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Envy-minimizing unemployment benefits

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Abstract

This paper offers an analysis of the optimality of unemployment benefits based on the concept of no-envy. Using a general equilibrium framework with uncertainty, we derive the conditions for a trade-off between the intensity of envy and the expected percentage of envious persons. If the government's sensitivity to the intensity of envy is not too strong (alongside conditions on households' utilities), the optimal benefit is positive and below the full-insurance level. We also show that, for a low enough sensitivity to the intensity of envy, the optimal replacement ratio decreases with unfavorable changes in the distribution of the technological shock.

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1. Introduction

In recent decades, there has been a surge of interest in formal criteria of *distributive justice*. There is now a quickly expanding literature on the axiomatics of equity and fair distribution (see, among many others, Thomson and Varian, 1984; Arnsperger, 1994; Thomson, 1994; Fleurbaey, 1996; Kolm, 1995; Thomson, 1995). A wealth of competing criteria have been elaborated, and their formal interconnections have been thoroughly investigated within specific, one might say 'canonical', economic environments such as exchange economies or one-input, one-output production economies (see, among many others, Moulin, 1990).

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What appears to be still missing for the moment, however, is the attempt to integrate this growing literature into more general economic frameworks – or, to put it differently, the attempt to actually *use* existing criteria of distributive justice to address economic problems developed in other segments of the profession. The literature on fair distribution could be very useful to gain understanding of some important issues in general political economy, but few or no attempts seem to have actually been launched.

The present paper aims at making such an attempt. It seeks to integrate some aspects of fair-distribution theory into the analysis of a crucial problem in contemporary economies: the determination of unemployment benefits. Our contribution, as will quickly become apparent, will be largely methodological. We will not claim to display a positive description of how unemployment benefits are actually determined. Rather, we want to show how such benefits *would* be determined *if* a government were to focus on fairness concerns of a certain type. The predictive power of our results is therefore relatively limited. However, we believe that our model makes an important methodological contribution because the literature contains, as yet, no thorough discussion of the potential role of fairness considerations in the determination of unemployment benefits.

It is a well-documented fact that unemployment benefits (henceforth UBs) are a powerful tool for the provision of *unemployment insurance*, but that they have to be set in a way that takes account of various *incentive problems* (see, among others, Baily, 1978; Easley et al., 1985; Brown and Wolfstetter, 1988; Wadsworth, 1991). In the present paper, we want to investigate another – in fact, complementary – view, namely that the government fixes UBs out of a *concern for distributive fairness*. There are signs that existing UB schemes also pursue objectives other than insurance against the loss of work. Indeed, UBs are, in some countries, linked to a resource condition (see OECD, 1988) [1] and/or are unrelated to past wages (OECD, 1991). [2]

Moreover, UBs are paid out in some countries to persons who have never worked (as in Belgium) or have worked only a few weeks (as in Canada). In addition, even when unemployment programs are purely insurance-oriented and are not concerned with distribution (as in the USA), other complementary programs are.

The question we address is the following: If a government is concerned about fairness, at what level should it set the UB, and how will this optimal UB react to unforeseen changes in the state of the world? We feel this is a very relevant question in the face of current employment problems in Western countries.

[1] These countries are Australia, Austria, Finland, France, Germany, Ireland, the Netherlands, New Zealand, Portugal, Spain and the United Kingdom.

[2] In Australia and the United Kingdom, there is only a lump-sum benefit. In other countries, such as Belgium, Denmark, France, Germany and Canada, the benefit is proportional but only up to a ceiling.

Governments are worried not only about insurance coverage, but also – and perhaps primarily these days – about social problems. It appears to us that social cohesion, especially in a context of economic uncertainty, can be secured only by maintaining a certain degree of fairness. Using the UB system as a tool to reach this goal seems to be a plausible option, and one we intend to analyze here in some detail.

The criterion of fairness which will be adopted is *envy-freeness*, which says that no individual should feel that he would be better off with one of his neighbors' income–leisure bundle than with his own. This criterion has received considerable attention in the axiomatic literature in recent years, and it is worth exploring in a more 'applied' perspective. Moreover, envy-freeness is a good criterion for judging the 'involuntariness' of unemployment. As pointed out for instance in Azariadis and Stiglitz (1983), involuntary unemployment can be viewed as a phenomenon of envy between unemployed and employed workers.

We model a government which sets the UB so as to trade off optimally (from its own point of view) two evils linked to involuntary unemployment. Put differently, the government sets the level of the UB so as to maximize its objective function, which depends (negatively) on two central variables: the intensity of envy between individual agents and the percentage of envious persons, i.e., the unemployment rate. The first of these variables is an indicator of the intensity of individual envy, whereas the second is an indicator of how widespread the envy is. What we are assuming is that the government, of course, cares about employment, but also gives weight to envy in itself: for any given level of employment, it aims at minimizing the *intensity* of envy of the unemployed vis-à-vis the employed.

This decision takes place in a stochastic context: the government's choice of the level of UBs is made under uncertainty about the state of the world, materialized here by an unknown productivity shock. This important aspect of the paper reflects the fact that the UB system aims at compensating those who lose their jobs, without knowing a priori the number of unemployed workers. The uncertainty affecting the unemployment rate is central in the analysis of UB systems.

Moreover, in actual economies, UB decisions are taken for long periods. They are part of an institutional set-up and are not revised after every temporary shock. Rather, casual observation shows that it is taxes more than UBs which adjust to such shocks. This feature of the UB system is captured in our model by the following simplified sequence of decisions. First, the government fixes the UB by maximizing its expected objective function without knowing the particular state of the world. Then, a particular state of the world materializes. Finally, unions and firms negotiate the wage and the level of employment given the UB and the state of the world. Taxes adjust to balance the government's budget. Firms produce and sell on a perfectly competitive market, and individuals consume.

But whereas UBs seem to be unaffected by temporary shocks, European economies appear to have experienced a permanent shift in their unemployment rate. This central feature of the current situation is modelled in our framework by a change in the distribution of shocks. In the face of such a change, governments are likely to reconsider the structure of the UB system, including the level of the benefits. This is reflected, for instance, in the recent analysis of the Swedish situation by Lindbeck et al. (1993). It is thus important to see how a UB system aimed at maintaining fairness would react to permanent changes in the economic environment.

Our main conclusions are the following. There are conditions under which there is a trade-off between the intensity of envy and the expected percentage of envious persons. We then show that, if (alongside a condition on households' utility functions) the government's sensitivity to the intensity of envy is weak enough at low envy and strong enough at high envy, the optimal UB is positive but lower than in the full-insurance case. A strong sensitivity to the intensity of envy at high envy could arise, for instance, if the government is strongly averse to envy or if the level of envy itself is very high when there is no UB. On the contrary, when the government is strongly sensitive to the intensity of envy at low envy (for instance, when it is not interested in reducing unemployment that has become nearly completely voluntary), it chooses the full-insurance UB. We also show that, for a low enough sensitivity to the intensity of envy at equilibrium, the optimal UB is positively correlated with the mean of the technological shock and negatively correlated with the variance. In other words, if economic conditions become more favorable, the optimal UB will actually increase under some patterns of government preferences.

Our essay is organized as follows. In Section 2, we describe the private economic agents and the bargaining framework in which employment and wages are determined. In Section 3, we offer a formalization of the objective function we think a 'fairness-oriented' government might plausibly possess. In Section 4, we discuss the existence of a strictly positive optimal UB without voluntary unemployment. Section 5 studies the influence on this optimal UB of shifts in the probability distribution of the productivity shock. Section 6 discusses various objections and possible extensions.

2. The individuals' optimization

The model we present in this paper has been specified in various particular ways. Making specific choices is necessary in order to get any conclusion. In the final section, we indicate our awareness that other choices could have been made, and we give some idea of how our analysis would be affected.

2.1. The agents

The economy is composed of a continuum of identical firms over [0, 1], each producing the same good. At each point on this continuum, there is a continuum of consumer–workers distributed uniformly over the interval $[0, N]$, each of them indexed by i, supplying their workforce to the corresponding firm. Each firm is owned by one capitalist, indexed by K. There is a decentralized bargaining process: The workers of each firm are all members of a trade union which negotiates with the firm. This symmetric framework allows us to work with a representative firm and a representative union.

All workers have the same utility function $U(\cdot,\cdot)$ defined over consumption and leisure, written as

$$U(c_i, h - x_i),$$

with

$$\frac{\partial U}{\partial c_i} = U_1'(c_i, h - x_i) > 0, \quad \frac{\partial U}{\partial (h - x_i)} = U_2'(c_i, h - x_i) > 0,$$

where $U(\cdot,\cdot)$ is assumed concave, continuous and twice continuously differentiable; c_i denotes i's consumption and x_i his work input. The amount $h > 1$ is the maximum level of available leisure.

We assume that, because of labor indivisibility, $x_i \in \{0, 1\}$, so that labor time is either 1 if a worker is employed or 0 if he is unemployed. This assumption is conceptually useful in order to have a clear split between employment and unemployment, which is central for our purpose. Indeed, UBs are rarely, if ever, paid on a part-time basis.

An employed worker receives a wage w and pays a lump-sum tax t. An unemployed worker receives a benefit b. Denoting income by I, we thus have for each i,

$$I_i = \begin{cases} w - t & \text{if} \quad x_i = 1, \\ b & \text{if} \quad x_i = 0. \end{cases}$$

The first-order condition for consumption is

$$c_i = I_i. \tag{1}$$

Since there are no savings in this model, consumption is equal to income. Each worker will accept to perform 1 unit of labor if the utility of working is no smaller than the utility of being unemployed, i.e. if

$$U(w - t, h^1) \geq U(b, h) \tag{2}$$

where $h^1 (= h - 1)$ denotes the leisure of an employed worker.

The technology displays decreasing returns and is subjected to an exogenous multiplicative shift representing a stochastic productivity component:

$$y = \alpha f(\ell), \quad f'(\ell) > 0, f''(\ell) < 0,$$

where ℓ denotes the number [3] of employed workers. The production function $f(\cdot)$ is assumed continuous and satisfies the so-called Inada conditions:

$$\lim_{\ell \to 0} f'(\ell) = \infty, \quad \text{and} \quad \lim_{\ell \to \infty} f'(\ell) = 0.$$

The stochastic term α is distributed according to a distribution function $G(\cdot \mid \mu, \sigma^2)$ over an interval $[\underline{\alpha}, \overline{\alpha}]$, where μ is the mean and σ^2 the variance. $G(\cdot)$ is continuous on $[\underline{\alpha}, \overline{\alpha}]$, and we shall write $g(\cdot)$ for the associated density function. The term α is the only stochastic component of the model. It therefore describes the state of the world.

The capitalists are risk neutral. For simplicity, we just assume that

$$U_K(c) = c.$$

The first-order condition for K is then simply $c_K = I_K$, and since the capitalist is the sole owner of the firm, $I_K = \alpha f(\ell) - w\ell$. (The role of the capitalists is very limited in this version of the model. Possible extensions are discussed later on in Section 6.)

2.2. The bargaining set-up

The workers are grouped into a trade union which aims at maximizing a function V, defined as the expected utility of its representative member (see Oswald (1985), for a discussion of union utility functions). Using Eq. (1), this can be written as

$$V(w, t, b, \ell) = \frac{\ell}{N} U(w - t, h^1) + \left(1 - \frac{\ell}{N}\right) U(b, h).$$

The bargaining process is similar to the one in McDonald and Solow (1981), but with taxes and UBs added. It aims at reaching a bilaterally efficient outcome in terms of the wage w and the employment level ℓ. The agents take t as given, i.e., they do not internalize the effect of their decisions on the government's budget constraint. This is most simply written as

$$\max_{w,\ell} \beta V(w, t, b, \ell) + \alpha f(\ell) - w\ell$$

subject to $\ell \leq N$ and (2). This latter constraint guarantees that, at the negotiated wage, all workers would agree to perform 1 unit of labor. The parameter β measures the trade union's relative bargaining power. (It also incorporates the

[3] We use the expression 'number of employed (resp. unemployed, envious)' as a substitute for 'size of the employed (resp. unemployed, envious) portion of the interval [0, N].'

scaling operation necessary to make the workers' and the capitalist's utilities comparable.) The first-order condition for the wage implies that either

$$U(w-t,h^1)=U(b,h),$$

yielding an implicit function

$$w=\phi_1(b)+t \quad \text{with} \quad \phi_1'=\frac{U_1'(b,h)}{U_1'(w-t,h^1)}>0, \tag{3}$$

or

$$U_1'(w-t,h)=\frac{1}{\beta} \quad \text{if} \quad U(w-t,h^1)>U(b,h),$$

yielding another implicit function

$$w=\phi_2(\beta)+t \quad \text{with} \quad \phi_2'=\frac{|U_{11}''(w-t,h^1)|}{\beta^2}>0. \tag{4}$$

In other words, if working is preferred to being unemployed, the net wage is an increasing function of union power, whereas if workers are indifferent between employment and unemployment, the net wage is equal to the reservation wage and becomes an increasing function of the UB. These two conditions can be summarized as

$$w-t=\max[\phi_1(b),\phi_2(\beta)]\equiv\phi.$$

We now introduce the following definition:

Definition 1. Let $b^0(\beta)$ the benefit for which $\phi_1(b^0)=\phi_2(\beta)$.

Since $\phi_1'>0$, this implies that $\phi=\phi_1(b)>\phi_2(\beta)$ for all $b\geq b^0$. The amount b^0 is the minimum UB such that unemployment is voluntary. In other words, it is the benefit that makes the workers indifferent between working and not working: b^0 *thus represents the full-insurance benefit.*

The first-order condition for employment yields either $\ell=N$ or

$$\phi+t-\beta[U(\phi,h^1)-U(b,h)]-\alpha f'(\ell)=0. \tag{5}$$

Given a level of the UB and a realization of the productivity shock, this equation defines a decreasing relationship between employment and taxes.

2.3. The government's budget constraint

Since our model is static, the government's budget has to be balanced:

$$t=b\frac{N-\ell}{\ell}, \tag{6}$$

which yields a second decreasing relationship between employment and taxes.

There is a fiscal externality because the union and the firm do not take into account the effect of their decisions on the government's budget constraint. Moreover, there is a 'strategic complementarity' in the sense of Cooper and John (1988) between the union–firm pair and the government. These two characteristics may lead to multiple equilibria which can be Pareto ranked. If the economy gets stuck in a dominated equilibrium, there is a 'coordination failure'. While the purpose of our paper is not to study the potential occurrence of coordination failures in employment and tax determination, we must emphasize that our subsequent analysis will be limited to studying one of potentially numerous equilibria. We also have to assume that at least one equilibrium exists.

Let $\ell^*(b, \alpha, \beta)$ denote one of the solutions of the system (5)–(6). The optimality condition for employment thus becomes

$$\ell = \min[\ell^*(b, \alpha, \beta), N]. \tag{7}$$

This makes it possible to define the realization of the stochastic shock such that the labor force constraint bites:

Definition 2. Let $\hat{\alpha}(b, \beta)$ be such that $\ell^*(b, \hat{\alpha}, \beta) = N$.

2.4. Goods market equilibrium

Total consumption is given by adding up the consumption levels of the capitalists, the workers and the unemployed:

$$c = \alpha f(\ell) - w\ell + (w - t)\ell + (N - \ell)b.$$

A balanced government budget (Eq. (6)) is necessary for the goods market to be in equilibrium, i.e., $c = \alpha f(\ell)$.

2.5. Unemployment benefits and employment

Let us now analyze the effect of UBs on ℓ^*, the employment level when the total labor force is not a binding constraint. This effect will differ according to whether the reservation wage constraint is binding or not. The following notation will be convenient:

Definition 3. Let ξ be an indicator function such that

$$\xi = \begin{cases} 1 & \text{if} \quad w - t = \phi = \phi_1(b) \quad \text{[unemployment is voluntary]} \\ 0 & \text{if} \quad w - t = \phi = \phi_2(\beta) \quad \text{[unemployment is involuntary]} \end{cases}$$

Using this notation, it follows that

Lemma 1. If and only if

$$\alpha|f''(\ell^*)| > \frac{bN}{(\ell^*)^2} \tag{P.1}$$

holds, $\partial\ell^*/\partial b < 0$.

Proof. The derivative of ℓ^* with respect to benefits is

$$\frac{\partial\ell^*}{\partial b} = \frac{\xi\frac{U_1'(b,h)}{U_1'(\phi,h^1)} + (1-\xi)U_1'(b,h) + \left(\frac{N}{\ell^*} - 1\right)}{\frac{bN}{\ell^{*2}} + \alpha f''(\ell^*)}.$$

Given our assumptions about the utility function, this expression is negative if and only if its denominator is negative.

Lemma 1 shows that raising the UB may not go hand in hand with the usual deterioration in employment because (P.1) may fail to hold. In that case, lowering t will lead to an increase in employment in such a way that tax revenues actually increase, allowing a rise in the UB. (This corresponds to a kind of Laffer effect.) If condition (P.1) does hold at the equilibrium, the function $t \propto \alpha f'(l)$ defined by (5) is steeper than the function $t = b(N - \ell)/\ell$, so that raising the UB implies raising the tax level and this has a detrimental effect on employment.

The negative effect of UBs on employment pervades the literature and can flow from a variety of mechanisms (see Atkinson and Micklewright (1991) for a survey). In our model, if (P.1) holds, UBs are detrimental to employment because they raise reservation wages and taxes, leading to a rise in the rate of equilibrium unemployment. To restrict the scope of our analysis, we shall remain in line with the literature and we thus focus our attention on equilibria at which (P.1) holds.

3. The government's problem

3.1. Envy

We now move on to the specification of the government's objective function, which is the central part of our analysis of fairness. Concern for fairness enters the government's objective via the notion of *envy-freeness*, which states that in a fair society individuals should envy one another as little as possible (see, e.g., Foley, 1967; Varian, 1974). More specifically, we assume that the government is sensitive to the magnitude of envy via two indicators taken from the social-welfare literature: the *proportion* of envious persons in the population and the *intensity* of the envy of any given envious person. Let us discuss these two components in turn.

Proportion of envious persons: Feldman and Kirman (1974) were the first authors to study the measure of envy. They proposed the simplest possible measure, which is the number of envious persons. This can be readily applied to our notation. Since $U(b, h) \leq U(\phi, h^1)$, every unemployed worker will, unless $b \geq b^0$, envy every employed worker. Provided $b < b^0$, the number of unemployed $N - \ell$ is thus the number of envious persons, and as a percentage of the population it is simply $(N - \ell)/N$, that is, the unemployment rate. In our model, unemployment is thus considered an indicator of the extent to which envy is widespread in the population. It is, in this relevant sense, *involuntary* unemployment: an unemployed worker would be prepared, despite the fact that he gets a benefit, to trade places with an employed worker.

Intensity of envy: To measure the intensity of envy between two individuals, we adopt the criterion proposed by Chaudhuri (1986). Chaudhuri's definition, as applied to our model, is the following:

Definition 4. Let $\varepsilon(b, \beta) \leq 1$ be such that $U(b, h) = U(\varepsilon\phi, \varepsilon h^1)$.

The scalar ε is an indicator of *distance*. It measures the percentage by which one would have to 'shrink' an employed worker's bundle so that, given the wage, tax and benefit, he is no longer envied by an unemployed worker. Since the utility functions are identical for all workers and hence interpersonally comparable, one might object to our use of Chaudhuri's measure on grounds that it might be much simpler to measure envy through the difference or ratio of utilities. However, this measure, which was initially constructed for ordinal preferences, makes our present model generalizable in subsequent research to the case of heterogeneous agents. Furthermore, note that, if U is linear homogeneous, ε is in fact the ratio $U(\phi, h^1)/U(b, h)$.

It is important to note that ϵ is a counterfactual magnitude, which can be defined even if the allocation associated with it is not feasible. If $\epsilon < 1$, and given the assumption of identical utility functions, all unemployment is involuntary. It is clear that since $w - t$ is equal to the function ϕ defined earlier ($\phi = \max[\phi_1(b), \phi_2(\beta)]$), ε is a function only of b and β and not of α. The relation between ε and b is therefore known with certainty by the government.

We can now measure the intensity of an unemployed's envy as follows:

Definition 5. The measure of the intensity of envy is a function

$$\lambda(b, \beta) = \frac{1}{\varepsilon(b, \beta)} - 1.$$

In particular, let $\bar{\lambda}(\beta) \equiv \lambda(0, \beta)$.

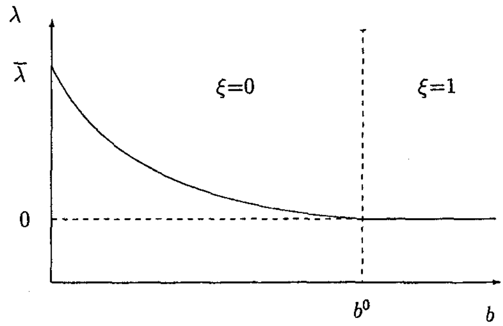

Fig. 1. The intensity of envy as a function of benefits.

This is the measure suggested by Chaudhuri. When $b = 0$, the envy of the unemployed worker is maximal ($\bar{\lambda}$). By definition, the intensity is equal to zero when $b \geq b^0$. More generally, the effect on the intensity of envy of a change in the UB is given by

Lemma 2. An increase in unemployment benefits reduces the intensity of envy as long as unemployment is involuntary:

$$\xi = 0 \Rightarrow \frac{\partial \lambda}{\partial b} < 0 \quad \textit{and} \quad \xi = 1 \Rightarrow \frac{\partial \lambda}{\partial b} = 0.$$

Proof. The derivative of the intensity of envy with respect to benefits is

$$\frac{\partial \lambda}{\partial b} = \frac{-U_1'(b,h)}{\varepsilon^2}\left[\frac{1 - \left[U_1'(\varepsilon\phi, \varepsilon h^1)/U_1'(\phi, h^1)\right]\varepsilon\xi}{U_1'(\varepsilon\phi, \varepsilon h^1) + h^1 U_2'(\varepsilon\phi, \varepsilon h^1)}\right] \leq 0.$$

It is clear that if $\xi = 0$, $\partial\lambda / \partial b < 0$ and if $\xi = 1$ (the reservation-wage constraint becomes binding), ε becomes equal to 1 by definition, implying $\lambda = 0$ and $\partial\lambda / \partial b = 0$.

The evolution of the intensity of envy as a function of UBs is shown in Fig. 1. (For the continuity of this function, see Appendix A.)

The government determines the level of b. Under condition (P.1), Lemmas 1 and 2 imply that both ℓ and λ decrease with b: in its decision about the level of *b, the government thus faces a tradeoff between the intensity of envy and the proportion of envious persons.*

This tradeoff is driven by the presence of unions. Indeed, in efficient bargaining, when the UB increases, the union is prepared to accept a lower employment level. [4]

3.2. The government's objective function

The government is averse to both intense envy, represented by λ, and to widespread envy, represented by $(N-\ell)/N$. We assume the objective function to be multiplicatively separable:

$$S\left(\frac{N-\ell}{N}, \lambda\right) = -\frac{N-\ell}{N}\Omega(\lambda), \quad \text{with} \quad \Omega(0) \geq 0,\ \Omega' \geq 0.$$

This particular form is continuous in ℓ and λ if the function $\Omega(\cdot)$ is continuous, which we shall assume. The main concern of the government is (realistically) the reduction of unemployment, so that the welfare function S reaches an absolute maximum at the full-employment point $\ell = N$. (This allows us to avoid an all too trivial setup in which the government would systematically maximize its welfare by setting $b = b^0$ regardless of the associated level of employment.) But as long as $\ell < N$, the government may consider it equivalent to have very few but very intensely envious persons, or to have numerous but only slightly envious persons. This seems to be a rather intuitive way to model a government's concern for fairness. [5]

Note that S is not a social welfare function. This is discussed further in Section 6. An objection may now be that minimizing envy may lead to a non-Pareto optimal result. Indeed, this is one reason why Diamantaras and Thomson (1990) introduce the Pareto criterion alongside their envy minimization criterion. In our case, however, things are much simpler because, as soon as there is at least one unemployed worker in expected terms (as will be shown to be the case in Lemma

[4] One referee stressed that a similar tradeoff could also be obtained without a union as a result of the moral hazard aspect to unemployment insurance. This requires some heterogeneity in worker preferences. Those who put a relatively high weight on leisure will withdraw from the labor market and thus reduce labor supply. If the draw of α is high, firms may want to employ all available workers and UBs may reduce the pool of available workers. Raising b closer to envy-free levels will at the same time increase unemployment.

[5] Notice that, since λ is an inverse function of ε, the government's welfare function can in some cases be viewed as representing a sensitivity to inequality rather than envy. This is the case, as mentioned earlier, if U is linear homogeneous. In that case, $\Omega \geq 0$ means that S is a decreasing function of ε, which is itself equal to the ratio $U(\phi, h^1)/U(b, h)$, i.e., an index of inequality of welfare. There is thus a possibility to link our analysis to some aspects of the theory of inequality indices, with however one important caveat: even under linear homogeneity, ε is not merely an indicator of *income* inequality, but rather of inequality in the overall quality of life (income plus leisure). Hence, ε represents a more extensive index of inequality than the indices of income inequality traditionally used in much of applied inequality analysis.

3), there is never a Pareto-dominated $b \leq b^0$. This is of course conditional on the structure of the model (bargaining, uncertainty, ...).

We now introduce the following definition:

Definition 6. The government's sensitivity to intense envy (henceforth SIE) is written

$$\omega(\lambda) = \frac{\partial S / \partial \lambda}{S} = \frac{\Omega'(\lambda)}{\Omega(\lambda)}$$

The SIE is the relative variation of the government objective function with respect to λ. It also represents the fraction of the gap between S and its maximum (which here is 0) that is due to the intensity of envy being non-zero. It measures the sensitivity of government utility to a change in the intensity of envy. A particular case arises when $\Omega(0) = 0$: the government is then not interested in unemployment as long as it is entirely voluntary, and it follows that S is also maximal at $\lambda = 0$. At this point, $\omega(\lambda)$ is 'infinite' since any rise in the intensity of envy creates a deviation from maximal government utility.

Before moving on, it is instructive to relate our form of S to the Chaudhuri (1986) model. He chooses to take as an indicator of envy in the society $\Sigma = (N - \ell)\lambda$, that is, the sum of 'individual envies'. (His notation is different from ours. This definition of Σ is adapted to our own model, which is simpler than Chaudhuri's because, as explained earlier, we assume there to be no heterogeneity in preferences.) The government objective function S which we postulate in our paper is rather similar to the Chaudhuri function:

$$S = -\frac{\Omega(\lambda)}{\lambda} \frac{\Sigma}{N}.$$

The only essential difference is that we take S to be non-linear in λ because we want to investigate the role of the government's 'sensitivity' to intense envy. (Chaudhuri implicitly assumes that the SIE is $1/\lambda$.)

3.3. Expectations

Since it does not know the realization of the productivity shock α at the time when it sets the level of b, the government maximizes $\mathrm{E}(S)$, its expected welfare. Since λ is not a function of α, this is written as

$$\mathrm{E}(S) = -\Omega(\lambda) \frac{N - \mathrm{E}(\ell)}{N}, \tag{8}$$

where

$$\mathrm{E}(\ell) = \int_{\underline{\alpha}}^{\hat{\alpha}(b)} \ell^*(b, \alpha)\mathrm{d}G(\alpha) + N\int_{\hat{\alpha}(b)}^{\overline{\alpha}} \mathrm{d}G(\alpha). \tag{9}$$

Let us now report a few simple properties of the function $\mathrm{E}(\ell)$:

Lemma 3. If

$$\forall b \geq 0, \exists \tilde{\alpha} \in [\underline{\alpha}, \overline{\alpha}] \text{ such that } \ell^*(\tilde{\alpha}, b) < N, \tag{P.2}$$

then $\mathrm{E}(\ell) < N$.

Proof. Direct using Eq. (9).

Corollary. *(P.2) implies* $\partial \mathrm{E}(\ell)/\partial b < 0$.

Proof. Direct using Eq. (9) and Lemma 1.

Condition (P.2) is very weak; it means that for every b there exists at least one possible realization of the shock $\tilde{\alpha}$ such that there is unemployment. This amounts to assuming that the distribution g is not 'too generous' in terms of the shocks it generates. Under (P.2), $\mathrm{E}(\ell)$ is always lower than N and is always decreasing in b. In what follows, we shall be assuming (P.2) to hold.

We shall later have use for the following technical result on the government's expected welfare function:

Lemma 4. The function $\mathrm{E}(S)$ *is continuous in b on* $]0, b^0]$.

Proof. See Appendix A.

4. The optimal unemployment benefit

Let us now move to the government's maximization. For any $b > b^0$, the function $\mathrm{E}(S)$ is non-increasing because $\lambda = 0$ while employment decreases due to rising taxes and benefits. Thus, if there exists an optimal benefit $b^* > b^0$, then b^0 is also optimal and indifferent to b^* for the government. Hence we can restrict our analysis to $[0, b^0]$ without loss of generality. [6]

[6] It must be noted that, because of the Inada conditions, Eq. (5) will never yield $\ell^*(b, \alpha) = 0$. Therefore, there is never any risk of having to stop short of b^0 because of financing constraints: even if there are very few workers ($\ell^* > 0$ but close to 0), their real wages will be so high that the bulk of unemployment benefits (equal to $\ell^* t$) will always be financed.

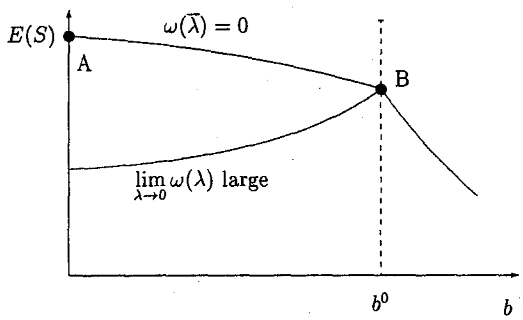

Fig. 2. Expected government utility: Corner solutions.

Since the derivative of Eq. (9) with respect to the integration limit $\hat{\alpha}$ is zero, the first-order condition for the benefit maximizing expected government welfare is

$$\omega(\lambda)[N - \mathrm{E}(\ell)] = \frac{\int_{\underline{\alpha}}^{\hat{\alpha}(b)}[\partial \ell^* / \partial b] \mathrm{d}G(\alpha)}{\partial \lambda / \partial b}. \tag{10}$$

The left-hand side of Eq. (10) is the government's marginal rate of substitution between intense and expected widespread envy. The right-hand side represents the way the private agents' reaction in the bargaining process transforms intense envy (connected with wages) into expected widespread envy (connected with expected employment) in the face of changes in UBs. The equality between the government's marginal rate of substitution and the private marginal rate of transformation is a necessary condition for the optimality of the UB. This is, however, not sufficient to ensure that there exists an optimal benefit located within the *open* interval $]0, b^0[$. Proposition 1 below provides conditions which, if satisfied to a 'sufficient' extent, guarantee an interior optimal benefit. If the conditions of Proposition 1 are not satisfied to a sufficient extent, the economy could end up in one of two situations: either $b = b^0$ and all unemployment is voluntary, or $b = 0$ and envy is at its most intense. Examples of these two extreme situations are presented in Fig. 2.

Proposition 1. There exists $b^* \in]0, b^0[$ *if (P.1) and (P.2) hold and if*

$$\lim_{\lambda \to \bar{\lambda}} \omega(\lambda) > 0 \tag{C.1}$$

and

$$\lim_{\lambda \to \bar{\lambda}} \omega(\lambda) \textit{ large enough and } \lim_{b \to b^0} U_1'(b, h) \textit{ small enough}; \tag{C.2}$$

and / or

$$\lim_{\lambda \to 0} \omega(\lambda) \textit{ small enough and } \Omega \textit{ convex enough}; \tag{C.3}$$

and / or

$$\lim_{\lambda \to 0} \omega(\lambda) \text{ small enough and } \bar{\lambda} \text{ large enough.} \tag{C.4}$$

Proof. See Appendix B.

Condition (C.1) is necessary. The three others are qualitative sufficient conditions. They are complementary, and they will ensure the result if they are satisfied to a 'sufficient' degree. Interpreting them gives a fuller understanding of the mechanisms at work in our model for the fixation of the UB. The four conditions in fact display the various forces which, in our simple model, can combine to diverse extents to drive the optimal UB away from the two extreme values, 0 and b^0. (Fig. 2 shows what could occur if some or all of the four conditions were not satisfied to a sufficient extent.)

Condition (C.1) says that, when the UB is close to zero, so that the intensity of envy is close to its maximum $\bar{\lambda}$, the government has to have a nonzero SIE. If not, $b = 0$ is welfare-maximizing for the government.

Condition (C.2) adds that the optimum has all the more chances to be an interior one if the government's SIE at $\bar{\lambda}$ is very strong – so that it will want to push the UB above zero – , and if, at the other extreme, the unemployed's marginal utility of income in the neighborhood of the full-insurance UB becomes very small – which means that they are close to being saturated at that point, so that a small decrease in the UB below b^0 would have a negligible effect on the intensity of envy, and hence on the government's welfare.

Condition (C.3) says that the optimum has all the more chances to be an interior one if the government's SIE is very small in the neighborhood of b^0 but very large in the neighborhood of 0, due to the strong convexity of the function Ω. This strong convexity can be interpreted as a strong aversion for intense envy.

Condition (C.4) says that a large value of the maximal intensity of envy itself, $\bar{\lambda}$, will add fuel to the forces driving the UB away from 0.

An example where the conditions of Proposition 1 are satisfied to a sufficient extent is given in Appendix C.

Proposition 1 does not require E(S) to be concave. It is possible, however, to show that E(S) is locally concave at the equilibrium:

Lemma 5. Under condition (P.1), the function E(S) *is concave at* b^*.

Proof. If the second derivative is well-defined at the optimum, then it should be negative and E(S) is concave. We just have to prove that the second derivative is well-defined. Tedious calculations show that this is indeed the case under the conditions

$$\alpha \neq \frac{bN}{|f''(\ell^*(b^*))|(\ell^*(b^*))^2} \quad \text{and} \quad \hat{\alpha} \neq \frac{b}{|f''(N)|N},$$

which are satisfied under (P.1).

5. Uncertainty and the sensitivity to envy

Since the presence of uncertainty at the time of the government's decision is a central feature of our framework, we now investigate how the optimal unemployment benefit derived in the previous section is affected by a change in the distribution of the productivity shock. More precisely, it is interesting to analyze how government preferences with respect to envy, summarized in the SIE, determine the reaction of the optimal UB in the face of a change in the mean μ and/or in the variance σ^2 of the distribution.

To answer this question, we need to specify a distribution function. Since the solution of the model is already rather complicated, we shall confine our analysis to the simplest possible function, i.e., a uniform distribution function:

$$G(\alpha) = \frac{\alpha - \underline{\alpha}}{\overline{\alpha} - \underline{\alpha}}, \quad \underline{\alpha} \leq \alpha \leq \overline{\alpha}.$$

We assume that α is low enough to meet (P.2). Notice that $dG(\alpha) = 1/(\overline{\alpha} - \underline{\alpha})d\alpha$, that $\mu = (\overline{\alpha} + \underline{\alpha})/2$ and that $\sigma^2 = (\overline{\alpha} - \underline{\alpha})^2/12$.

We first consider a shift of the distribution consisting in a small variation in $\overline{\alpha}$ and $\underline{\alpha}$ of the same magnitude, let us say $d\mu$, so that the mean of α varies by $d\mu$ and the variance of α remains unchanged. Under these conditions, we have

Proposition 2. Suppose the productivity shocks are distributed uniformly. If the government has a low enough SIE at the equilibrium, the optimal unemployment benefit is positively correlated with the mean of the distribution.

Proof. Under (P.1), $\partial \mathrm{E}(S(b^*))^2/(\partial^2 b)$ is negative by Lemma 5. Differentiating the first-order condition (10) with respect to $\overline{\alpha}, \underline{\alpha}$, and b yields

$$(\overline{\alpha} - \underline{\alpha})\left|\frac{\partial \mathrm{E}(S(b^*))^2}{\partial^2 b}\right|db^*$$
$$= \left[-\omega(\lambda(b^*))(N - \ell^*(b^*, \underline{\alpha})) + \frac{\partial \ell^*(b^*, \underline{\alpha})/\partial b}{\partial \lambda(b^*)/\partial b}\right]d\underline{\alpha}. \qquad (11)$$

Knowing that $d\mu = d\underline{\alpha}$, we obtain

$$\frac{db^*}{d\mu} > 0 \Leftrightarrow \omega(\lambda(b^*)) < \frac{1}{N - \ell^*(b^*, \underline{\alpha})} \frac{\partial \ell^*(b^*, \underline{\alpha})/\partial b}{\partial \lambda(b^*)/\partial b},$$

which completes the proof.

Before giving an interpretation to this result, we consider a change in the variance of the distribution, i.e., a variation in $\overline{\alpha}$ and $\underline{\alpha}$ of the same magnitude, let us say $d\underline{\alpha}$, but in opposite directions. This amounts to modifying the variance while leaving the mean unchanged. Assuming this shift not to violate (P.1) and (P.2), we obtain

Proposition 3. Suppose the productivity shocks are distributed uniformly. If the government has a low enough SIE at the equilibrium, the optimal unemployment benefit is negatively correlated with the variance of the distribution.

Proof. Differentiating the first-order condition (10) with respect to $\overline{\alpha}$, $\underline{\alpha}$ and b yields Eq. (11). Knowing that $d\sigma^2 = -(d\underline{\alpha})^2/3$, we obtain

$$\frac{db^*}{d\sigma^2} < 0 \Leftrightarrow \omega(\lambda(b^*)) < \frac{1}{N - \ell^*(b^*, \underline{\alpha})} \frac{\partial \ell^*(b^*, \underline{\alpha})/\partial b}{\partial \lambda(b^*)/\partial b},$$

which completes the proof.

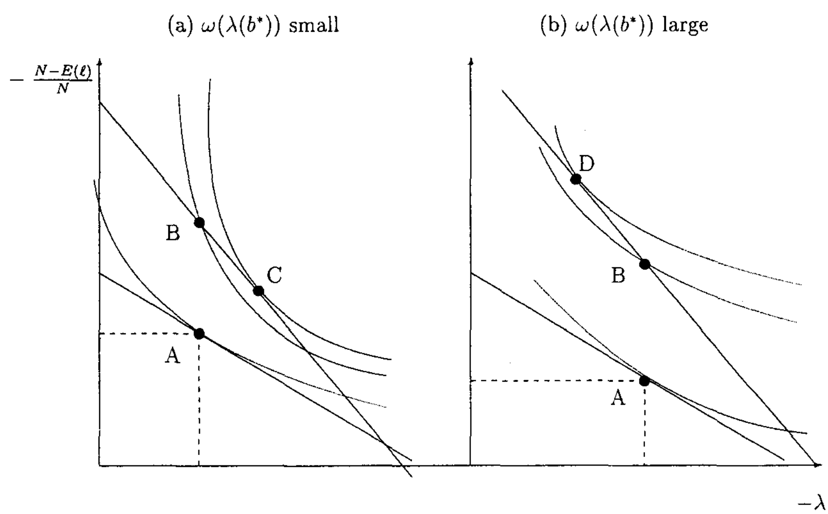

Fig. 3. Effect of distribution shifts.

Propositions 2 and 3 can be better understood by considering Fig. 3:

– We have drawn two possible indifference maps of the government. On the left, the SIE at b^* is small enough to satisfy Propositions 2 and 3. On the right it is too large. The straight lines are a stylized representation of the right-hand side of Eq. (10). Each line gathers the combinations of intense envy and expected widespread envy resulting from the bargaining process (at given β and $G(\alpha)$) for various possible values of b.

– At point A, the government's marginal rate of substitution between intense and expected widespread envy equals the slope of the bargaining reaction function. In the case of a low SIE at b^* (panel (a)), the government's initial optimum is characterized by relatively low unemployment and high intense envy. In the case of a high SIE at b^* (panel (b)), the initial situation is characterized by relatively high unemployment and low intense envy.

– The shift of the line represents a favorable shock, i.e., either an increase in the mean of the distribution of the shocks or a drop in its variance. Such a shock implies higher expected employment for any given value of λ. If the government were to keep b^* constant, the economy would move to point B: the favorable shift of the distribution would profit employment exclusively.

– In case (a), unemployment becomes so low that it is optimal for the government to increase the UB, thus reallocating part of the gain in employment to reduce the intensity of envy. In case (b), the government will amplify the effect of the shock by reducing the UB. In the case of a negative shift, the opposite holds.

– This shows that, when the distribution of the shocks becomes more favorable, the difference between a government with low or high SIE subsides. When the distribution of the shocks becomes less favorable, the difference is exacerbated: the low-SIE government will choose a lower UB, and the high-SIE government will choose a higher UB.

Let us now consider a government whose SIE satisfies Propositions 1, 2 and 3. [7] We then have the following implications.

- Due to the specific bargaining set-up we have chosen (other possible frameworks are discussed in the next section), the wage is not affected by the productivity shock neither directly through the bargaining system, nor indirectly through the government budget constraint (because net wages are not affected by taxes). Although this is a particular case, it is consistent with the very low statistical correlation between observed wages and output (see e.g. Blanchard and Fischer, 1989, ch. 1). Moreover, under the conditions of Propositions 1 and 2, i.e., a low SIE, UBs are positively correlated with the

[7] Technically, this means the following. We focus on a government whose Ω function is such that, at b^*, the conditions of Propositions 2 and 3 hold. Moreover, this function has to be such that, at λ close to zero and close to $\bar{\lambda}$, the conditions of Proposition 1 are also satisfied to a sufficient extent.

mean of the productivity shocks. This implies that the replacement ratio (the ratio of UBs to wages) should vary positively with shifts in the mean. Under the same assumptions, the response of employment to such shifts is affected by the fact that at least part of the 'employment effect' of the shift is offset by the change in UBs.

- With respect to the full-insurance case, it is clear that since wages are not sensitive to productivity and since net wages are not affected by taxes, b^0 is not affected by $d\mu$. This means that even if the full-insurance UB is constant, the envy-minimizing UB is affected by shifts in the distribution of the shocks. Because this envy-minimizing UB reduces the response of employment to such shocks, it is also likely that employment will be less volatile in our model than in the full-insurance case.
- Proposition 3 implies that periods characterized by higher uncertainty should display a tendency for governments to pay lower UBs.

6. Open problems and possible extensions

The aim of this paper has been to make some steps in the direction of an analysis of the optimality of unemployment benefits (UBs) based on the concept of envy-freeness. Using a general equilibrium framework, we have derived the conditions under which there is a trade-off between the intensity of envy and the expected percentage of envious persons. If (alongside a condition on households' utility functions) the government's sensitivity to intense envy (SIE) is weak enough at low envy (λ close to 0) and strong enough at high envy (λ close to $\bar{\lambda}$), the optimal UB is positive and lower than in the full-insurance case. The strong SIE can come from the fact that the government might be strongly averse to intense envy or that the intensity of envy itself might be very high when there is no UB. We have also shown that, for a low enough SIE at equilibrium, the optimal replacement ratio decreases with unfavorable changes in the distribution of the technological shock.

We realize that the predictive power of our results is relatively limited. However, we believe that our model makes an interesting methodological contribution because the literature contains, as yet, no discussion of the potential role of fairness considerations in the determination of UBs. [8] Given this predominantly methodological aspect, it is useful to discuss the specific assumptions we introduced, to point out their role in our results and to assess the relevance of alternative hypotheses.

[8] Two exceptions are Marceau and Boadway (1994) and Phipps (1991).

6.1. Other non-competitive wage formation structures

Due to the specific bargaining set-up we have chosen (efficient bargaining with representative member's expected utility), the wage is not affected by the productivity shock. The interest of this assumption is to give a simple wage reaction function which, in addition, is consistent with the observed non-cyclicity of wages. Of course, it may be desirable to have a wage function affected by productivity. Other wage formation structures would change this result. One may for instance use a Nash product instead of a utilitarian criterion for the efficient solution, use a noncooperative bargaining framework (right-to-manage bargaining) instead of a cooperative framework, or use an efficiency-wage model instead of a bargaining model. All these are a priori possible but will make the analysis of the reaction of UBs to changes in the distribution of shocks substantially more complicated. This is because, in addition to employment, the intensity of envy too will be affected by such changes. Whatever the case may be, the results of Lemmas 1 and 2, which are necessary to get a tradeoff between the intensity of envy and the number of envious persons, are likely to be consistent with a large variety of non-competitive alternative wage formation structures. [9]

It is worth noting, furthermore, that we have assumed that the union is not concerned by envy as such, or more generally about issues of distribution among the workers. As a result, the government's and the union's objectives are of a very different nature. Our rationale for this is that we have wanted to focus on envy minimization as a public objective when private agents behave in a standard fashion. However, introducing fairness considerations in union objectives could also be a fruitful avenue of research (see de la Croix (1994) for a survey on wage envy in bargaining models).

6.2. Other modes of financing of the UBs

Our analysis assumes a balanced budget with lump-sum taxes paid by the workers. Other possible frameworks would include proportional wage taxation, taxation of profits in addition to wages, etc. In these cases, the wage would be affected by any changes in the level of taxes, and the intensity of envy would therefore be affected by changes in the tax base, i.e., by employment.

[9] It is nevertheless possible to build a simple counterexample in which the tradeoff does not arise by removing unions and by keeping the assumption that labor supply is vertical above the reservation wage. Assume that firms commit to a wage equal to the expected marginal product of labor under full employment and that the government sets the UB and taxes to balance the budget in ex-ante, expected terms. Unemployment may result from a bad draw of α. The utility of the unemployed increases with b, but the number of these unemployed is unaffected. It is then always envy-minimizing to set $b = b^0$. However, in this framework, the government budget is not balanced ex-post, so that it might be necessary to introduce money or bonds into the model.

The choice of other wage formation processes or other modes of financing would make individual envy stochastic and therefore unknown by the government at the time of its decision. The government's first-order condition would then include the covariance between aggregate and individual envy, making the analysis richer but also more complicated.

6.3. Heterogeneous workers

The assumption of identical utility functions is rather strong. In particular, does it not take the interest out of the very notion of envy? It is true that with a single good, identical utilities make envy-freeness a bit trivial: everyone should receive the same fraction of the available quantity of the good. With more than one good, however, this is no longer the case. What envy-freeness tells us then is that all individuals should be located *on the same indifference curve*; this leaves considerable room for combining income and labor. Some heterogeneity is lost by assuming a single utility function, but not all of it. Envy still makes sense: there is envy as soon as two individuals are located on two different indifference curves. The fact that we assumed labor time to be indivisible (one either has h or h^1 hours of leisure) additionally restricts us to two points in the commodity space, $(w-t, h^1)$ and (b, h). The comparison of these two points yields our particularly simple measure of envy, λ. If there were many types of preferences, we would have a large number of coefficients λ_{ij} (where i and j designates any pair of preference types), and we would have to aggregate them in one way or another, for instance via the summation rule suggested by Chaudhuri (1986). Defining a set $M=\{(i, j) | \lambda_{ij} \geq 0\}$, this would then yield an envy indicator

$$\lambda = \sum_{(i,j)\in M} \lambda_{ij}.$$

Such an extension could be useful if one wanted to study the design of UBs as a function of individual characteristics. However, making this operational would require identifying, at any particular allocation, the size of the set M, and the analysis would become much more complex. Additional complexity would be introduced through the use of the more elaborate envy measure suggested by Diamantaras and Thomson (1990), which relies on a lexicographic operation on the λ_{ij}'s rather than a summation, thus avoiding the potentially very uneven distributions of envy that Chaudhuri criterion makes possible.

It is certainty worthwhile to analyze the case of heterogeneous agents. However, let us point out one advantage of our homogeneity assumption, beyond its tractability. It turns out that, with identical utility functions, our conclusions apply to other equity notions. We are thinking here, in particular, of the main alternative to envy-freeness, namely egalitarian-equivalence (Pazner and Schmeidler, 1978). Indeed, in this case, the two notions coincide, so that our index λ measures departures from egalitarian-equivalence as well.

6.4. Capitalists included in envy calculation

Another objection is that we have excluded the capitalist agent from the calculation of envy. Could profits not be so low that the capitalists might envy the employed workers, in particular if the union power parameter β is large enough? Is the exclusion of the capitalists not the sign of an unjustified 'dictatorship of the proletariat'? Focusing on solely workers – and thus leaving the capitalists essentially in the role of 'firms' which are neutral in terms of individual welfare – seemed to us relevant enough in the first step represented by this paper. However, including a profitability constraint in the form of envy-freeness would surely have an impact on the level of the UB that can be distributed. However, if N is much larger than 1 (i.e. the density of the capitalist interval is much smaller than that of the worker interval), the impact on government welfare of adding the capitalists is likely to be negligible anyway.

6.5. Risk averse government

In our analysis, we have assumed the government to be risk neutral: its objective function S is linear in the stochastic component ℓ. It might be relevant to assume instead risk aversion in the form of non-linearity in ℓ. One simple way to introduce this would be to slightly modify the objective function into

$$S((N-\ell)\ell, \lambda) = -(N-\ell)\ell\Omega(\lambda).$$

The rationale for such a modified form could be as follows. Instead of being sensitive to widespread envy in the form of the proportion of envious persons as a proportion of the population, the government might, rather, be sensitive to the number of envy relations: if there are $N-\ell$ unemployed, each of them envies each of the ℓ workers, and $(N-\ell)\ell$ is thus the number of 'occurrences' of envy in the society. Thus, slightly modifying the indicator of aggregate envy could allow us to work with a risk averse government. However, this would introduce substantial complications in the derivation of the first-order condition (10).

6.6. Why not a social welfare function?

Another objection that can be raised against our analysis is that the government's objective function, S, is not explicitly derived from the aggregation of individual preferences: it is not, in any relevant sense, a 'social welfare function' (henceforth SWF). In other words, there appears to be no easy way to write

$$-\frac{N-\ell}{N}\Omega(\lambda) = W\big(U(w-t, h^1), U(b, h)\big),$$

where W would be the SWF. Indeed, as we already emphasized when discussing the function S earlier, it is a rather specific function designed to capture the trade-off between two envy magnitudes, the 'number' of envious persons and the 'intensity' of each envious person's envy. To the best of our knowledge, there is nothing in the literature on the derivation of SWF's from envy considerations, and since our focus in this paper is explicitly on the role of UB's in the alleviation *of envy*, we have chosen to neglect the SWF aspect.

However, some may raise an even more fundamental objection: why take envy as an indicator of fairness? Why not assume that the government fixes b on the basis of some *other* criterion of fairness? This, again, is a broad debate which we can address only very sketchily. Rather than maximize S, the government could maximize, for example, the utilitarian SWF (see, e.g., Moulin, 1984):

$$\max_b \mathrm{E}(\ell)U(\phi, h^1) + (N - \mathrm{E}(\ell))U(b, h).$$

This would yield the following first-order condition:

$$(N - \mathrm{E}(\ell))U_1'(b, h) + \int_0^{\hat{\alpha}(b)} \frac{\partial \ell^*(b, \alpha)}{\partial b} \mathrm{d}G(\alpha)\left[U(\phi, h^1) - U(b, h)\right]$$

$$+ \xi \mathrm{E}(\ell)U_1'(\phi, h^1)\frac{\partial \phi}{\partial b} = 0,$$

which would imply $b^* \to \infty$: since the government would no longer care about unemployment, it would raise b (which would lead to a rise in w) as much as possible, even at the cost of having (expected) employment tend to zero. Although this result would be formally coherent with the utilitarian framework, it does not seem appealing to us.

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Appendix A. Proof of Lemma 4

The proof essentially relies on the fact that a differentiable function is continuous. (This is only a *sufficient* condition for continuity, but it will be enough for our purpose.) Let us proceed in steps.

1.a. From Definition 4, it is obvious that

$$\frac{\mathrm{d}\varepsilon}{\mathrm{d}b}=\frac{U_1'(b,h)}{\phi U_1'(\varepsilon\phi,\varepsilon h^1)+h^1U_2'(\varepsilon\phi,\varepsilon h^1)},$$

which is defined for all $b \neq 0$. Hence ε is differentiable on $]0, b^0]$. This implies continuity on $]0, b^0]$.

1.b. As a consequence, from Definition 5, λ is continuous on $]0, b^0]$.

2.a. Define the antiderivative

$$P(b,\alpha)=\int \ell^*(b,\alpha)g(\alpha)\mathrm{d}\alpha.$$

By assumption, g is continuous. Moreover, it is clear from Eq. (5) that

$$\frac{\partial \ell^*}{\partial \alpha}=-\frac{f'(\ell^*)}{bN/\ell^{*2}+\alpha f''(\ell^*)},$$

which is defined for all α under (P.1). Hence under (P.1), $\ell^*(\cdot|b)$ is differentiable and thus continuous in α. Consequently, $P(\cdot|b)$ is also continuous in α.

2.b. This allows us to write

$$\mathrm{E}(\ell)=P(b,\hat{\alpha}(b))-P(b,0)+N\left[G(\bar{\lambda})-G(\hat{\alpha}(b))\right].$$

2.c. G is continuous by assumption, and by definition $G(\bar{\lambda})=1$. By definition also,

$$\frac{\mathrm{d}\hat{\alpha}}{\mathrm{d}b}=-\frac{\partial \ell^*/\partial b}{\partial \ell^*/\partial \alpha}.$$

Since $\partial \ell^*/\partial \alpha > 0$ under (P.1), this is defined whenever $\partial \ell^*/\partial b$ is defined, i.e., for all $b > 0$. (For $b = 0$, the expression $U_1'(b, h)$ may not be finite, so that it is better to exclude this border value of b here.) Consequently, $\hat{\alpha}$ is differentiable, and hence continuous, on $]0, b^0]$.

2.d. Since $\partial \ell^*/\partial b$ exists for all $b > 0$, $\ell^*(\cdot|\alpha)$ is continuous on $]0, b^0]$. Hence, $P(\cdot|\alpha)$ is continuous in b.

2.e. From 2.c and 2.d, $\mathrm{E}(\ell)$ is continuous on $]0, b^0]$.

3. From 1.b and 2.e, $\mathrm{E}(S)$ is continuous on $]0, b^0]$. This completes the proof.

Appendix B. Proof of Proposition 1

Since $\mathrm{E}(S)$ is continuous on $]0, b^0[$ by Lemma 4, it is sufficient to prove that

$$\lim_{b \overset{>}{\to} 0} \mathrm{dE}(S)/\mathrm{d}b > 0 \quad \text{and} \quad \lim_{b \overset{<}{\to} b^0} \mathrm{dE}(S)/\mathrm{d}b < 0.$$

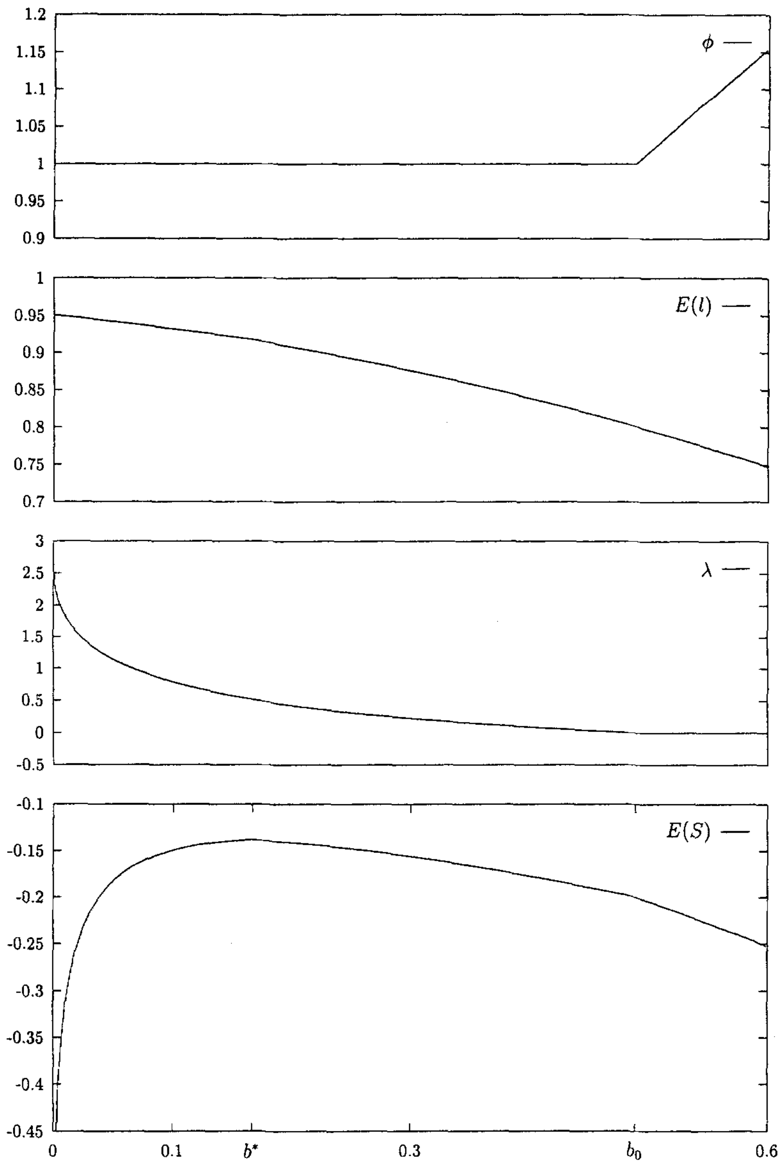

Fig. 4. Example.

After some elementary manipulations, these two conditions read

$$\frac{(1+\bar{\lambda})^2\omega(\bar{\lambda})[N-\mathrm{E}(\ell^*(0,\alpha))]}{\phi_2 U_1'(\phi_2/(1+\bar{\lambda}),h^1/(1+\bar{\lambda}))+h^1U_2'(\phi_2/(1+\bar{\lambda}),h^1/(1+\bar{\lambda}))}$$
$$+\int_{\underline{\alpha}}^{\hat{\alpha}(0)}\frac{1+[(N/\ell^*(0,\alpha))-1]/U_1'(0,h^1)}{\alpha f''(\ell^*(0,\alpha))}\mathrm{d}G(\alpha)>0$$

and

$$\frac{\omega(0)[N-\mathrm{E}(\ell^*(b^0,\alpha))]}{\phi_2 U_1'(\phi_2,h^1)+h^1U_2'(\phi_2,h^1)}$$
$$+\int_{\underline{\alpha}}^{\hat{\alpha}(b^0)}\frac{1+((N/\ell^*(b^0,\alpha))-1)/U_1'(b^0,h^1)}{b^0N/(\ell^*(b^0,\alpha))^2+\alpha f''(\ell^*(b^0,\alpha))}\mathrm{d}G(\alpha)<0.$$

In both inequalities, the integral is negative from condition (P.1). Therefore, a necessary condition for the first inequality to hold is $\omega(\bar{\lambda})>0$. The three other conditions given in Proposition 1, if satisfied to a sufficient extent, will allow the two inequalities to hold simultaneously.

Appendix C. Example

It is easy to build an example of an optimal UB strictly between 0 and b^0. Let us take $U(c_i,h-x_i)=\sqrt{c_i}+3(h-x_i)/10$ and $h=2$. The technology is $y=\alpha\sqrt{\ell}$. Union power is $\beta=2$ and population is $N=1$. Government preferences are $-(1-\ell)e^{\lambda}$. The technological shock can take one of two values: $\Pr[\alpha=1.8]=3/4$, $\Pr[\alpha=2]=1/4$. Numerically, we obtain $b^0=0.49$ and $b^*=0.167$. Fig. 4 displays graphs of the key variables for $0\le b\le 0.6$. The first panel confirms that the net wage is constant up to the full insurance point and then rises monotonically. In the second panel, we see that, as expected from Lemma 1, expected employment decreases with the UB. The third panel shows that the intensity of envy is monotonically decreasing (Lemma 2) and converges to zero for large UBs. Finally the fourth panel displays a strictly concave expected government welfare function, which is a particular case of the result proven in Proposition 1.

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