

The q Theory of Investment under Unit Root Tests *

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We test a q investment model for Belgium using a multivariate co-integration approach. The introduction of the degree of capacity utilization duc , in addition to investment and average q , is necessary to determine the co-integration space. This supports the idea that marginal q differs from average q by a factor which is a function of duc .

Average q and Marginal q

According to the q theory of investment, in the presence of convex adjustment costs for capital input, investment depends upon the ratio between the discounted value of all expected future profits generated by the installation of an additional unit of capital and the purchase price of investment goods. This ratio, called marginal q , is equal to the average q under the conditions derived by HAYASHI (1982). In that case, the discounted value of future profits is equal to the average stock market value of capital. The main interest of this approach is that expectations about future profits are contained in the q term.

Various authors have tried to test empirically the influence of stock market value on firms' investment (cf. ABEL and BLANCHARD (1986)). Average q turns out to be highly significant in explaining investment, although it is unsatisfactory for various reasons: Lagged values of q also

* We are grateful to Frédéric Aprahamian, Luc Bauwens and to two anonymous referees for their useful comments. Paul Olbrechts kindly provided the statistical material.

have significant coefficients (while the theory suggests that current q includes all available information about future profits); moreover, the residuals are autocorrelated. These elements suggest that marginal q differs from average q and that this difference is a function of missing variables (see GALEOTTI and SCHIANTARELLI (1991)).

Following an idea initially proposed by MALINVAUD (1987), D'AUTUME (1988), LICANDRO (1992) and to some extent PRECIOUS (1987) have shown that the difference between marginal q and average q could be explained by the ratio of production to capacities, called the degree of capacity utilization and denoted duc . The idea is that if firms face demand uncertainty at the time of their input (and possibly also price) decision, the predetermined capacity can be underutilized. Together with average profitability, the intensity of this underutilization determines the marginal value of the firm. In this case, average q differs from marginal q by a factor which depends on the sequence of all expected future duc . In the long run, investment depends on average q and duc .¹

In order to investigate the relevance of these contributions, we test the existence of two different relations. Since the series that we will use exhibit clear non-stationary behaviour (this is discussed in the next section), standard asymptotic theory do not apply and co-integration analysis will turn out to be necessary. The two relations (1) and (2) can therefore be seen as co-integration relations. The first relation is derived from the usual classical q investment model, with the simplifying assumption that the adjustment cost function is exponential.² In this case, the investment rate α is simply a function of the logarithm of q , which is the average value of the firm (measured by the stock market index) divided by the price of investment. This equation assumes the equality

¹ This framework can be compared to another recent contribution by SCHIANTARELLI and GEORGOUTSOS (1990) who stress the role of imperfect competition: Contrary to LICANDRO'S (1992) model, their firm can always adjust employment in order to fully utilize its capacity. However, monopolistic competition allows them to include output per unit of capital as an additional explanatory variable in investment equation.

² The installation function is equal to $k\Omega(\alpha)$ with k for the capital stock and α for the investment rate. Ω is given by $\Omega(\alpha) = \delta - \frac{1}{b} + \frac{1}{b} \exp\{b(\alpha - \delta)\}$ where δ is the depreciation rate and b is a parameter. Ω satisfies $\Omega(\delta) = \delta$, $\Omega' > 0$ and $\Omega'(\delta) = 1$.

between average q and marginal q :

$$(1) \quad \alpha = b_0 + b_1 \ln q$$

The second specification we want to test is taken from LICANDRO (1992) and de la CROIX and LICANDRO (1992). It is directly comparable to the classical model and takes the following long-run form:

$$(2) \quad \alpha = b_0 + b_1 \ln q + b_2 \ln duc$$

where b_2 is positive: the closer the firm is to the full utilization of capacity, the more it invests.

The presence of duc in the long-run equation (2) reflects simply the fact that

- Because of demand uncertainty and investment irreversibility the degree of utilisation of capacities is, in general, lower than one, even in the long-run.
- Therefore, each additional unit of capital has a positive probability of non-utilisation. Its marginal productivity is lower than in the deterministic case and marginal q is smaller than average q .
- This gap between marginal and average profitability is a function of the probability of non-utilisation and therefore of duc .

The Data

The Belgian data we use are quarterly and extend from 1971:3 to 1990:2 (76 observations). The investment rate α is defined as i_t/k_{t-1} where i_t is nominal investment in manufacturing industries according to VAT declarations deflated by the wholesale price of the investment goods sector. The capital stock k has been built using the investment series with an annual depreciation rate of 10%. Its initial value has been chosen in accordance with the aggregate evaluation of the capital stock by the Central Planning Bureau in 1971. Average q is measured by the stock market index of industrial values divided by an index of the wholesale

price of the investment goods sector. The *duc* comes from the business surveys of the National Bank of Belgium and is related to the manufacturing sector only. Finally, note that the investment rate series has been seasonally adjusted.

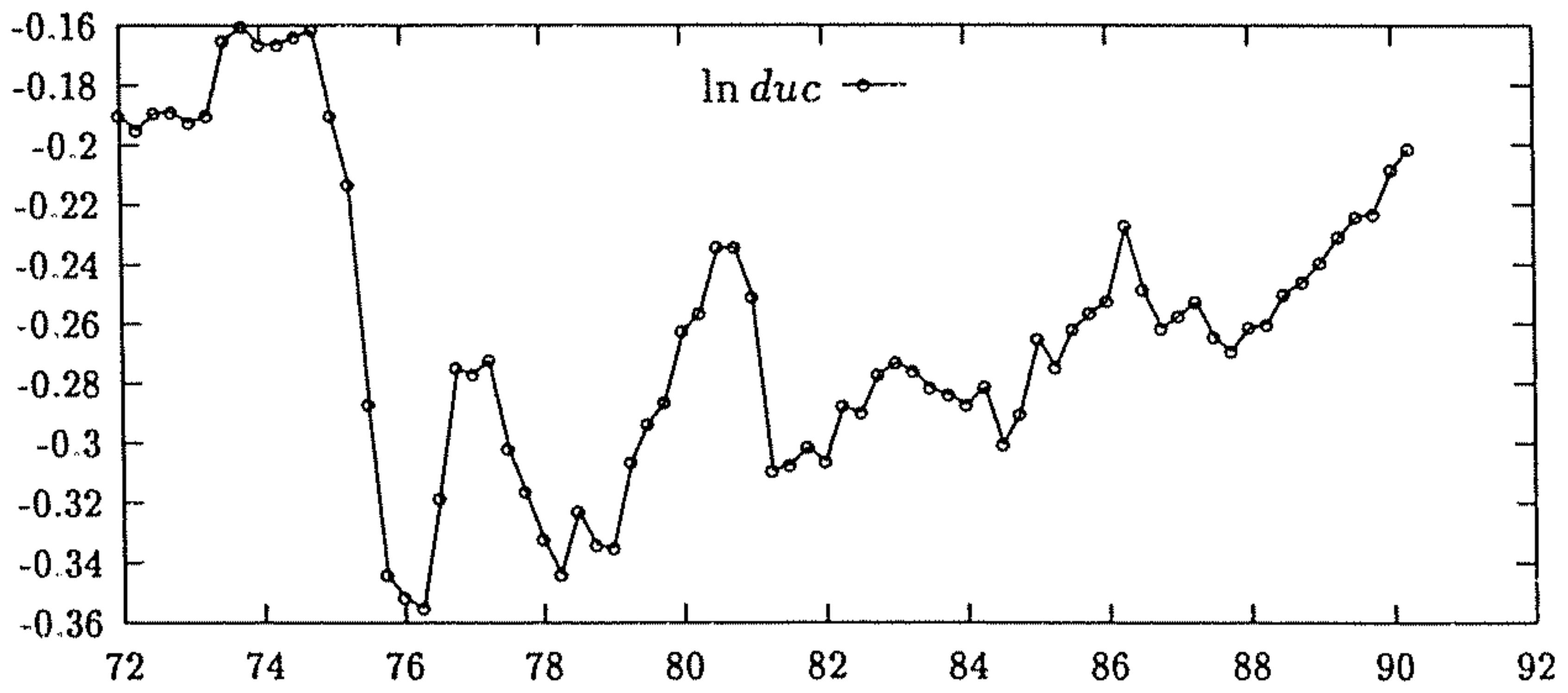
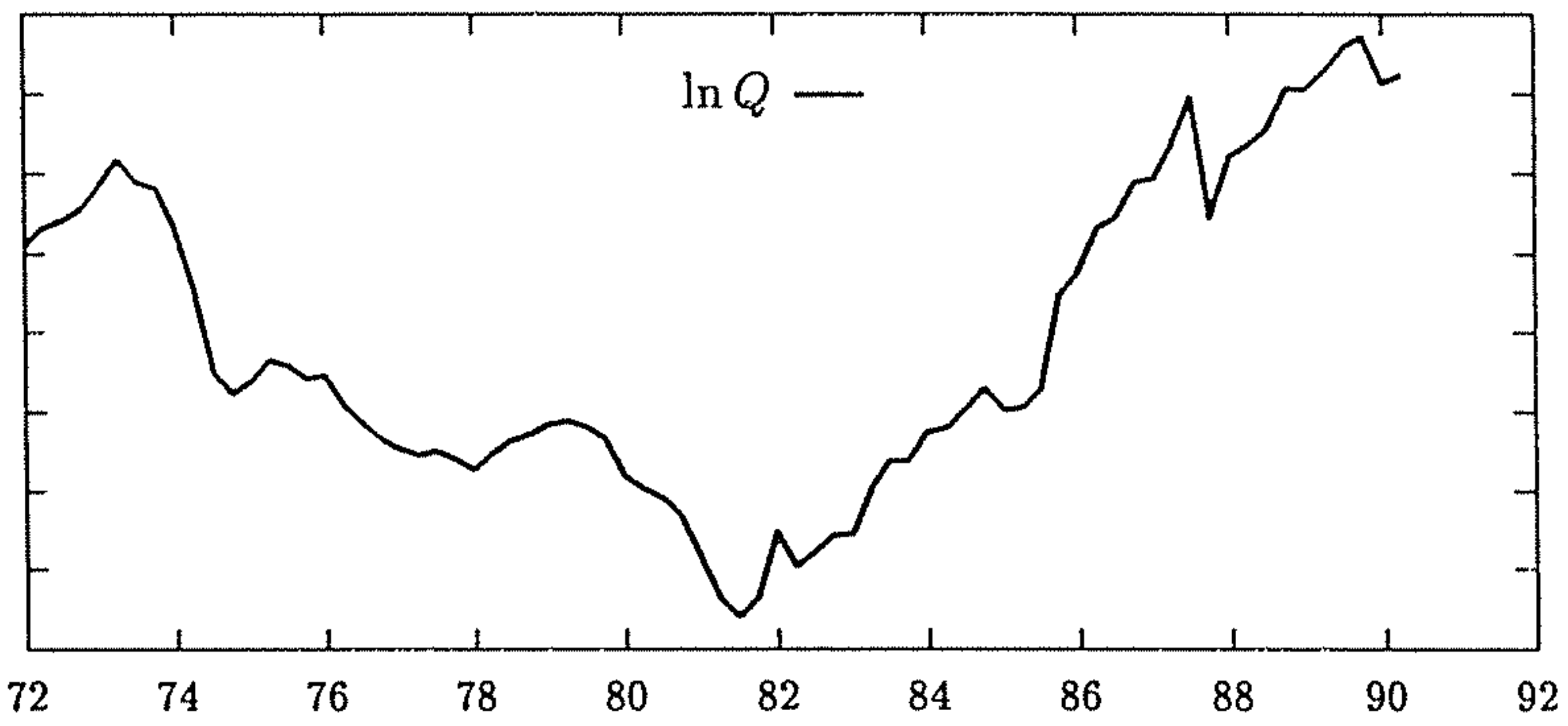
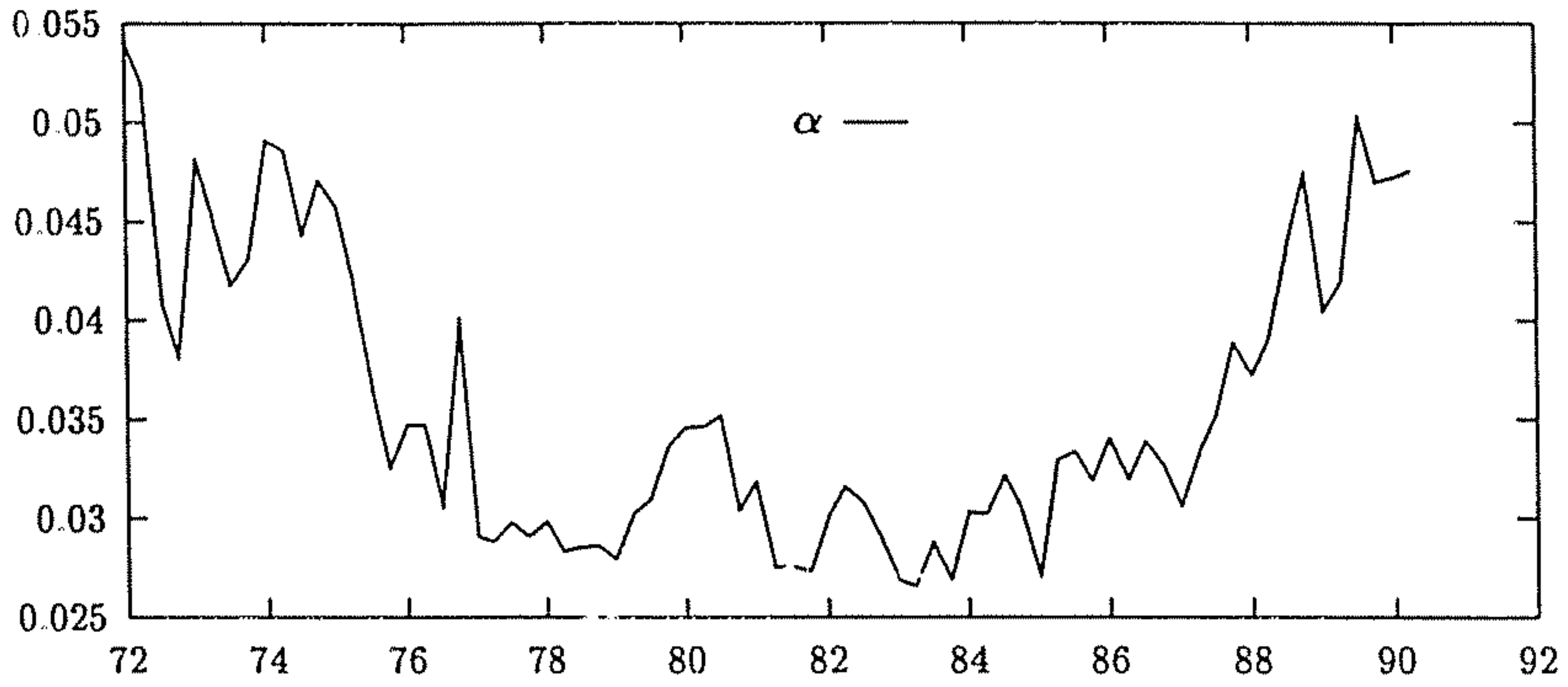
Before doing any multivariate analysis, we have to test the order of integration of the variables α , $\ln q$ and $\ln duc$. Note that the classical theory suggests that these variables are stationary since the endogenous response of investment should ensure that *duc* is driven to its "normal" level and *q* to unity in the long run. To test the level of integration of any variable *y*, we use a standard ADF test selecting carefully the number of lags *p* in the following regression:

$$\Delta y_t = \mu + \alpha y_{t-1} + \sum_{i=1}^p \Delta y_{t-i} + \epsilon_t$$

The role of these lags is to cope with remaining autocorrelation in the residuals. However, if their number is too large the power of the test is weakened. Therefore, careful inspection of whether the residuals are white noise and whether the lags are significant, as well as the use of some information criterion, can give important guidance. The estimations are presented in Table 1.

TABLE 1 : ADF Unit Root Tests

	α				$\ln q$				$\ln duc$			
	τ_a	AIC	AR4	HET	τ_a	AIC	AR4	HET	τ_a	AIC	AR4	HET
DF	-2.69	-11.24	6.85	0.06	-0.14	-4.97	5.13	1.82	-1.72	-7.92	19.53	0.09
ADF1	-2.32	-11.20	10.4	0.49	-0.43	-4.95	3.5	4.81	-2.54	-8.07	7.21	4.08
ADF2	-1.15	-11.32	10.8	2.25	-0.77	-4.92	3.06	0.38	-2.74	-8.03	7.26	5.92
ADF3	-0.68	-11.32	7.07	3.42	-1.01	-4.88	2.65	0.13	-2.65	-7.98	7.05	5.56
ADF4	-1.25	-11.34	6.32	0.01	-0.92	-4.82	1.04	0.17	-1.96	-8.00	1.77	1.81
ADF5	-1.45	-11.31	6.78	0.01	-1.2	-4.79	3.52	0.79	-2.08	-7.95	5.53	2.21
ADF6	-1.16	-11.31	5.01	0.03	-1.13	-4.73	3.19	0.76	-2.42	-7.96	2.3	2.94
ADF7	-1.08	-11.25	6.3	0.03	-1.12	-4.67	15.5	0.91	-2.4	-7.90	4.94	2.86
ADF8	-1.58	-11.27	3	0.11	-0.68	-4.64	6.03	0.9	-2.32	-7.83	5.31	2.68



Together with the number of lags and the value of the ADF test (τ_a) we give the Akaike information criterion (AIC), the LM test for the autocorrelation of residuals up to order 4 (whose critical value at 95% is a $\chi_4^2 = 9.49$) and an LM test of heteroscedasticity regressing the squared residuals on the squared levels of the estimated dependent variable (whose critical value at 95% is a $\chi_1^2 = 3.84$). The best model with respect to these criteria is given in italics. From MacKINNON (1990), the critical value below which stationarity is rejected is -2.90 for τ_a , implying that our three variables are not stationary in levels. The optimal lag length is 4 for α and $\ln duc$ while the best representation for $\ln q$ is a pure random walk.

An argument against this result could be that there is very slow mean reversion in all three variables, which is not adequately captured by the ADF test. It would be too optimistic to hope that twenty years of data would reveal such slow mean reversion. To cope with this argument we have computed the non-parametric test due to PHILLIPS and PERRON (1988) which should allow for a larger class of error term distributions. Their statistic $Z(\tau_a)$ are presented in Table 2 for different values of the truncation lag parameter l (cf. the article of Phillips and Perron for details). The critical value is the same as in the parametric ADF test. This test raises some doubts about the robustness of the conclusion concerning investment rate, since for $l = 1$ the unit root hypothesis is rejected. However, since we have no good reason for selecting any particular value of l , we are not able to draw a definitive conclusion. Since for all other values of l the test does not reject the unit root hypothesis, we treat the investment rate as an integrated variable. Non-reported tests on the first differences of the variables show that the three variables are $I(0)$ in variations.

Of course, in general, investment rate, duc and q are expected to be $I(0)$. Nevertheless we are forced to accept that, during the 20 years under consideration, they are not stationary (as it is often the case for the unemployment rate, the real interest rate etc.). This is an interesting illustration of the fact that, although the series are expected on a large sample period to be stationary, their evolution in the selected sample is not distinguishable from $I(1)$ processes.

We do not believe that this non-stationnarity rejects the theory(ies), but, to accept equation (1) or (2), these variables should at least move not too far apart, i.e. they should be cointegrated. Consequently, we now try to find some co-integration relationship between our three variables.

TABLE 2 : Non-Parametric Unit Root Tests

	DF	l=1	l=4	l=7	l=10
α	-2.69	-3.11	-2.54	-2.57	-2.41
$\ln q$	-0.14	-0.13	-0.12	-0.11	-0.11
$\ln duc$	-1.72	-1.29	-1.39	-2.76	-2.17

The Co-integration Space

Using the methodology of JOHANSEN (1988) and JOHANSEN and JUSELIUS (1990), we estimate a system composed of three autoregressive error correction equations. The determination of the dimension of the co-integration space can be found by a procedure which is a kind of multivariate generalization of the augmented Dickey-Fuller test. Once this rank is determined, standard asymptotic theory can be used for testing on the cointegrating vectors and analyzing the shape of the error correction terms. We are interested in identifying the co-integration space, if any, and testing whether $\ln duc$ is important in this identification. The number of lags in the VAR has been chosen looking at the remaining autocorrelation of the residuals. It turns out that three lags are necessary to eliminate autocorrelation (up to order 8) in the investment equation.³ Concerning the choice of the deterministic part of the VAR, it seems wise to restrict the constant to lie solely in the co-integration space, since the use of an unrestricted constant would imply the existence of deterministic linear trends in the data, which is not the case.⁴ The following model is tested:

³ Box-Pierce tests BP(8) are 15.746, 6.657 and 9.294 for 2 lags and 3.082, 5.684 and 9.592 for three lags with a critical value of 15.51.

⁴ Anyway, the unrestricted estimation is very close to the restricted one.

$$\Delta x_t = \sum_{i=1}^2 \gamma_i \Delta x_{t-i} + \xi(\beta', \beta_0)(x_{t-3}, 1) + \epsilon_t$$

where $x_t = [\alpha_t \ln q_t \text{ Induc}_t]'$, μ is a 3 x 1 vector of constants, γ_i are a 3 x 3 matrices, ξ and β are 3 x r matrices, β_0 is a r x 1 vector, r being the dimension of the co-integration space.⁵

Table 3 presents the co-integration tests used to determine r, the number of co-integration vectors. Two statistics are provided: the first is based on the maximal eigenvalue of Johansen's stochastic matrix; the second uses the trace of this matrix.⁶ The last column gives the threshold above which the null hypothesis is rejected (from OSTERWALD-LENUM (1992)).

TABLE 3 : Dimension of the Co-integration Space

	null	alternative	statistics	95% threshold
(eigenvalue)	r = 0	r = 1	26.46	22.00
	r ≤ 1	r = 2	10.56	15.67
(trace)	r = 0	r ≥ 1	38.15	34.91
	r ≤ 1	r = 2	11.68	19.96

The two statistics unanimously allow us to conclude that there is only one cointegration relationship in the model. This unique co-integration vector, normalized to have a unit weight for α , implies the following long-run relationship:

$$\alpha = 0.049 + 0.0099 \ln q + 0.084 \ln \text{duc}$$

The signs are compatible with the theoretical model. This long-run vector can be compared to the one we would have obtained using the

⁵ Note that in this way of writing the VAR the long-run impact matrix is already decomposed into $\xi(\beta', \beta_0)$. This should not makes us forget that there are first a searching for a long-run matrix of reduced rank, in which case this decomposition allows the columns of β to be interpreted as cointegrating vectors.

⁶ The eigenvalues in descending order are 0.311, 0.138 and 0.016.

ENGLE and GRANGER (1987) estimates:⁷

$$\alpha = 0.038 + 0.0092 \ln q + 0.049 \ln duc$$

Johansen's procedure also allows to test the presence of cointegrating vectors in each equation of the system, using a likelihood ratio test whose critical value is distributed as a χ_1^2 . This aims at analyzing whether the variations of the variables adjust to the long run of the model. This test is presented in Table 4: At a 10% critical level, only the investment rate and the degree of utilization of capacity adjust in order to satisfy the long run relationship. At a 5% level, only the investment rate do adjust. This can be interpreted in terms of long-run Granger non-causality (see e.g. URBAIN (1992)): Since average q do not adjust to the long-run, it is not long-run Granger caused by the two other variables and can be considered as weakly exogenous for the long-run parameters.

Note that the adjustment speed of investment to the long-run is relatively rapid since the corresponding ξ parameter is around 0.45.

TABLE 4 : Absence of Cointegrating Vectors

equation	$\Delta\alpha$	$\Delta\ln q$	$\Delta\ln duc$
test	30.16	0.21	2.75

We now turn to our main question, and test whether duc is important in the determination of the co-integration space. If duc is of no importance in the long run, its weight in the co-integration relation should be zero. We therefore test the exclusion of duc from the co-integration space using a likelihood ratio test whose critical value is distributed as a χ_1^2 . The result is presented in Table 5 together with the exclusion tests for the other variables. From this, it is clear that duc helps significantly to determine the co-integration space.

⁷ The cointegration tests in this framework are the following: CRDW=0.674, DF=-4.23, ADF1=-3.64, ADF4=-2.7.

TABLE 5 : Exclusion from the Co-integration Space

variable	α	$\ln q$	$\ln duc$
test	11.04	5.24	9.31

This leads to the conclusion that $\ln duc$ improves significantly the determination of the co-integration space. This results give some support to the idea that, in the long run, average q diverges from marginal q and that this difference can be explained by the ratio of effective output to total capacity.

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