relations between the π_D^k of the various sectors. Using the demand equation (9.4), we can compute pairwise ratios of demands:

$$\frac{yd^x}{yd^y} = \frac{\alpha^x \, p^y}{\alpha^y \, p^x} \tag{15}$$

Relative demand is a function of the relative propensity to consume (α^x/α^y) and of relative prices. From the Appendix, using price and wage equations, (15) implies that:

$$\frac{\lambda^x \left(\frac{1}{\pi_D^x} - 1\right)^{1/\rho^x}}{\lambda^y \left(\frac{1}{\pi_D^y} - 1\right)^{1/\rho^y}} = \frac{\alpha^x \left(1 - \frac{1}{\varepsilon^x \pi_D^x}\right)}{\alpha^y \left(1 - \frac{1}{\varepsilon^y \pi_D^y}\right)}, \; x \neq y \tag{16}$$

This equation defines a positive relationship between the π_D^k of any two sectors. It is independent of ϕ because ϕ is the same in all sectors. In a K-sector economy, there are K − 1 relations of type (16), called DD hypersurfaces. These positive relationships between the output of two sectors can be interpreted in the following way: if the output in one sector increases, the output in the other sector has to increase in order to keep the budget shares constant. The intersection of these DD hypersurfaces gives a curve in $\{\pi_D^k\}$ space. In the two-sector case, we have only one relation which is drawn as the DD curve in Figure 2.

The *equilibrium* vector $\{\pi_D^k\}$ at E is given by the intersection of the DD hypersurfaces (16) with the SS hypersurface (14). Unfortunately, (14)-(16) does not yield a simple determination of the π_D^k due to non-linearities. However, the differentiation of this system allows to determine the sign of the effect of parameter changes on the $\{\pi_D^k\}$ vector. This differentiation is presented in the appendix: the signs of the variation of π_D^k as a function of the variations of β^x, β^y, ϕ and α^y/α^x, for all x and y are:

$$\frac{d\pi_D^x}{d\beta^x} > 0 \quad \frac{d\pi_D^x}{d\beta^y} > 0 \quad \frac{d\pi_D^x}{d\phi} > 0 \quad \frac{d\pi_D^x}{d\left(\alpha^y/\alpha^x\right)} > 0$$

These will be discussed in the next section "comparative statics".

Moreover, we may compute an approximation of the equilibrium in the following way. For reasonable values for the standar error of the shocks u_i^k, ρ^k is large; this implies that $(1/\pi_{\mathrm{D}}^k - 1)^{1/\rho^k}$ is close to 1. Using the approximation $(1/\pi_{\mathrm{D}}^k - 1)^{1/\rho^k} \simeq 1$, the non-linearities disappear and we are able to compute the equilibrium vector of $\{\pi_{\mathrm{D}}^k\}$ from (14) and (16). For sector k we have:

$$\text{(17)} \qquad \pi_{\mathrm{D}}^k \simeq \frac{1}{\varepsilon^k}\left[\frac{1-\phi\sum_z(1-\beta^z)\lambda^z}{1-\phi\sum_z(1-\beta^z)\lambda^z-\phi\lambda^k\sum_z\beta^z\alpha^z/\alpha^k}\right] \geq \frac{1}{\varepsilon^k}$$

Clearly, we retrieve (12) if $\phi = 0$ [16]. Through this approximation, we see that the equilibrium unemployment rate is a function of union power and of relative demand (α^z/α^k)only in the presence of externalities.

The E equilibrium is less efficient than the E* equilibrium in the sense that aggregate output is lower and thus unemployment is higher:

PROPOSITION 2: Unemployment is increasing with the magnitude of externalities.

Proof: From the appendix, $d\pi_{\mathrm{D}}^k/d\phi > 0$. Since the sectorial unemployment rate is a positive function of π_{D}^k (10), unemployment is a positive function of ϕ.

Figure 2 shows clearly the loss of efficiency due to the externalities: this is represented by the distance between E* and E, which is itself a function of the position of the SS curve. If $\phi \to 0$, the SS curve moves closer to E*, the firms are less constrained by demand and unemployment is lower in all sectors. If ϕ increases, SS move north-east and unemployment increase [17].

4 Comparative Statics

The result stating that (non-internalised) externalities increase unemployment is already implicitly present in the existing literature on wage

16. This equation shows also that a strong determinant of π_{D}^k is the elasticity of substitution between goods ε^k. This can explain why, when we look at the π_{D}^k data from business surveys, some sectors experiment systematically higher proportions of demand-constrained firms. Our model interprets this as resulting from low value of ε^k which characterises sectors with high monopoly power.
17. Note that there is a maximum limit for ϕ beyond which the condition $w^k > \phi\,\tilde{w}$ is violated. In that case there is no production and no employment. If all sectors are the same, this limit is $(1-1/\varepsilon)/(1-(1-\beta)/\varepsilon) < 1$.

interdependence. The contribution of our framework is to integrate the role of externalities in a general equilibrium model with bargaining which helps us in analysing the influence of various parameters of the model on unemployment.

4.1. Union Power and Unemployment

PROPOSITION 3: In the presence of externalities, unemployment in all sectors is increasing with union power in any sector.

Proof: From the appendix, $d\pi_{\mathrm{D}}^{k}/d\beta^{z} > 0 \quad \forall k, z.$

Graphically, from the SS curve (14), a rise in β^k has to be compensated by a rise in π_{D}^{k} (at given π_{D}^{j}, $j \neq k$). This shifts the SS curve to the North-East (Figure 3).

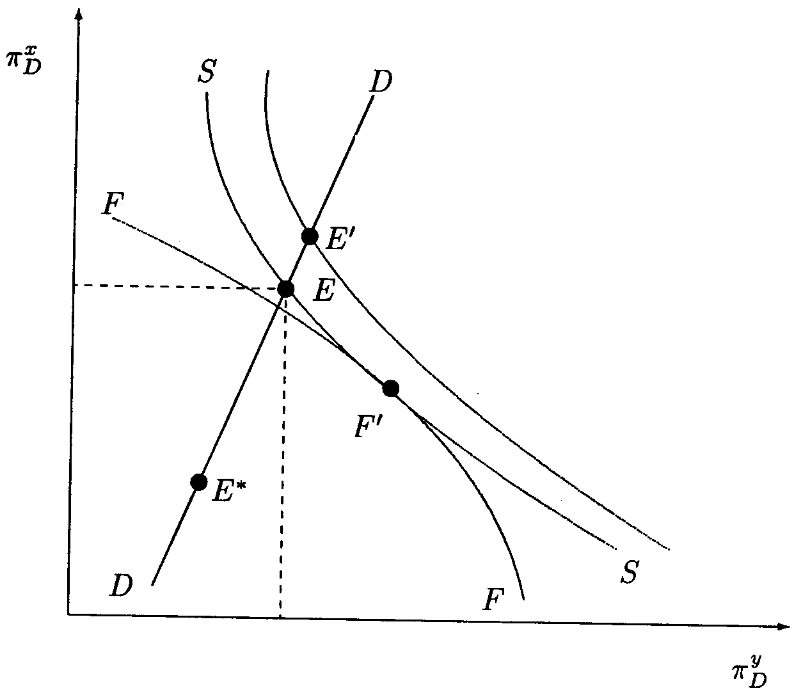

FIGURE 3

Union Power and Unemployment

From the literature we know that decentralised bargaining leads to sub-optimal equilibrium. However, the link between union power and this sub-optimality has not been made explicit. Therefore Proposition 3, although intuitive, is important. It goes also against a usual critique that has been addressed to efficient bargaining models which is that the positive relation between union power and employment implied by efficient contracts is not conform to empirical studies. In our case, we find a *negative* relationship even in the presence of efficient bargaining: this is due to the underestimation by the agents of the effect on employment of a wage increase through the "wage-wage" spiral (9.2)-(13) [18].

Proposition 3 implies also that a rise in union power in one sector "contaminates" the other sectors by increasing their labour share. This "contagion" effect seems very intuitive. However, it is not so clear that we have a systematic contagion of any sectorial shock to the other sectors. For instance, a rise in the productivity of labour a^k in sector k affects neither the SS hypersurface not the DD hypersurfaces. It increases the real wage only in sector k in order to keep the labour share unchanged.

4.2. The Role of Demand

Of course, money m^0 is neutral in this model. Moreover, any change in demand [19] which affects all sectors symmetrically is also neutral: a change in the propensity to consume $(1-\sum_{k=1}^{\mathrm{K}} \alpha^k)$ which does not affect any of the α^x/α^y ratios has no effects on the level of activity. The DD hypersurfaces (16) are only affected by the ratio of the two propensities to consume. If both are reduced by a same factor, nothing happens except an increase in prices.

However, the presence of externalities allows for a role of demand through relative changes:

PROPOSITION 4: Changes in the ratio α^x/α^y modify the allocation of output across sectors in the presence of externalities.

Proof: In the presence of externalities, from the appendix, $\frac{d\pi_{\mathrm{D}}^y}{d\,(\alpha^x/\alpha^y)} > 0$. This is true as long as $\phi \neq 0$: If $\phi \rightarrow 0$, we know that $\pi_{\mathrm{D}}^k \rightarrow 1/\varepsilon^k$ and $\frac{d\pi_{\mathrm{D}}^y}{d\,(\alpha^x/\alpha^y)} \rightarrow 0$.

The economic intuition behind this result is the following: a change in relative demand implies a change in relative prices. As long as there are no externalities, the efficient bargaining outcome implies a proportional change in prices in order to kept the real variables unchanged [20]. In the presence of externalities, this is no longer true. The change in prices will not completely offset the change in relative demand: in the rising-demand sector, the price has to rise, but it will rise less because the wage will be attracted downward by the falling wage in the falling-demand sector. In this case, part of the adjustment will be made through quantities.

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18. Note that, in our model, we have assumed the same ϕ for all sectors. If the ϕ's were different, we need only two non-zero ϕ_i's to obtain Proposition 3.
19. By "change in demand" we mean change in the α^k parameters.
20. As in standard monopolistic competition models, there is only one level of real demand compatible with the sectorial equilibrium; if the real demand were higher, all producers would want to choose a nominal price higher than the others, and that is impossible.

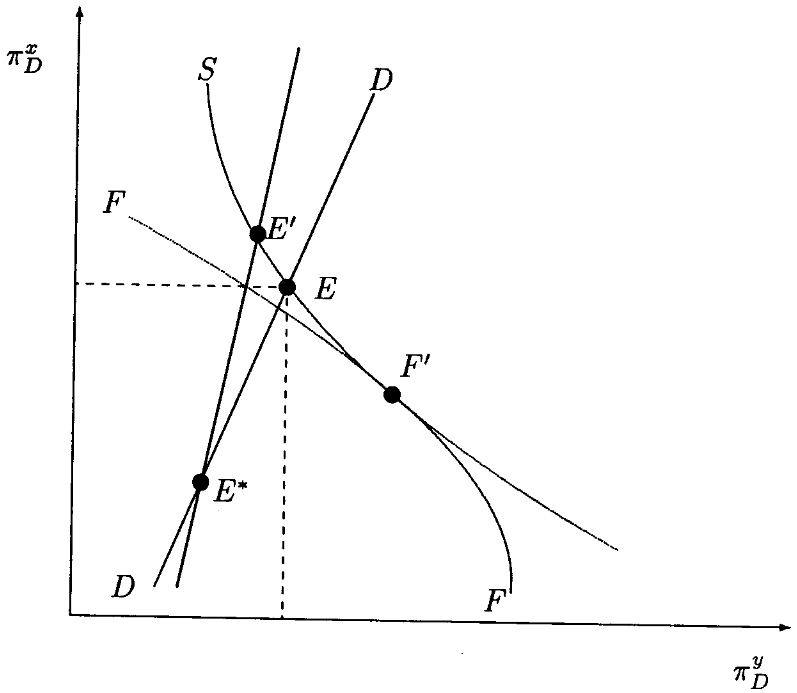

FIGURE 4

Relative Demand and Unemployment

Graphically, the DD hypersurfaces (16) always pass through the E* point defined in (12). Changes in the relative propensity to consume α^x/α^y make the surface rotate with the E* point as a fixed point. This implies that the redistribution of activities from one sector to another is larger if the SS hypersurface is located far from the fixed point. This is very clear for the two sector case on Figure 4, where a drop in α^x has been simulated.

In our model, any given constellation of relative demands selects a specific point on the SS hypersurface but the position of this hypersurface itself is not affected. However, demand shifts may move the economy closer (or farer) from point F′ of Figure 4, which is the best point on the SS hypersurface with respect to unemployment: therefore, *changes in any ratio α^x/α^y affect total employment* [21].

In the two-sector model of DIXON [1988], we find something similar to Proposition 4. In his model, the government can increase employment

21. Similarly, if an economy is not far from point F′, a shock which affects the relative demand may imply employment losses. For instance, a rise in α^y/α^x in a two sector model implies a rise in employment in sector y which may not offset the drop in employment in sector x. This situation may be compared with the drop in world demand of 1975 implying heavy losses in manufacturing employment in Europe together with a steadily increasing employment in services.

by manipulating relative demands through its spendings but its action is constrained to shifts along a kind of supply curve. However, Dixon does not make the link between the *intensity* of wage rivalry and the effectiveness of demand policy.

Concerning the correlation between the labour share of different sectors, we have that a fall in α^x (leading to decreases in the α^x/α^y ratios $\forall y$) increases the labour share in x and decreases the labour share in all other sectors [22]. In the two sector case, the rotation of the DD curve implied by the change in α^x/α^y forces the sector which is hitten by the demand shock to increase its labour share in order to make the firm-union claims compatible with the new π_D [23]. The inverse is true for the sector which is negatively affected.

An important thing to note is the variation of labour share in all sectors in the face of an adverse shock in one sector: if union power increases in one sector, the labour share increases in all sectors. If demand decreases in one sector, the labour share increases in this sector and decreases in all others sectors. Consequently, labour shares tend to be positively correlated in the face of union power shocks and negatively correlated in the face of (sector- specific) demand shocks.

TABLE 2

Evolution of Labour Shares as a Function of Shocks

Shock	LS^k	$LS^j, \quad \forall j \neq k$
$\Delta^+ \beta^k$	+	+
$\Delta^- \alpha^k$	+	–

5 Conclusion

We have provided a multi-sector general equilibrium model with unemployment where wages and prices result from the behaviour of optimising agents (firms, households and unions). The main framework is inspired by previous work on quantity constrained equilibrium, monopolistic

22. This is because $\dfrac{d\pi_D^y}{d(\alpha^x/\alpha^y)} > 0 \,\forall y$ and because the mark-up rule implies a positive relation between the labour share and π_D (from (9.1) $\dfrac{dLS^y}{d\pi_D^y} > 0$.).
23. In other words, using Figure 1, the WW curve is moving along the PP curve whose position is not affected by the shock: a drop in π_D requires a rise in the labour share.

competition and efficient wage bargaining. Externalities arise because the workers evaluate their gain from working by comparing their wage with the average wage in the economy. This model corresponds to the wage determination process described by Keynes where each labour group is restrained by the temporary fixity of the wages of the other labour groups. Consequently, sectorial wages are a weighted sum of the average wage and of the firm's labour productivity. Contrary to the rivalry model of GYLFASON and LINDBECK [1984], the weigths of the two elements in the wage equation are a function of union power: if union power is high, the wage is determined to a large extent by labour productivity in the firm. If it is low, the wage depends much more on the wages of the other sectors.

We have analysed the properties of the macroeconomic equilibrium. This shows very different results from the ones at the firm level, since we incorporate the effect of the "wage-wage" spiral. Unions and firms are not able to internalise the fact that their reference wage is modified by the decision they take; this introduces a source of inefficiency: the outcome of sectorial level bargaining is a non-cooperative equilibrium, where the resulting wages are sub-optimal. This sub-optimality can be characterised by the difference in the unemployment rate with and without externalities. To conclude we show how two well-known concepts of unemployment (structural unemployment, natural rate of unemployment) are present in our framework and stress some micro-founded determinants of these traditional concepts (summarized in Table 3).

TABLE 3

Determinants of the Unemployment Rate

	ρ^k	ε^k	β^k	ϕ	α^x/α^y	$\sum_{k=1}^{K} \alpha^k$
SURE	+	0	0	0	0	0
$UR^*_{\phi=0}$	+	−	0	0	0	0
$UR^*_{\phi>0}$	+	−	+	+	?	0

The basic concept of unemployment that has been defined (Definition 3) is the structural unemployment rate, SURE, which prevails when sectorial demand and sectorial supply are equal. This concept is taken from the literature on quantity rationing models [24]. The SURE defines an unemployment rate which is due to the presence of a segmented labour market together with stochastic shocks affecting the location of each firm in the distribution of demand. These two characteristics imply that, even if sectorial labour supply equals sectorial labour demand, unemployment exists because the distribution of demand is not equal among firms and because unemployed people are not able to find a job in the high-demand firms. Therefore, this unemployment rate is only a function of the variance of the demand distribution function, which is related to the parameter ρ^k.

24. See SNEESSENS [1987], SNEESSENS and DRÈZE [1986] and ARNSPERGER and DE LA CROIX [1992].

Concerning the nature of this structural unemployment, it was initially thought that the underlying demand shocks could model structural problems coming from the fact that e.g., "firms supply iron rods while consumers buy Japanese video sets" (DRÈZE [1987]). This interpretation reflects shifts of activity between countries (or at least between sectors). However, this interpretation is not fully consistent with the conception of demand shocks (the u_i^k) as being deviations from the average. Structural unemployment linked to ρ^k (also called structural mismatch) is more a question of intra-sectoral problems of labour mobility (across regions, skills or ages) in the face of firm-specific uncertainty than a global demand problem. For this reason, an interest of our sectorial model along with the quantity rationing set-up is to take explicitly into account inter-sectoral demand shifts together with structural mismatch.

The second concept of unemployment is the one resulting from the firm-union agreement through an efficient bargain. Despite its cooperative nature, the unemployment rate can be assimilated to the equilibrium rate of unemployment [25] (or the natural rate of unemployment), denoted here UR*, to which we have added uncertainty considerations (the parameter ρ^k).

• In the absence of externalities, the equilibrium unemployment rate $\mathrm{UR}^*_{\phi=0}$ is determined by the intensity of the competition among firms, ε^k: lower competition on the goods market implies a higher unemployment rate. As we have already stressed, it does not depend on union power because households are risk neutral.

• In the presence of externalities, price and wage claims alone (called the SS relation) define a range of feasible equilibrium unemployment rate rather than a single level. Equilibrium unemployment may therefore vary between two limits, $\mathrm{UR}_{\min}$ and $\mathrm{UR}_{\max}$. From the comparative statics, a rise in union power β^k or a rise in the intensity of the externalities ϕ move the SS relation, leading to higher levels of $\mathrm{UR}_{\min}$ and $\mathrm{UR}_{\max}$ [26]. The equilibrium unemployment rate in the presence of externalities, $\mathrm{UR}^*_{\phi>0}$ is a point between $\mathrm{UR}_{\min}$ and $\mathrm{UR}_{\max}$ which is also a function of relative demand parameters α^x/α^y but is affected neither by monetary policy nor by shocks to the average propensity to consume.

25. The basic reference is LAYARD and NICKELL [1986]. See NICKELL [1990] for a survey.
26. The parameters which determine the location of the SS relation also determine these two limits. DIXON [1988] and BHASKAR [1990] call the range implied by wage and price claims the natural range of unemployment.

APPENDIX

Derivation of the SS hypersurface:

Putting (9.2) into (13), we obtain the reference wage as a function of prices:

$$\tilde{w} = \frac{\sum_k \beta^k \lambda^k a^k p^k}{[1 - \sum_k \lambda^k \phi (1 - \beta^k)]}$$

Putting (9.2) into (9.1), we express the prices as a function of the reference wage:

$$p^k = \frac{\phi}{1 - \frac{1}{\varepsilon^k \pi_{\mathrm{D}}^k}} \left(\frac{\tilde{w}}{a^k} \right)$$

Solving now the system of the last two equations, we get:

$$1 = \phi \sum_{k=1}^{\mathrm{K}} \lambda^k \left(1 + \frac{\beta^k}{\varepsilon^k \pi_{\mathrm{D}}^k - 1} \right)$$

which is called the SS hypersurface.

Derivation of the DD hypersurfaces:

Using equation (9.4) we compute the pairwise ratio of demands:

$$\frac{yd^x}{yd^y} = \frac{\alpha^x p^y}{\alpha^y p^x}$$

Putting (9.2) into (9.1), we express the prices as a function of the reference wage:

$$p^k = \frac{\phi}{1 - \frac{1}{\varepsilon^k \pi_{\mathrm{D}}^k}} \left(\frac{\tilde{w}}{a^k} \right)$$

Using this the price ratio can be written as:

$$\frac{p^y}{p^x} = \frac{a^x \left(1 - \frac{1}{\varepsilon^x \pi_{\mathrm{D}}^x} \right)}{a^y \left(1 - \frac{1}{\varepsilon^y \pi_{\mathrm{D}}^y} \right)}$$

We now compute the ratio of the two demands using this new price ratio:

$$\frac{yd^x}{yd^y} = \frac{\alpha^x a^x \left(1 - \frac{1}{\varepsilon^x \pi_{\mathrm{D}}^x} \right)}{\alpha^y a^y \left(1 - \frac{1}{\varepsilon^y \pi_{\mathrm{D}}^y} \right)}$$

From (9.3) and (9.6) we know that

$$yd^k = a^k\, ls^k \left(\frac{1}{\pi_{\mathrm{D}}^k} - 1\right)^{1/\rho^k}$$

which allows to transform the demand ratio into

$$\frac{ls^x \left(\frac{1}{\pi_{\mathrm{D}}^x} - 1\right)^{1/\rho^k}}{ls^y \left(\frac{1}{\pi_{\mathrm{D}}^y} - 1\right)^{1/\rho y}} = \frac{\alpha^x \left(1 - \frac{1}{\varepsilon^x \pi_{\mathrm{D}}^x}\right)}{\alpha^y \left(1 - \frac{1}{\varepsilon^y \pi_{\mathrm{D}}^y}\right)}$$

which is one DD hypersurface linking together a pair of π_{D}^k. We may find K − 1 independent surfaces of this type.

Differentiation of (14) and (16):

Differentiating the SS hypersurface leads to:

$$\phi \sum_k -\lambda^k \frac{\beta^k \varepsilon^k}{(\varepsilon^k \pi_{\mathrm{D}}^k - 1)^2}\, d\pi_{\mathrm{D}}^k + \phi \sum_k \lambda^k \frac{1}{\varepsilon^k \pi_{\mathrm{D}}^k - 1}\, d\beta^k$$

$$+ \sum_{k=1}^{\mathrm{K}} \lambda^k \left(1 + \frac{\beta^k}{\varepsilon^k \pi_{\mathrm{D}}^k - 1}\right) d\phi = 0$$

Differentiating each DD hypersurface leads to:

$$-\frac{1}{(\pi_{\mathrm{D}}^x)^2} \left[\frac{\lambda^x \alpha^y}{\rho^x \alpha^x} \left[1 - \frac{1}{\varepsilon^y \pi_{\mathrm{D}}^y}\right] \left(\frac{1}{\pi_{\mathrm{D}}^x} - 1\right)^{1/\rho^x - 1} - \frac{\lambda^y}{\varepsilon^x} \left(\frac{1}{\pi_{\mathrm{D}}^y} - 1\right)^{1/\rho^y}\right] d\pi_{\mathrm{D}}^x$$

$$+ \frac{1}{(\pi_{\mathrm{D}}^y)^2} \left[\frac{\lambda^y}{\rho^y} \left[1 - \frac{1}{\varepsilon^x \pi_{\mathrm{D}}^x}\right] \left(\frac{1}{\pi_{\mathrm{D}}^y} - 1\right)^{1/\rho^y - 1} - \frac{\lambda^x \alpha^y}{\varepsilon^y \alpha^x} \left(\frac{1}{\pi_{\mathrm{D}}^x} - 1\right)^{1/\rho^x}\right] d\pi_{\mathrm{D}}^y$$

$$+ \lambda^x \left[1 - \frac{1}{\varepsilon^y \pi_{\mathrm{D}}^y}\right] \left(\frac{1}{\pi_{\mathrm{D}}^x} - 1\right)^{1/\rho^x} d(\alpha^y/\alpha^x) = 0 \quad \forall y = 1, \ldots, \mathrm{K}, \quad y \neq x$$

Putting everything together, we find the signs of the variation of π_{D}^x as a function of the variations of the parameters:

$$\frac{d\pi_{\mathrm{D}}^x}{d\beta^x} > 0 \qquad \frac{d\pi_{\mathrm{D}}^x}{d\beta^y} > 0 \qquad \frac{d\pi_{\mathrm{D}}^x}{d\phi} > 0 \qquad \frac{d\pi_{\mathrm{D}}^x}{d(\alpha^y/\alpha^x)} > 0$$

• References

AKERLOF, G. (1969). – "Relative Wages and the Rate of Inflation", *Quarterly Journal of Economics*, 83, pp. 353-374.

ARNSPERGER, C., De la CROIX, D. (1992). – "Bargaining and Equilibrium Unemployment: Narrowing the Gap Between New Keynesian and "Disequilibrium" Theories", *European Journal of Political Economy*, forthcoming.

AUKRUST, O. (1977). – "Inflation in the Open Economy: A Norwegian Model", in L. Krause and W. Salant, eds., *Worldwide inflation*, The Brookings Institution, Washington DC.

BENASSY, J.-P. (1975). – "Neokeynesian Disequilibrium Theory in a Monetary Economy", *Review of Economic Studies*, 42, pp. 502-523.

BENASSY, J.-P. (1982). – *The Economics of Market Disequilibrium*, Academic Press.

BHAKSAR, V. (1990). – "Wage Relativities and the Natural Range of Unemployment", *Economic Journal*, 100, pp. 60-66.

BINMORE, K., RUBINSTEIN, A., WOLINSKI, A. (1986). – "The Nash Bargaining Solution in Economic Modelling", *Rand Journal of Economics*, 17, pp. 176-188.

BLANCHARD, O., KIYOTAKI, N. (1987). – "Monopolistic Competition and the Effects of Aggregate Demand", *American Economic Review*, 77, pp. 647-666.

COOPER, R., JOHN, A. (1988). – "Coordinating Coordination Failures in Keynesian Models", *Quarterly Journal of Economics*, 53, pp. 441-463.

DIXIT, A., STIGLITZ, J. (1977). – "Monopolistic Competition and Optimum Product Diversity", *American Economic Review*, 67, pp. 297-308.

DIXON, H. (1988). – "Unions, Oligopoly and the Natural Range of Employment", *Economic Journal*, 98, pp. 1127-1147.

DRÈZE, J. (1987). – "Underemployment Equilibria: From Theory to Econometrics and Policy", *European Economic Review*, 31, pp. 9-34.

GYLFASON, T., LINDBECK, A. (1984). – "Competing Wage Claims, Cost Inflation, and Capacity Utilization", *European Economic Review*, 24, pp. 1-21.

JACOBS, J., JANSSEN, M. (1990). – "Coordinating Unions, Wages and Employment", *De Economist*, pp. 321-339.

JACOBSEN, H., SCHULTZ, C. (1990). – "A General Equilibrium Macro Model with Wage Bargaining", *Scandinavian Journal of Economics*, 92, pp. 379-398.

KEYNES, J. M. (1936). – "The General Theory of Employment, Interest and Money", Macmillan, London.

LAMBERT , J.-P. (1988). – "Disequilibrium Macroeconomic Models", Cambridge University Press.

LAYARD, R., NICKELL, S. (1987). – "The Labour Market", in R. Dornbusch and R. Layard, (eds.), *The Performance of the British Economy*, Clarendon Press, Oxford.

LICANDRO, O. (1991). – "Monopolistic Competition and Aggregation over Micromarkets in Disequilibrium", *mimeo*, Carlos III, Madrid.

MACDONALD, I., SOLOW, R. (1981). – "Wage Bargaining and Employment", *American Economic Review*, 71, pp. 898-908.

MALINVAUD, E. (1977). – "The Theory of Unemployment Reconsidered", Basil Blackwell.

MITCHELL, D. B. J. (1980). – "Unions, Wages, and Inflation", Brookings Institution, Washington DC.

NICKELL, S. (1990). – "Unemployment: A Survey", *Economic Journal*, 100, pp. 391-439.

SNEESSENS, H. (1987). – "Investment and the Inflation-Unemployment Trade off in a Macroeconomic Rationing Model with Monopolistic Competition", *European Economic Review*, 31, pp. 781-815.

SNEESSENS, H., DRÈZE, J. (1986). – "A Discussion of Belgian Unemployment, Combining Traditional Concepts and Disequilibrium Econometrics", *Economica*, 53, S89-S119.