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Income growth in the 21st century: Forecasts with an overlapping generations model

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Abstract

We forecast income growth over the period 2000–2050 in the US, Canada, and France. To ground the forecasts on relationships that are as robust as possible to changes in the environment, we use a quantitative theoretical approach which involves calibrating and simulating a general equilibrium model. Compared to existing studies, we allow for life uncertainty and migrations, use generational accounting studies to link taxes and public expenditures to demographic changes, and take into account the interaction between education and work experience. Forecasts show that growth will be weaker over the period 2010–2040. The gap between the US and the two other countries is increasing over time. France will catch-up and overtake Canada in 2020. Investigating alternative policy scenarios, we show that increasing the effective retirement age to 63 would be most profitable for France, reducing the gap between it and the US by one third. A decrease in social security benefits would slightly stimulate growth but would have no real impact on the gap between the countries.

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1. Introduction

In most industrialized countries, the rise in life expectancy and the drop in fertility rates progressively increase the share of the elderly in the population. In the next few decades, this phenomenon will be particularly strong as large baby boom cohorts reach the retirement

age. As illustrated in Cutler, Poterba, Sheiner, and Summers (1990), this demographic transition really starts in Japan. The European Union (especially western countries such as Germany, Italy, Belgium and France) and Canada seem to age before the US and after Japan, right in the middle of the wave of aging. In the US, the elderly dependency ratio (the population aged 65+/the population 15–64) will increase from 18% in 1990 to a peak of 32% by 2035. In France, the share of elderly should double between 1995 and 2045. These movements are stronger in Canada, where the

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dependency rate starts at the US level in 2000, but is expected to reach 42% in 2050. Even if demographic institutes usually provide alternative projections, the extent of these population trends is now estimated with a relatively high level of accuracy by demographers. Population aging appears to be an inescapable ending, and its magnitude in the next fifty years cannot be affected by policy.

Economists and policymakers have been concerned with quantifying the potential effects of demographic changes upon the economy. In this paper, we focus on the potential effects of aging on GDP per capita in three important developed countries, Canada, France and the USA. The ways in which demography affects the growth prospect of industrialized economies are manifold. First, population changes have a direct impact on the size of the labor force. After 2000, the growth rate of the population aged 15–64 is expected to decrease in most industrialized countries (especially in France and Canada where the growth rate will become negative). Second, aging also modifies saving rates and capital accumulation, which in turn impacts on wage rates and interest rates. Since the seminal overlapping generations model of [Auerbach and Kotlikoff \(1987\)](#), a large number of studies¹ have concluded that aging will stimulate the capital stock per worker and wages rates, while reducing interest rates². Third, aging also has large effects on the characteristics of the labor force. As argued by [Wasmer \(2001b\)](#), demographic changes in the last 35 years induced a substitution of “low education–high experience” workers with “high education–low experience” workers. The stock of experience is now increasing at a fast pace. However, as the baby boom generations leave the labor force, the stock of experience will decrease after the year 2020, while the average education level is expected to peak around 2040. Finally, the public finance consequences (and

their incidence in terms of intergenerational transfers) have raised a considerable amount of anxiety amongst scholars. The financial sustainability of the social security system is obviously the most important political and economic issue related to aging. Can or should current benefits be maintained? How far can we increase social security contributions?

The consequences of aging described above are still subject to uncertainty, and the net effects cannot be determined a priori. They must be simulated within a fully articulated and calibrated model. The purpose of our study is to integrate the mechanisms sketched above in a general equilibrium, OLG framework. Compared to existing studies, our model builds on a strong demographic block which allows for life uncertainty and migratory movements. In each country, the evolution of the population size and the structure per age corresponds to official projections. Particular attention is also devoted to public sector modelling. Using generational accounting studies³, we closely link the evolution of taxes and expenditures to demographic changes. Finally, we take into account the influence of the interactions between education and experience on the labor market. We also derive the impact of population changes and the educational decisions of the young on the stock of experience and the stock of education.

We use this structural model to forecast the GDP per capita, using the demographic movements as a key source of changes. Our central projection is based on constant policies. Retirement age and the debt–GDP ratio are kept constant, while public transfers and government expenditures are indexed on the total factor productivity. The wage tax rate (including social security contribution and income tax) is adjusted to meet the debt–GDP ratio target.

Under these assumptions, the forecasted incomes display three main features. In all three countries, the growth rate will be weaker over the period 2010–2040. The gap between the leading country (the USA) and the two other countries (France and Canada) increases over the period 2000–2050. France will catch up and overtake Canada in 2020.

The following elements are found to be essential in generating these results. The potential labor forces

¹ See e.g. [Auerbach, Kotlikoff, Hagemann, and Giuseppe \(1989\)](#), [Börsch-Supan \(1996\)](#) or [Hviding and Mérette \(1998\)](#) on various OECD countries, [Chauveau and Loufir \(1997\)](#) on the G7 countries, or more recently [Hirte \(2002\)](#) on Germany, [Miles \(1999\)](#) on the UK and the [INGENUE Team \(2001\)](#) on the World economy.

² One exception is [Fehr, Jokisch, and Kotlikoff \(2003\)](#), who argue that the rise in wages tax (particularly payroll taxes) reduces saving and capital accumulation. Over the course of the century, capital shortages progressively lower real wages per unit of human capital and stimulate interest rates.

³ See [Gokhale, Kotlikoff, Sefton, and Weale \(2001\)](#) for the USA, [Crettez, Feist, and Raffelüschen \(1999\)](#) for France and [Hicks \(1998\)](#) for Canada.

shrink in France and Canada between 2020 and 2050, but continue to increase in the USA. The dynamics of education capital are favorable to France and Canada, since they start from a lower level than the USA. The dynamics of experience capital are relatively more favorable in the USA.

Next we evaluate the robustness of our results to policy choices. Two different policy measures are simulated. First, we consider the increase of the effective retirement age to a similar level (age 63) in all three countries. Such a policy induces major changes in the French economy where the current retirement age is lower than in the other countries. The gap between France and the USA is reduced by one third. Then we consider a scenario where the level of pension benefits is adjusted downward so as to keep wage taxes constant over the period 2000–2050. This policy change has a small impact on the gap between the different countries.

The rest of this paper is organized as follows. Section 2 describes the model. The calibration is presented in Section 3. Section 4 provides the baseline forecasts. The sensitivity of GDP forecasts to policy changes is examined in Section 5. Finally, Section 6 concludes.

2. The model

We consider a model economy where households and firms maximize their objectives subject to resource constraints, and where a government levies taxes and issues debt to finance spending and transfers.⁴ All demographic variables, i.e., fertility, mortality and migration, are exogenous. We will then use the model to map the official demographic forecast into the predicted level of GDP per capita and other economic variables.

2.1. Population

Model time is discrete and goes from 0 to $+\infty$. At each date, some individuals die and a new generation appears. Households reaching age 15 at year t belong

to generation t . The size of the young generation increases over time at an exogenous growth rate, where $N_{0,t}$ measures the initial size of generation t and m_t is one plus the demographic growth rate, including both fertility and migration,

$$N_{0,t+1} = N_{0,t}m_t.$$

Each household lives a maximum of 8 periods ($a=0, \dots, 7$), but faces a cumulative survival probability decreasing with age. The size of each generation declines deterministically through time. However, this decline is affected by net migration flows:

$$N_{a,t+a} = N_{0,t}\beta_{a,t+a} + M_{a,t+a},$$

where $0 \leq \beta_{a,t+a} \leq 1$ is the fraction of generation t alive at age a (and hence, at period $t+a$) and $M_{a,t+a}$ is the stock of migrants of age a . We also have $\beta_{0,t}=1$. Obviously, the total population at time t amounts to $N_t = \sum_{a=0}^7 N_{a,t}$.

The demographic growth rate, immigration flows and survival probability vector vary over time. Taking immigration flows into account enables us to use official demographic observations and projections.

2.2. Households

Individuals have a positive probability of dying during each period of their life. In the spirit of Arrow–Debreu, we postulate the existence of a market for every contingent consumption. These markets open before the resolution of uncertainty: each individual has the possibility of insuring himself against uncertainty at the beginning of his life. Hence, the problem of agents born at time t is to select a consumption contingent plan and his education in order to maximize his expected utility, subject to his intertemporal budget constraint, given the sequence of contingent wages and prices. The expected utility function is time-separable and logarithmic:

$$E(U_t) = \sum_{a=0}^7 \beta_{a,t+a} \ln(c_{a,t+a}) \quad (1)$$

where $c_{a,t+a}$ is the consumption of generation t at age a . The life-cycle budget constraint requires equality between the expected value of expenditures and the

⁴ Compared to our previous work (Bouzahzah, de la Croix, & Docquier, 2002), both demographic variables and public finance are much more detailed, and we distinguish between experience and education in the modeling of human capital.

value of income. For a household born with age 0 at time t , it is written as

$$\begin{aligned} & \sum_{a=0}^7 p_{a,t+a} [c_{a,t+a}(1 + \tau_{t+a}^c) - T_{a,t+a}] \\ & = \sum_{a=0}^7 (\omega_{a,t+a}^L + \omega_{a,t+a}^E e_{a,t+a} + \omega_{a,t+a}^H h_{a,t+a}) \ell_{a,t+a}, \end{aligned} \quad (2)$$

where τ_{t+a}^c is the consumption tax rate at period $t+a$; $p_{a,t+a}$ is the discounted price of one unit of good in case he/she is alive at age a ; $T_{a,t+a}$ denotes the amount of transfer received at age a , including education benefits, pensions and other transfers (health care, family allowances, social benefits, ...); $\ell_{a,t+a}$ measures labor supply at age a ; and raw labor, education and experience are supplied at net-of-taxes contingent discounted wages $\omega_{a,t+a}^L$, $\omega_{a,t+a}^H$ and $\omega_{a,t+a}^E$.

As in [Fehr et al. \(2003\)](#), we assume that migrants of each generation have the same characteristics as natives, i.e., they have the same implicit wealth, experience and education.

The vector of labor supply for generation t (defining labor supply at all ages) is

$$\bar{\ell}_t = (q_t(1 - u_t), q_{t+1}, q_{t+2}, q_{t+3}, q_{t+4}(1 - \alpha_{t+4}), 0, 0, 0), \quad (3)$$

where q_t is the exogenous participation rate at time t , $0 \leq u_t \leq 1$ measures the endogenous time invested in education in the first period of life, and α_{t+4} stands for the (exogenous) time spent in retirement in the fifth period of life (i.e. between age 55 and age 65). The variable q_t is introduced to capture the rise in women's participation rates in the labor market.

In this model we consider labor supply as exogenous, except in the first period where education decisions occur. In [Auerbach and Kotlikoff \(1987\)](#) and many subsequent works, labor supply was endogenized. It is then necessary to calibrate the age-specific parameter of preference for leisure so as to match the age-profile of participation rates observed in the most recent period. However, all these models fail to match the time path of participation rates, especially the rise in women's participation rates over the last three decades. One way to address this issue would be to assume that preference parameters have changed over time. Such changes can then be calibrated to match observations. To avoid modeling changes in preference parameters and to

keep our forecast assumptions as transparent as possible, we decided to keep participation rates exogenous.

The education decisions made in the first period of life completely determine the vectors of experience, skills, education subsidies and public transfers for generation t . Following [Wasmer \(2001b\)](#), the individual stock of experience sums up past participation rates on the labor market. The stock of education transforms the education investment when young into labor efficiency later according to a decreasing return function. Public transfers sum up education subsidies, pension benefits and other transfers. These vectors are written as

$$\begin{aligned} \bar{e}_t = & (0, (1 - u_t)q_t, (1 - u_t)q_t + q_{t+1}, (1 - u_t)q_t \\ & + q_{t+1} + q_{t+2}, (1 - u_t)q_t + q_{t+1} + q_{t+2} \\ & + q_{t+3}, 0, 0, 0), \end{aligned} \quad (4)$$

$$\bar{h}_t = (0, \epsilon u_t^\psi, \epsilon u_t^\psi, \epsilon u_t^\psi, \epsilon u_t^\psi, 0, 0, 0), \quad (5)$$

where $\epsilon > 0$ and $\psi \in (0, 1)$ are two parameters of the educational technology; and

$$\begin{aligned} \bar{T}_t = & (v_t q_t u_t \omega_{0,t}^L \\ & + \gamma_0 g_t, \gamma_1 g_{t+1}, \gamma_2 g_{t+2}, \gamma_3 g_{t+3}, \alpha_{t+4} b_{t+4} \\ & + \gamma_4 g_{t+4}, b_{t+5} + \gamma_5 g_{t+5}, b_{t+6} \\ & + \gamma_6 g_{t+6}, b_{t+7} + \gamma_7 g_{t+7}), \end{aligned} \quad (6)$$

where v_t is the rate of subsidy of the cost of education, b_t is the pension benefit allocated to each full-time retiree at period t , and $\gamma_a g_t$ is the amount of age-related transfer made by the government to agents of age a . The parameter γ_a determines the share of the total transfer g_t in favor of age a .

Since there is no utility of leisure, the problem of households is separable. Individuals first maximize the expected value of income – the right hand side of Eq. (2) – with respect to educational investment u_t . Then, in a second step, they maximize the expected utility function and select the optimal contingent plan subject to the budget constraint in which the education investment is set to its optimal value. The optimal education investment is given by

$$u_t^* = \left(\frac{\epsilon \psi \sum_{a=1}^4 [\omega_{a,t+a}^H \ell_{a,t+a}]}{(1 - v_t) q_t \omega_{0,t}^L + \sum_{a=1}^4 [\omega_{a,t+a}^E \ell_{a,t+a}]} \right)^{\frac{1}{1-\psi}}. \quad (7)$$

Obviously, the optimal education investment balances the marginal gain of education in the numerator (future path of net returns on education) and the marginal cost of education in the denominator; the marginal cost includes both the foregone wage when young and foregone net returns on experience.

Maximizing utility with respect to the levels of consumption determines the law of motion of contingent consumption expenditures over the lifetime:

$$c_{a+1,t+a+1} = \frac{(1+r_{t+1})(1+\tau_t^c)}{(1+\tau_{t+1}^c)} c_{a,t+a} \quad \forall a = 0, \dots, 6. \quad (8)$$

Substituting Eqs. (7) and (8) into the budget constraint (2) gives the optimal level of consumption in the first period of life. The aggregated consumption at period t then amounts to $C_t = \sum_a N_{a,t} c_{a,t}$.

2.3. Firms

At each period of time, a representative firm uses labor in efficiency units (Q_t) and physical capital (K_t) to produce a composite good (Y_t). We assume a Cobb–Douglas production function with constant returns to scale:

$$Y_t = A_t K_t^{1-\varphi} Q_t^\varphi \quad (9)$$

where φ measures the share of wage income in the national product, and A_t represents the total factor productivity (TFP), growing at the rate G_{t-1} . We consider an autoregressive process for the growth factor:

$$\frac{A_t}{A_{t-1}} \equiv G_t = (1-\lambda)\bar{G} + \lambda G_{t-1} + \varepsilon_t, \quad (10)$$

where \bar{G} is the long run growth factor of the TFP, λ is a parameter of inertia in productivity shocks and ε_t is a iid shock process.

The quantity of efficiency units of labor combines physical labor supply and human capital according to a Cobb–Douglas transformation function. Human capital is itself a linear combination of experience and education. Formally, we have

$$Q_t = L_t^{1-\delta} [\mu E_t + (1-\mu)H_t]^\delta, \quad (11)$$

where L_t measures the input of manpower at time t ; E_t measures the input of experience; H_t is the input of

education; δ is a parameter representing the importance of human capital in the determination of labor income; and μ is a parameter of preference for experience. In another paper (de la Croix & Docquier, 2003), we explore the importance of complementarities between education and experience to help us understand how the skill and experience premium has changed in the past. Here we use a simplified production function where education and experience are perfect substitutes, after having checked that forecasts are not very sensitive to the assumed elasticity of substitution between the two.

The representative firm behaves competitively on the factor markets and maximizes profits:

$$Y_t = (r_t + d)K_t - w_t^L L_t - w_t^H H_t - w_t^E E_t, \quad (12)$$

where d is the depreciation rate of physical capital. This behavior requires the marginal productivity of each factor to be equal to its rate of return. They may be written as

$$r_t = (1-\varphi)A_t Y_t / K_t - d \quad (13)$$

$$W_T^L = \varphi(1-\delta)A_T Y_T / L_T \quad (14)$$

$$w_t^E = \varphi\delta\mu A_t K_t^{1-\varphi} Q_t^{\varphi-1} L_t^{1-\delta} [\mu E_t + (1-\mu)H_t]^{\delta-1} \quad (15)$$

$$w_t^H = \varphi\delta(1-\mu)A_t K_t^{1-\varphi} Q_t^{\varphi-1} L_t^{1-\delta} [\mu E_t + (1-\mu)H_t]^{\delta-1}. \quad (16)$$

2.4. The public sector

The government issues bonds and levies taxes on labor earnings (τ_t^w), consumption expenditures (τ_t^c) and capital income (τ_t^k) to finance public transfers and general public consumption. Five types of spending are distinguished: education subsidies, social security benefits, other transfers (health care, family allowance, social benefits), non age-specific general consumption expenditure, and interest payments on public debt. The government budget constraint may be written as

$$\begin{aligned} \tau_t^w (w_t^L L_t + w_t^E E_t + w_t^H H_t) + \tau_t^c C_t + \tau_t^k r_t K_t + D_{t+1} - (1+r_t)D_t \\ = N_{0,t} v_t q_t u_t w_t^L (1-\tau_t^w) + \sum_a N_{a,t} \gamma_a g_t + \vartheta_t Y_t + (N_{4,t} \alpha_t + \sum_{a=5}^7 N_{a,t}) b_t, \end{aligned} \quad (17)$$

where D_t denotes the public debt at the beginning of period t ; ϑ_t is the share of non-transfer public consumption in GDP, and $\gamma_a g_t$ is the amount of transfer per capita allocated to individuals of age a .

Several scenarios can be considered to balance this budget constraint. The budget can be balanced through tax adjustments, expenditure adjustments or changes in the public debt. We assume in the sequel that the path of debt is given and the tax rate τ_t^w adjusts to meet the debt–GDP ratio target.

2.5. Equilibrium

At each date, the composite good is taken as numeraire. The spot price is thus normalized to one: $p_t=1$. We denote the interest rate between dates t and $t+1$ by r_{t+1} , and the appropriate discount factor applied to age- a income and spending is given by

$$R_{a,t+a} \equiv \prod_{s=t+1}^{t+a} (1 + r_s(1 - \tau_s^k))^{-1},$$

where, by convention, $R_{0,t}=1$. Spot gross wages at time $t+a$ are denoted by w_{t+a}^L , w_{t+a}^H and w_{t+a}^E . They correspond to the marginal productivities of labor components, as shown above.

Since there is perfect competition in the insurance market, the contingent prices are related to the spot prices through a set of no arbitrage conditions. The equilibrium (discounted) contingent prices of the consumption good at time t are given by:

$$p_{a,t+a} = R_{a,t+a} \beta_{a,t+a} p_{t+a} = R_{a,t+a} \beta_{a,t+a}. \quad (18)$$

Equilibrium (discounted) contingent net wages are:

$$\begin{aligned} \omega_{a,t+a}^L &= R_{a,t+a} \beta_{a,t+a} w_{t+a}^L (1 - \tau_{t+a}^w), \\ \omega_{a,t+a}^E &= R_{a,t+a} \beta_{a,t+a} w_{t+a}^E (1 - \tau_{t+a}^w), \text{ and} \\ \omega_{a,t+a}^H &= R_{a,t+a} \beta_{a,t+a} w_{t+a}^H (1 - \tau_{t+a}^w), \end{aligned} \quad (19)$$

where τ_{t+a}^w denotes the rate of tax on labor income at time t . The originality of the model is that labor income consists of three components: manpower, experience and education. Equivalently, individual gross wages are the sum of these three elements, so that a tax on labor income τ_t^w affects all wage components similarly.

The equilibrium condition on the goods market is written as follows:

$$Y_t + K_t^* = \underbrace{\sum_{a=0}^7 N_{a,t} c_{a,t}}_{C_t} + \underbrace{K_{t+1} - (1-d)K_t}_{I_t} + \underbrace{\vartheta_t Y_t}_{G_t}, \quad (20)$$

where K_t^* represents the asset holdings brought into the country by migrants. The labor market equilibrium equalizes the demand for labor from firms L_t , E_t and H_t to the sum of the individual supplies:

$$\begin{aligned} L_t &= \sum_{a=0}^7 N_{a,t} \ell_{a,t}, & E_t &= \sum_{a=0}^7 N_{a,t} \ell_{a,t} e_{a,t}, \\ H_t &= \sum_{a=0}^7 N_{a,t} \ell_{a,t} h_{a,t}. \end{aligned} \quad (21)$$

3. Calibration

Three model economies are calibrated, following exactly the same methodology in order to make the forecasts comparable. Calibration implies using the data from observed exogenous variables, fixing some constant parameters, and choosing paths for the unobserved exogenous variables in order to match a series of characteristics. Calibration is not focused on reproducing the characteristics of a given steady state, where all of the interesting information on population history, experience stocks and skills per age group would be lost. Instead, the equilibrium is computed as a transition from one steady state in 1900 to one another in 2250. By starting in 1900, the stocks of education and experience around 1960 reflect the correct history of the population.

3.1. Simulation method

Our dynamic model is characterized by a set of non-linear equations of the form:

$$f(z_{t-4}^1, \dots, z_t^1, z_t^2, \dots, z_{t+4}^2, x_t) = 0,$$

where z_t^i denotes an endogenous variable at time t , which can be either predetermined (z_t^1), or forward looking (z_t^2). The maximum lead or lag in the model is 4 periods, which is related to the length of active life. f is a function representing our dynamic model, and x_t is the vector of exogenous variables and parameters.

The problem also includes initial conditions on predetermined variables, which here correspond to an initial steady state in 1900:

$$z_{-3}^1, \dots, z_0^1 = z_{init}^1.$$

As far as terminal conditions are concerned, we assume that exogenous variables stay constant after a given date. Hence, if the corresponding steady state is a saddle-point, the economy should converge to this steady state, as long as it does not start too far away from it (because the saddle-point stability only applies locally).

Since f is non-linear, it is not possible in general to solve the model analytically. The general problem is to solve a system of finite difference equations with initial and terminal conditions. Approximating the infinite horizon by a finite one (meaning that we assume that the steady state is actually reached at the end of the horizon of simulation), the complete system has as many equations as the number of equations at each period, multiplied by the simulation horizon, plus the initial and terminal conditions:

$$\begin{cases} z_{-3}^1, \dots, z_0^1 & = z_{init}^1 \\ f(z_{-3}^1, \dots, z_1^1, z_1^2, \dots, z_5^2, x_1) & = 0 \\ \vdots & \vdots \\ f(z_{T-4}^1, \dots, z_T^1, z_T^2, \dots, z_{T+4}^2, x_T) & = 0 \\ z_{T+1}^2, \dots, z_{T+4}^2 & = z_{steady\ state}^2 \end{cases} \quad (S)$$

The system (S) is then solved for z_t using a Newton–Raphson relaxation method put forward by Laffargue (1990) and Boucekkine (1995) for solving dynamic nonlinear models with perfect foresight. With this technique, the Newton–Raphson improvement at each iteration is computed by triangulation (instead of inversion) of the matrix of the first derivatives of the system. As Boucekkine (1995) shows, this method allows us to characterize the nature of the dynamics of the model (explosivity, saddle-point trajectory or an infinite number of stable solutions) without having to linearize it and compute the eigenvalues of the linearized system. In particular, it is easy to determine whether the convergence of the algorithm is due to the existence of saddle-point trajectory or not. Indeed, the algorithm is characterized by an explosivity property in the case where an infinite number of stable solutions exist (see Boucekkine & Le Van, 1996). This explosivity

property is in fact common to all convergent relaxation methods. The explosive behavior is put forward by a simple numerical procedure relying on the initialization of the relaxation. Initializing the relaxation with values slightly different from the steady state leads to an explosive behavior at the first Newton–Raphson improvement. This routine is implemented using the package Dynare, from Juillard (1996).

3.2. Observed exogenous variables

3.2.1. Demography

Past and future survival probabilities, $\beta_{a,t}$, and population shares per age are taken from official demographic institutes. For France, we use observations and forecasts from INED and INSEE. For the US, data and forecasts are taken from the Population Division of the US Census Bureau. For Canada, we use observations from the Department of Human Resources and Skills Development, and forecasts are taken from the United Nations database. As for demographic projections, we always use the central scenario.

Figs. 1, 2 and 3 show the demographic transitions experienced in all three countries. There are important differences in the dynamics of the populations. After 2000, the population size keeps increasing in the USA. Consequently, the growth rate of the labor force (the population aged 15–64) remains positive. In France and Canada (from 2030 onwards), the population stagnates and the labor force decreases. The magnitude of aging is also different. As shown in Fig. 3, the old-

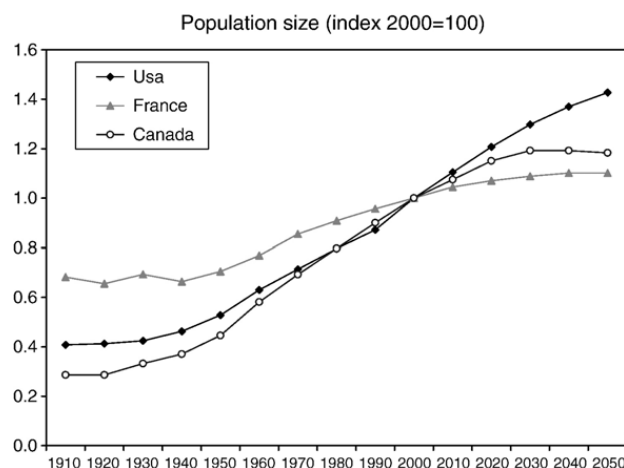


Fig. 1. Population size (index 2000=100).

age dependency increases considerably in Canada (from 18 to 42% between 2000 and 2050) and France (from 25 to 50%), and moderately in the US (from 18 to 32%). Hence, the population of Canada is likely to age more drastically than the other two countries.

3.2.2. Education and participation rates

The time invested in education is computed using school attendance and educational attainment measures. For France, the time invested in education is computed using school attendance measures for the 15–24 year old population reported in Estrade and Minni (1996). For the US, we use the data and projections of educational attainment by Cheeseman Day and Bauman (2000). For Canada, data on educational attainments are taken from Laroche and Mérette (2000), who split the Canadian population by age and by the highest diploma obtained between 1971 and 1996.

The old age participation rate α is computed using the effective retirement age data from Blondal and Scarpetta (1997). The overall participation rates q_t are normalized to 1 in 2000, and computed from Wasmer (2001a) for the three countries.

3.2.3. Public finance

Regarding public finance, our model distinguishes three proportional taxes: the labor income tax, the capital income tax and indirect taxes. For France, the indirect tax rate and the rate of tax on capital income are estimated by Carey and Tchilinguirian (2000) to be 18% and 24% respectively for 1995. The labor income tax rate is endogenous in the model but needs a target value:

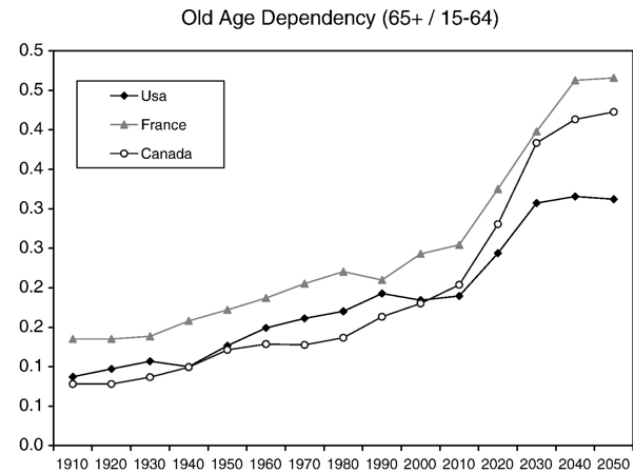


Fig. 3. Old Age Dependency (65+ / 15–64).

it was estimated by Eurostat (1999) to be 44% in 1995. For the US, we calibrate these tax rates in such a way that the shares of revenues in GDP correspond to the estimations of Gokhale, Page, and Sturrock (1999), i.e., 8% for labor income, 7% for indirect taxes, and 5% for capital income in 2000⁵. For Canada, we build on Charbonneau (1997), who provides complete time series of implicit tax rates on labor income, capital income and consumption between 1961 and 1995. We distinguish two types of government spending (net of debt charges): non-age-specific public consumption, and age-specific transfers. For the composition of these categories and for the age profiles of transfers, we build on previous generational accounting exercises, i.e., Crettez et al. (1999) for France, Gokhale et al. (1999) for the US, and Hicks (1998) for Canada. The history of non-age-specific spending is based on OECD data for the period 1960–1995. The history of public debt is taken from official statistics.

3.3. Parameters

A set of parameters is set a priori, the same for all three countries. By doing so, we minimize the number of assumed differences between them. The labor share of the output, φ , is set to 0.7. (This value is commonly

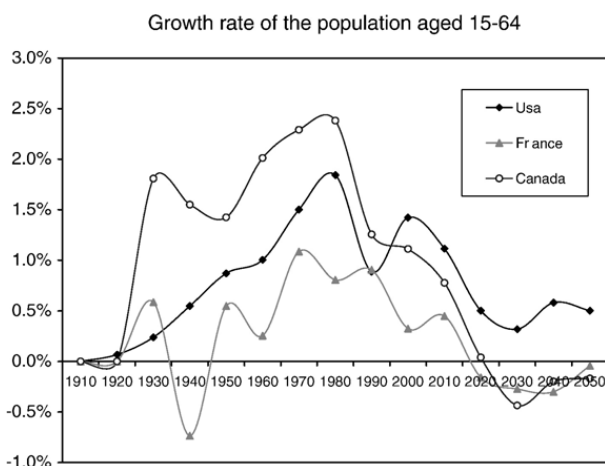


Fig. 2. Growth rate of the population aged 15–64.

⁵ In our simulations, we consider (i) that the French tax rate on consumption expenditures increases linearly from 10% in 1940 to 18% in 1990, and (ii) that the tax rate on capital income increases from 15% to 24% over the same period. The same evolution is applied to the US tax rates (from 8% to 14% for the indirect tax rate and from 10% to 28% for the capital income tax rate).

used in calibrating models of the US economy. In France, the labor share equaled 0.693 in 1995, according to INSEE. For Canada, Fougère and Mérette (1999) use a share of 0.66.) The depreciation rate of capital d equals 0.4. This value implies an annual depreciation rate of 5%. The parameter μ in the production function is a scale parameter which is set to 0.5. Two parameters are important in shaping the wage profile by age: the share of raw labor in labor income $(1 - \delta)$ is set to 0.4, and the scale parameter in the production function of human capital ϵ is set to 2.1. They will both deliver an adequate wage profile (see below). The parameter ψ is the elasticity of education capital to investment in education. It should be calibrated using the estimated elasticity of future earnings with respect to additional schooling (see e.g., Psacharopoulos, 1994, for a survey of these elasticities); we give it the value $\psi=0.6$, which is in accordance with a return of 11.5% to an additional year of schooling, assuming that this additional year of schooling raises education expenditure by 20%.

3.4. Identification of Unobserved Exogenous Variables

The model contains some exogenous variables for which time series data are not available: total factor productivity, A_t ; the rate of subsidy on education expenditures, v_t ; the level of pension benefit, b_t ; and the scale of the age-specific transfer profile, g_t . These four exogenous processes are chosen to match the available time series data for four endogenous variables which are closely related to the unknowns: the GDP growth rate, the share of social security and

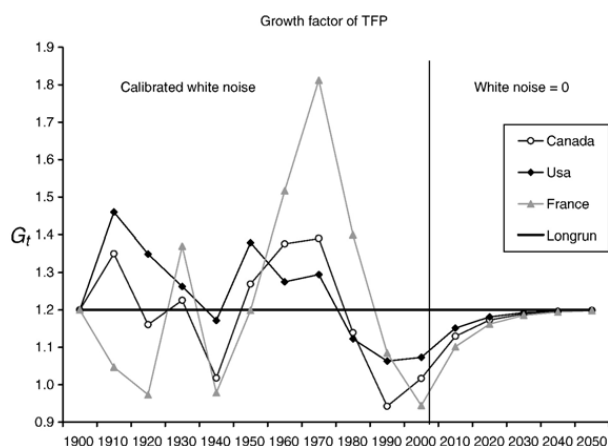


Fig. 4. Growth factor of TFP.

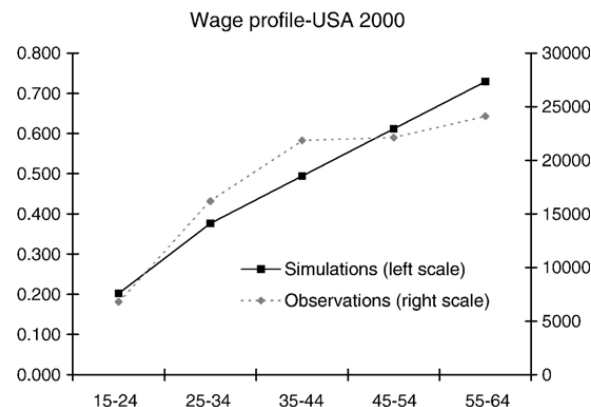


Fig. 5. Wage profile — USA 2000.

other transfers in GDP, and the education investment of young cohorts.

Basically, our identification process implies swapping four exogenous variables for four endogenous variables, and solving the identification step using the algorithm proposed by Laffargue (1990) and Boucek-kine (1995). Our identification process resembles the backsolving method of Sims (1990) for stochastic general equilibrium models. We use a similar idea of treating exogenous processes as endogenous, not to solve a model, but as a calibration device in a deterministic framework.

The autoregressive process (10) is calibrated on the resulting TFP dynamics by pooling country results. The average growth factor is set to $\bar{G}=1.2$, which corresponds to a long-run growth rate of 1.84% per year. The correlation between $G_t - \bar{G}$ and $G_{t-1} - \bar{G}$ then determines $\lambda=0.384$. The iid shock process ϵ_t is estimated as a residual. Hence, we consider a similar growth process for all three countries, except for the shock term. Fig. 4 depicts the growth factor of total factor productivity for the three countries.

Notice that our backsolving identification procedure allows us to calibrate the model “dynamically”. This is much better and more rigorous than calibrating on a hypothetical steady state (in 1900 or in 2250) and re-scaling exogenous variables to obtain reasonable outcomes at a given date, as has usually been done in the Auerbach and Kotlikoff (1987) tradition.

3.5. Wage and Assets Profile per Age

The quality of our model depends on its ability to match individual profiles by age. Let us focus on the

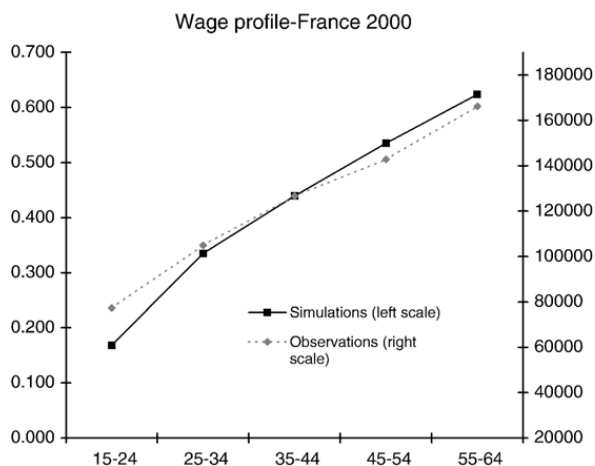


Fig. 6. Wage profile — France 2000.

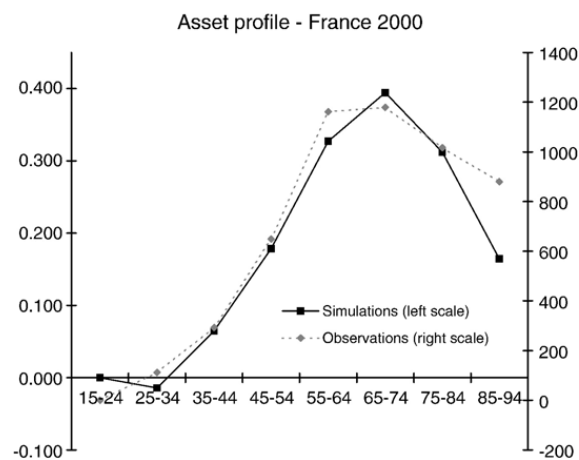


Fig. 8. Asset profile — France 2000.

wage and wealth profiles for the US and France. Fig. 5 provides the wage profile by age for the year 2000 in the US, comparing the model's outcome with PSID data. Fig. 6 conveys the same information for France, where the actual data were obtained from INSEE. The shape of the profile by age is fully determined by the accumulation of experience; there is no need to assume an exogenous profile as in Auerbach and Kotlikoff (1987). This figure comforts us in our choice of values of δ and ϵ . However, we overestimated the wages for old workers in the US; to correct for this, one could assume a positive depreciation rate of experience in the US, but we preferred to keep the same specification for all countries.

It is usually argued that the standard life cycle model with selfish households does not provide a good description of wealth accumulation after retirement. Figs. 7 and 8 report asset profiles by age at time 2000,

together with their empirical counterparts from PSID for the US and from INSEE for France. It appears that our model matches the profile, except for the very old people (85–94). Hence, there is no need to suppose a pure time preference parameter on top of the mortality rate. The annuity market is also helpful for avoiding poverty in old age.

4. Forecasts

Once the model has been calibrated, we run a baseline simulation over the period 1900–2250, where future values of policy variables are kept at the level observed in 2000. We concentrate our attention on the determinants of economic growth. From Eq. (9), growth is driven by total factor productivity, the accumulation of physical capital, and the dynamics of employment in efficiency units.

Regarding employment, Fig. 2 indicates that the labor force grows at a slower pace in all countries between 2000 and 2040. This phenomenon is particularly important in France and Canada. The characteristics of the labor force also change. The next three figures describe the average education level and the average experience of workers. On the one hand, the average education level will rise in all countries. However, the rise is stronger in France and Canada, since they start from a lower level in 2000 compared to the USA. On the other hand, the dynamics of the experience capital are more favorable in the USA. In each country, the average experience of workers increases between 2000 and 2010, then decreases

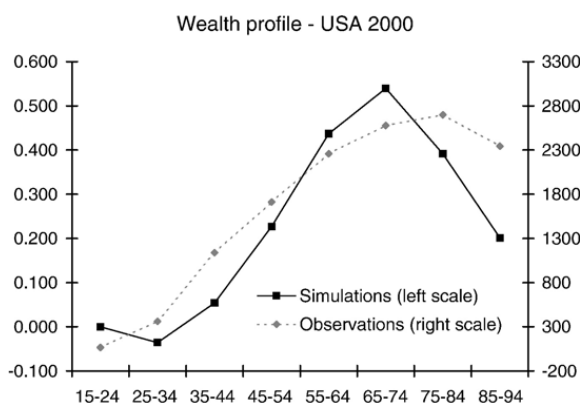


Fig. 7. Wealth profile — USA 2000.

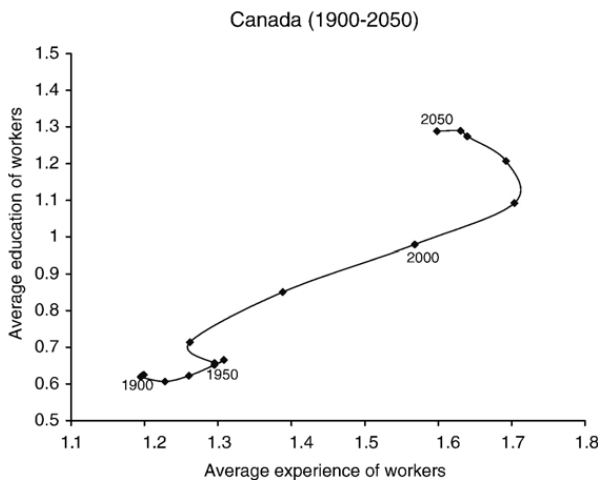


Fig. 9. Education and experience.

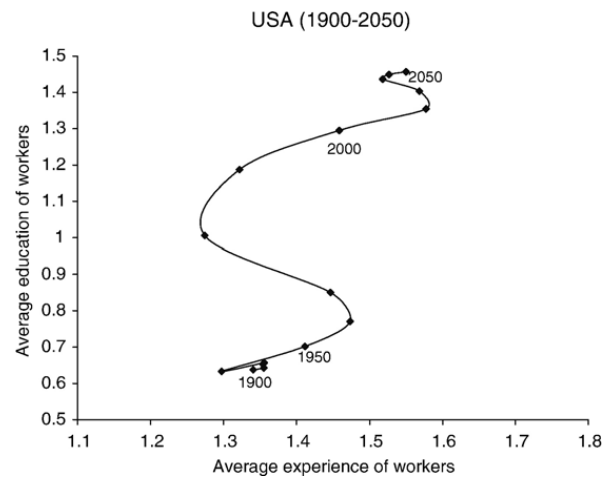


Fig. 11. Education and experience.

(until 2050 in Canada, and until 2030 in France and the USA). In 2050, the net gain in experience is important in the US, but not significant in France or Canada (Figs. 9, 10, and 11).

Regarding capital accumulation, it is usually argued that aging raises the stock of capital compared to the supply of labor, hence expanding real wages while decreasing interest rates. The prospect of such a capital deepening arises from the fact that the oldsters are the main suppliers of capital, while the young are the main suppliers of labor (more oldsters relative to youngsters means more capital per worker). Fehr et al. (2003) recently questioned this assertion: if benefits are paid as promised, increased taxes may undermine capital

intensity (as in their simulations). Similarly, the rise in education and/or experience boosts workers' human capital, and may reduce the capital per efficiency unit of labor. Hence, the net impact on capital intensity cannot be determined a priori. Our simulations reveal that capital intensity will rise between 2000 and 2030 and decrease (very slightly) thereafter. Such a trend translates into an opposite movement in the interest rates, as shown in Fig. 12. Note that, in 2050, the Canadian interest rate is roughly equal to its current level, whilst the French and US interest rates are below the current level. This reflects the fact that there will be a net increase in capital intensity between 2000 and 2050 in France and the US, while the net effect in

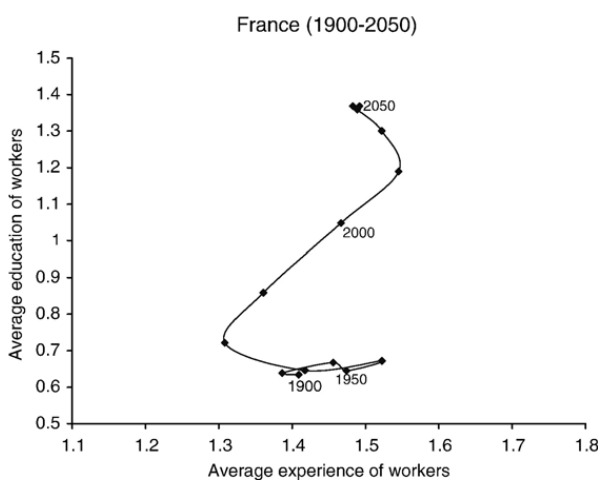


Fig. 10. Education and experience.

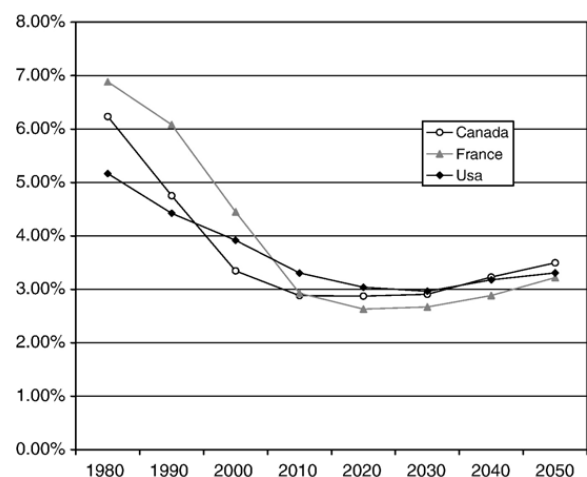


Fig. 12. Annual interest rate.

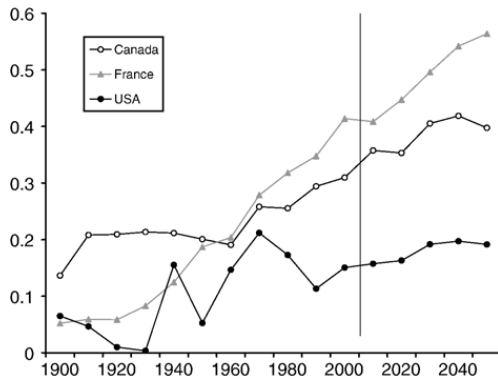


Fig. 13. Income tax.

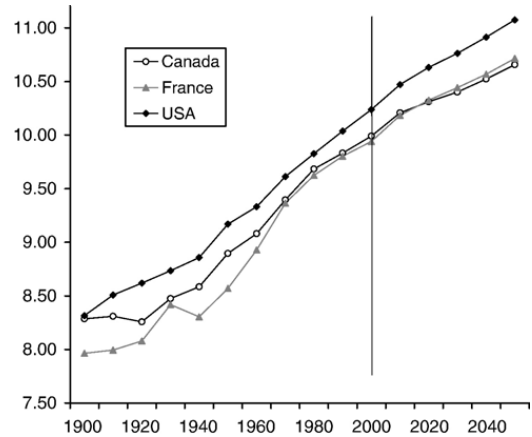


Fig. 15. GDP per capita.

Canada will be negligible (the rise in 2000–2020 is compensated for by the drop between 2020–2050).

Changes in capital intensity closely depend on public policies. In our baseline forecast, we assume that all transfers (including social security benefits) are perfectly adjusted on the total factor productivity. The government budget is balanced via the wage income tax. As appears in Fig. 13, this policy induces large changes in the income tax rate. Between 2000 and 2050, the tax rates increase from 41 to 56% in France, from 31 to 40% in Canada, and from 13 to 16% in the US. These evolutions are mainly driven by the share of social security benefits in GDP (Fig. 14). In France, the share of public pensions amounts to 12%, and is expected to reach 18% in 2050. In the US, the share will only move from 6 to 8%. In Canada, the size of the “Bismarckian” Canadian Pension Plan (CPP) will rise from 4 to 7% of the GDP. However, the “Beveridgean” Old Age Security (OAS) scheme is also included in

our simulations (in other individual transfers). On the whole, the Canadian share of total transfers will rise from 13 to 21% of the GDP.

Our forecasts of GDP per capita combine all these elements. The GDP per capita between 1900 and 2050 is represented in Fig. 15 (in logarithms). Tables 1 and 2 give the simulated levels and annual growth rates for the next five decades (in USD). In 2000, the US leadership appears: the Canadian and French levels are respectively 22 and 26% below the US level. Despite the adverse demographic shock, the 2050 level will be roughly twice as important as in 2000. However, population changes should reinforce the leadership of the US. In 2050, the Canadian and French levels will be 34 and 30% below the US level, respectively. Hence, France will catch-up and overtake Canada (in 2020).

Demographic movements have a stimulating effect on growth between 2000 and 2010. Thereafter, annual growth rates of GDP per capita are positive, but become lower than the TFP growth after 2010. The minimal growth rates are experienced between 2010 and 2030. In Canada, the GDP per capita increases by

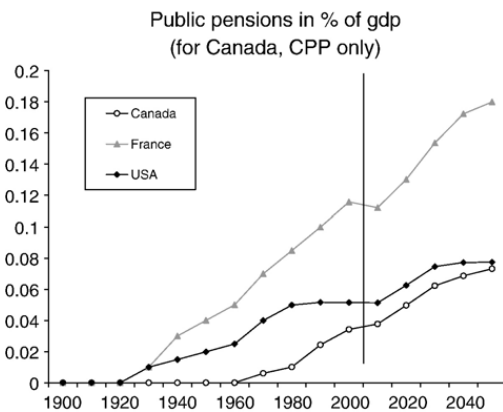


Fig. 14. Public pensions in % of gdp (for Canada, CPP only).

Table 1
Forecasted levels of GDP per capita (USD)

	Canada	France	USA
2000	21,843	20,769	27,954
2010	27,126	26,490	35,292
2020	30,091	30,539	41,466
2030	32,908	34,289	47,277
2040	37,175	38,899	54,944
2050	42,522	45,035	64,554

Table 2
Forecasted annual growth rates of GDP per capita

	Canada	France	USA
2000	1.61%	1.39%	2.02%
2010	2.19%	2.46%	2.36%
2020	1.04%	1.43%	1.63%
2030	0.90%	1.16%	1.32%
2040	1.23%	1.27%	1.51%
2050	1.35%	1.48%	1.63%

less than 1% per year over the period 2020–2030 (despite a TFP growth rate of 1.84% per year).

5. Alternative Policies

The evolution of GDP per capita depends on both socio-demographic changes (size and structure of the population, participation rates of men and women, education choices of future young cohorts) and public policy. If most socio-demographic changes can be estimated with a relatively high level of accuracy, policy changes are quite uncertain. More precisely, there is a great deal of uncertainty about the evolution of social security replacement rates (will the current level of benefit be maintained?) and about the labor participation rates of old workers (will the retirement age increase?).

Intuitively, these alternative social policies aim to reduce the fiscal burden of aging. Their impact on the economy is likely to be important. Reducing benefits should stimulate saving and capital intensity. Increasing the retirement age impacts on the size of the labor force and on the stock of experience. Lowering income tax levels alters the return and cost of education.

In this section, we evaluate the robustness of our forecasts to policy choices. Two different policy changes are simulated. First, we consider an increase of the

Table 3
The gain in growth rates obtained by raising the effective retirement age to 63 years

	Canada	France	USA
2000	−0.01%	0.00%	−0.01%
2010	0.20%	0.24%	0.10%
2020	0.35%	0.32%	0.04%
2030	0.06%	0.39%	0.00%
2040	0.04%	0.33%	0.00%
2050	0.05%	0.17%	0.01%

Table 4
The gain in growth rates obtained by adjusting pension benefits to keep the income tax constant

	Canada	France	USA
2000	0.05%	0.04%	0.02%
2010	0.06%	0.05%	0.03%
2020	0.05%	0.07%	0.03%
2030	0.07%	0.09%	0.03%
2040	0.08%	0.11%	0.03%
2050	0.08%	0.13%	0.03%

effective retirement age to 63 in all three countries. Such a policy induces major changes in the French economy, where the current effective retirement age is lower than 59. For Canada and the US, the effect is expected to be smaller (as the current retirement ages are respectively 61.5 and 62.5).⁶ Second, we consider a scenario where the level of pension benefits is adjusted downward so as to keep wage taxes constant over the period 2000–2050.

Table 3 depicts the growth gain associated with a rise in the effective retirement age. As expected, such a policy change has a deep impact on the French economy. Between 2020 and 2040, the annual growth rate increases by 0.3–0.4 percentage points. The impact is less important in Canada (except in 2020), and is negligible in the US. Consequently, a convergence in retirement age would mainly reduce the gap between France and the USA (by one third). It would not strongly modify the Canadian position.

Table 4 gives the growth gains associated with a budget adjustment through social security benefits. Such a scenario induces growth gains in all periods and for all countries. By stimulating capital accumulation, the GDP per capita increases, especially in countries where the fiscal pressure reaches historic levels. However, the gains are quite small (usually less than 0.1 percentage points). This policy change has only a small impact on the gap between the different countries.

6. Concluding remarks

Statistical methods for performing long-run forecasts of economic data are strongly subject to the

⁶ We are aware that there is no systematic mandatory retirement age in the US and Canada. For these two countries, our “policy change” can be seen as resulting from a private decision to increase the participation rate when old. By considering exogenous labor supply, we leave the modeling of incentives to work longer for future research.

Lucas critique. Indeed, it is very unlikely that the estimated relationships will stay stable over such long period of time, given that changes in the environment and/or policy will be large. Methods based on quantitative theory are likely to be more robust.

Accordingly, we use a general equilibrium overlapping generations model to forecast the level of income per capita in three major developed countries, Canada, France and the USA. Our model is built to integrate the most important sources of economic growth: it reproduces the accumulation of physical capital and human capital (distinguishing between education and experience), increasing participation rates, and technical changes. Such a tool can be calibrated on detailed data, with special attention paid to demography and the public sector.

Our baseline forecast is based on constant policies (the current level of benefits will be paid, and the effective retirement age and the debt–GDP ratio will remain constant). We show that the annual growth rate of GDP per capita will be positive, but lower than the total factor productivity growth over the period 2010–2040.

The population aging process differs across countries, in both intensity and timing. It is interesting to study the implications of these differences in terms of income convergence or divergence. Our results suggest that the gap between the leading country (the USA) and the two other countries (France and Canada) will increase significantly in the next four decades. Moreover, France will catch-up and overtake Canada in 2020.

A change in social policy could alter these conclusions. It is shown that a rise to 63 in the retirement age would be very profitable for France (the gap between France and the USA would be reduced by one third). A decrease in social security benefits would slightly stimulate growth, but would have no real impact on the gap between the countries.

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