The Dynamics of Unemployment, Capacity Constraints and Demand Shortages

David de la CROIX*, Jean-François FAGNART*

ABSTRACT. – This paper presents an intertemporal model in which the agents may face different quantity constraints resulting from technological rigidities, microeconomic uncertainty and market segmentation. Its main purpose is to derive the path of the unemployment rate, the rate of capacity utilisation and the proportion of firms reporting demand constraints from a union-firm optimisation programme. The model is calibrated and the impact of different unfavourable shocks is simulated using a new algorithm for solving dynamic non-linear models. The present exercice underlines the need for caution in interpreting business surveys indicators.

Dynamiques du chômage, des capacités et des contraintes de débouchés

RÉSUMÉ. – Ce papier présente un modèle intertemporel où les agents peuvent faire face à différentes contraintes quantitatives résultant de rigidités technologiques, d'incertitude microéconomique et de segmentation des marchés. Son objectif est de dériver le sentier du taux de chômage, du degré d'utilisation des capacités et de la proportion d'entreprises reportant des contraintes de débouchés à partir du comportement optimisateur des entreprises et syndicats. Le modèle est calibré et l'impact de différents chocs défavorables est simulé en utilisant un nouvel algorithme de résolution des modèles dynamiques non-linéaires. Cet exercice souligne qu'il y a lieu d'interpréter avec prudence les indicateurs des enquètes de conjoncture.

---

* D. de la CROIX: National Fund for Scientific Research (Belgium) and University of Louvain, IRES; J.-F. FAGNART: University of Louvain, IRES. We thank the participants to the conference on the macroeconomics of imperfect competition held in Paris, January 94, and particularly Alan Manning for his comments. We are grateful to Jean-Pascal Benassy, Pierre Malgrange and an anonymous referee for their discussions and suggestions on an earlier draft. The financial support of the "Projet d'Action Concertée" 93/98-162 of the ministery of scientific research is gratefully aknowledged.

1 Introduction

Several stylised facts suggest that the economic agents in market economies may experience quantity constraints. The persistence of high unemployment rates in European countries is certainly the most striking one. The business surveys also shed light on other quantity constraints which are possibly related to unemployment. In particular, it is frequently reported that a lot of firms sell a level of output lower than what their installed productive capacity would allow to produce. Like the available labour force at a macroeconomic level, the productive equipments are persistently underused (see Figure 1 and 2 for Belgium). It is noticeable on these Figures, that, after each of the oilshocks, the rise in the unemployment rate U was accompanied by a drop in the rate of utilisation of productive capacities D. The ulterior rise in D without a major reduction in unemployment suggests an increasing gap between the existing productive capacities and the work-stations that would be necessary to reach full-employment (see e.g. Bean [1989] and Burda [1986]).

Demand shortages on the goods market are another stylised fact accompanying the underutilisation of the productive capacities: business surveys point out persistently that a lot of firms put forward an insufficient demand as the explanation of the fact they do not produce more (see Figure 3)). Strikingly, this proportion of demand constrained firms (hereafter Π_d) rose strongly after the first oilshock of 1973 and did not diminish before the counteroilshock of 1986.

Matching problems in the labour market are another well-known feature of our economies. As Figure 4 shows (for Belgium), a significant number of firms encounter problems when recruiting and seem unable to hire as many workers as they wish even when the unemployment rate is very high. In particular, during the growth phase following the counter-oilshock, the proportion of firms in such a situation has rapidly increased although the unemployment rate was still at a high level.

This paper aims at building a dynamic model in which these facts are understandable under the assumption that the agents behaviour is consistent with their individual optimising programme. The quantity rationing models (hereafter QRM) offer a natural framework for analysing the behaviour of the stylised facts mentioned above. Using an explicit aggregation over heterogeneous micromarkets, Sneessens [1987], Licandro [1992] and de la Croix [1993] show that technological rigidities and market segmentation may account for the underutilisation of the productive factors (when wages and prices are imperfectly competitive). Our paper analyses the dynamics of the quantity constraint indicators (U, D and Π_d) in an intertemporal QRM model with imperfect competition.

As the optimal formation of prices and wages in the model is only responsible for real rigidities, there is no Keynesian multiplier in the model: with a fixed labour supply, an anticipated demand policy will only have price effects. Our concept of unemployment is thus close to an equilibrium unemployment rate à la Layard et al. [1991] in an intertemporal context. The

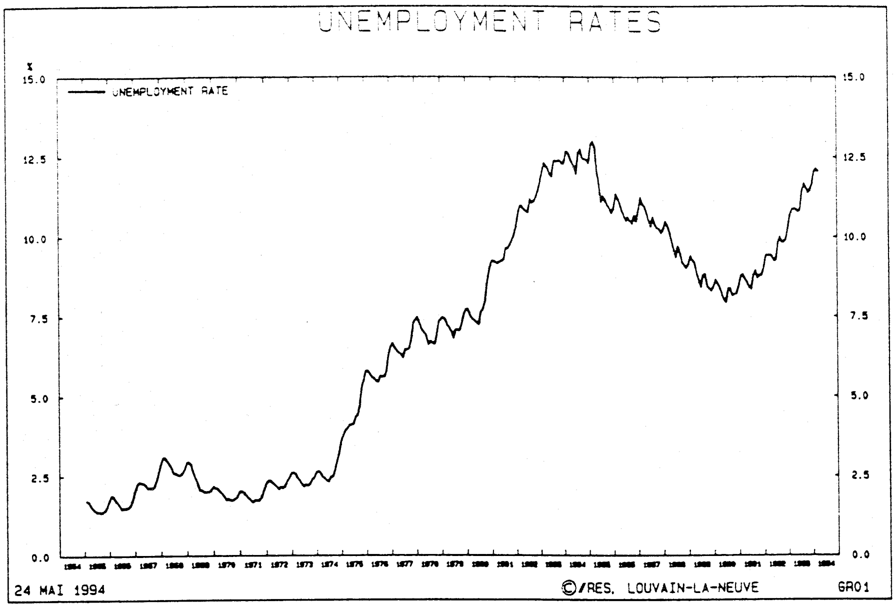

FIGURE 1

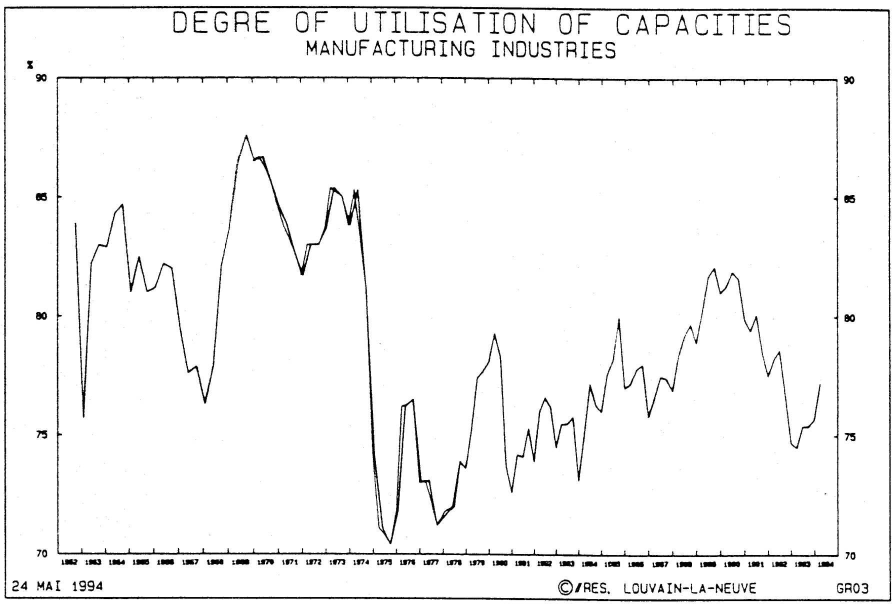

FIGURE 2

U, D and Π_d dynamics are accounted for on the basis of real perturbations only. We do thus neglect short run phenomena which could influence the behaviour of these variables and focus on the role of structural factors.

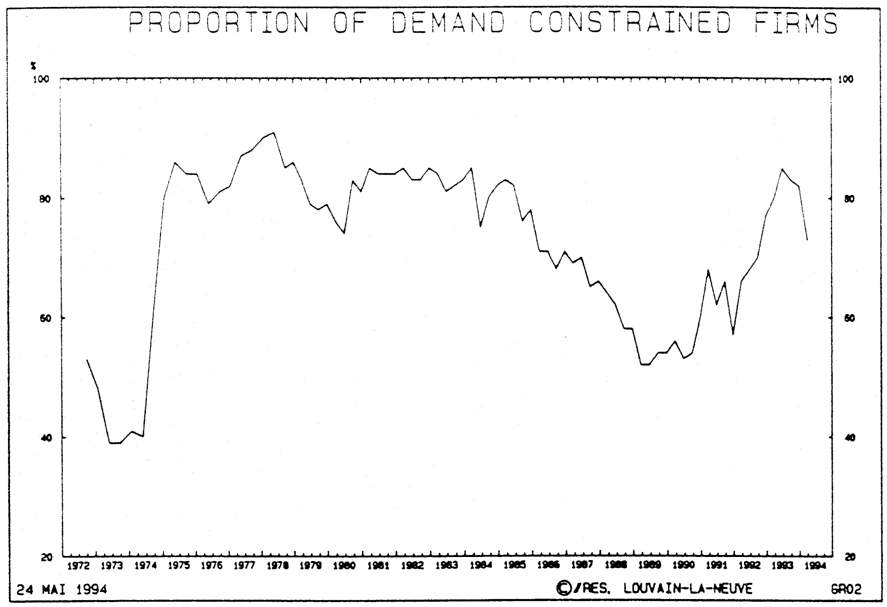

FIGURE 3

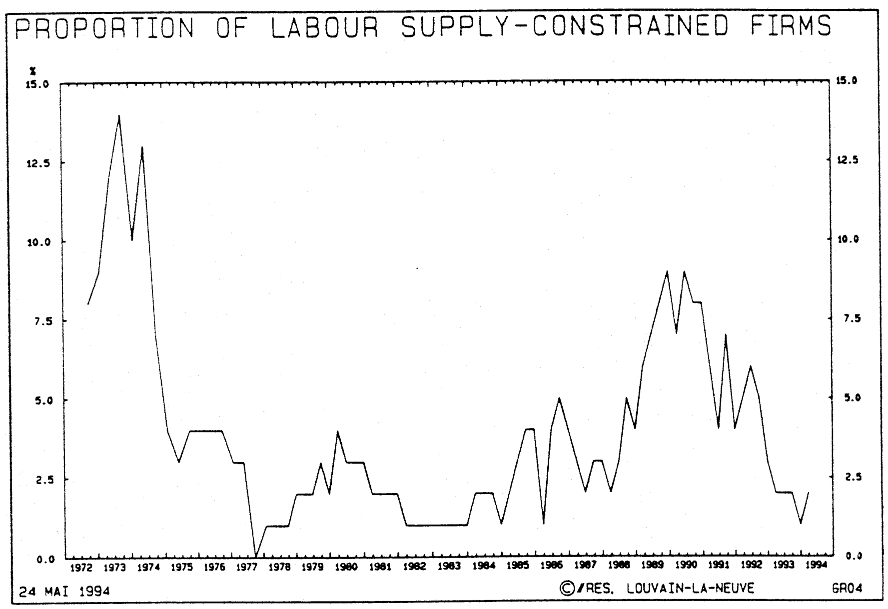

FIGURE 4

This gives us the opportunity to show that the meaning of some indicators given in business surveys is ambiguous. In particular, a high proportion of firms reporting demand shortages does not reflect in itself any sensitivity of output to the level of demand. [1]

The paper is organised as follows: the theoretical model is presented in the second section. After having derived the microeconomic first-order conditions, we present the aggregate dynamic model and analyse its steady state. The third section presents numerical simulations describing the response of the model to unexpected changes in the environment analogous to those experienced by European economies during the last twenty years.

2 The Theoretical Model

The economy consists of an infinite number of firms, each of measure zero, indexed on the interval $[0, 1]$ and producing differentiated products. Firm level variables are denoted by lowercase letters. Aggregate variables are denoted by uppercase letters.

2.1. Technology

As mentioned in the introduction, business surveys suggest that the existing capital stock or the labour supply may put a limit to the productive capacity of the firm. Clearly, these two facts could not be accounted for without the presence of short run technical rigidities. Two types of rigidities are assumed here. First, building new production units takes one unit of time and the investment decisions at t increases the productive capacity at $t+1$. Secondly, the firm uses a putty-clay technology à la MALINVAUD [1987]. Although it is possible to substitute capital for labour when deciding to install new productive equipments, such a flexibilily does not exist anymore once the equipments are installed. This implies that the firm chooses simultaneously a capacity level and the labour/capital ratio. In particular, we assume here that at each time t, the firm designs its new productive installations Δy_t^c by combining investment goods i_t and labour according to a Cobb-Douglas technology with constant return to scale:

$$\Delta y_t^c = {n_t}^{\alpha} \, {i_t}^{1-\alpha} \tag{1}$$

where n_t represents the additional work-stations. (1) can easily be recast as a function of the labour-capital ratio x_t (i.e. n_t/i_t):

$$\Delta y_t^c = {x_t}^{\alpha} \, i_t \tag{2}$$

1. As far as this last point is concerned, a more detailed discussion on the respective role of nominal and real rigidities in a QRM model with aggregation can be found in Sneessens (1992).

where ${x_t}^{\alpha}$ is the technical coefficient of the new equipments (accordingly, ${x_t}^{\alpha-1}$ is the technical coefficient of labour associated to the new work-stations). The total number of production units in the firm y_t^c at the end of period t is therefore

$$(3) \qquad y_t^c \;=\; y_{t-1}^c\,(1-\delta) \;+\; \Delta y_t^c$$

where δ is the average depreciation rate. This formulation amounts to assuming that the firm uses all its production units at a mean rate of utilisation.

Given the one-period time-to-build assumption, the new production units Δy_t^c become productive only in $t+1$. At time t, the output y_t^p corresponding to the full employment of the available production units y_{t-1}^c may thus be defined as

$$(4) \qquad y_t^p \;=\; y_{t-1}^c\;\tau\;\nu_{1t}$$

where τ is a productivity parameter which can be interpreted as the total factor productivity. This full capacity output y_t^p is partially influenced by a random component ν_{1t} (a i.i.d shock of unitary mean) reflecting different technical hazards like the rate of breakdowns around a unitary mean.

At t, the total number of work-stations in the firm is y_t^c/a_t (a_t being the average technical coefficient of labour) and is equal to

$$(5) \qquad \frac{y_t^c}{a_t} \;=\; \frac{y_{t-1}^c}{a_{t-1}}\,(1-\delta) \;+\; x_t\;i_t$$

where $x_t\;i_t$ represents the new work-stations. This identity allows us to compute the evolution of the average technical productivity of labour a_t. Assuming a firm-specific labour market, the firm cannot hire more workers than the number of workers in its segment of the labour market l_t^s. The output corresponding to the full employment of the available workforce is then

$$(6) \qquad y_t^s \;=\; \tau\;a_{t-1}\;l_t^s\;\nu_{2t}$$

where ν_{2t} is a i.i.d random shock of mean one representing the uncertainty on the size of the segment l^s. Note that the relevant technical coefficient is a_{t-1} since the new work-stations at t are only productive at $t+1$.

2.2. **Demand and Output**

At time t, the firm faces a demand y_t^d equal to a share of total real demand $\bar{C}_t/P_t$. This share depends on the ratio of the firm's price p_t to the general price index P_t and on a specific random shock η_t, reflecting the uncertainty on the firm's market share.

$$(7) \qquad y_t^d = \left(\frac{p_t}{P_t}\right)^{-\epsilon}\,\frac{\bar{C}_t}{P_t}\;\eta_t, \qquad \epsilon > 1$$

The random shock η_t is i.i.d. of mean 1. This demand function is implicitly derived from a Dixit-Stiglitz utility function with a constant elasticity of substitution between the different goods (ϵ). In this Dixit-Stiglitz framework, the random shock can be interpreted as a preference parameter unknown by the firm and weighting the consumption of the particular good i in the CES consumption index (see for instance LICANDRO [1991]).

We suppose that the customers of the firm do not send their purchasing order before knowing perfectly the price they will have to pay for the purchased products. Consequently, each firm has to announce its price p_t in advance (i.e. before knowing its demand and the technological shocks prevailing during the production process). Given the technical rigidities (putty-clay production function, one period time to build, fixed labour supply and segmented labour market), the firm can thus be constrained after the realisation of the shocks either by demand or by its productive capacity or by labour availability. Output is then the minimum of the three potential constraints:

$$y_t = \min(y_t^d, y_t^s, y_t^p), \tag{8}$$

The effective employment level is given by

$$l_t = \frac{y_t}{\tau \, a_{t-1}} \tag{9}$$

As some decisions are taken under uncertainty, the firm has to form an expectation over its future output. Under the assumption that η_t, ν_{1t} and ν_{2t} are i.i.d. and lognormally distributed with a variance-covariance matrix with identical variances and identical covariances, [2] the expected output is an appealing CES function of the three expected constraints (see SNEESSENS [1983]). At time s,

$$E_s(y_t) = \left[\left(E_s(y_t^d)\right)^{-\rho} + (E_s(y_t^s))^{-\rho} + (E_s(y_t^p))^{-\rho}\right]^{-1/\rho} \tag{10}$$

where ρ is a function of the variances and covariances of the shocks. [3] The CES expression allows to compute easily the elasticities of the expected output with respect to demand (π_{dt}), productive capacity (π_{p_t}) and full employment output (π_{lt})

$$\begin{gathered} \pi_{dt} \;=\; \left[\frac{E_s(y_t)}{E_s(y_t^d)}\right]^{\rho}, \\ \pi_{p_t} \;=\; \left[\frac{E_s(y_t)}{E_s(y_t^p)}\right]^{\rho} \quad and \quad \pi_{lt} \;=\; \left[\frac{E_s(y_t)}{E_s(y_t^s)}\right]^{\rho} \end{gathered} \tag{11}$$

2. This particular hypothesis on the variance-covariance matrix is made for analytical simplicity. More general cases are possible but lead to less tractable functional forms for expected output. See e.g. Entorf et al. (1991).
3. As the random shocks have a unitary mean, the expectations of the different constraints are

$$E_s(y_t^p) \;= \tau\, y_{t-1}^c, \; E_s(y_t^s) \;=\; a_{t-1}\, l_t^s \qquad and \qquad E_s(y_t^d) = \left(\frac{p_t}{P_t}\right)^{-\epsilon} \frac{\bar{C}_t}{P_t}.$$

It is easy to verify that the three elasticities add up to one. As it is shown is SNEESSENS [1983], these elasticities can also be interpreted as the probabilities of the firm being constrained by demand, capacities or labour.

2.3. Union-Firm Contract

The l_t^s households who supply their workforce to the firm are grouped in a union specific to the firm. The union's utility function is defined over employment l_t and over the real wage w_t/P_t:

$$u\left(l_t, \frac{w_t}{P_t}\right) = l_t \;\; \frac{1}{\nu} \;\; \left(\frac{w_t}{P_t} - \omega\right)^{\nu} \tag{12}$$

ω is to interpret as the pecuniary evaluation of the disutility of work and is assumed constant. ν measures the aversion of the workers with respect to changes in the real wage net of the disutility of work.

In a dynamic model like ours, the agents have an incentive to conclude a long-term agreement bearing here on wages and employment. An analogy can be made with the efficient bargaining models in a dynamic framework. ESPINOZA and RHEE [1989] and STRAND [1989] analyse the conditions under which the efficient outcome is time consistent in a repeated game. They show that when the agents are sufficiently concerned by their future welfare, the welfare effect of the future losses caused by any deviation from the contract [4] would offset the immediate benefit associated to the deviation. Therefore, sufficiently high discount rates makes the cooperative outcome consistent.

As the agreement takes place before knowing the realisation of demand and productivity shocks, a contract about effective employment would not exclude a possibility of negative profits for the firm if it experienced a defavorable micro-shock. We suppose therefore the bargaining bears on wages and *expected employment*. [5]

In our framework, an *ex ante* agreement on wages and expected employment determines simultaneously the price p_t, the investment i_t and the technology x_t on top of w_t. In other words, even though the effective employment level is optimally fixed by the firm *ex post*, the *ex ante* decisions about prices and technology are more favourable to effective employment than what they would have been if the agreement had only born on wages. As furthermore the two agents have the same discount rate, a sequence of repeated contracts over p_t, w_t, x_t and i_t at each t is equivalent to a contract bearing on the complete path of these variables. This equivalence relies on the assumption that the idiosyncratic random shocks are i.i.d. The objective function to maximise is the weighted sum of all future utilities and profits:

$$\max_{w_t, p_t, i_t, x_t} E_s \left\{ \sum_{t=s}^{\infty} R_t \left[\beta\, u\left(l_t, \frac{w_t}{P_t}\right) + \frac{1}{P_t} \left(p_t y_t - w_t l_t - P_t^i i_t\right) \right] \right\} \tag{13}$$

$$\text{such that for all } t \text{ the equalities (3) and (6) hold and } i_t \geq 0$$

---

4. punishment and/or return to the non-cooperative outcome.
5. At the firm level, expected employment is nothing else than the number of work-stations in the firm times their probability of utilisation.

where l_t, i_t, y_t and x_t have been defined before. β represents the union's relative power in the process and P_t^i is the price of investment goods.

R_t is a discounting factor given by

$$R_t = \prod_{j=s}^{t} \left(\frac{1}{1+r_j} \right) \tag{14}$$

where r_j is the union's time-preference parameter and the firm's discount rate.

As the uncertainty only bears on output and employment, the expectation operator may be distributed inside the objective function. Replacing the utility function by its expression in (12), associating Lagrange's multipliers λ_t and μ_t to the technological constraints (3) and (5), and imposing the positivity of investment, one could recast the problem in the following way

$$\begin{aligned}
\max_{w_t,p_t,i_t,x_t} \; & \sum_{t=s}^{\infty} \frac{R_t}{P_t} \left[\left(p_t - \frac{w_t}{\tau a_{t-1}} + \frac{\beta}{\nu} \left(\frac{w_t}{P_t} - \omega \right) \frac{p_t}{\tau a_{t-1}} \right) E_s(y_t) - P_t^i \, i_t \right] \\
& + \sum_{t=s}^{\infty} R_t \, \gamma_t \, i_t + \sum_{t=s}^{\infty} R_t \\
& \times \left[\lambda_t \big(y_t^c - y_{t-1}^c (1-\delta) - (x_t)^\alpha \, i_t \big) + \mu_t \left(\frac{y_t^c}{a_t} - \frac{y_{t-1}^c}{a_{t-1}} (1-\delta) - x_t \, i_t \right) \right]
\end{aligned}$$

2.4. **Optimality Conditions**

The first order condition for price is

$$p_t = \left[1 - \frac{1}{\epsilon \pi_{dt}} \right]^{-1} \left[w_t - \frac{\beta}{\nu} \left(\frac{w_t}{P_t} - \omega \right)^{\nu} P_t \right] \frac{1}{\tau \, a_{t-1}} \tag{15}$$

According to this price equation, the price is a mark-up over the "social" marginal cost of labour (i.e., the shadow price of labour). The mark-up rate depends positively on the firm market power and negatively on union power. Moreover, it decreases with the endogenous probability of facing a demand constraint. Since, in (15), the gap between the social marginal cost of labour and the private cost w_t is a function of union power β, the mark-up rate of prices over money wages is a negative function of β. Hereafter, the real social marginal revenue per unit of output is written as

$$m_t = \frac{1}{P_t} \left(p_t - \left[w_t - \frac{\beta}{\nu} \left(\frac{w_t}{P_t} - \omega \right)^{\nu} P_t \right] \frac{1}{\tau \, a_{t-1}} \right) \tag{16}$$

The first order equation for wages is:

$$\frac{w_t}{p_t} = \beta^{1/(1-\nu)} + \omega \tag{17}$$

Real wages are positively related to union power.

The Slatter's condition for the positivity of investment is

$$\gamma_t\, i_t \;=\; 0 \tag{18}$$

The first order condition on y_t^c can be rewritten [6] as

$$\begin{aligned}
(19)\quad & \underbrace{\frac{1+r_{t+1}}{\tau\, x_t^{\alpha}}\;\frac{P_t^i}{P_t}\;-\;\frac{1-\delta}{\tau\, x_{t+1}^{\alpha}}\;\frac{P_{t+1}^i}{P_{t+1}} \;=\; m_{t+1}\,(\pi_{p_{t+1}})^{\frac{\rho+1}{\rho}}}_{(A)} \\
& + (1+r_{t+1})\;\frac{\mu_t}{\tau}\left(x_{t+1}^{1-\alpha}-x_t^{1-\alpha}\right) \\
& + \left[\underbrace{m_{t+1}\,(\pi_{l_{t+1}})^{\frac{\rho+1}{\rho}}\;\tau\, a_t\, ls_{t+1}}_{(B)} + \underbrace{\left(\frac{p_{t+1}}{P_{t+1}}-m_{t+1}\right)E_s(y_{t+1})}_{(C)}\right] \\
& \times\;\frac{1-a_t\, x_{t+1}^{1-\alpha}}{\tau\, y_t^c}
\end{aligned}$$

In order to interpret this optimality condition, let us first suppose that the firm does not change its technical coefficients ($x_{t+1}^{1-\alpha} = x_t^{1-\alpha} = a_t$). In this case, the optimality condition is reduced to the condition (A). Its left hand side is the net cost of installing one unit of productive capacity in t (*i.e.* purchasing $1/(\tau\, x_t^{\alpha})$ investment goods at price P_t^i), taking into account that this unit undergoes some depreciation δ during its use in $t+1$. The right hand side is simply the expected marginal revenue generated by this capacity unit in $t+1$ i.e. the extra marginal revenue m_{t+1} times the probability of using this new capacity unit.

When the labour intensity of the equipment is modified, investing in new capacity units allows to transform the technical coefficients and the optimality condition is much less simple. A change in the technical productivity of labour modifies both the level of production corresponding to the full employment output and the average cost of a work-station. In particular, an increase in a_t (induced by a lower x_t) raises the profit of the firm if the employment constraint becomes binding but increases the wage bill paid by the firm at given ouput. In expected terms, the first effect is measured by the expression (B) giving the extra marginal revenue multiplied by the probability of a labour constraint. The induced economy in wage bill (*i.e.* the expected employment times the wage rate) is measured by the expression (C). Of course, the more the labour capital labour ratio is changed (in the condition (19) the more $a_t\; x_t^{1-\alpha}$ is far from one), the larger these effects will be.

---

6. For simplicity we suppose that the investment i_t is strictly positive so that the positivity constraint is not binding and $\gamma_t = 0$. The complete expression is given in the appendix.

The optimality condition for the labour-capital ratio is

$$\tau\, m_{t+1} \left\{ \underbrace{\alpha(\pi_{p_{t+1}})^{\frac{\rho+1}{\rho}}}_{(A')} - \underbrace{(\pi_{l_{t+1}})^{\frac{\rho+1}{\rho}} \left(a_t x_{t+1}^{1-\alpha} - \alpha\right) \frac{l_{t+1}^s a_t}{y_t^c}}_{(B')} \right\} \tag{20}$$

$$= \underbrace{\left(\frac{p_{t+1}}{P_{t+1}} - m_{t+1}\right) \frac{E_s(y_{t+1})}{y_t^c} \left(a_t x_{t+1}^{1-\alpha} - \alpha\right)}_{(C')}$$

$$- (1+r_t)\mu_t \left(x_{t+1}^{1-\alpha} - x_t^{1-\alpha}\right)$$

The labour capital ratio has to be fixed in such a way that the expected marginal revenue of the increase in this ratio (first line) is equal to the increase in the wage cost linked to the corresponding decrease in the labour productivity C'. The expected marginal revenue is the difference between the expected revenue generated by the higher productivity of investment goods in the case of a capacity constraint A' and a lower one due to the lower productivity of labour in the case of a labour constraint B'.

Finally, the multiplier μ_t must satisfy the constraint

$$\frac{y_t^c}{a_t}\left((1+r_t)\,\mu_t \;-\; (1-\delta)\,\mu_{t+1}\right) \tag{21}$$

$$= m_{t+1}\,(\pi_{l_{t+1}})^{\frac{\rho+1}{\rho}} \tau\, a_t l_{t+1}^s + \left(\frac{p_{t+1}}{P_{t+1}} - m_{t+1}\right) E_s(y_{t+1})$$

μ_t is thus a measure of the discounted effect of an increase in the productivity of labour in t on future profits.

2.5. **Aggregation**

Since all the union-firm couples are in a symmetric situation at the time of contracting, their optimal decisions are the same and the equilibrium at each time t is a symmetric one. The aggregate price level is then

$$P_t = p_t = P_t^i$$

Furthermore, assuming an infinite number of firms allows to link interestingly the microeconomic problem to the macroeconomic aggregates (see SNEESSENS [1987] and Arnsperger and de la CROIX [1993]). In particular, aggregate output (resp. employment) takes the same functional form as firm's expected output (resp. employment), and may thus be written as :

$$Y_t = \left[\left(Y_t^d\right)^{-\rho} + \left(\tau\, Y_{t-1}^c\right)^{-\rho} + \left(\tau\, A_{t-1}\, L_t^s\right)^{-\rho}\right]^{-1/\rho} \quad \text{with } \; Y_t^d = \frac{\bar{C}_t}{P_t} \tag{22}$$

As the different firms have experienced different micro-shocks, [7] each of the three possible constraints is binding *ex post* for a subset of firms. More precisely, the proportions of firms in the three "regimes", Π_d, Π_p or Π_l, are equal to the *ex ante* individual elasticities π_d, π_p or π_l. Furthermore, the unemployment rate, $U_t = 1 - L_t/L_t^s$ and the macroeconomic degree of capacity utilisation $D_t = Y_t/\tau Y_{t-1}^c$ may be recast in terms of these macroeconomic proportions:

$$D_t = (\Pi_{p_t})^{1/\rho} \tag{23}$$

$$U_t = 1 - (\Pi_{l t})^{1/\rho} = 1 - (1 - \Pi_{d_t} - \Pi_{p_t})^{1/\rho} \tag{24}$$

The complete aggregate model is detailed in the Appendix. As there is no aggregate uncertainty, the expectation operator can be removed.

2.6. **The Steady State**

In a stationary state, technical coefficients have reached their optimal value: the technical coefficients are respectively $A = X^{\alpha-1}$ for labour and X^{α} for investment goods. There is furthermore no net investment. Without the short run dynamics due to the accumulation process and the technological change, the aggregate model may be reduced to the following equations and solved for $\{Y, D, \Pi_d, W/P, X, Y^c, I\}$:

$$Y = \left[\frac{1 - \Pi_d}{(\tau Y^c)^{-\rho} + (\tau L^s X^{\alpha-1})^{-\rho}}\right]^{1/\rho} \tag{$s1$}$$

$$D = \frac{Y}{\tau Y^c} \tag{$s2$}$$

$$1 - \frac{1}{\epsilon \Pi_d} = \frac{1}{\tau X^{\alpha-1}} \left(\frac{W}{P} - \frac{\beta}{\nu}\left(\frac{W}{P} - \omega\right)^{\nu}\right) \tag{$s3$}$$

$$\frac{W}{P} = \beta^{1/(1-\nu)} + \omega \tag{$s4$}$$

$$\frac{r + \delta}{D} = D^{\rho} \frac{X^{\alpha} \tau}{\epsilon \Pi_d} \tag{$s5$}$$

$$\frac{D^{\rho}}{\Pi_d} = (1-\alpha)(\epsilon - 1) \tag{$s6$}$$

$$I = \delta \frac{Y^c}{X^{\alpha}} \tag{$s7$}$$

Equations $(s1)$ and $(s2)$ are definitions. $(s3)$ and $(s4)$ form the price-wage block of the model. (s5) is the steady state condition on the productive

---

7. It is worth repeating that the fact that the firms differ *ex post* at each period only influences the contemporary output and employment but does not affect the decisions of the union-firm couples in the subsequent periods (this is due to the assumption of i.i.d. shocks).

capacities: As each unit of equipment is used at a rate D, the effective user cost is $(r+\delta)/D$. This marginal cost must be equal to the marginal revenue, i.e., the physical productivity of equipment $\tau\ X^{\alpha}$ times the margin per unit of output $1/(\epsilon \Pi_d)$ times the elasticity of output with respect to capacities D^{ρ}. The long run level of productive capacities determines the stationary value of investment $(s7)$. $(s6)$ is obtained by symplifying the long run optimality condition on the labour capital ratio W. At α, ϵ and ρ constant, D and Π_d are positively correlated in the long-run. Any shock increasing the unit cost of equipment leads the firms to increase their capacity utilisation and their prices implying higher D and Π^d.

The price and wage formation defines the steady state proportion of firms facing demand shortages:

$$(s8) \qquad \Pi_d \ = \ \frac{1}{\epsilon}\left[1 \ - \ \frac{\omega \ + \ \beta^{1/(1-\nu)}\ (1-1/\nu)}{\tau\ X^{\alpha-1}}\right]^{-1}$$

Note that the relative importance of this demand constraint cannot be interpreted in a Keynesian fashion. Here, firms and unions voluntarily accept the possibility of a demand constraint: given the prevailing uncertainty, the firm sets price optimally, *i.e.*, chooses an optimal probability to face an insufficient demand. An *ex post* proportion of firms Π_d reporting such a shortage is simply the macroeconomic implication of these microeconomic choices. This does not mean at all that an anticipated boom in demand would produce a rise in output. [8] Rather, firms would change their prices in order to keep the same optimal π_d, implying the same macroeconomic Π_d. Stated differently, a (permanent) shock that would increase the proportion Π_d would not make the output more sensitive to aggregate demand [9] This points out the ambiguity of such a business cycle indicator in a medium run perspective.

The core of the steady state model is given by $(s5)$, $(s6)$ and $(s8)$ which form a system of three nonlinear equations with three unknowns Π_d, D and X. This system can still be reduced by isolating $X^{\alpha-1}$ at the left hand side of both $(s5)$ and $(s8)$. Equating the two equations and using $(s6)$, the steady state D is solution of

$$(s9) \qquad D\left[1-\frac{(1-\alpha)\ (1-1/\epsilon)}{D^{\rho}}\right]^{\frac{\alpha}{1-\alpha}} = \ \frac{r+\delta}{\tau}\ \frac{\epsilon(1-\alpha)}{\epsilon-1}\left[\frac{\omega+\beta^{1/(1-\nu)}\ (1-1/\nu)}{\tau}\right]^{\frac{\alpha}{1-\alpha}}$$

8. Note however that an unanticipated boom would have a temporary effect on output since the firms would not have changed their price accordingly. In this case the larger Π_d at the time of the shock, the greater the effect on output would be. Furthermore, since some firms would then produce in the vertical part of their marginal cost function, the macroeconomic mark-up rate would be contracyclical in the case of an unanticipated macro-shock.
9. Monopolistic competition models à la Benassy could be seen as an extreme case of this: all the firms would be willing to produce more at the prevailing prices if demand was higher (in our framework this means that $\Pi_d = 1$). However, in the absence of nominal rigidities an increase in nominal demand would simply produce a rise in prices instead of a rise in output.

This equation determines a unique positive value of D as a function of the parameters of the model and allows to compute the steady state Π_d using $(s6)$. U is then obtained as

$$U = 1 - (1 - \Pi_d\,(\epsilon(1-\alpha)+\alpha))^{1/\rho}$$

At ρ, α and ϵ constant, the long-run unemployment is positively related to the proportion of firms reporting a demand shortage. The labour-capital ratio is

$$X = \frac{(r+\delta)/D}{\omega + \beta^{1/(1-\nu)}(1-1/\nu)}\left[\frac{\epsilon}{(\epsilon-1)(1-\alpha)} - D^{-\rho}\right]$$

The labour-capital ratio is thus a function of the relative factor cost (wherein D is present since it increases the cost of equipment). The other variables of the model Y, Y^c, I, P are determined subsequently.

Note that, if all goods were perfect substitute ($\epsilon \to \infty$), without unions ($\beta \to 0$) and without uncertainty ($D \to 1$), the steady state value of X would be the same as in a perfect competition model, *i.e.*,

$$X = \frac{r+\delta}{\omega}\ \frac{\alpha}{1-\alpha}.$$

Although it is not possible to obtain a closed form solution for D and hence Π_d and so on, the signs of the partial derivatives of these variables with respect to parameters of the model can however be precisely determined.

	D	Π_d	X	U
competition ϵ	?	−	?	−
productivity τ	−	−	?	−
interest rate r	+	+	+	+
micro uncertainty $1/\rho$	−	?	+	+
disutility of work ω	+	+	−	+

More intuition on the effects of changes in the first four parameters is provided in the next section. Concerning the effect of the disutility of work, a rise in ω obviously leads firms to substitute capital for labour. However, the net effect on the unit labour cost is positive inducing a rise in prices and hence in Π_d. Both effects raise unemployment.

3 Simulations

In order to illustrate the dynamics of the model, we propose some simulations of a calibrated version. The resolution of the dynamic model relies on a Newton-Raphson relaxation method proposed by LAFFARGUE [1990] for solving dynamic nonlinear models with rational expectations. Approximating the infinite horizon by a finite one, [10] the complete system has got as many equations as the number of equations at each period times the simulation horizon plus the initial and terminal conditions:

$$(S) \qquad \begin{cases} z_0^1 & = & z_{\text{init}} \\ f(z_1, z_0^1, z_2^2, \kappa) & = & 0 \\ \vdots & \vdots & \vdots \\ f(z_T, z_{T-1}^1, z_{T+1}^2, \kappa) & = & 0 \\ z_{T+1}^2 & = & z^2_{\text{steady state}} \end{cases}$$

where $z_t = (z_t^2, z_t^3, z_t^1)$ is the vector of endogenous variables at t, including the predetermined variables (z_t^1), the non-predetermined variables (z_t^2) and the static variables (z_t^3). f is a vector function representing our aggregate dynamic model (see the Appendix) and κ is the vector of exogenous variables and parameters. The system (S) is solved using a Newton-Raphson algorithm in which the Newton-Raphson improvement at each iteration is computed by triangulation (instead of inversion) of the matrix of the first derivatives of the system.

As BOUCEKKINE [1993] shows, this method allows to characterise the nature of the dynamics of the model (explosivity, saddle-point trajectory or infinite number of stable solutions) without having to linearise it and to compute the eigenvalues of the linearised system. In particular, it is easy to determine whether the convergence of the algorithm is due to the existence of saddle-point trajectory or not. Indeed, the algorithm is characterised by an explosivity property in the case where an infinity of stable solutions exist. [11]

3.1. **Simulation Exercices**

As the dynamic analysis aims at qualitative rather than quantitative illustrations, the model has been calibrated in an elementary way. It is supposed to replicate the magnitude of Belgian aggregate data for the year 1973. ($Y = 3.456423$ billions of Belgian francs at prices of 1985, $L = 3.74019$ millions of workers, $L^s = 3.89382$ millions of workers, $W/P = 0.72538$ millions of francs 1985, capital stock (*i.e.* Y^c/X^α) = 13.72452 billions of francs 1985). The depreciation rate (0.04) and the discount rate (0.01) have been chosen in order to simulate a half-yearly model. The other parameters of the model have been fixed at plausible values. The coefficient of labour in the production function α has been put to 0.76 which should be equal to the labour share in value-added when perfect

---

10. That means that the transversality conditions on anticipated variables are replaced by the steady-state values of these variables at the end of the horizon of simulation.
11. This explosivity property is in fact common to all convergent relaxation methods, see Boucekkine-Le Van 1993. The explosive behaviour is put forward by a simple numerical procedure relying on the initialisation of the relaxation. Initialising the relaxation with values slightly different from the steady state leads to an explosive behaviour at the first Newton-Raphson improvement.

competition prevails. The total productivity of factors has been computed in order to retrieve the magnitude of Belgian GDP given the other parameters ($\tau = 0.69$). The mismatch parameter ρ is set to 40 in 1973 reflecting a low structural unemployment rate. We have chosen 6.4 for ϵ in order to retrieve a credible mark-up rate of about 20% and a proportion of demand-constrained firms around 40%. It is worth mentioning that different calibrations have shown us that the model responses are particularly sensitive to the value of ϵ. The parameter ν has been fixed to be 0.65 (for an estimation exercise see de la Croix, PALM and PFANN [1993]). The value of β is not interpretable since it also includes a normalisation factor of the utility function of the union. β has been set equal to 0.5. Finally, given β and ν, ω has been chosen to replicate the magnitude of real wage. This gives a disutility of work of about 0.587 millions of 1985 francs.

We propose hereafter 4 simulation exercises in order to understand better the properties of the model and to see its ability to generate changes in the endogenous variables comparable to the stylised facts presented in the introduction. Each simulation describes the optimal response of the agents in the face of an unexpected change in their environment. Changes are expected to be permanent once they have occured. The four different changes are:

• A drop in the price elasticity of demand ϵ. This is a crude way to model a decrease in the degree of competition between the firms. Our aim is to clarify the message of our model with respect to the literature on variable markups (see, e.g., ROTEMBERG and WOODFORD [1993]).

• A drop in the total factor productivity (τ). The effects of τ on economic activity are extensively investigated in perfect competition models by the Real Business Cycle school. We want here to see the effect of technological shocks on quantity constraints indicators. Moreover, a negative change in τ can be interpreted as an oil shock if energy is a complementary factor to capital and labour in the production function. This sufficient (but not necessary) assumption seems realistic.

• A rise in the real interest rate r, supposed to reflect the tight monetary policies of the eighties. This shock decreases the weight of future profits and utility in the objective function of the agents.

• A rise in the uncertainty faced by the firms. This shift decreases ρ and leads to higher heterogeneity across firms and higher mismatch. Such a shock is analysed in DRÈZE and BEAN [1990] in an *ad hoc* dynamic structure. In empirical works, this lower ρ appears as an aftermath of the first oil shock.

For each shock, we concentrate our attention on five variables: output (Figure 5), mark-up rate (Figure 6), unemployment rate (Figure 7), degree of utilisation of capacities (Figure 8), and proportion of firms reporting demand constraints (Figure 9). The intensity of the shocks has been chosen in order to produce a 5 % decrease in output at the new steady-state. The simulation has been carried out on an horizon of 100 periods although only the first twenty are reported in the Figures.

• Demand Elasticity Shock

We simulate here an unexpected change in ϵ from 6.4 to 4.4. In the face of such a shock, the firms increase their prices. The consequent drop in real

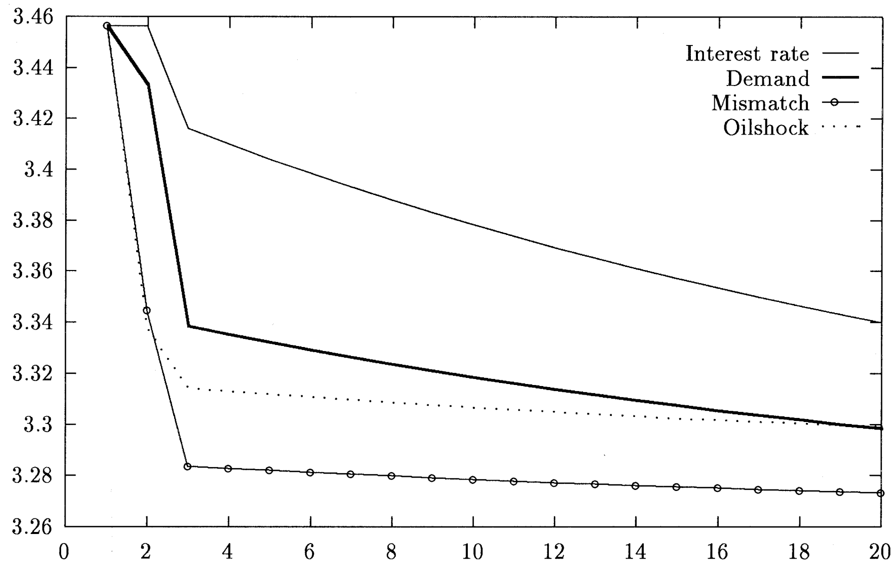

FIGURE 5

Output

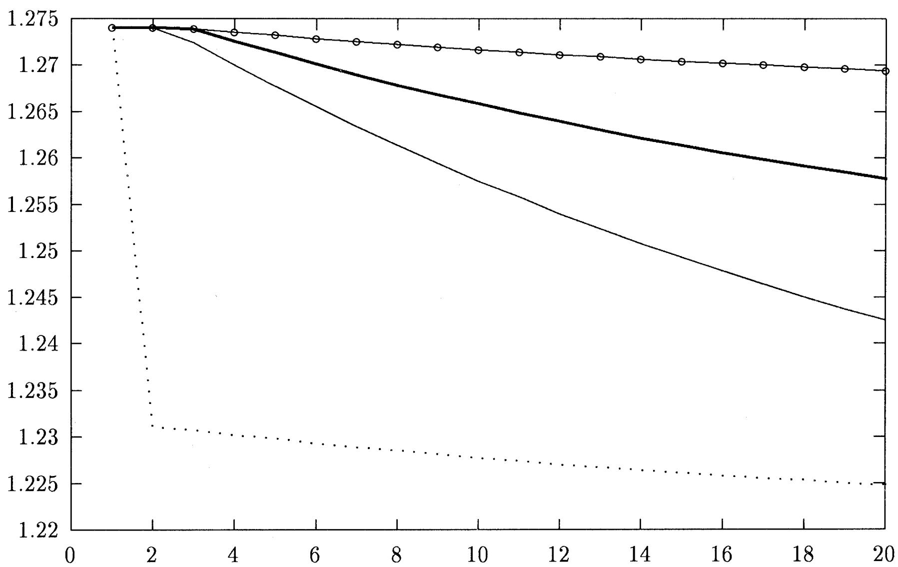

FIGURE 6

Mark-up Rate

demand induces a significant and immediate rise in the proportion of firms reporting demand shortages. Output drops and unemployment increases from 4% to more than 7% within three periods (one and a half years). The effect on the degree of utilisation of capacities is negative in the short-run because

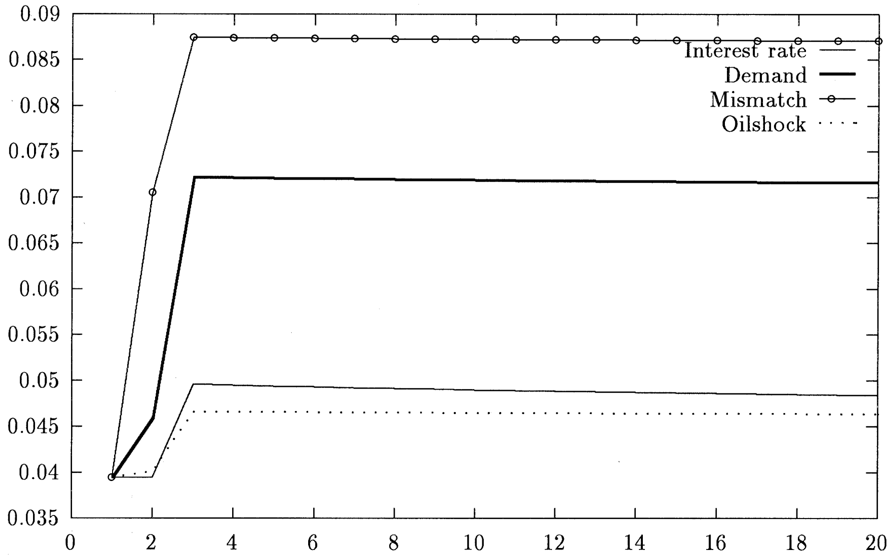

FIGURE 7

Unemployment Rate

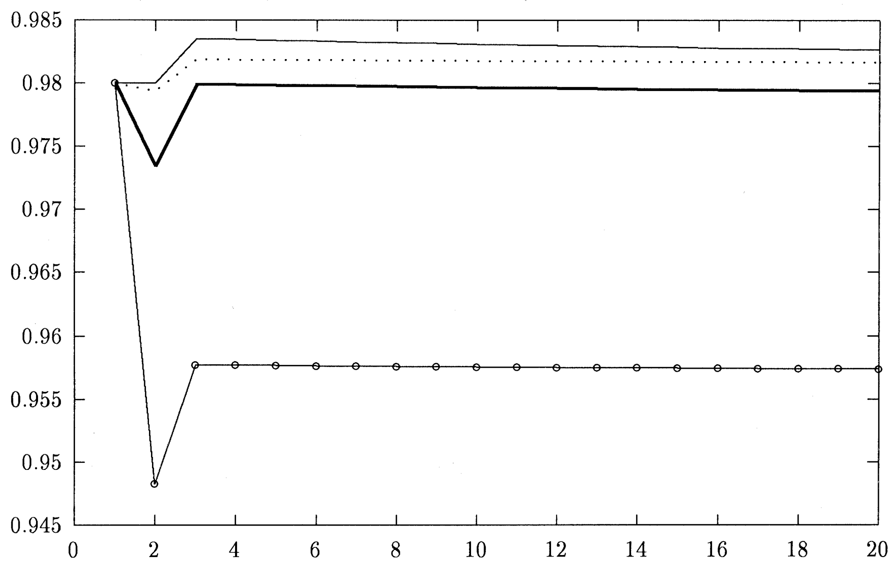

FIGURE 8

Degree of Utilisation of Capacities

of the drop in demand, but vanishes with the contraction of capacities after two periods. Surprisingly, the mark-up rate is not significantly affected. Indeed the drop in ϵ is entirely compensated by the rise in the probability of a demand shortage, so that $\epsilon\pi_d$ remains constant. This simulation shows that a price-elasticity shock may generate three stylised facts of the year

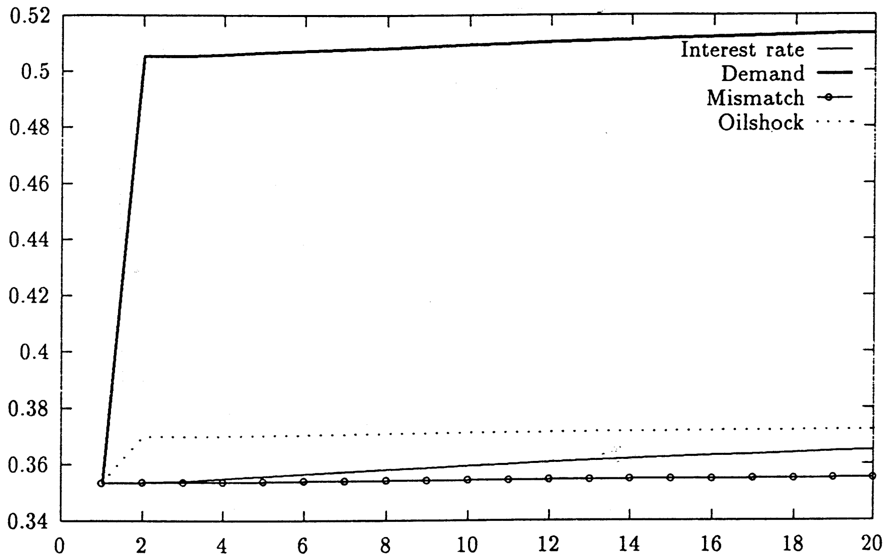

FIGURE 9

Firms Reporting Demand Shortages

1973, *i.e.* a rise in the proportion of demand-constrained firms together with a rise in unemployment and a temporary drop in capacity utilisation.

• Productivity Shock

Total factor productivity τ drops from 0.69 to 0.66. (This change is comparable to a permanent change in the deterministic part of the Solow residual.) This lower productivity induces an immediate drop in output. Unemployment, however, does not rise immediately since the productivity of labour is reduced by the shock. Moreover, in response to the higher unit cost of labour, the firms raise their prices. The resulting fall in real demand explains the slight drop in D during the first period following the shock and the rise in Π_d. This latter does not seem quantitatively important, but is enough to lower very much the mark-up rate. Contrary to the previous shock, this one is responsible for an immediate drop in profit margins, *i.e.*, an increase in the labour share in value-added. During the following periods, the fall in productivity of factors and the loss in profits depress investment. This destruction of productive capacities explains the accompanying rise in U and D. The effects on U and D are quantitatively small. However, it is worth noting that magnitude of the effects on U and D is heavily dependent on the chosen calibration. With a lower ϵ the effects of a drop in τ are quite more important.

• Interest Rate Shock

The real interest rate is raised from 1% to 1.9% on a semiannual basis. The main effect is to increase the user cost of capital and to reduce the optimal level of productive capacities. Since the shock affects the economy essentially through capital formation, the initial effect on output is weak and the total 5% drop in output is spread over a very long period. The negative effect on productive equipments is strengthened by the fact that the firms increase their rate of capacity utilisation. Indeed, the firms revise their trade-off between their interest to accumulate excessive capacities because of the uncertainty and the cost of this investment. This leads to destructions of work-stations and to a higher equilibrium unemployment rate. After an initial increase, unemployment stops increasing although output continues to drop. The underlying capital/labour substitution effect is however insufficient to overcome the negative impact of the lower profitability. The drop in labour productivity linked to the substitution effect induces a progressive erosion of the mark-up rate. Note finally that, with respect to the three other simulations, the economy takes more time to converge to its new steady state.

- **Increased Uncertainty**

The simulated change in ρ is a drop from 40 to 18. This reflects a rise in the uncertainty faced by the firms, for instance through the rise in the variance of one of the idiosyncratic shocks. In the aftermath, the distribution of demand, capacities and labour supply among micro-markets is more unequal, and the mismatch increases. From the microeconomic side, the rise in uncertainty reduces expected output at given prices and productive capacity. This is reflected at the macroeconomic level by an initial sharp drop in output and in D. Subsequently, the firms are compelled to reduce both the optimal level of their capacities and the optimal rate of capacity utilisation. The macroeconomic impact of these micro-perturbations is a rise in U. This increase in U can be interpreted as a rise in structural unemployment. In the unemployment-vacancies space, it can be shown that the corresponding Beveridge curve shifts outward. In our case, this shift is directly linked to an optimal response to increasing uncertainty. On the demand side, the drop in ρ has no significant effects either on the proportion of firms reporting demand shortages nor on the mark-up rate.

4 Conclusion

This paper shows that one can construct a intertemporal macromodel with shares many features with traditional quantity-constrained models – in particular, regime switches from excess supply to excess demand in the goods and labour markets – yet which is not at all Keynesian in its policy implications. This is because the rigidities which give rise to the regime-switching behaviour are all real, not nominal. These rigidities result from technological rigidities (i.e., a putty-clay technology), microeconomic uncertainty and market segmentation. In this framework we derive the path of the unemployment rate, the rate of capacity utilisation and the proportion of firms reporting demand constraints. The resulting unemployment rate looks very much like an equilibrium unemployment rate (or NAIRU) with a focus on capacity accumulation and technical rigidities.

The model is then calibrated and simulated using a new algorithm for solving dynamic non-linear models with perfect foresight. We have successively analysed the optimal response of the agents in unfavourable changes in the price elasticity of demand, total factor productivity, interest rate and uncertainty. The four shocks induce a rise in unemployment but different patterns for capacity utilisation, markups and demand shortages.

More research is needed for assessing the empirical relevance of the model and distinguishing the relative importance of the various shocks. Nevertheless, the present exercice shows how cautious the interpretation of business survey indicators should be. For instance, a rise in the proportion of firms reporting demand shortages does not imply *per se* any Keynesian phenomena. A rise in the interest rate increases the long-run value of the rate of capacity utilisation: A persistently high utilisation rate may thus reflects a need for lowering the interest rate and not the contrary.

The Aggregate Dynamic Model

A) Accumulation equations

$$\begin{cases} Y_t^c = Y_{t-1}^c \,(1-\delta) \;+\; X_t^\alpha \; I_t \\ Y_t^c/A_t = (Y_{t-1}^c/A_{t-1})\,(1-\delta) \;+\; X_t \; I_t \end{cases}$$

B) Output and related business cycle indicators

$$\begin{cases} Y_t = \left[\left(\bar{C}_t/P_t\right)^{-\rho} + \left(\tau \, Y_{t-1}^c\right)^{-\rho} + \left(\tau \, A_{t-1} \, L_t^s\right)^{-\rho} \right]^{-1/\rho} \\ D_t = \dfrac{Y_t}{\tau \, Y_{t-1}^c} \\ \Pi_{dt} = \left(\dfrac{Y_t}{\bar{C}_t/P_t} \right)^{\rho} \end{cases}$$

C) Price and wage formation

$$\begin{cases} 1 - \dfrac{1}{\epsilon \Pi_{dt}} = \dfrac{W_t/P_t}{\tau \, A_{t-1}} - \dfrac{\beta}{\nu} \; \dfrac{1}{\tau \, A_{t-1}} \left(\dfrac{W_t}{P_t} - \omega \right)^{\nu} \\ \dfrac{W_t}{P_t} = \beta^{1/(1-\nu)} + \omega \end{cases}$$

D) FOC's on capacities, investment and labour-capital ratio

$$\begin{cases} \begin{aligned} (1+r_{t+1}) \frac{1-\gamma_t}{\tau \, X_t^\alpha} =& (1-\delta) \frac{1-\gamma_{t+1}}{\tau \, X_{t+1}^\alpha} \\ &+ (1+r_{t+1}) \, \mu_t \; \frac{(X_{t+1})^{1-\alpha} - (X_t)^{1-\alpha}}{\tau} + \frac{(D_{t+1})^{\rho+1}}{\epsilon \Pi_{dt+1}} \\ &+ \left(1 - \frac{1}{\epsilon} - \frac{D_{t+1}^{\rho}}{\epsilon \Pi_{dt+1}}\right) D_{t+1} \left(1 - \frac{A_t}{(X_{t+1})^{\alpha-1}}\right) \\ \gamma_t \; I_t =& 0 \\ \frac{D_{t+1}^{\rho+1}}{\epsilon \, \Pi_{dt+1}} =& (1+r_{t+1}) \; \frac{\mu_t}{\tau \, \alpha} \; \left((X_t)^{1-\alpha} - (X_{t+1})^{1-\alpha} \right) \\ &- \; \left(1 - \frac{1}{\epsilon} - \frac{D_{t+1}^{\rho}}{\epsilon \Pi_{dt+1}}\right) D_{t+1} \left(1 - \frac{A_t}{\alpha \, (X_{t+1})^{\alpha-1}}\right) \\ (1+r_{t+1}) \, \mu_t \frac{Y_t^c}{A_t} =& \mu_{t+1} \, (1-\delta) \; \frac{Y_t^c}{A_t} \\ &+ \frac{\tau \, A_t \, L_{t+1}^s}{\epsilon \, \Pi_{dt+1}} \; \left[\frac{Y_{t+1}}{\tau \, A_t \, L_{t+1}^s} \right]^{\rho+1} + \left(1 - \frac{1}{\epsilon \, \Pi_{dt+1}}\right) Y_{t+1} \end{aligned} \end{cases}$$

• References

ARNSPERGER C., de la CROIX D. (1993). – "Bargaining and Equilibrium Unemployment: Narrowing the Gap between New Keynesian and "Disequilibrium" Theories", *European Journal of Political Economy*, 9, pp. 163-190.

BEAN C. (1989). – "Capital Shortages and Persistent Unemployment", *Economic Policy*, pp. 12-53.

BLANCHARD O., KAHN C. (1980). – "The Solution of Linear Difference Models under Rational Expectations", *Econometrica*, 48, pp. 1305-1311.

BOUCEKKINE R. (1993). – "An Alternative Methodology for Solving Nonlinear Forward-Looking Models", forthcoming in *The Journal of Economic Dynamics and Control*.

BOUCEKKINE R., LE VAN C. (1993). – "Checking for Saddlepoint Stability: An Easy Test", forthcoming in *The Journal of Economic Dynamics and Control*.

BURDA M. (1988). – "Is There a Capital Shortage in Europe", *Weltwirtschaftliches Archiv*, pp. 38-57.

de la CROIX D. (1992). – "Union-Firm Bargaining, Investment and Equilibrium Unemployment", *Research Memorandum 92-048*, Limburg University, Maastricht.

de la CROIX D., FAGNART J. F. (1993). – "Underemployment of Labour and Equipment in a Bargaining Model with Forward-looking Bahaviour", Working paper CEPREMAP.

de la CROIX D., PALM F., PFANN G. (1993). – "A Dynamic Contracting Model for Wages and Employment", *Discussion Paper IRES*, University of Louvain.

DRÈZE J., BEAN C. (1990). – "European Unemployment: Lessons from a Multi-Country Econometric Exercise", in Drèze J. et al., *Europe's Unemployment Problem*, MIT Press.

ENTORF H., KONIG H., POHLMEIER W. (1992). – "Labor Utilization and Nonwage Labour Costs in a Disequilibrium Macro Framework", *Scandinavian Journal of Economics*, 94, pp. 71-83.

ESPINOZA P., RHEE C. (1989). – "Efficient Wage Bargaining as a Repeated Game", *Quarterly Journal of Economics*, 104, pp. 565-588.

LAFFARGUE J. P. (1990). – "Résolution d'un modèle macroéconomique à anticipations rationnelles", *Annales d'Economie et de Statistique*, 17, pp. 97-119.

LAYARD, NICKELL, JACKMAN (1991). – *Unemployment - Macroeconomic Performance and the Labour Market*, Oxford University press.

LICANDRO O. (1991). – "Monopolistic Competition and Aggregation over Micromarkets in Disequilibrium", *mimeo*, Universidad Carlos III.

LICANDRO O. (1992). – "Investment Dynamics and Capacity Utilization in Monopolistic Competition", *Annales d'Economie et de Statistique*, 27, pp. 91-113.

MALINVAUD E. (1987). – "Profitability and Factor Demands Under Uncertainty", *De Economist*, 137, pp. 1-15.

ROTEMBERG J., WOODFORD M. (1993). – "Dynamic General Equilibrium Models with Imperfectly Competitive Product Markets", paper presented at the conference on the macroeconomics of imperfect competition, Paris, 1994.

SNEESSENS H. R. (1983). – "Aggregation In Quantity Rationing Models", Département des Sciences Economiques, Université Catholique de Louvain, *mimeo*.

SNEESSENS H. R. (1987). – "Investment and the Inflation- Unemployment Trade off in a Macroeconomic Rationing Model with Monopolistic Competition", *European Economic Review*, 31, pp. 781-815.

SNEESSENS H. (1992). – "Contraintes de débouchés, capacités de production et chômage dans un modèle macroéconomique avec concurrence imparfaite", *L'Actualité Economique*, 68, pp. 140-174.

STRAND J. (1989). – "Monopoly Union versus Efficient Bargaining, a Repeated Game Approach", *European Journal of Political Economy*, 5, pp. 473-486.