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Life expectancy and endogenous growth

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Abstract

We consider an overlapping generations model with uncertain lifetime and endogenous growth. Individuals have to choose the length of time devoted to schooling before starting to work. We show that it depends positively on life expectancy. Moreover, the effect of life expectancy on growth is positive for economies with a relatively low life expectancy, but could be negative in more advanced economies. The positive effect of a longer life on growth could indeed be offset by an increase in the average age of the workers. Dynamics are characterised by a delay differential equation and human capital converges with oscillations to a balanced growth path.

Keywords: Endogenous growth; Human capital; Life expectancy; Lambert W function; Delay differential equation

JEL classification: O41; J10

1. Introduction

Much attention has been paid to the factors responsible for the secular increase in longevity, and the importance of the economic growth process has been stressed (Fogel, 1994). Little interest has been shown on the causal relationship going in the other direction, that is, from increased longevity to economic growth.[1] Still, in their empirical study of the determinant of growth, Barro and Sala-I-Martin (1995) find that life expectancy is an important factor for growth: a 13 year increase in life-expectancy is estimated to raise the annual growth rate by 1.4 percentage points. The authors think that it is likely that life expectancy has such a strong, positive relation with growth as it proxies for features other than good health that reflect desirable performance of a society. There is however

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[1]Exceptions are Ehrlich and Lui (1991) and Grossman and Helpman (1993), p. 122.

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several channels through which life expectancy affects growth directly: for instance, when the probability of dying young is high, the discount rate is also high making it optimal for people to start working early in their life and not to stay at school too long (part of this effect should be captured in Barro and Sala-I-Martin (1995) by the variables ''male/female secondary and higher education''). Moreover, when life expectancy is short, the depreciation rate of human capital is high, making its accumulation more difficult. If the human capital accumulated at school is an important engine of growth, we should thus expect that the growth rate depends upon life expectancy. We thus want to investigate this question in an overlapping generations model à la Blanchard (1985), in which we assume that agents decide the length of time devoted to schooling before starting to work.[2] This contrasts with Lucas (1988) which has the unrealistic implication that people invest a share of their time in education over all their life. Moreover, in our model, the aggregate human capital is built from a sequence of generations having different human capital (''vintage human capital''). The resulting dynamics will be described by a delay differential equation (DDE).[3]

2. The model

Time is continuous and the equilibrium is evaluated from time 0 onward. At each point in time there is thus a continuum of generations indexed by the date at which they are born, t. The set of individuals born in t is assumed to be $[0,\pi]$. Denoting $V_{t,z}$ the set of individuals born in t still living in z, the key assumption is that its measure $\mu(V_{t,z})$ satisfies

$$\mu(V_{t,z}) = \mathrm{e}^{-\beta(z-t)}\pi \quad \pi > 0,\ \beta > 0.$$

The parameter π is the measure of a new cohort. The parameter β can be seen as the rate at which members of a given generation die. Although the measure of each generation declines deterministically through time, each agent is uncertain about the time of his death. For an individual born in t, $\mu(V_{t,z})/\pi$ is the expectancy at time t to live at least until time z. Hence, the probability of living exactly a period of length $z-t$ is $\beta\ \mathrm{e}^{-\beta(z-t)}$ and the life expectancy is $1/\beta$ which does not depend on age. The size of the total population is π/β.

There is a unique material good, the price of which is normalized to 1, that can be used for consumption. This good is produced from a technology using labour as the only input.

An individual born at time t, $\forall t \geq 0$, derives from his consumption stream the following expected utility:

$$\int_t^\infty c(z,t)\,\mathrm{e}^{-(\beta+\theta)(z-t)}\,\mathrm{d}z \quad \theta > 0, \tag{1}$$

[2] Another extension of Blanchard (1985) model to allow for endogenous growth is in Saint-Paul (1992) who assumes an AK technology.

[3] A recent and comprehensive theoretical analysis of DDEs arising from growth models is due to Benhabib and Rustichini (1991).

in which θ is the subjective discount rate. To simplify the resolution of the model, the instantaneous utility function is assumed linear.[4] $c(z, t)$ is consumption of generation t member at time z. We assume the existence of perfect insurance markets. All lending and borrowing contracts between generations are insured by competitive life insurance companies. The intertemporal budget constraint of the agent born in t is:

$$\int_t^{\infty} c(z, t)R(z, t)\,\mathrm{d}z = \int_{t+T(t)}^{\infty} \omega(z, t)R(z, t)\,\mathrm{d}z \quad \text{with} \quad R(z,t) \equiv \mathrm{e}^{-\int_t^z (r(s)+\beta)\mathrm{d}s}, \tag{2}$$

in which $r(s)$ is the risk-free interest rate and $R(z, t)$ is the discount factor. The left-hand side is the expected discounted flow of spending on consumption goods. The right-hand side is the expected discounted flow of earnings. The agent is assumed to go to school until time $t + T(t)$. After this education period, he earns a wage $\omega(z,t)$ per unit of time. Notice that the young agent has to borrow from the older agents to finance his consumption. The life insurance company would pay his debt in case of death.

Wages depend on individual human capital, $h(t)$:

$$\omega(z, t) = h(t)w(z),$$

where $w(z)$ is the wage per unit of human capital. The individual's human capital is a function of the time spent at school $T(t)$ and of the *average* human capital $\overline{H}(t)$ at birth:[5]

$$h(t) = A\ \overline{H}(t)T(t) \quad A > 0. \tag{3}$$

The parameter A is a productivity parameter. The presence of $\overline{H}(t)$ introduces an externality as in Lucas (1988) and Azariadis and Drazen (1990): the cultural ambiance of the society at the time of the birth influences positively the future quality of the agent (through for instance the quality of the school). It is as if each individual receives a private tutor at birth with the average human capital. Education depends on the private tutor's human capital.

The problem of the agent is to select a consumption flow and the duration of his education in order to maximize his expected utility subject to his intertemporal budget constraint and given the aggregate human capital and the sequence of wages and interest rates. The equilibrium condition is

$$r(z) = \theta$$

reflecting the fact that, with a linear utility function, the equilibrium interest rate should be equal to the subjective discount rate at all points in time. This can be established by means of a reductio ad absurdum. If $r(z) > \theta$ at some date z, consumption should be zero for all generations; as current workers are not allowed (by assumption) to leave the labour market to go to school and as consumption cannot be transformed into another good (like capital), a zero consumption level for all

[4] This is a common assumption in general equilibrium models generating a DDE system (with endogenous delays), because it allows for an analytical characterization of the equilibrium delays. See for example Boucekkine et al. (1997a).

[5] We do not explicitly introduce obsolescence of $h(t)$, although this would not change the results as long as the individual's human capital never becomes fully depreciated.

generations would violate the equilibrium condition on the goods market.[6] If $r(z) < \theta$ consumption would be infinite which is again not compatible with goods market equilibrium.

Using this, we may rewrite the discount factor as

$$R(z,t) = \mathrm{e}^{-(\theta+\beta)(z-t)}.$$

The first order condition for $T(t)$ is

$$\int_{t+T(t)}^{\infty} \mathrm{e}^{-(\theta+\beta)(z-t)} w(z)\,\mathrm{d}z = T(t)\,\mathrm{e}^{-T(t)(\theta+\beta)} w(t+T(t)). \tag{4}$$

The left-hand side is the marginal gain of increasing the time spent at school by one unit. The right hand side is the marginal cost, i.e. the loss in wage income if the entry on the job market is delayed.

The production function is assumed to allow one unit of efficient labour to be transformed into one unit of good. Total output is given by

$$Y(t) = H(t), \tag{5}$$

where $H(t)$ is the aggregate stock of human capital. The equilibrium in the labour market thus implies that the wage per unit of human capital is constant through time and equal to one: $w(t) = 1$.

Since wages are constant over time, Eq. (4) becomes

$$T(t) = T \equiv \frac{1}{\theta+\beta}, \tag{6}$$

$\forall t \geq 0$. The optimal time spent on education is thus negatively affected by the instantaneous rate of death β. If $\beta \to 0$ (infinitely lived agents), the optimal time spent on education remains finite and converges to $1/\theta$. Notice that the income of an individual does not grow over time; growth in the economy is linked to the appearance of new generations. Hence, the objective function of an individual is always finite.

The aggregate human capital stock is computed from the capital stock of all generations currently at work:

$$H(t) = \int_{-\infty}^{t-J(t)} \pi\,\mathrm{e}^{-\beta(t-z)} h(z)\,\mathrm{d}z, \tag{7}$$

where $t - J(t)$ is the last generation that entered the job market at t. Function $T(.)$ evaluated at birth gives the interval of time spent at school for any generation. Then, $J(t) = T(t - J(t))$.

To evaluate $H(t)$, for $t \geq 0$, we need to know an entire span of initial conditions for $h(t)$, from $-\infty$ to $J(0)$. However $J(0)$ is endogenously determined at $t = 0$. Let us assume that $J(t) = J_0(t)$, $\forall t < 0$,

[6] In a standard Ramsey model with linear utility, the initial capital stock can be smaller than the corresponding balanced growth value, implying that the interest rate is initially greater than the individual discount factor. Consequently consumption is zero and physical capital accumulates until the balanced growth path is reached, which arrives after a finite number of periods. Consumption then jumps from zero to steady-state. Such ''bang–bang'' solutions cannot happen in our model.

with $\lim J_0(t)_{t\to 0^-} = J_0$, finite and strictly positive. Additionally, we assume $h(t) = h_0(t)$, $\forall t < -J_0$. Generations older than $-J_0$ have already entered the labour market before time $t = 0$.[7]

Concerning existing generations still at school at $t = 0$, two possible cases could arise. If $J_0 \geq T$, all generations $t \in [-J_0, -T[$ enter the labour market at $t = 0$ with an education $T(t) = -t$. Generations $t \in [-T,0[$ choose a schooling time of T and will enter the labour market at period $t + T > 0$. Otherwise, if $T \geq J_0$, all generations $t \in [-J_0,0[$ decide $T(t) = T$ and nobody enters the labour market until time $t = T - J_0 > 0$.

In the next, we assume that $J_0 \geq \frac{1}{(\theta + \beta)}$. From (7) and the initial conditions on $h(t)$ and $J(t)$, we can compute initial conditions for $H(t)$, $\forall t < 0$,

$$H(t) = H_0(t) \equiv \int_{-\infty}^{t-J(t)} \pi \, e^{-\beta(t-z)} h_0(z) \, dz.$$

Using Eqs. (3), (6) and (7), the initial conditions and the fact that $\overline{H}(t) = H(t)\beta/\pi$, we rewrite the aggregate stock of human capital, $\forall t \geq 0$, as:

$$H(t) = H(0) \, e^{-\beta t} + \int_{-T}^{t-T} \beta \, e^{-\beta(t-z)} [A \;\; T \;\; H(z)] dz, \tag{8}$$

where

$$H(0) = \lim_{t\to 0^-} H_0(t) + \int_{-J_0}^{-T} \beta \, e^{\beta z} [A(-z) H_0(z)] dz. \tag{9}$$

As generations $T \in [-J_0, -T[$ enter the labour market together, human capital jumps at $t = 0$, implying that $H(0) > \lim_{t\to 0^-} H_0(t)$.

Differentiating (8) with respect to time, we find the following DDE, $\forall t \geq 0$:

$$H'(t) = AT\beta \, e^{-\beta T} H(t - T) - \beta H(t), \tag{10}$$

with $H(0)$ given by (9). Aggregate human capital decreases at a rate β as time passes and people die. This is compensated by the entry of new generations in the job market. At time t, $\pi \, e^{-\beta T}$ individuals of generation $t - T$ enter the job market with human capital $A \;\; T \;\; H(t - T)\beta/\pi$.

The steady state growth rate of human capital γ is the solution to

$$\gamma + \beta = AT\beta \, e^{-(\beta+\gamma)T}.$$

Solving for γ leads to

$$\gamma = -\beta + \frac{W(T^2 A\beta)}{T} \tag{11}$$

[7] We implicitly assume that people on the labour market never restart school, which is the optimal rule for all generations born at $t = 0$ or later.

where $W(.)$ is the Lambert W function that satisfies $W(z)\,e^{W(z)} = z$, see Corless et al. (1996). Since T is positive, $W(T^2A\beta)$ gives a real solution, amongst other complex solutions, which is unique and positive. As we show next, dynamics depend crucially on the complex solutions. Notice that the growth rate does not depend on the size of the new cohorts π, as the externality has been specified in terms of average human capital.

Using the fact that $\partial W(x)/\partial x = W(x)/(x(1+W(x)))$, the effect of the instantaneous probability of death β on the growth rate γ is given by

$$\frac{d\gamma}{d\beta} = -2 + \frac{\theta}{\beta} + W(T^2A\beta) + \frac{\beta-\theta}{\beta(1+W(T^2A\beta))},$$

and the sign is indeterminate.

Intuitively, the total effect of an increase in life expectancy results from combining three factors: (a) agents die later on average, thus the depreciation rate of aggregate human capital decreases; (b) agents tend to study more because the expected flow of future wages has risen, and the human capital per capita increases; (c) the economy consists more of old agents who did their schooling a long time ago. The two first effects have a positive influence on the growth rate but the third effect has a negative influence.

Notice that the two last effects would still be effective even if we had introduced a fixed retirement age (which does not change with life expectancy) or assumed that human capital becomes fully depreciated after a given age. This is due to the fact that a rise in life expectancy reduces the probability of dying during the activity period.

Numerical computations show that, when life expectancy is below a certain threshold, or when the discount rate is above a certain threshold, the two first effects dominate. From Fig. 1, we observe that the effect of $1/\beta$ on γ is hump shaped. Starting from a situation in which agents have a short horizon (low $1/\beta$), a rise in $1/\beta$ first leads to an increase in the growth rate. After some point, the sign of the effect changes and a rise in $1/\beta$ leads to a drop in γ. From an empirical point of view, we should thus

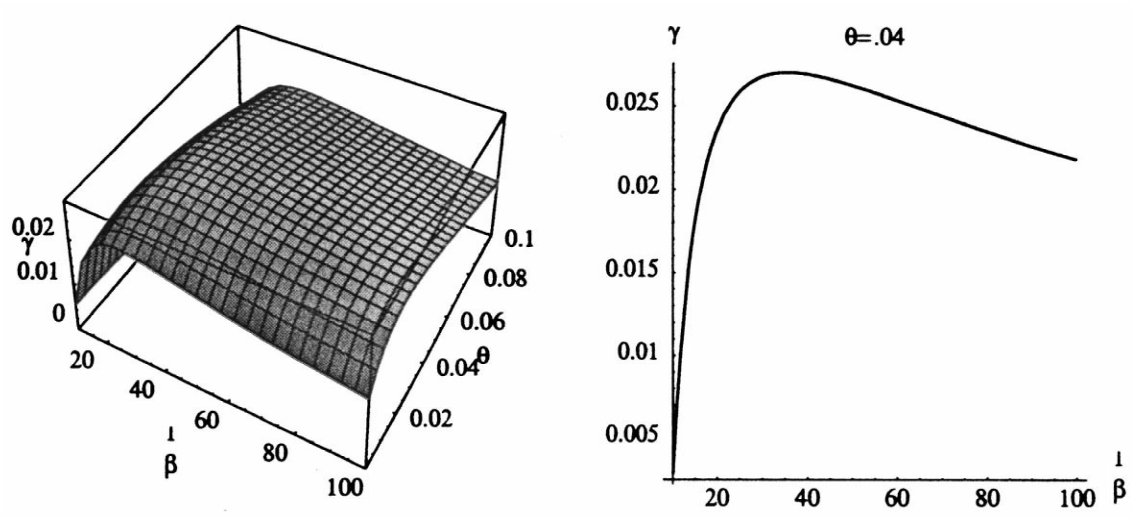

Fig. 1. Life expectancy and steady state growth rate ($A = 0.3$).

observe that the effect of life expectancy on growth is positive for countries with a relatively low life expectancy, but could be negative in more advanced countries.

To study the dynamics of this economy, we define detrended human capital as

$$z(t) = H(t)\,\mathrm{e}^{-\gamma t}.$$

The DDE (10) becomes, $\forall t \geq 0$,

$$z'(t) = (\beta + \gamma)(z(t - T) - z(t)), \tag{12}$$

with $z(0) = H(0)$. To solve it, we follow Bellman and Cooke (1963). We guess that $z(t) = \mathrm{e}^{st}$ is a solution. Then,

$$s = (\beta + \gamma)(\mathrm{e}^{-sT} - 1). \tag{13}$$

If s_k is a solution of Eq. (13), by linearity

$$z(t) = \sum p_k e^{s_k t}.$$

Eq. (12) is identical to the one handled by Boucekkine et al. (1997b). Using the results of the later authors, we know that any root s_k of Eq. (13) has non-positive real part. The only root with zero real part is $s_k = 0$. Then, (12) is asymptotically stable. Moreover, the solution path is oscillatory because except for the origin, the system only admits complex non-real roots.

An example of this oscillatory dynamics is provided in Fig. 2. The initial conditions are $h_0(t) = \mathrm{e}^{\gamma_0 t}$

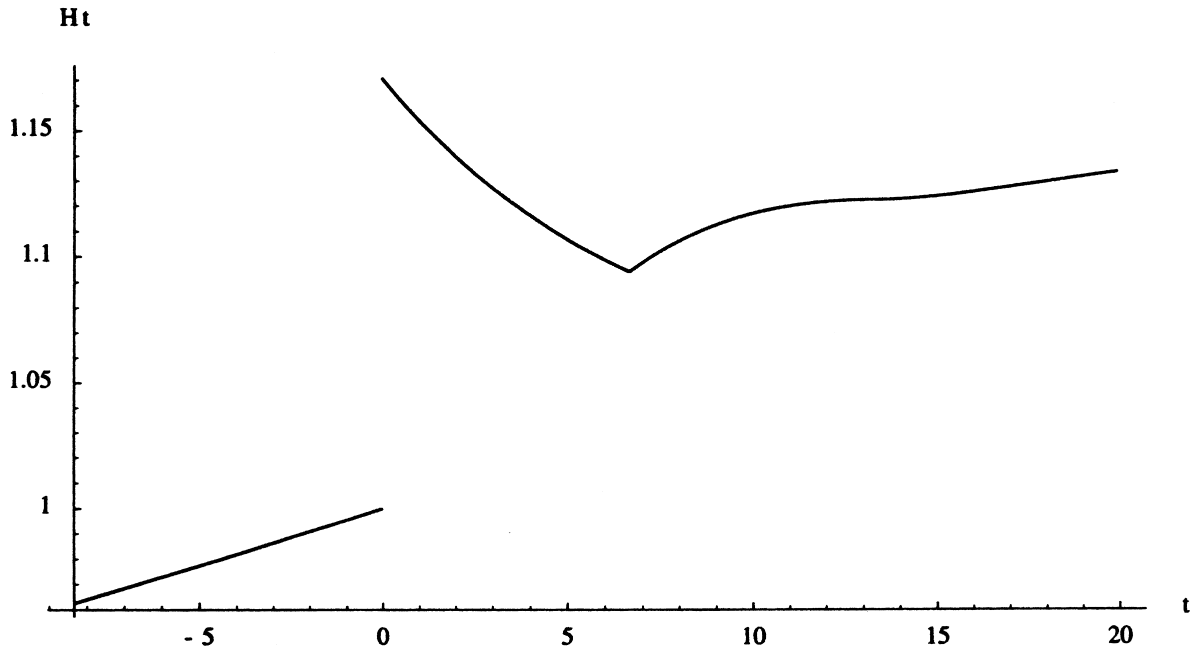

Fig. 2. Dynamics.

in which γ_0 is the stationary growth rate of an economy defined by the parameters (A, β, θ_0), with $A = 0.3$, $\beta = 0.1$ and $\theta_0 = 0.02$. These parameters imply that $\gamma_0 = 0.45\%$ and that the optimal time spent at school is $J_0 = 8.33$. We assume that there is a permanent unexpected change in θ at $t = 0$. The new θ is now equal to 0.05. The new stationary growth rate is $\gamma = 0.16\%$ and the new optimal length of schooling is $T = 6.66$.

This unexpected and permanent increase in θ, at $t = 0$, decreases T instantaneously. This implies that generations from $-J_0$ to $-T$ enter the labour market at $t = 0$, producing a discrete positive jump in the average human capital.[8] New generations (those born just during and after the shock) do not enter the labour market until $t = T$. During the interval $[0,T[$ only old generations enter the labour market and aggregate human capital decreases because initial conditions are relatively low. At time $t = T$, new generations start entering the labour market. As human capital has jump upwards at $t = 0$, new generations enter the labour market with a relatively high human capital and this increases the aggregate stock. After some time, human capital follows its new balanced growth path with a lower growth rate.

This example thus shows that the transitory dynamics from one balanced growth path to another can be characterised by damped oscillations with a discrete jump in the stock of capital at the time of the shock.

3. Concluding comments

When households have to decide the moment at which they will leave school to work, life expectancy is a central factor that affects positively the optimal length of education, and hence, the growth rate of the economy. However, the positive effect of a longer life on growth could be offset by an increase of the average age of the working population.

To derive this result in a simple way, we have made the assumption of a linear utility function and of the absence of physical capital. A further investigation of the role of life expectancy on the dynamics of growth should include a more general utility function and introduce physical capital. This enrichment of the model makes the problem significantly more difficult to solve, because the optimal length of education would be no longer constant over time. The dynamics will then be described by a mixed-delay differential system with endogenous leads and lags (Boucekkine et al., 1997c) propose a shortcut to deal with this type of model). The solution to this problem is on our research agenda.

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[8] This initial condition at $t = 0$ is computed accordingly.

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