ELSEVIER

Economics Letters 53 (1996) 89–96

economics letters

The dynamics of bequeathed tastes

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Received 25 March 1996; accepted 25 June 1996

Abstract

If children inherit life standard aspirations from their parents, then their savings are affected and cycles may appear in overlapping generations (OLG) models with production. At some point of an expansion, aspirations grow faster than wages, savings decrease, and a contraction begins.

Keywords: Non-separable preferences; Aspirations; OLG; Cycles; Hopf

JEL classification: E32; O41

1. Introduction

The main objective for studying models which are able to generate endogenous cycles is to set up a theory of fluctuations that can compete with the dominant paradigm, i.e. the linear stochastic model. A variety of mechanisms may be responsible for self-driven oscillatory phenomena. As stressed by Boldrin and Woodford (1990) in their survey, the construction of examples that allow endogenous cycles in the case of empirically realistic mechanisms and parameters is one of the main challenges of this line of research. In particular, a body of this literature has looked for mechanisms that can be responsible for the persistence of fluctuations in the absence of exogenous shocks to fundamentals, in frameworks where agents optimize with perfect foresight.

Using a scalar overlapping generations (OLG) model, Grandmont (1985) shows that complicated cycles may occur if savings are sufficiently decreasing in the interest rate. An extension of this model to account for elastic labor-supply shows that cycles are possible even though savings are not a decreasing function of the interest rate. In this case, the production factors should be highly complementary (Reichlin, 1986). Another planar OLG model is proposed by Farmer (1986) in which cycles arise only if the government pursues a particular policy, namely a policy of fixing the value of the deficit. As in Reichlin (1986), low values of the elasticity of substitution in production are required for cycles to occur.

PII S0165-1765(96)00888-9

Some critics may argue that these examples are of limited interest because they rely on implausible assumptions. The aim of our paper is to provide a further example of limit cycles based on a different mechanism than those already pointed out, namely the fact that the agents may inherit life standard norms from their parents. Indeed, we show that oscillations are the rule rather than the exception in a very simple OLG model in which young workers evaluate their consumption by comparison with what the parents consumed when they were still living with them. The presence of oscillations requires neither a negative effect of the interest rate on savings nor low values of the elasticity of substitution in production. It simply results from the fact that consumption is evaluated within a specific context, i.e. the life standard during the youth. If the effect of these aspirations on utility is strong enough then limit cycles appear.

To stress the role of these bequeathed tastes, our set-up will be such that, in the case of no effect of life standard aspirations on utility, the equilibrium path is always characterized by monotonous convergence to a single steady state.

2. The model

The model is a simple extension of the basic example of a Diamond (1965) economy without outside money. At each date a single good is produced. This good can be either consumed during the period or accumulated as capital for future production. Population grows at a rate n. Each generation lives three periods. The young generation has no decision to take and only inherits life standard aspirations, h_t, from their parents. The adult generation sells one unit of labor inelastically at any real wage, w_t, consumes the quantity c_{1t} and saves s_t for next period consumption by holding capital. The old generation spends all its savings, from the previous period and consumes c_{2t+1}. The intertemporal utility of the typical adult has a specific functional form of the Stone–Geary type:

$$U(c_{1t}, c_{2t+1}, h_t) = (c_{1t} - \gamma h_t)^{\theta}(c_{2t+1})^{1-\theta} ,$$

where $\theta \in]0, 1[$ is a parameter of the utility function and $\gamma \in]0, 1[$ measures the intensity of the effect of the intergenerational spill-over. We thus assume that bequeathed tastes determine a frame of reference against which consumption when adult is judged and that the depreciation rate of aspirations (i.e. forgetting) is high so that they no longer affect the evaluation of consumption when old. A typical adult faces the following problem:

$$\begin{aligned} \max \quad & (c_{1t} - \gamma h_t)^{\theta}(c_{2t+1})^{1-\theta} \\ \text{s.t.} \quad & c_{1t} + s_t \leq w_t , \\ & c_{2t+1} \leq (1 + r_{t+1})s_t , \\ & c_{1t} \geq \gamma h_t, c_{2t+1} \geq 0 , \end{aligned}$$

where r_{t+1} is the real rate of interest. w_t, r_{t+1} and h_t are given to the consumer. Assuming an interior solution, the above decision problem has a unique solution characterized by the following saving function:

$$s_t = (1-\theta)(w_t - \gamma h_t) . \tag{1}$$

Savings do not depend on the interest rate because the utility function is Cobb–Douglas, there is no wage income in the last period of life and aspirations are fully forgotten after one period. We thus avoid the potential negative effect of the interest rate on savings that can be responsible for complex dynamics. Eq. (1) also shows that aspirations affect savings negatively. When aspirations are low, the adult generation has a sober lifestyle and savings are high. When aspirations are high compared with wage income, adults spend much on consumption to maintain a life standard similar to the one of their parents and their propensity to save is low.

Production is made through a Cobb–Douglas constant returns to scale technology. Net output per capita y_t is a function of capital intensity k_t: $y_t = \tau k_t^\alpha - \delta k_t$ with $\tau > 0$ and $\alpha \in]0, 1[$. $\delta \in [0, 1]$ is the depreciation rate of capital. The competitive behavior of firms leads to the equalization of the marginal productivity of each factor to its marginal cost:

$$r_t = \alpha\tau k_t^{\alpha-1} - \delta , \tag{2}$$

$$w_t = (1-\alpha)\tau k_t^\alpha . \tag{3}$$

Finally, aspirations are based on the life standard of the adult of the previous generation. This reflects the idea that children become habituated to a certain life standard when they still live with their parents. Thus,

$$h_t = c_{1t-1} . \tag{4}$$

At date zero, the economy is endowed with a fixed quantity of capital per capita, k_0, and a level of aspirations, h_0. A perfect foresight equilibrium is a sequence $(k_t)_{t>0}$, $(h_t)_{t>0}$ verifying at each date $t \geq 0$

$$(1+n)k_{t+1} = (1-\theta)[(1-\alpha)\tau k_t^\alpha - \gamma h_t] , \tag{5}$$

$$h_{t+1} = \theta(1-\alpha)\tau k_t^\alpha + (1-\theta)\gamma h_t . \tag{6}$$

Eq. (5) is the clearing condition of the asset market, given that the labor market is in equilibrium (i.e. that (3) holds). It reflects the fact that savings are to be equal to the capital stock of the next period. Eq. (5) is the aspiration rule (4) given that the asset and the labor markets are in equilibrium. It appears from the system above that the equilibrium can be characterized by using the following forward dynamic planar map:

$$\begin{pmatrix} k \\ h \end{pmatrix} \mapsto \begin{pmatrix} (1+n)^{-1}(1-\theta)(1-\alpha)\tau k^\alpha - (1+n)^{-1}(1-\theta)\gamma h \\ \theta(1-\alpha)\tau k^\alpha + (1-\theta)\gamma h \end{pmatrix} . \tag{7}$$

This map has two non-negative fixed points, one at $(0, 0)$ and one at $(\bar{k}(\gamma), \bar{h}(\gamma))$, which depends on the parameter of interest, γ, and is defined by

$$\bar{k}(\gamma) = \left(\frac{\tau(1-\theta)(1-\alpha)(1-\gamma)}{(1+n)(1-\gamma(1-\theta))} \right)^{1/(1-\alpha)} , \tag{8}$$

$$\bar{h}(\gamma) = \tau(1-\alpha)\bar{k}^{\alpha} - \bar{k}(1+n)\,. \tag{9}$$

It can be checked that $\partial\bar{k}/\partial\gamma < 0$, implying that the stationary capital stock per head is lower in the economy with bequeathed tastes than in the standard Diamond economy. This is essentially due to the fact that aspirations affect savings negatively. This result holds even if aspirations are not completely forgotten after one period, provided that their effect is stronger on the adult than on the old. The presence of bequeathed tastes introduces here an intergenerational externality, implying that the decentralized equilibrium is sub-optimal compared to the equilibrium that would maximize the planner's utility.[1]

The following proposition establishes that for all possible values of the parameters in their admissible domain, there is only one value of γ in which the fixed point is a non-hyperbolic equilibrium, i.e. in which at least one of the eigenvalues of the Jacobian matrix of the linearized system has unit modulus. In that case, the linear approximation cannot be used to determine stability. In all other cases the proposition establishes that if γ is smaller (resp. greater) than a certain value $\hat{\gamma}$ determined by the parameters α and θ, then the fixed point is stable (resp. unstable). The remaining parameters, i.e. τ, δ and n, have no importance for the determination of $\hat{\gamma}$. This proposition means also that, if the effect of aspirations on utility is strong enough, then this makes the fixed point of the map unstable.

Proposition 1. The fixed point $(\bar{k}(\gamma), \bar{h}(\gamma))$ is hyperbolic if $\gamma \neq \hat{\gamma}$ where

$$\hat{\gamma} = \frac{1+\alpha-\sqrt{(1+\alpha)^2 - 4\alpha(1-\theta)}}{2\alpha(1-\theta)}\,.$$

It is stable if $\gamma < \hat{\gamma}$ and it is unstable if $\gamma > \hat{\gamma}$.

The proof is provided in the appendix. In addition to the result on stability, it is possible to define a domain for γ inside which the dynamics of the system is characterized by oscillations.

Proposition 2. Orbits around $(\bar{k}(\gamma), \bar{h}(\gamma))$ are oscillatory if $\gamma \in \,]\underline{\gamma}, \hat{\gamma}[\, \cup \,]\hat{\gamma}, \bar{\gamma}]$, where

$$\underline{\gamma} = \frac{1}{2} - \sqrt{\frac{\alpha^2\theta}{(\theta-1)^2} + \frac{\alpha(1+\theta)}{2(1-\theta)}} - \frac{1}{2}\sqrt{\frac{(\alpha-1)(\alpha(1+6\theta+\theta^2) - (1-\theta)^2 - 4(1-\theta^2)\sqrt{\alpha^2\theta/(\theta-1)^2}}{(\theta-1)^2}}\,,$$

$$\bar{\gamma} = \frac{1}{2} - \sqrt{\frac{\alpha^2\theta}{(\theta-1)^2} + \frac{\alpha(1+\theta)}{2(1-\theta)}}$$

[1] The computation of this first-best solution and the question of its decentralization by, for example, taxation, is left for future research.

$$+\frac{1}{2}\sqrt{\frac{(\alpha-1)(\alpha(1+6\theta+\theta^2)-(1-\theta)^2-4(1-\theta^2)\sqrt{\alpha^2\theta/(\theta-1)^2}}{(\theta-1)^2}}\,.$$

The proof is provided in the appendix. The values of $\hat{\gamma}$, $\underline{\gamma}$ and $\overline{\gamma}$ are plotted in Fig. 1 as a function of α and θ; the other parameters, n, τ and δ, not affecting these critical values of γ. The whole region below $\hat{\gamma}$ and above $\underline{\gamma}$ is characterized by (local) stable spirals. Notice that the case $\gamma = 0$ corresponds to the basic example of OLG with production, which is characterized by monotonous convergence to the unique stable state (see Azariadis, 1993, p. 203).

Let us now analyze the properties of the model at the non-hyperbolic equilibrium. The following proposition establishes that a Naimark–Sacker[2] bifurcation arises when $\gamma = \hat{\gamma}$. The Naimark–Sacker theorem is one powerful tool for the detection of limit cycles in discrete maps. Indeed, in a neighborhood of the bifurcation point, a limit cycles around $(\hat{k}, \hat{h})$ appears either on the low or on the high side of the critical parameter value.

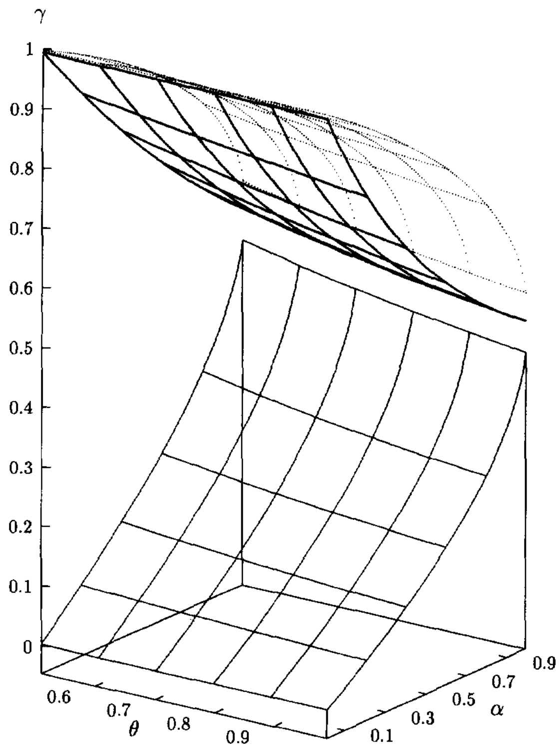

Fig. 1. Bifurcation diagram. Key: $\underline{\gamma}$—; $\overline{\gamma}\cdots$; $\hat{\gamma}$—.

[2] This bifurcation is often called 'Hopf bifurcation for maps'. As stressed by Wiggins (1990), the bifurcation theorem, first elaborated by Poincaré, Andronov and Hofp for the continuous time case, was next extended (which is far from being obvious) to the discrete setting by Naimark (1959) and Sacker (1965).

Proposition 3. (Naimark–Sacker bifurcation). *Let* $(\hat{k}, \hat{h})$ *be the fixed point of the map for* $\gamma = \hat{\gamma}$. *There is a neighborhood* U *of* $(\hat{\gamma})$ *for which there is, either for the case* $\gamma > \hat{\gamma}$ *or for* $\gamma < \hat{\gamma}$, *a closed invariant curve* $\Lambda(\gamma)$ *which encircles* $(\hat{k}, \hat{h})$ *with* $\Lambda(\hat{\gamma}) = (\hat{k}, \hat{h})$.

The proof is provided in the appendix. In addition, it is theoretically possible to check whether the limit cycle (the invariant curve $\Lambda(\gamma)$) appears on the unstable side of the fixed point and is attracting, or whether it encircles the fixed point on the stable side and is repelling. However, the computations involved are very lengthy (see Hale and Koçak, 1991) and the expressions depend on third derivatives of utility and production functions. As is stressed by Farmer (1986), since economic theory does not place a priori restrictions on third derivatives, one might expect that either repelling or attracting cycles may exist. Note also that both cases are of interest, since, according to Grandmont (1985), if the agents' learning mechanism is taken explicitly into account when modeling the dynamic evolution of an economy, the stability result may well be reversed.

3. Interpretation

The spill-over from one generation to the next has two components: (a) savings of the old generation finance the capital stock required to produce and to pay the wages of the young generation; this process that transforms the income/savings of the old into income for the young displays decreasing returns; and (b) past consumption levels of the parents generate life standard aspirations for the young generation, leading them to spend more on consumption; this process displays constant returns. At one point, owing to decreasing returns in the production process, the bequest in terms of higher wages is not sufficient to cover the bequest in terms of higher aspirations. This leads to a drop in savings to maintain the life standard and induces a recession. When the consecutive impoverishment is strong enough, aspirations have reverted to lower levels, allowing a rise in savings and the start of an expansion period. Depending on the relative strength of the two effects and on the current state of the economy compared with its stationary state, this process could either converge, or explode, or lead to ever-lasting cycles.

4. Conclusion

The idea that some norms can be (involuntarily) passed from one generation to the next has, to our knowledge, not been formalized in economics.[3] One exception is in Jones (1984) who analyzes traditions of behaviour within the context of a workplace. Jones analyzes a

[3] The idea that there exist voluntary transfers and exchanges between generations taking place outside markets is generally treated in the literature on altruism, as in Stark (1995). These voluntary transfers could even be designed by the parents so as to obtain something from their children, as in Cremer and Pestieau (1993). Involuntary intergenerational spill-overs are present in the literature on endogenous growth models with human capital. In that case, children inherit a part of their parents' human capital. See Azariadis and Drazen (1990).

model in which "It is through conformism between neighbouring generations that we generate traditions passed down from one generation to the next."

Besides its intergenerational aspects, the idea at the basis of our example reflects the fact that past decisions affect the perception of current outcomes. In the context of consumption, this clearly refers to the models of habit formation initiated by Duesenberry (1949) and developed afterwards by many others. Moreover, there is a strong experimental support for supposing an habituation mechanism by which the most salient events are progressively absorbed into the new baseline against which further events are judged (see Brickman et al., 1978).

Our example shows that the inclusion of this highly realistic assumption in a simple general equilibrium model may generate endogenous oscillations, making interesting further studies on stabilization policies and intergenerational equity in this context. Another interesting extension could be to consider a two-good model in which habituation plays a role only for one good. In that case, the magnitude of the habituation effect depends on the allocation of consumption between the two goods and the intergenerational spill-over is made endogenous.

Acknowledgements

The financial support of the PAC programme 93/98-162 of the Ministry of Scientific Research (French Speaking Community, Belgium) is gratefully acknowledged. I am grateful to R. Anderson, A. d'Autume, P. Dehez, J.-F. Fagnart, P. Michel, P. Pestieau and C. Wampach for their comments on an earlier draft.

Appendix: Proofs of propositions

Proof of Proposition 1 (following Hirsch and Smale, 1974, p. 96, and Azariadis, 1993). The determinant, D, of the Jacobian matrix of the linearization of (7) is $\alpha\gamma(1-\gamma(1-\theta))/(1-\gamma)$. Its trace T is $\gamma(1-\theta)+\alpha\gamma(1-\gamma(1-\gamma(1-\theta))/(1-\gamma)$. In planar maps, non-hyperbolicity may arise only if there is at least one eigenvalue equal to 1 (case (a)), or if there is at least one eigenvalue equal to -1 (case (b)), or if the two eigenvalues are complex conjugates with modulus 1 (case (c)). A necessary condition for case (a) is that $D=T-1$. This may only happen if $\gamma=1/(1-\theta)$, which is excluded given the domain of these parameters. A necessary condition for case (b) is that $D=1-T$. This may only happen if $\gamma=((1-\alpha)\theta \pm \sqrt{4(\alpha+1)^2(1-\theta)+(\alpha-1)^2\theta^2})/(2(1+\alpha)(\theta-1))$, which is also excluded given the domain of the parameters. Case (c) arises if $D=1$ and $T\in[-2,2]$, which is true only if $\gamma=\hat{\gamma}$ (notice that cases in which $\gamma=\hat{\gamma}$, $D=1$ and $T>2$ arise when some parameters take their value outside their domain). Computing the eigenvalues, it appears that their modulus is strictly lower (resp. larger) than one when $\gamma<\hat{\gamma}$ (resp. $\gamma>\hat{\gamma}$) for all possible α and θ. □

Proof of Proposition 2. The discriminant of the characteristic polynomial of the Jacobian matrix is $(\gamma(1-\theta)+\alpha(1-\gamma(1-\theta))/(1-\gamma))^2-4\alpha\gamma(1-\gamma(1-\theta)/(1-\gamma)$ and it is negative if the above condition is verified. In that case, the corresponding eigenvalues are complex numbers, leading to (local) oscillations around $(\bar{k}(\gamma), \bar{h}(\gamma))$. □

Proof of Proposition 3. Following Hale and Koçak (1991), it can be checked that, at the non-hyperbolic equilibrium point $(\hat{k}, \hat{h})$, (i) the discriminant of the Jacobian matrix is negative (i.e. the two eigenvalues are complex conjugates), that (ii) the two non-real eigenvalues cross the unit circle at a non-zero speed when γ changes around $\hat{\gamma}$, and that (iii) none of them may be of the first four roots of unity (excluding cases of weak resonance). These three conditions are sufficient to identify a Naimark–Sacker bifurcation. □

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