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RESEARCH ARTICLE

David de la Croix · Philippe Michel

Education and growth with endogenous debt constraints

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Abstract When future human capital cannot be alienated, households are allowed to borrow up to the point where it is in their own interest not to default. In such a framework, endogenous borrowing limits arise as the outcome of individual rationality constraint. In a model where education is the engine of growth, we show that endogenous borrowing constraints imply global indeterminacy. Comparing outcomes across the various equilibria we show that the relation between growth and yields is hump-shaped. Maximum growth can arise in an equilibrium with binding borrowing constraints, specially if the elasticity of human capital to education spending is large. Deepening financial markets promotes long-run growth in the case of a poverty trap, but not necessarily otherwise. On the methodological side, our approach stresses the importance of studying borrowing limits in general equilibrium, not only in small open economies.

Keywords Financial depth · Borrowing constraints · Indeterminacy · Incentive compatibility

JEL Classification Numbers O410 · 0160 · J240 · D310

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Philippe Michel passed away on July 22, 2004. His death is a great loss for his friends and for the overlapping generations and optimal control community.

D. de la Croix (✉)
Department of Economics and CORE, Université Catholique de Louvain,
Place Montesquieu 3, 1348, Louvain-la-Neuve, Belgium
E-mail: delacroix@ires.ucl.ac.be

P. Michel
GREQAM, Marseilles, France

1 Introduction

The framework proposed by Kehoe and Levine (1993) to model imperfect credit markets is becoming the benchmark to treat borrowing limits and has replaced the old-styled set-ups where an ad-hoc liquidity constraint was imposed on the agents. In their model, default is associated to a penalty consisting in the seizure of the tangible assets of the person who has defaulted. As a consequence, this person will be excluded from financial markets for the time during which his/her assets can be seized.[1]

Defaulting has thus a benefit – not reimbursing the loan, and a cost – of being excluded from financial markets. There is in this context a borrowing limit below which it is in the interest of the households to reimburse. Lending more than this limit would inevitably lead to default. The borrowing limit depends on endogenous variables, including current and future yields. Since endogenous borrowing limits arise as the outcome of individual rationality constraints which prevent individuals from defaulting at equilibrium, enforcement of loan contracts is left to the self-interest of borrowers.

So far, the Kehoe and Levine (1993) concept was mostly used in pure exchange frameworks where savings finance consumption by other agents. For example Azariadis and Lambertini (2003) study an overlapping generations model in which endowments (which can be seen as labor income) are inalienable. Middle-aged households are net borrowers while old-aged households are net lenders. In their set-up, changes in current and future rates of interest affect the borrowing constraints. This mechanism leads to multiple steady states, and to indeterminacy. Moreover, Azariadis and Lambertini (2003) show that complex dynamics are consistent with endogenous debt limits but not with exogenous liquidity constraints. Endogenous debt constraints is not simply a nicer way of modeling credit market imperfections, it makes a real difference compared to exogenous liquidity constraints.

The implications of endogenous borrowing limits for education funding have not yet been analyzed in general equilibrium, and this is the subject of this paper. We aim at modeling carefully credit market imperfections and deriving their role on education funding. This is obviously an important issue, since education is a key factor determining the ability of poor countries to grow, but it is often constrained by current resources, given the inability of students to borrow against future human capital. So far, investment in education subject to exogenous liquidity constraints has been studied by De Gregorio (1996), Buiter and Kletzer (1995), and Azariadis and de la Croix (2006). A first attempt to introduce endogenous constraints is made by Andolfatto and Gervais (2006), but they do so in an small open economy, where prices (wages and interest rates) are exogenous. They show that the usual policy scheme involving education subsidies, income taxes and pensions is welfare reducing, because pensions make borrowing constraints more binding. This is because pensions make savings for old age less useful, thereby reducing the incentive to reimburse the student loan. A richer model with four-period life agents is developed by Lochner and Monge (2002) to quantitatively assess the importance of having

[1] The length of exclusion from financial markets is exogenous in Kehoe and Levine (1993). Bond and Krishnamurthy (2004) go a step further by determining the level of exclusion which is required to sustain active credit markets.

endogenous borrowing constraints instead of exogenous ones in the face of policy changes for the US economy. They conclude that the role of initial wealth and government subsidies is more important when borrowing constraints are endogenous.

On the empirical side, several authors have estimated the importance of borrowing constraints on education decisions. For the USA, none of the methods proposed by Cameron and Taber (2004) produces evidence that borrowing constraints generate inefficiencies in the market for schooling. For developing countries, the picture is quite different. Glewwe and Jacoby (2000) show that borrowing constraints are paramount as far as private schooling expenditures are concerned (Vietnam 1993–1998). Jacoby (1994) provides some evidence of borrowing constraints in Peru. Additional references are provided by Andolfatto and Gervais (2006).

The study of education funding in a Kehoe and Levine (1993) framework gives rise to interesting questions. Having in mind a small open economy, the effect of the exogenous interest rate on education and growth is no longer straightforward, because of its complex effect on the incentive to reimburse the loan. We show that too low interest rates are bad for growth because they lead people to stay away from assets market and not reimburse their loan. Too high interest rates are bad too, because they make credit expensive. In a closed economy where total savings fund education spending, imperfect credit markets are responsible for indeterminacy of the balanced growth paths. The equilibrium where education is set at its individually optimal level may coexist with equilibria where households are credit constrained. Surprisingly, the maximum growth rate is not necessarily achieved in the situation where education is unconstrained.

In a first section we present the model. The study of the incentive constraints is provided in Sect. 3. The optimal education level is determined in Sect. 4. The above results have implications for the relationship between the interest rate and growth which are examined in Sect. 5. Endogenizing the interest rate, equilibrium steady states and dynamics are studied in Sects. 6 and 7. Section 8 discusses the main assumptions of the model, robustness of the results and possible extensions. A last section concludes.

2 The model

Each generation is composed by a continuum of agents of measure one. Population is stationary. The typical household lives for three periods. An agent born in $t-1$ draws utility from consumption when middle-aged c_t and old d_{t+1}:

$$u\left(c_t,\ d_{t+1}\right) \tag{1}$$

We assume that $u(\cdot)$ is increasing in its arguments and concave; it is homogeneous of degree one (homothetic preferences) and satisfies the Inada conditions.

Individuals borrow b_{t-1} amount of good when young to build up their human capital for the next two periods. The workers enjoy h_t unit of human capital when middle-aged and δh_t when old. The parameter δ defines the income growth ability over life, and is determined by different factors: health when old, determining the old-age endowment in efficient labor; retirement age; effect of experience on human capital.

Human capital depends on individual investment b_{t-1} and on the human capital of the previous generation h_{t-1}. This latter effect may reflect either the influence of parents or the society as a whole on education. For analytical tractability we assume a Cobb-Douglas function:

$$h_t = A\ b_{t-1}^{\lambda}\ h_{t-1}^{1-\lambda} \tag{2}$$

where A is a productivity parameter and $0 < \lambda < 1$ is the elasticity of human capital to investment in education. The function is assumed homogeneous of degree one to be consistent with balanced growth. Defining the ratio $e_{t-1} = b_{t-1}/h_{t-1}$, human capital growth is given by:

$$\frac{h_t}{h_{t-1}} = A\ e_{t-1}^{\lambda}.$$

In the above framework, b_{t-1} is understood as a spending on education good. One can alternatively interpret b_{t-1} as a spending on physical capital: in that case, young households build their own firm by investing b_{t-1} in it, and the production of this firm at time t is given by h_t, and δh_t at time $t+1$. The externality then reflects the influence on past production on the productivity of current capital. In this case, though, one needs to assume that future production is inalienable for the individual rationality constraints to make sense. Since human capital better fits the assumption on inalienability, we shall always interpret b_{t-1} as investment in education.

When adult (middle-aged), an agent may choose to repay his load b_{t-1} or not. If he defaults, he is kept out of the credit market for the remaining period of his life. The budget constraints in case of repayment are

$$c_t = h_t - s_t - R_{t-1} b_{t-1} \tag{3}$$

$$d_{t+1} = R_t s_t + \delta h_t \tag{4}$$

h_t and δh_t represent income from labor, where the wage per unit of human capital is constant and equal to 1. R_{t-1} and R_t represents interest factors.

In case of default, the agent is excluded from credit markets, and the budget constraints are

$$c_t = h_t$$
$$d_{t+1} = \delta h_t$$

The problem of the consumer born in $t-1$ is to maximize its utility (1) subject to the human capital accumulation technology (2), the budget constraints (3) and (4) and the following individual rationality constraints:

1. IRC old-age: The middle-age agent are not allowed to borrow because they would never reimburse their debt when old. Hence savings should be non-negative:

$$s_t \geq 0 \tag{5}$$

2. IRC middle-age: The utility of repaying the debt and saving should be larger than the utility obtained from consuming labor income in each periods. This constraint can be written:

$$\max_{s} u(h_t - s - R_{t-1}b_{t-1}, R_t s + \delta h_t) \geq u(h_t, \delta h_t) \tag{6}$$

with h_t given by (2).

The two conditions (5) and (6) are equivalent to the following condition bearing on b_{t-1}:

$$\max_{s\geq 0} u(h_t - s - R_{t-1}b_{t-1}, R_t s + \delta h_t) \geq u(h_t, \delta h_t) \tag{IC}$$

In this constraint, R_{t-1} is observed, while R_t is anticipated.

Firms produce a quantity Q_t of physical goods by using efficient labor L_t as the only input. The production function is linear: $Q_t = L_t$. The productivity of efficient labor is normalized to 1, without loss of generality given the assumptions made on technology and preferences. Perfect competition implies that marginal productivity is equal to marginal cost. The labor market equilibrium requires

$$L_t = h_t + \delta h_{t-1}$$

The assets market equilibrium requires

$$b_{t-1} = s_{t-1}, \tag{7}$$

which determines an endogenous level for the interest rate R_{t-1}.

3 Education choices under incentive constraints

In this section we analyze the incentive constraints in order to determine an upper bound on borrowing which will constrain – or not – the education choice. We temporarily take the interest rates R_{t-1} and R_t as given. In a first step, we analyze the constraint of non-negative savings (5) for given incomes, which amounts to study the indirect utility function. We define the value function of an unconstrained agent (indirect utility function):

$$V(\omega_1, \omega_2, R) = \max_{s} u(\omega_1 - s, \omega_2 + Rs)$$

where ω_1 is the first period income, and ω_2 is the second period income. We also define the value function taking the constraint (5) into account:

$$V^{+}(\omega_1, \omega_2, R) = \max_{s\geq 0} u(\omega_1 - s, \omega_2 + Rs)$$

The following Lemma characterizes the (unconstrained) life-cycle arbitrage conditions under the assumptions made on utility.

Lemma 1 *The life-cycle arbitrage condition*

$$u_1'(c,d) = R u_2'(c,d)$$

is equivalent to

$$d/c = \mu(R),$$

where μ is an increasing, differentiable, function from $\mathbb{R}_{++}$ onto $\mathbb{R}_{++}$: $\mu(0) = 0$, $\mu(+\infty) = +\infty$ and $\mu' > 0$.

Proof The life-cycle arbitrage conditions writes

$$u_1'(1,\rho) = R u_2'(1,\rho),$$

with $\rho = d/c$ (from homogeneity of degree 0 of the first-order derivatives of u). The marginal rate of substitution

$$\mathrm{MRS}(\rho) = \frac{u_1'(1,\rho)}{u_2'(1,\rho)} = \frac{u_1'(1/\rho,1)}{u_2'(1,\rho)}$$

is increasing in ρ (since $u_{11}'' < 0$ and $u_{22}'' < 0$). The Inada conditions imply $\mathrm{MRS}(0) = 0$ and $\mathrm{MRS}(+\infty) = +\infty$. The inverse function $\mu(.)$ of $\mathrm{MRS}(.)$ satisfies the properties of Lemma 1. □

We can now characterize the function V^+ as follows.

Proposition 1 *If $\omega_2/\omega_1 > \mu(R)$, the constraint $s \geq 0$ is binding and $V^+(\omega_1, \omega_2, R) = u(\omega_1, \omega_2)$.*
If $\omega_2/\omega_1 \leq \mu(R)$, savings are given by

$$s^\star = \frac{\mu(R)\omega_1 - \omega_2}{R + \mu(R)} \geq 0,$$

and the function $V^+(\omega_1, \omega_2, R) = V(\omega_1, \omega_2, R)$ is increasing in R in the set $\{\mu(R) > \omega_2/\omega_1\}$.

Proof Optimal savings (maximizing V) can be obtained by solving for s the following identity:

$$\frac{\omega_2 + Rs}{\omega_1 - s} = \frac{d}{c} = \mu(R).$$

We obtain

$$s^\star = \frac{\mu(R)\omega_1 - \omega_2}{R + \mu(R)}.$$

the constraint $s \geq 0$ in the definition of V^+ will not bind when $\omega_1/\omega_2 > \mu(R)$. The function $V^+ = V$ is increasing in R in the set $\{\mu(R) > \omega_2/\omega_1\}$ because $s^\star > 0$, and from the envelope theorem $\partial V/\partial R = s^\star u_2'$. □

Corollary 1 *With the solution $c^\star = \omega_1 - s^\star$, $d^\star = \omega_2 + Rs^\star$ maximizing V, we have:*

$$V(\omega_1, \omega_2, R) = (R\omega_1 + \omega_2)u_2'. \tag{8}$$

Proof Using the homogeneity of u

$$V(\omega_1, \omega_2, R) = u(c^\star, d^\star) = c^\star u_1' + d^\star u_2' = (Rc^\star + d^\star)u_2' = (R\omega_1 + \omega_2)u_2'.$$

□

Given ω_1 and ω_2, Proposition 1 determines a threshold for the interest rate below which constrained savings are zero (unconstrained savings are negative or nil). The Corollary gives a useful link between V and u_2'.

We now turn on attention to the incentive constraint (IC) by making incomes ω_1 and ω_2 explicit: $\omega_1 = h_t - R_{t-1}b_{t-1}$ and $\omega_2 = \delta h_t$. Defining the debt repayment as a share of income as:

$$x_t = \frac{R_{t-1}b_{t-1}}{h_t} = \frac{R_{t-1}}{A}\, e_{t-1}^{1-\lambda}, \tag{9}$$

we have $\omega_1 = h_t(1 - x_t)$ and the constraint (IC) can be written as:

$$V^+(h_t(1 - x_t), \delta h_t, R_t) \geq u(h_t, \delta h_t)$$

which is equivalent to

$$V^+(1 - x_t, \delta, R_t) \geq u(1, \delta). \tag{10}$$

Proposition 2 *The constraint (IC) is equivalent to an upper bound $\bar{x}_t$ on the income share of debt repayment x_t, given by*

$$\bar{x}_t = 1 - g(R_t) \quad \textit{with} \quad g(R_t) = \left[\frac{u(1, \delta)}{u_2'(1, \mu(R_t))} - \delta\right]\frac{1}{R_t}.$$

This upper bound is a function of the future interest factor R_t. It is equal to zero for low values of R_t – satisfying $R_t \leq R_{\min}$ – and it is positive for large values – satisfying $R_t > R_{\min}$ – with $R_{\min}$ such that

$$\mu(R_{\min}) = \delta.$$

The function $g(\cdot)$ and the threshold $R_{\min}$ only depend on preferences and δ.

Proof Applying Proposition 1 to $V^+(1 - x_t, \delta, R_t)$ we observe the following.

If $\mu(R_t) \leq \delta$, then $\delta/(1 - x_t) > \mu(R_t)$ for all $x_t > 0$, and we have $V^+ = u(1 - x_t, \delta) < u(1, \delta)$. Hence the borrowing constraint (IC) defines a maximum borrowing level of $\bar{x}_t = 0$, implying $x_t = 0$.

In the opposite case, when $\mu(R_t) > \delta$, there are positive borrowing levels x_t such that $\delta/(1 - x_t) < \mu(R_t)$, and thus $V^+ = V$. The incentive constraint (10) will determine a borrowing limit $\bar{x}_t$. We rewrite Eq. (10) using the result (8):

$$V = (R_t(1 - x_t) + \delta)u_2'(1, \mu(R_t)) \geq u(1, \delta).$$

This can be expressed as a condition on x_t:

$$(1 - x_t) \geq \left[\frac{u(1, \delta)}{u_2'(1, \mu(R_t))} - \delta \right] \frac{1}{R_t} \equiv g(R_t).$$

The function $g(\cdot)$ allows to compute the borrowing limit:

$$x_t \leq 1 - g(R_t) \equiv \bar{x}_t.$$

It remains to show that the condition $x \leq 1 - g = \bar{x}$ is always more restrictive than the condition $1 - x > \delta/\mu$ for which $V^+ = V$. It is straightforward to prove that at $\bar{x}$ this latter condition holds: Indeed, since $u(1, \mu) = (\mu + R)u_2'(1, \mu)$, with $\mu = \mu(R_t)$ and $R = R_t$,

$$Rg + \delta = \frac{u(1, \delta)}{u(1, \mu)}(\mu + R).$$

Since for $\delta < \mu$ we have

$$\frac{u(1, \delta)}{u(1, \mu)} = \frac{\delta u(1/\delta, 1)}{\mu u(1/\mu, 1)} > \frac{\delta}{\mu}$$

we have $Rg + \delta > (\mu + R)\delta/\mu$ which leads to $g > \delta/\mu$, and thus $V^+ = V$. □

Proposition 2 provides an interesting link between the borrowing constraint and the future interest rate. Small interest rates exclude borrowing and, hence, education spending. Indeed, for small interest rates, households optimal consumption profile will be flatter than their income profile (condition $\delta > \mu(R_t)$), implying that their optimal level of saving is non positive. In this case, it would be optimal for them not to reimburse their education loan. As a consequence, they will not be granted access to borrowing.

The above reasoning gives intuition with the case where the third period individual rationality constraint binds. When the second period individual rationality constraint binds (which is the case on which Azariadis and Lambertini (2003) concentrate on by construction), the importance of δ becomes very transparent for the analysis: when δ is small, say because individuals are retired, then financing education is not a problem. Indeed, with δ small enough, the punishment to default gets very harsh (no consumption when old). As δ increases, defaulting becomes profitable at the point where the gain from additional resources in the default allocation compensates the loss of consumption smoothing.

The above results are robust to the specification of the education technology. According to the chosen education technology, no education spending leads to zero human capital. We could of course consider that there is a positive minimum level $\underline{h}$ of human capital reached even if households do not educate themselves, assuming for example that the human capital production function is given by:

$$h_t = \max \left[\underline{h}, A e_{t-1}^{\lambda} h_{t-1} \right].$$

Since $g(\cdot)$ and $R_{\min}$ do not depend on the education technology, Proposition 2 would still hold.

Using the link between x_t and e_{t-1} given by Eq. (9), the borrowing limit $\bar{x}$ can be translated in terms of education spending $\bar{e}$ through

$$\bar{e}_{t-1} = \left(\frac{A}{R_{t-1}}\bar{x}_t\right)^{\frac{1}{1-\lambda}}. \tag{11}$$

Notice that $\bar{x}_t$ is independent from the education technology while $\bar{e}_{t-1}$ is not.

Let us study the function linking the borrowing limit to the interest rate for large interest rate, $\bar{x}(R) = 1 - g(R)$.

Proposition 3 *The borrowing limit function $\bar{x}(R) = 1 - g(R)$ is increasing from 0 to 1 when R goes from $R_{\min}$ to $+\infty$. Its slope at $R_{\min}$ is equal to 0. For given $R > R_{\min}$, $\bar{x}(R)$ decreases with respect to δ.*

Proof $\bar{x}(R)$ is defined by

$$V(1-\bar{x}, \delta, R) \equiv u(1-\bar{x}-s^\star, \delta + Rs^\star) = u(1, \delta)$$

We can show that $\bar{x}$ is increasing in R, i.e. $g(R)$ is decreasing in R:

$$\frac{\mathrm{d}\bar{x}}{\mathrm{d}R} = \frac{\bar{s}^\star u_2'}{u_1'} = \frac{\bar{s}^\star}{R} > 0,$$

using the envelope theorem for $\partial\bar{s}^\star$. When $R \to R_{\min}$, we have $\bar{x}(R_{\min}) = 0$ and $s^\star(R_{\min}) = 0$. As a consequence of the latter expression,

$$\frac{\mathrm{d}\bar{x}(R_{\min})}{\mathrm{d}R} = 0.$$

To compute the limit of $\bar{x}$ when $R \to +\infty$, we use the fact that the function g is bounded above by the following expression:

$$g < \frac{u(1,\delta)}{Ru_2'(1,\mu(R))} = \frac{u(1,\delta)}{u_1'(1,\mu(R))} = \frac{u(1,\delta)}{u_1'(1/\mu(R),1)}$$

using the life-cycle arbitrage condition and the homogeneity of degree zero of the marginal utility. Since $\mu(+\infty) = +\infty$ and $u_1'(0,1) = +\infty$, we conclude that g goes to zero when R goes to infinity, and

$$\lim_{R\to\infty} \bar{x}(R) = 1.$$

Moreover, given $R > R_{\min}$, $g(R)$ is increasing with respect to δ, i.e.

$$\frac{\partial g(R)}{\partial \delta} = \frac{1}{R}\left[\frac{u_2'(1,\delta)}{u_2'(1,\mu(R))} - 1\right] > 0,$$

since $\delta < \mu(R)$ and $u_2'(1,\delta) > u_2'(1,\mu(R))$. We also have $g(R) \geq 0$ and $g(R) < 1$ since $(R+\delta)u_2'(1,\mu) = V(1,\delta,R) > u(1,\delta)$ thanks to the incentive constraint, which implies

$$R > \frac{u(1,\delta)}{u_2'(1,\mu)} - \delta. \qquad \square$$

From this proposition we conclude that the borrowing limit decreases with the steepness of the labor income profile over time: when future labor income prospects are high, δ is high, households can borrow less.

4 The optimal education level

The *unconstrained optimal level of education* maximizes life-time income. Using the notation in de-trended terms $e_{t-1} = b_{t-1}/h_{t-1}$, the maximization problem can be written:

$$\max_{e_{t-1}} \left(1 + \frac{\delta}{R_t}\right) A e_{t-1}^{\lambda} - R_{t-1}\, e_{t-1}$$

The optimal education is solution to the first order condition:

$$A\lambda (e_{t-1}^{\star})^{\lambda-1} = \frac{R_{t-1}}{1 + \delta/R_t}.$$

This equation determines education spending as depending negatively on both interest rates R_{t-1} and R_t.

Knowing the optimal level of education spending $e_{t-1}^{\star}$ we can compute the corresponding level of reimbursement as a share of income:

$$x_t^{\star} = \frac{R_{t-1}\left(e_{t-1}^{\star}\right)^{1-\lambda}}{A} = \lambda \left(1 + \frac{\delta}{R_t}\right) \equiv x^{\star}(R_t). \tag{12}$$

$x_t^{\star}$ is simply proportional to the factor $(1 + \delta/R_t)$ which transforms human capital into life-cycle income.

To determine the *constrained optimal level of education*, it is sufficient to compare $x_t^{\star}$ to $\bar{x}_t$ to determine whether the unconstrained solution prevails or not. Both $x_t^{\star} = x^{\star}(R_t)$ and $\bar{x}_t = \bar{x}(R_t)$ depend on the next period interest rate. If $x_t^{\star} \leq \bar{x}_t$, we have necessarily

$$\frac{\delta}{1 - x_t^{\star}} \leq \frac{\delta}{1 - \bar{x}_t} < \mu(R_t).$$

In this case savings are positive, $V^{+} = V$ and the optimum satisfies both incentive constraints. On the contrary, if $x_t^{\star} > \bar{x}_t$, the constrained optimal level is $\bar{x}_t \geq 0$.

Accordingly, we can write the constrained optimal level of x_t as

$$x_t = \min\left\{x^{\star}(R_t), \bar{x}(R_t)\right\} \equiv x(R_t). \tag{13}$$

The corresponding growth factor of the economy is:

$$\frac{h_t}{h_{t-1}} = A\, e_{t-1}^{\lambda} \text{ with } e_{t-1} = \min\left\{e_{t-1}^{\star}, \bar{e}_{t-1}\right\}.$$

Proposition 4 *The constrained optimal level of borrowing is given by* (13). *There exists a unique level $\hat{R} > R_{\min}$ equalizing the optimal income share of borrowing to the borrowing limit, i.e.*

$$\bar{x}(\hat{R}) = x^{\star}(\hat{R}) \equiv \hat{x}. \tag{14}$$

The borrowing constraint restricts the education choice of households if and only if $R < \hat{R}$.

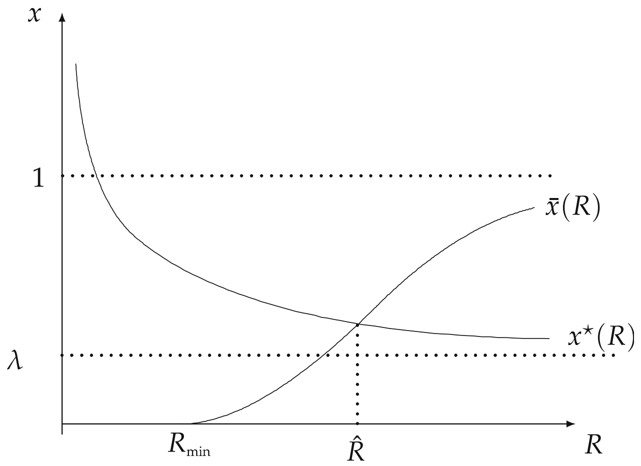

Fig. 1 The functions $\bar{x}(R)$ and $x^{\star}(R)$

Proof From Proposition 3, the function describing the borrowing limit function $\bar{x}(R)$ is increasing from $R_{\min}$ to $+\infty$. The function $x^{\star}(R)$ given by (12) decreases from $+\infty$ to λ when R goes from 0 to $+\infty$. The decreasing function $x^{\star}(R)$ cuts only once the increasing function $\bar{x}(R)$ at some point $\hat{R} > R_{\min}$. □

The two functions $\bar{x}(R)$ and $x^{\star}(R)$ are plotted in Fig. 1.

5 The effect of the interest rate on growth

Proposition 4 characterizes the share of first-period income devoted to the reimbursement of the student loan. In particular, it shows that this share is increasing in R_t in the interval $[R_{\min}, \hat{R}]$ and does not depend on the education technology. Constrained education itself is related to both R_{t-1} and R_t through [from Eq. (11)]:

$$\bar{e}_{t-1} = \left(\frac{A\ \bar{x}(R_t)}{R_{t-1}}\right)^{\frac{1}{1-\lambda}}.$$

To study the effect of interest rate on growth, we consider the case of a constant interest rate $R_t = R\ \forall t$.[2] This leads to:

$$\bar{e} = \left(\frac{A\ \bar{x}(R)}{R}\right)^{\frac{1}{1-\lambda}}.$$

The effect of R on $\bar{e}$ is of the same sign as the effect of R on $\bar{x}(R)/R$. This function admits the limit 0 both for $R \to R_{\min}$ and for $R \to +\infty$. Thus it reaches a maximum at some point $\bar{R} > R_{\min}$. $\bar{R}$ only depends on preferences and δ (since this is the case for $\bar{x}$).

[2] We can see this case as the one of a small open economy.

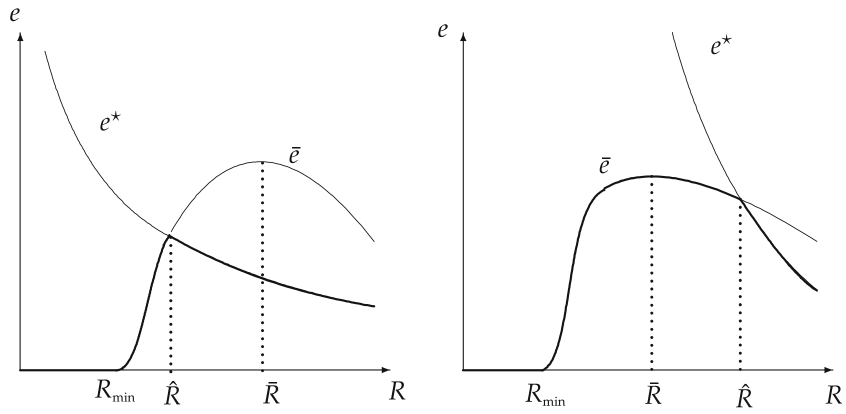

Fig. 2 The maximum growth rate

Let us now consider the constrained growth rate of the economy and look at the value of the interest rate that maximizes growth. We already know that too low interest rates ($<R_{\min}$) will go together with economic stagnation because they are incompatible with borrowing.

The maximum of the constrained growth rate cannot be reached for $R > \hat{R}$, i.e. in the interior of the unconstrained regime, because

$$e^{\star} = \left(\frac{\lambda A(1+\delta/R)}{R}\right)^{\frac{1}{1-\lambda}} = \left(\frac{Ax^{\star}(R)}{R}\right)^{\frac{1}{1-\lambda}}$$

is decreasing in R. Hence, high interest rates are detrimental to growth because they depress optimal education investment. As a consequence, the growth maximizing interest rate is either equal to $\hat{R}$ (optimal investment coincides with the borrowing limit and the borrowing limit reaches a maximum at $\bar{R} \geq \hat{R}$, left panel of Fig. 2), or it belongs to the interval $]R_{\min}, \hat{R}[$, and equals $\bar{R}$ (right panel of Fig. 2). The general result can be stated as follows.

Proposition 5 *If the elasticity of earnings to education, λ, is large enough, the maximum growth rate is attained in the interior of the constrained regime* (*right panel of Fig.* 2). *Otherwise, it is attained at the frontier between the unconstrained and constrained regimes* (*left panel of Fig.* 2). *The lower bound on λ only depends on preferences and δ.*

Proof Notice first that $\bar{R}$ is independent from λ, since it corresponds to the maximum of $\bar{x}(R)/R$, itself independent from λ. The condition $\bar{R} < \hat{R}$ is equivalent to

$$\bar{x}(\bar{R}) < x^{\star}(\bar{R}) = \lambda\left(1 + \frac{\delta}{\bar{R}}\right).$$

This condition is equivalent to a lower bound on λ, which only depends on preferences and δ. □

The higher the value of the elasticity λ, the larger the optimal education, and the more likely is the maximum of the borrowing limit to be binding. This is why, when λ is large, the constrained regime is binding over a large range of values for the interest rate, and the maximum growth rate is achieved in this zone.

5.1 Earnings profile and growth

Let us now consider the effect of another important parameter which determines the slope of the earnings over life, δ. As we already mentioned above, the parameter δ, which defines the income growth ability over life, depends on by health, retirement age, and on the effect of age and experience on human capital. Most of the empirical literature devoted to estimate the impact of education of wages finds that age is an important factor (see Psacharopoulos (1994) for a survey).

If δ increases, optimal investment $x_t^\star$ increases at given R_t Eq. (12):

$$\frac{\partial x_t^\star}{\partial \delta} = \frac{\lambda}{R_t} > 0.$$

According to Proposition 3, when δ is higher, households can borrow less, i.e. rationing will be more likely/severe:

$$\frac{\partial \bar{x}_t}{\partial \delta} = -\frac{\partial g(R)}{\partial \delta} < 0.$$

The effect of δ on growth is therefore uncertain. If the economy is credit-constrained, growth will be hampered by a rise in δ, while if the constraint is not binding, growth will be enhanced by higher δ.

This result may lead to interesting policy implications. For example, a policy designed to postpone the legal retirement age increases the labor endowment during the second period of life, and thereby increases δ. The effect of such a policy is nonetheless uncertain, depending on the extent of borrowing constraints. Indeed, for the households who are credit-constrained, rising the retirement age would further increase the severity of the constraints, because they will be less incited to reimburse their loans if they work longer. Indeed, the penalty of being excluded from financial markets harms them less in that case. In the economy as a whole, if households are credit-constrained, long-run growth can be negatively affected by postponed retirement. This analysis can be enriched if we assume an heterogeneous population of households with different δ as in De Gregorio and Kim (2000) and Azariadis and de la Croix (2006).

6 The steady state curve

In the previous sections we have concentrated our attention on the household decision problem, keeping the interest rate exogenous. We now consider the equilibrium condition on financial markets and look at the implied dynamics. We express the dynamics in terms of the variable R_t starting from the definition of μ:

$$d_{t+1} = \mu(R_t)c_t.$$

Using the budget constraints (3) and (4) and the equilibrium condition (7) we get:

$$R_t b_t + \delta h_t = \mu(R_t)\left((1-x_t)h_t - b_t\right).$$

Dividing by h_t and rearranging, we obtain:

$$\frac{b_t}{h_t} = e_t = \frac{\mu(R_t)}{R_t + \mu(R_t)}\left(1 - x_t - \frac{\delta}{\mu(R_t)}\right).$$

Using the relationship between e and x given by (9) and the definition of the effective education spending (13), the dynamics of the interest rate are described by:

$$x(R_{t+1}) = \frac{R_t}{A}\left(\frac{\mu(R_t)}{R_t+\mu(R_t)}\left[1 - x(R_t) - \frac{\delta}{\mu(R_t)}\right]\right)^{1-\lambda} \equiv \frac{1}{A}\,\phi(R_t, x(R_t)). \tag{15}$$

This relationship holds for all $t \geq 0$. The dynamics of the economy are thus described by a first-order difference equation. R_t is a current variable and R_{t+1} is a forward looking variable. There is no pre-determined variable.[3] Any path satisfying (15) is an equilibrium. There is no requirement in terms of initial condition(s). Hence, steady states are always equilibria.

Any steady state x of Eq. (15) should satisfy:

$$x = \frac{R^{\lambda}}{A}\left(\frac{\mu(R)R}{R+\mu(R)}\left[1 - x - \frac{\delta}{\mu(R)}\right]\right)^{1-\lambda} = \frac{1}{A}\,\phi(R, x). \tag{16}$$

This relationship implicitly defines a function $\tilde{x}(R)$, for $R \geq R_{\min}$ (i.e. $\mu(R) \geq \delta$). which describes the combinations R, x compatible with a steady state. Let us study this function.

Lemma 2 *The steady state function $\tilde{x}(R)$ is increasing from 0 to 1 when R goes from $R_{\min}$ to $+\infty$ and its derivatives satisfies:* $\mathrm{d}\tilde{x}(R_{\min})/\mathrm{d}R > 0$.

Proof The left hand side of (16) is increasing in x, while the right hand side is decreasing in x. Hence $\tilde{x}(R)$ is increasing. We also have $\tilde{x}(R_{\min}) = 0$ and $\tilde{x}(+\infty) = 1$.
To evaluate the slope at $R_{\min}$ we rewrite Eq. (16) as:

$$x^{\frac{1}{1-\lambda}} + xN(R) = D(R)$$

with

$$N(R) = \frac{R^{\frac{\lambda}{1-\lambda}}}{A^{\frac{1}{1-\lambda}}}\frac{R\mu(R)}{R+\mu(R)}$$
$$D(R) = N(R)\left(1 - \frac{\delta}{\mu(R)}\right).$$

[3] This simplication arises because there is no first-period consumption in the model, i.e., children do not decide separately from their parents how much to consume.

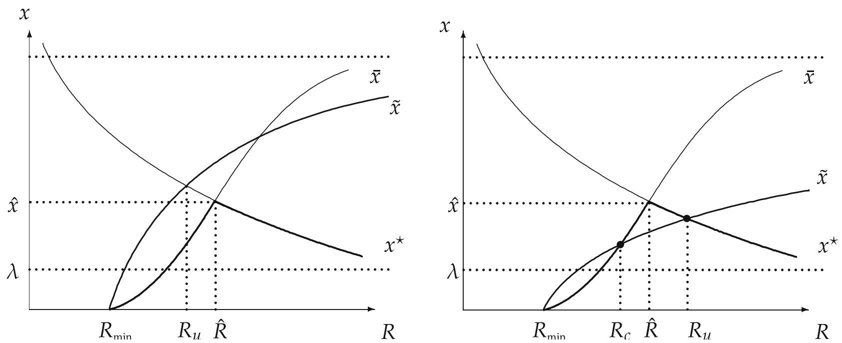

Fig. 3 The steady state curve $\tilde{x}$

Differentiating the function we obtain:

$$\left(\frac{1}{1-\lambda}x^{\frac{1}{1-\lambda}-1} + N(R)\right) \mathrm{d}x = \big(D'(R) - xN'(R)\big)\,\mathrm{d}R$$

Evaluating this expression at $R = R_{\min}$, this simplifies to (since $x = 0$):

$$N(R_{\min})\mathrm{d}x = D'(R_{\min})\mathrm{d}R \quad \text{with } D'(R_{\min}) = N(R_{\min})\frac{\delta\mu'(R_{\min})}{\mu(R_{\min})^2}.$$

This finally leads to (with $\mu(R_{\min}) = \delta$)

$$\frac{\mathrm{d}\tilde{x}(R_{\min})}{\mathrm{d}R} = \frac{\mu'(R_{\min})}{\delta} > 0. \qquad \square$$

We can then analyze graphically the existence of steady states by reporting the functions $\tilde{x}(R)$, $\bar{x}(R)$ and $x^\star(R)$ on the same figure, see Fig. 3. The increasing function $\tilde{x}(R)$ meets the decreasing function $x^\star(R)$ at a unique point R_u. This point is an equilibrium steady state if and only if $R_u > \hat{R} \Leftrightarrow \tilde{x}(\hat{R}) < \hat{x}$. The right panel of Fig. 3 represents a case with $R_u > \hat{R}$; the left panel represents a situation with $R_u < \hat{R}$. When the condition $R_u > \hat{R}$ holds, there also necessarily exists a steady state R_c with $R_{\min} < R_c < \hat{R}$ because the slope of $\tilde{x}(R)$ at $R_{\min}$ is positive (Lemma 2) while the slope of $\bar{x}$ is zero (Proposition 3). One can distinguish the two cases of Fig. 3 by a condition on the parameters, as it will be clear in the next section.

7 Equilibrium dynamics

We first concentrate on the simple case where the equilibrium interest rate is below $R_{\min}$, implying that savings and investment in education are both zero.

Proposition 6 (Dynamics in the poverty trap) *If at date* 0 *all newborn households anticipate an interest factor $R_1 \leq R_{\min}$, their constrained investment will be nil, and the equilibrium R_1 is self-fulfilled.*

Proof At date 0, old households consume their income, $d_0 = R_{-1}s_{-1} + \delta h_{-1}$, where s_{-1} and h_{-1} are part of the initial conditions. Middle-aged households have a given net income $h_0 - R_{-1}s_{-1}$ where human capital h_0 and borrowing b_{-1} are also part of the initial conditions. They choose s_0, c_0, and d_1 observing R_0. Young people chose their borrowing level b_0 observing R_0 (cost of borrowing) and anticipating R_1 (return on their future savings). If they anticipate any $R_1 \leq R_{\min}$, they will borrow $b_0 = 0$. As a consequence, R_0 will ensure $s_0 = b_0 = 0$. For the future, incomes are 0, and $R_1 \leq R_{\min}$ is an equilibrium. □

We now turn our attention to the case with positive savings, and study the two dynamics separately. This will allow us to characterize the local stability of the potential steady states, and to derive a condition on the parameters under which we have $R_u > \hat{R}$, ensuring the existence of non-trivial steady states. To keep the analysis tractable we do not consider dynamics with regime shifts.

From Eq. (15), the dynamics in the unconstrained regime are given by: $x^\star(R_{t+1}) = \phi(R_t, x^\star(R_t))/A$, which defines a function

$$R_{t+1} = \psi^\star(R_t) \quad \text{with} \quad \psi^\star(R) = {x^\star}^{-1}\left(\frac{1}{A}\,\phi(R, x^\star(R))\right).$$

provided that $x^\star(.)$ can be inverted. Since $x^\star$ is invertible on the interval $]\lambda, +\infty[$, the function $\psi^\star(R)$ is defined on $[\hat{R}, +\infty[$ if and only if

$$A < \frac{1}{\lambda}\,\phi(\hat{R}, \hat{x}) \equiv A_\lambda. \tag{17}$$

Proposition 7 (Dynamics in the unconstrained regime) *The function $\psi^\star(R)$ is decreasing. At $\hat{R}$, $\psi^\star(\hat{R})$ is larger than, equal to, or smaller than $\hat{R}$, if A is respectively larger than, equal to, or smaller than $\hat{A}$ with*

$$\hat{A} = \frac{\phi(\hat{R}, \hat{x})}{\hat{x}},$$

with $\hat{x}$ given by Eq. (14). A unique steady state $R_u > \hat{R}$ exists if and only if $A > \hat{A}$. This steady state is unstable.

Proof The function $\phi(R, x^\star(R))$ is defined if

$$D(R) \equiv 1 - x^\star(R) - \frac{\delta}{\mu(R)} = 1 - \lambda\left(1 + \frac{\delta}{R}\right) - \frac{\delta}{\mu(R)} > 0.$$

This inequality holds for $R \geq \hat{R}$. Indeed $D(\hat{R}) = 1 - \bar{x}(\hat{R}) - \delta/\mu(R) > 0$, since $g(\hat{R}) > \delta/\mu(\hat{R})$ by Proposition 2. Let us study the derivative of $\phi(R, x^\star(R))$.

$$\frac{1}{\phi}\frac{\mathrm{d}\phi}{\mathrm{d}R} = \frac{1}{R} + (1-\lambda)\left(\frac{\mu'(R)}{\mu(R)} - \frac{1+\mu'(R)}{R+\mu(R)} + \frac{D'(R)}{D(R)}\right).$$

$D'(R)/R$ is positive and

$$\frac{\mu'(R)}{\mu(R)} - \frac{1+\mu'(R)}{R+\mu(R)} > -\frac{1}{R+\mu}$$

since $\mu'(R) > 0$ and $\mu(R) > 0$. Hence

$$\frac{1}{\phi}\frac{\mathrm{d}\phi}{\mathrm{d}R} > \frac{1}{R} - \frac{1}{R+\mu(R)} = \frac{\mu(R)}{R(R+\mu(R))}.$$

The function $\phi(R, x^\star(R))$ is thus increasing in $[\hat{R}, +\infty)$.

The condition under which $\psi^\star$ is defined in $[\hat{R}, +\infty)$ can be written: $\phi(\hat{R}, \hat{x})/A > \lambda$, i.e. $A < A_\lambda$. Moreover, since $x^\star(R)$ is decreasing and $\phi(R, x^\star(R))$ is increasing, $\psi^\star(R)$ is decreasing. At the point $\hat{R}$ we have $x^\star(\psi^\star(\hat{R})) = \phi(\hat{R}, \hat{x})/A = \hat{A}\hat{x}/A = \hat{A}x^\star(\hat{R})/A$. Hence,

$$\psi^\star(\hat{R}) \gtreqless \hat{R} \Leftrightarrow x^\star(\psi(\hat{R})) \lesseqgtr x^\star(\hat{R}) \Leftrightarrow A \gtreqless \hat{A}.$$

The steady state $R_u > \hat{R}$ exists if $\psi^\star(\hat{R}) > \hat{R}$, i.e. $A > \hat{A}$.

We now study the local stability of R_u. Linearizing $x^\star(R_t{+}1) = \phi(R_t, x^\star(R_t))/A$, we get

$$\frac{1}{x^\star(R_u)}\frac{\mathrm{d}x^\star}{\mathrm{d}R}(R_u)\mathrm{d}R_{t+1} = \frac{1}{\phi}\frac{\mathrm{d}\phi}{\mathrm{d}R}(R_u, x^\star(R_u))\mathrm{d}R_t.$$

Since we have

$$\frac{1}{x^\star}\frac{\mathrm{d}x^\star}{\mathrm{d}R} = \frac{-\delta/R^2}{1+\delta/R} = -\frac{\delta}{R(R+\delta)} \quad \text{and} \quad \frac{1}{\phi}\frac{\mathrm{d}\phi}{\mathrm{d}R} > \frac{\mu}{R(R+\mu)}$$

we can use $\mu(R_u) > \delta$ to obtain

$$\left|\frac{\mathrm{d}R_{t+1}}{\mathrm{d}R_t}\right| > \frac{\mu(R_u+\delta)}{(R_u+\mu)\delta} > 1. \qquad \square$$

We now turn to the dynamics in the constrained regime. They are given by: $\bar{x}(R_{t+1}) = \phi(R_t, \bar{x}(R_t))/A$, which defines a function

$$R_{t+1} = \bar{\psi}(R_t) = \bar{x}^{-1}\left(\frac{1}{A}\,\phi(R, \bar{x}(R))\right).$$

Since $\bar{x}$ is invertible on the interval $]0, 1[$, the function $\bar{\psi}(R)$ is defined on $[R_{\min}, \hat{R}]$ if and only if

$$A > \phi(\hat{R}, \hat{x}) \equiv A_1. \tag{18}$$

Proposition 8 (Dynamics in the constrained regime) *The function $\bar{\psi}(R)$ is increasing. It satisfies $\bar{\psi}(R_{\min}) = R_{\min}$ and $\bar{\psi}(R) > R$ for R near $R_{\min}$. At $\hat{R}$, $\bar{\psi}(\hat{R})$ is larger than, equal to, or small than $\hat{R}$, if A is respectively smaller than, equal to, or larger than $\hat{A}$. A largest steady state $R_c < \hat{R}$ exists if $A > \hat{A}$. This steady state is stable.*

Proof We have that

$$g(R) = \left[\frac{u(1,\delta)}{u(1,\mu)}(R+\mu) - \delta\right]$$

is defined for $R \geq R_{\min}$ from Proposition 2. We deduce from

$$\frac{\mu R}{R+\mu}\left(g(R) - \frac{\delta}{\mu}\right) = \frac{\mu u(1,\delta)}{u(1,\mu)} - \delta = \frac{u(1,\delta)}{u(1/\mu,1)} - \delta$$

that

$$\phi(R,\bar{x}(R)) = R^{\lambda}\left(\frac{u(1,\delta)}{u(1/\mu(R),1)} - \delta\right)^{1-\lambda}.$$

The function $\phi(R,\bar{x}(R))$ is thus increasing in $[R_{\min},\hat{R}]$ and $\phi(R_{\min},\bar{x}(R_{\min}))=0$. The function $\bar{\psi}$ is thus defined and increasing in $[R_{\min},\hat{R}]$ if and only if $1/A < \phi(\hat{R},\hat{x}) < 1$, i.e. $A > \hat{A}$. In a neighborhood of $R_{\min}$, for $R > R_{\min}$, we have $\tilde{x}(R) > \bar{x}(R)$ by Lemma 2. We deduce that $\bar{x}(\psi(R)) = \phi(R,\bar{x}(R))/A > \phi(R,\tilde{x}(R))/A = \tilde{x}(R) > \bar{x}(R)$, since $\phi(R,x)$ is increasing with respect to x. Hence $\psi(R) > R$. For $A > \hat{A}$ the increasing curve $\bar{\psi}(R)$ starts from $R_{\min}$ with $\psi(R) > R$ and ends at $\psi(\hat{R}) < \hat{R}$. Hence, it crosses the 45 degrees line at least once. At the largest intersection point, R_c, the slope is smaller than 1. Hence, $R_{\min}$ is locally unstable, and R_c is locally stable. □

7.1 Interpretation

Propositions 7 and 8 define a threshold value for the productivity of education $\hat{A}$. Actual productivity can be either above or below. When $A < \hat{A}$, (right panel), the productivity of education is weak, and there is no non-trivial steady state. The economy does not grow. This is the standard inescapable poverty trap result, see for example de la Croix and Michel (2002).

In the case on the left, the productivity index of the learning technology A is large and there are two non-trivial steady states. R_c is in the constrained regime and it is locally stable, implying that there is an infinite number of trajectories converging to it (local indeterminacy). In addition to this steady state, there is R_u at which investment is unconstrained. R_u is locally unstable, hence there is a unique trajectory leading to it, the one amounting to select R_u from the initial period onward. On the whole, there is global indeterminacy since the equilibrium can either be any $R < R_{\min}$ or any trajectory leading to R_c, or R_u from date 0 onward.

Comparing outcomes across the various equilibria we can use the results derived in Sect. 5 and show that the relation between growth and the interest rate is hump-shaped. According to Proposition 5, maximum growth can be observed in an equilibrium where borrowing constraints are binding, specially if the elasticity of human capital to education is large.

We can now investigate what would happen if financial markets are suddenly made perfect and there is no longer borrowing constraints. Looking at Fig. 3, this experiment amount to remove the curve $\bar{x}$ from the picture. Then, using Lemma 2, It is straightforward to show that there is one point for which $\tilde{x} = x^{\star}$. This steady state is by Proposition 7 unstable. This implies that when there are no borrowing limits, there is a unique equilibrium converging instantaneously to the steady state. At this "perfect market" steady state, growth is higher than in the poverty trap, but not necessarily than in the "imperfect market" constrained steady state. Considering the two cases of Fig. 3 and the condition on A that separates them (Propositions 7 and 8), it appears that moving towards perfect markets promotes growth for sure in the case on the left (poverty trap, A low enough), while not necessarily in the case on the right (global indeterminacy).

8 Possible extensions and robustness

The model we have studied is very stylized, and simplifying assumptions were needed to obtain a sharp analytical characterization of the results. Compared to the existing literature, this characterization is one important contribution of the paper. Adding more realistic features to the model would generally not alter the main properties of the equilibrium but would often require to rely on numerical methods to solve for equilibrium. In this section we discuss several possible extensions.

8.1 Life-cycle modeling

We have here abstracted from young-age consumption, as it is common in the literature where households borrow to finance education (Michel 1993; Boldrin and Montes 2005). This amounts to assume that the consumption of the young is included in the one of his parents. If, on the contrary, we had assumed that households have preferences defined over consumption when young, middle-aged, and old, they would need to borrow non only to finance their education spending but also to consume when young. This would make the equation describing the dynamics of the interest rate (15) depending on R_{t-1}. The dynamics of the economy would now be described by a second-order difference equation, with one predetermined variable, R_{t-1}, one current variable and one forward looking variable. Most of the results of the paper are expected to hold in this set-up, with two differences: first, the borrowing limit is expected to be binding more often, since part of the loan will be consumed instead of being used to built reimbursement capacity (future human capital). Second, the indeterminacy result in the poverty trap regime would not survive to the inclusion of a predetermined variable in the dynamics, but the global indeterminacy result would still hold, as shown in Azariadis and Lambertini (2003) in a world without education but with first period consumption.

Another assumption made to simplify the model and obtain a general characterization of the results is to limit the life of households to three periods, implicitly of the same length. This has the undesirable consequence that young workers use the financial sector – conditional on they having repaid the education loan – to save, while it seems more plausible that young workers repay education loan to gain access to credit. To capture the different behaviors of young workers-borrowers and old workers-savers, one would need a model with an additional period of life, which would imply to rely on numerical simulations to solve the model. In the present set-up, we aggregate young borrowers and old savers into one category of middle-aged net savers.

8.2 Physical capital

Another simplifying assumption we have introduced is the absence of other forms of capital, such as physical capital. With physical capital, the equilibrium condition on the asset market (7) would become:

$$b_{t-1} + K_t = s_{t-1}$$

where K_t is the stock of capital which is productive at t. The above equations reflects that the two forms of capital – human and physical capital – compete for funding.

Introducing physical capital also requests to discuss the production function which will combine human and physical capital to produce the final good. Assuming a function $F(H_t, K_t)$ which is homogeneous of degree one, we can rewrite output per unit of human capital as $f(k_t)$ where $k_t = K_t/H_t$. The optimization problem of the firm will lead to equalize wages w_t and interest rate R_t with marginal productivity:

$$\begin{aligned} w_t &= f(k_t) - k_t f'(k_t) \\ R_t &= f'(k_t) \end{aligned}$$

As long as we reason at a given interest rate (say in a small open economy), the inclusion of physical capital would not change the results. Hence, all the results from Sects. 3 to 5 would remain unchanged. It is only for Sects. 6 and 7 that the inclusion of physical will affect the results. It will have two main effects. First, the dynamic Eq. (15) will become more complex, including terms reflecting the dependence of wages and interest rate on capital. Hence, the steady state curve will be different. Second, the presence of physical capital introduces an initial condition K_0. This will affect the shape of equilibrium trajectories, preventing for example instantaneous jumps to the steady state value. Obtaining a sharp characterization of the dynamics would be much more cumbersome than in the present paper, probably not feasible.

8.3 Externality and Pareto efficiency

The accumulation rule of human capital (2) introduces into the model the usual human capital externality, reflecting the idea that either the general level of knowledge in the society, or the quality of teachers, has a positive influence on the education outcome of the new generation. The presence of this externality implies that private education choices will lead to too few investment because individuals do not take into account the positive influence their decision has on future generations. Hence, the allocation of resources arising from competitive equilibria, with or without complete markets, is likely to be inefficient and the growth rate will be too low.

Focusing in this paper on maximum growth amounts to bring our economy “closer” to the first best in terms of growth, but the gap between the equilibrium and the first best cannot be closed without government intervention. Indeed, decentralizing an optimal allocation when there is human capital externality generally requires using an investment subsidy on top of the usual intergenerational transfers needed to reach the adequate saving rate (see Docquier and Michel 1999). It could be worthwhile in future work to analyze how the presence of incomplete financial markets modify these prescriptions.

9 Conclusion

We have introduced endogenous borrowing limits à la Kehoe and Levine (1993) in a otherwise standard OLG model with human capital. With respect to the small literature on the subject who assumes exogenous prices (Andolfatto and Gervais 2006) or solves numerically for the equilibrium (Lochner and Monge 2002), we have derived a set of useful analytical results.

If the productivity index of the learning technology is low, the economy can be caught in an inescapable poverty trap. In this case, implementing perfect credit markets makes the economy escape from stagnation.

On the contrary, if the productivity index of the learning technology is high enough, multiple steady states and global indeterminacy arise as a consequence of endogenous debt limits. This results complement the paper by Azariadis and Lambertini (2003) who restricted attention to endowment economies.

Comparing outcomes across the various equilibria we show that the relation between growth and the interest rate is hump-shaped. When interest rates are low, people stay away from assets market and would not reimburse their loan. This is why banks do not lend to them, and there is no investment. With high interest rates, credit is expensive, and investment is low too. With interest rates in a medium range, households are credit constrained, but still invest positive quantities.

If the elasticity of human capital to education is high enough, maximum growth is achieved in an equilibrium where borrowing constraints are binding. In this situation, implementing a financial reform leading to perfect credit markets would reduce economic growth.

On the methodological side, our approach stresses first that the endogeneity of borrowing limits plays an important role, and second, that taking it into account in general equilibrium, and not only in small open economies, matters.

With respect to the first point, Andolfatto and Gervais (2006) already showed that the design of fiscal policy to improve human capital investments by replacing missing capital markets with an intergenerational transfer scheme depends crucially on the nature of the credit constraint. In this paper, we add another result in this line. We showed that, when assessing the effect of financial deepening on growth, the way borrowing constraints are modeled is key as well. The literature based on exogenous borrowing limits (see for example Aghion et al. 2004) defends the view that there is a monotonic relationship between financial depth and long-term economic growth. This is consistent with the fact that there is a set of countries with little financial depth and slow or no growth (Africa) and another set with much better financial markets and sustained growth (OECD). Still, there is a group a countries such as China and Thailand with relatively weak financial markets but strong and sustained growth. This is often viewed as a simple catching-up effect. Alternatively, theory says that maximum growth can be achieved in a situation where agents cannot borrow all what they want to.

With respect to the second point, we have shown that modeling endogenous debt constraint in general equilibrium lead to global indeterminacy, stressing the importance of expectations; this aspect is completely absent from the frameworks with exogenous debt limits (De Gregorio 1996) or small open economies (Andolfatto and Gervais 2006).

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