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Information technologies, embodiment and growth

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Abstract

This paper studies the conditions under which an IT revolution may occur and have permanent effects on long-term growth. To this end, we construct a multi-sectoral growth model with endogenous embodied technical progress. The R&D sector expands the range of softwares. The capital sector produces efficient capital combining hardware with available softwares. Technological progress is therefore embodied: New softwares can only be run on the most recent generations of hardware. The new softwares are copyrighted during a fixed period of time. First, we analytically characterize the balanced growth paths of the model. Then we focus on the dynamic response of the economy to technological shocks. Substitution effects favorable to the IT sectors are shown to arise when positive supply shocks affect the production of efficient capital and/or the creation of new softwares. Positive shocks specific to the capital sector are unable to produce effects on long-term growth, in contrast to the shocks specific to the R&D sector.

JEL classification: E22; E32; O40; C63

Keywords: Information technology; Vintage capital; Embodied technological progress; Endogenous growth

1. Introduction

The sector of information technologies has been recently invoked to crucially matter in the recent trends and performances of national economies (see for example Gordon, 1999; Greenwood and Jovanovic, 1998; Whelan, 2000; Jorgenson and Stiroh, 2000). There is now a common view according to which we are entering in a “new economy”,

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the age of information technologies (IT hereafter). The huge productivity growth figures registered for the durable goods sector, and in particular for the computer sector (around 42% per year from 1995 to 1999 following Gordon, 1999), makes it difficult to argue against such a view. However, some issues are still debated and will be debated until a more substantial historical experience is available. The main debated issue concerns the status of this IT age from an historical perspective. Some authors like Greenwood, Yorukoglu or Jovanovic (see Greenwood and Yorukoglu, 1997; Greenwood and Jovanovic, 1998) have argued that we are witnessing the Third Industrial Revolution: After an adoption period along which the productivity slowdown takes place due to learning costs and slow diffusion, the IT are now driving the rest of the sectors. The productivity gains should accordingly spread over the economy exactly as the major discoveries affected the pace of economic activity during the nineteenth century's Industrial Revolution.

Gordon (1999, 2000) has argued against this strongly optimistic view of the recent recovery of the US economy. While the huge figures for productivity growth in the computer sectors are out of question, thus assigning a particular weight to IT in the recovery of total factor productivity growth in the USA, Gordon observes that no significant spillovers are taking place from IT to the rest of the economy. More precisely Gordon found using NIPA accounts that after correcting for the cycle there is no acceleration in total factor productivity outside the hardware sector. Even the productivity slowdown has worsened for the non-durable goods manufacturing sectors over the period 1995–1999 compared with the period 1972–1995. Therefore, according to Gordon, if undeniably something important is changing in the US economy, there is a reasonable doubt about the long-term viability of the observed IT-driven growth regime. Moreover, he argues, the so-called digital revolution is by no way comparable with the major discoveries of the nineteenth century. More doubts can be raised about the validity of comparing IT, and especially Internet, with the great inventions of the past. Interestingly, Gordon notes that much of the use of Internet involves substitution of existing means. This view is shared by Jorgenson and Stiroh (1999) who interpret the current boom of IT as "a vast and continuing substitution of IT equipment for other forms of capital and labor". This massive substitution is fundamentally due to a relative price effect: As properly shown by Gordon in his 1990's influential book, the relative price of durable goods, including hardware and communication, has considerably decreased since the mid-1970s, and even more sharply in the recent years. The so-called Solow paradox according to which productivity gains due to IT are showing up everywhere except in the statistics, can be therefore solved easily: "...This substitution generates substantial returns for the economic agents who undertake IT investments and restructure their activities in order to increase the role of IT. There is little evidence however that substitution is accompanied by technical change as this term is used by economists".

While the methodological issues are far from settled, the theoretical debate about the determinants, implications and the viability of the IT revolution is even more open. In this paper, we argue that there is much to learn from endogenous growth theories regarding the determinants and long-term viability of an IT driven economic expansion. To this end, we build up a multi-sectoral model with some specific characteristics in order to feature with some fine details the IT-based economy. In particular, the following characteristics are taken into account:

(i) *The embodied nature of technological progress*: The technological innovations occurring in the IT sectors are typically embodied in the new capital goods (hardware, robots, etc.). Therefore, the technological progress conveyed by these sectors is essentially embodied and should be modeled as such.

(ii) *The preeminent role of the R&D sector*: As documented by Segerstrom (2000) in the case of Intel Corporation, aggressive R&D policies are typical in the IT sector, in particular for the companies aiming to maintain their leadership. The amount of resources devoted to research activity is consequently impressive, especially in the USA, which leads some analysts to argue that the IT revolution has just begun. Indeed, the US corporate R&D has increased by an annual average of 11% in the past five years, and there is no evidence that this trend will be reversed in the next years.

(iii) *The crucial link between innovation and market power*: The Microsoft trial makes it clear that the information and communication technologies' markets are typically non-competitive. The Schumpeterian link between innovation and market power is even more crucial in these markets. In effect, most start-ups sell information, and information is a good which is costly to produce but which reproduction can be achieved at a very low cost. Therefore, modeling the IT sector requires a careful treatment of the property rights accruing to the innovators.

In this paper, these three basic characteristics are met within a Romer-like research based growth model with an endogenous embodied technological progress. The role of embodiment in explaining the growth performances of the US economy since the end of the second world war has been put forward by some mainstream macroeconomists in the recent years. A recent and fundamental empirical contribution underlying this role is due to Greenwood et al. (1997) (see also Hulten, 1992). According to Greenwood and Yorukoglu (1997), the rate of embodied technical change has even accelerated after 1974, as reflected in the observed higher rate of decline of the relative price of equipment (reported by Gordon, 1990, for example). This is especially true for IT equipment as emphasize Jorgenson and Stiroh (2000): The price of computer investment fell around 17% per year from 1990 to 1996 while the price of IT equipment to households fell 24% annually.

Within a computable general equilibrium setup, Greenwood and Yorukoglu (1997) introduce these features by assuming that the rate of embodied technological change has *exogenously* accelerated suddenly and permanently from 1974. As the pre-existing firms are unable to immediately use the new techniques at their full potential, a relatively long adoption period takes place which duration depends upon different endogenous costs (fundamentally skilled labor to facilitate the adoption). Therefore, the story told here does not provide any explanation for the acceleration in the rate of embodied technological progress in the mid-1970s, rather it assumes it. And it does not deliver any particular insight into the long-term viability of an IT-driven growth episode like the one experienced by the USA in the recent years. This paper is intended to remedy these two shortcomings. Using a computable general equilibrium approach, we endogenize embodied technological progress: Instead of assuming an exogenous initial shift in the latter, we study whether some specific supply shocks, mostly in the hardware and R&D sectors according to the very recent empirical literature of the digital revolution, are able to affect the pace of embodied technical change in the economy and under which conditions this may have permanent effects on long-term growth.

The paper is organized as follows. The next section is devoted to a detailed description of the model and derives the corresponding dynamical equations. Section 3 provides an analytical characterization of the balanced growth paths and the calibration procedures. In particular, it is shown that a permanent positive shock on the productivity of the hardware sector has no permanent effect on the growth rate, while a similar productivity shock in the R&D sector, expanding the range of softwares, does. Section 4 presents the results of the dynamic simulations conducted on the calibrated model. Particular attention is paid to the interaction of the innovation dynamics with the accumulation of physical capital along the growth process. Several other interesting questions are also raised (such as the *productivity slowdown*, the skill premium, etc.). Section 5 concludes.

2. The model

Here comes a short description of our model. The economy includes 4 sectors. The final good sector production technology relies on a vintage capital specification à la Solow (1960). It uses two types of labor (raw and skilled) and efficient capital: Technological progress is embodied in the new capital goods. The capital sector produces efficient capital using the final good and the available intermediate inputs. We interpret the final good used by this sector as *hardware* and the intermediate inputs as *softwares*. The R&D sector increases the range of softwares (horizontal differentiation), it uses skilled labor and its productivity depends on the stock of public knowledge as in Romer (1990). A last sector produces the softwares using raw labor. The innovators are rewarded by a market power (copyrights) so as to stimulate innovation and growth. This is a typical specification in research-based growth models. Finally, while the new softwares are copyrighted during a fixed period of time, they become public knowledge at a certain point in time, which generates positive externalities in the rest of the economy. In this sense, the information technology may be a powerful engine of growth.

2.1. The producer of physical goods

The final good sector produces a composite good that is used either to consume or to invest in physical capital. It uses efficient capital and two types of labor as inputs. Efficient capital is built from physical capital and immaterial capital in the equipment sector. Let $K_{t,s}$ represent the efficient capital stock bought at time t (i.e., the vintage t) and still in use at time $s \geqslant t$. We assume that the depreciation rate, δ, is constant so

$$K_{t,s} = K_{t,t}(1-\delta)^{s-t}. \tag{1}$$

At time $s \geqslant t$, the vintage t is operated by a certain amount of unskilled labor, say $L_{t,s}$, and skilled labor, say $H_{t,s}$. Let $Y_{t,s}$ be the output produced at time s with vintage t.

Under the following Cobb–Douglas technology we have

$$Y_{t,s} = z_s K_{t,s}^{\gamma} L_{t,s}^{\alpha} H_{t,s}^{\beta} \tag{2}$$

with $\alpha, \beta \in [0,1]$ and $\gamma = 1 - \alpha - \beta$. The variable z_s represents *disembodied* technological progress. An increase in z_s rises the marginal productivity of all vintages, independently of the age structure of the stock of efficient capital. In sharp contrast, we will see in the next sub-section that there is *embodied* technological progress in $K_{t,s}$, which is specific to the equipment of vintage t.

The discounted profits of investing $K_{t,t}$ in vintage t are given by

$$\Pi_t = \sum_{s=t}^{\infty} [Y_{t,s} - b_s L_{t,s} - w_s H_{t,s}] R_t^s - d_t K_{t,t},$$

where

$$R_t^t = 1 \quad \text{and} \quad R_t^s = \prod_{\tau=t+1}^{s} \left(\frac{1}{1+r_\tau} \right)$$

is the discounted factor at time s and r_τ is the interest rate at time τ. b_s and w_s are, respectively, the wages for unskilled and skilled labor input at time s. d_t is the price of efficient capital.

The representative firm chooses efficient capital and the labor allocation across vintages in order to maximize its discounted profits taking prices as given and subject to its technological constraint:

$$\max_{K_{t,t},\, \{L_{t,s}\}_{s=t}^{\infty},\, \{H_{t,s}\}_{s=t}^{\infty}} \Pi_t.$$

The first-order conditions characterizing an interior maximum for Π_t are

$$\gamma K_{t,t}^{-\alpha-\beta} \sum_{s=t}^{\infty} R_t^s z_s (1-\delta)^{\gamma(s-t)} L_{t,s}^{\alpha} H_{t,s}^{\beta} = d_t, \tag{3}$$

$\forall s \geqslant t$:

$$\alpha z_s K_{t,s}^{\gamma} L_{t,s}^{\alpha-1} H_{t,s}^{\beta} = b_s, \tag{4}$$

$$\beta z_s K_{t,s}^{\gamma} L_{t,s}^{\alpha} H_{t,s}^{\beta-1} = w_s. \tag{5}$$

Eq. (3) determines investment at time t by equalizing marginal returns to marginal costs. Eqs. (4) and (5) determine the labor allocation at time s to vintage t.

Solving Eqs. (4) and (5) for labor inputs, one obtains that

$$L_{s,t} = \left(\frac{\beta^{\beta} \alpha^{1-\beta} z_t}{b_t^{1-\beta} w_t^{\beta}} \right)^{1/\gamma} K_{s,t}, \tag{6}$$

$$H_{s,t} = \left(\frac{\alpha^{\alpha} \beta^{1-\alpha} z_t}{b_t^{\alpha} w_t^{1-\alpha}} \right)^{1/\gamma} K_{s,t}. \tag{7}$$

Defining aggregate variables as

$$K_t = \sum_{s=-\infty}^{t} K_{s,t}, \quad L_t = \sum_{s=-\infty}^{t} L_{s,t}, \quad H_t = \sum_{s=-\infty}^{t} H_{s,t}, \quad Y_t = \sum_{s=-\infty}^{t} Y_{s,t}.$$

The aggregate demand for unskilled employment and skilled employment, respectively, can be written as

$$L_t = \left(\frac{\beta^\beta \alpha^{1-\beta} z_t}{b_t^{1-\beta} w_t^\beta} \right)^{1/\gamma} K_t, \tag{8}$$

$$H_t = \left(\frac{\alpha^\alpha \beta^{1-\alpha} z_t}{b_t^\alpha w_t^{1-\alpha}} \right)^{1/\gamma} K_t. \tag{9}$$

Replacing now $L_{s,t}$ and $H_{s,t}$ in (2) by their value taken from Eqs. (6) and (7), and computing total production, one obtains

$$Y_t = z_t \left(\frac{\beta^\beta \alpha^{1-\beta} z_t}{b_t^{1-\beta} w_t^\beta} \right)^{\alpha/\gamma} \left(\frac{\alpha^\alpha \beta^{1-\alpha} z_t}{b_t^\alpha w_t^{1-\alpha}} \right)^{\beta/\gamma} \sum_{s=-\infty}^{t} K_{s,t}. \tag{10}$$

Eqs. (8)–(10) jointly imply that

$$Y_t = z_t K_t^\gamma L_t^\alpha H_t^\beta. \tag{11}$$

Hence we retrieve a Cobb–Douglas production function as in Solow (1960).

2.2. *The producer of efficient capital*

The producer of efficient capital uses physical capital (or hardware) bought from the final good producers, and immaterial capital sold by the software producers. It builds efficient capital from these two inputs and sell it to the final good firm. Efficient capital $K_{t,t}$ is built following a constant return to scale technology:

$$K_{t,t} = e_t\, Q_t^\lambda I_t^{1-\lambda}. \tag{12}$$

The parameter λ belongs to (0,1). The productivity variable e_t will be used to model productivity shocks specific to the IT industry. The variable Q_t is the immaterial capital embodied in the vintage $K_{t,t}$. This immaterial capital is built from a series of specialized intermediate goods, following a Dixit–Stiglitz (1977) CES function

$$Q_t = \left(\int_0^{n_t} x_{i,t}^{(\sigma-1)/\sigma} \,\mathrm{d}i \right)^{\sigma/(\sigma-1)}, \tag{13}$$

where n_t is the number of varieties available in t, $x_{i,t}$ is the quantity of input used in t of variety i and $\sigma > 1$ is the elasticity of substitution between two varieties.

The maximization program of this sector is static in nature. Profits are time t are:

$$d_t e_t \left(\int_0^{n_t} x_{i,t}^{(\sigma-1)/\sigma} \,\mathrm{d}i \right)^{\lambda\sigma/(\sigma-1)} I_t^{1-\lambda} - I_t - \int_0^{n_t} p_{i,t} x_{i,t} \,\mathrm{d}i,$$

where d_t is the price of efficient capital and $p_{i,t}$ is the price of software of variety i. In the sequel, it is convenient to denote the software–hardware ratio as

$$q_t = Q_t/I_t.$$

The first-order conditions with respect to I_t and $x_{j,t}$ are

$$(1-\lambda)e_t d_t q_t^{\lambda} = 1, \tag{14}$$

$\forall j \in [0, n_t]$:

$$\lambda e_t d_t q_t^{\lambda-1} \left(\frac{Q_t}{x_{j,t}}\right)^{1/\sigma} = p_{j,t}. \tag{15}$$

Using Eqs. (14) and (15) the demand for intermediate input j by the firms of the final good sector can be rewritten as

$$\frac{x_{j,t}}{Q_t} = \left(\frac{\phi}{q_t}\right)^{\sigma} p_{j,t}^{-\sigma} \tag{16}$$

with $\phi = (1-\lambda)/\lambda$. The price elasticity of demand is thus $-\sigma$.

2.3. *The producer of immaterial capital*

The intermediate good sector produces a number of immaterial products (or softwares) that are sold to the equipment sector. It uses unskilled labor to produce the goods and skilled labor to research for new varieties.

2.3.1. *The production activity*

The sector $[0, n_t]$ producing the intermediate goods is divided into a competitive sector $[0, n_t^{\mathrm{c}}]$ and a monopolistic sector $]n_t^{\mathrm{c}}, n_t]$. The market power is given by the presence of copyrights which have a lifetime of T. Hence, after a span of time T, monopolistic firms become competitive and we have

$$n_t^{\mathrm{c}} = n_{t-T}.$$

The intermediate good of type $i \in [0, n_t]$ is produced with a constant return to scale technology involving unskilled labor as the only input:

$$x_{i,t} = \tau \tilde{L}_{i,t}, \tag{17}$$

where $\tilde{L}_{i,t}$ denotes unskilled labor employed in the intermediate sector and τ measures labor productivity.

In the side of the sector that behaves competitively, the output price is equal to the marginal cost:

$$p_{i,t} = \frac{b_t}{\tau}, \quad \forall i \in [0, n_t^{\mathrm{c}}]. \tag{18}$$

In the side of the sector that behaves monopolistically, the output price is chosen so as to maximize profits subject to the demand formulated by the final good sector

$$\max \left(p_{i,t} - \frac{b_t}{\tau}\right) x_{i,t} \quad \text{s.t. (16).}$$

This leads to

$$p_{i,t} = \mu \frac{b_t}{\tau}, \quad \forall i \in]n_t^c, n_t] \text{ with } \mu = \left(1 - \frac{1}{\sigma}\right)^{-1} \tag{19}$$

and the price is a mark-up over unit labor costs, whose mark-up rate depends on the price elasticity of demand.

2.3.2. The research activity

Following Grossman and Helpman (1991) and Michel and Nyssen (1998), the research activity requires labor and public knowledge. The stock of public knowledge m_t that is used in the production of new types of input consists in the inputs being in the public domain $[0, n_t^c]$ but is also influenced by the inputs covered by copyrights. This latter influence is moderated by the parameter $\theta < 1$:

$$m_t = n_t^c + \theta(n_t - n_t^c). \tag{20}$$

The production of new inputs is made with skilled labor, according to the following constant return to scale technology:

$$\Delta n_t = n_t - n_{t-1} = a m_t \tilde{H}_t$$

and the unit cost of research v_t is given by

$$v_t = \frac{w_t}{a m_t}. \tag{21}$$

The unit cost increases with the skilled wage and decreases with the level of public knowledge.

There will be entry of new firms until this cost is equal to the discounted flow of profits linked to one invention. This equilibrium condition that determines the number of new firms n_t can be written as

$$v_t = \sum_{z=t}^{t+T-1} R_t^z \frac{1}{\sigma - 1} \frac{b_z}{\tau} x_{i,z}. \tag{22}$$

Note that by (16) the discounted flow of profits depends on the investment made by the firms in the final goods sector. This is the main consequence of embodiment in our model: The return to research is related to investment in the final goods sector as in Boucekkine et al. (2000).[1]

Finally, the demand for skilled labor by the research sector is given by

$$\tilde{H}_t = \frac{\Delta n_t}{a m_t}. \tag{23}$$

[1] Note that we can have an equilibrium situation where it is not profitable to invest in research. In this case we have

$$v_t > \sum_{z=t}^{t+T-1} R_t^z \frac{1}{\sigma - 1} \frac{b_z}{\tau} x_{i,z}$$

and the consequence is $n_t = n_{t-1}$. In this paper, we concentrate on the case where (22) holds.

2.4. Household behavior

There are two types of households, skilled and unskilled. They both consume, save for future consumption and supply labor inelastically. The households savings are invested either in physical capital or in the research activity.

We model these households as one representative household supplying H units of skilled labor and L units of unskilled labor. He/she maximizes the discounted sum of instantaneous utility:

$$\sum_{t=0}^{\infty} \rho^t \ln C_t,$$

where ρ is the psychological discount factor and the utility function is logarithmic. The budget constraint is

$$A_{t+1} = (1 + r_{t+1})A_t + w_t H + b_t L - C_t,$$

where A_t stands for the assets detained by households. The first-order necessary condition for this problem is

$$\frac{C_{t+1}}{C_t} = (1 + r_{t+1})\rho \tag{24}$$

which, together with the usual transversality condition, is sufficient for an optimum.

2.5. Market equilibrium

Equilibrium on the efficient capital market implies that the price d_t is such that the production of efficient capital (12) equals its demand (3).

Equilibrium on the skilled labor market implies the skilled labor force is employed in the final good sector or in the research sector:

$$H = H_t + \tilde{H}_t. \tag{25}$$

Equilibrium on the unskilled labor market implies that the unskilled labor force is employed in the final good sector or in the intermediate good sector:

$$L = L_t + \int_0^{n_t} \tilde{L}_{i,t}\, \mathrm{d}i. \tag{26}$$

Equilibrium on the final good market implies

$$Y_t = C_t + I_t \tag{27}$$

which, after using the budget constraints of the agents, is equivalent to

$$\Delta A_{t+1} = I_t + v_t \Delta n_t,$$

i.e. savings finance either investment in physical capital or in research.

3. The equilibrium

In this section we characterize the equilibrium and give some analytical characterization of a balanced growth path.

3.1. Characteristics

We begin by stating a proposition summarizing the equilibrium and optimality conditions of the model. Eqs. (28)–(30) describe the equilibrium on the unskilled labor, skilled labor and final goods markets respectively. The equilibrium interest rate obtains from (31). Optimal consumption is given in Eq. (32). Eq. (33) is the accumulation rule of capital. Eq. (34) links the embodied technological progress to the expansion in the varieties of intermediate products. Eq. (35) is derived from the free entry condition.

Proposition 1. *Given the initial conditions K_{-1} and $\{n_t\}_{t=-T..-1}$, an equilibrium is a path $\{w_t, b_t, q_t, I_t, K_t, r_{t+1}, n_t, C_t, m_t\}_{t\geqslant 0}$ that satisfies the following conditions*:

$$L = \left(\frac{\beta^\beta \alpha^{1-\beta} z_t}{b_t^{1-\beta} w_t^\beta}\right)^{1/\gamma} K_t + (n_{t-T} + \mu^{-\sigma}(n_t - n_{t-T}))\left(\frac{\tau\phi}{b_t q_t}\right)^\sigma q_t I_t, \tag{28}$$

$$H = \left(\frac{\alpha^\alpha \beta^{1-\alpha} z_t}{b_t^\alpha w_t^{1-\alpha}}\right)^{1/\gamma} K_t + \frac{\Delta n_t}{a m_t}, \tag{29}$$

$$z_t^{1/\gamma} K_t \left(\frac{\alpha}{b_t}\right)^{\alpha/\gamma} \left(\frac{\beta}{w_t}\right)^{\beta/\gamma} = C_t + I_t, \tag{30}$$

$$\gamma z_t^{1/\gamma}(1-\lambda) e_t\, q_t^\lambda \left(\frac{\alpha}{b_t}\right)^{\alpha/\gamma} \left(\frac{\beta}{w_t}\right)^{\beta/\gamma} = 1 - \frac{1-\delta}{1+r_{t+1}}\left(\frac{q_t}{q_{t+1}}\right)^\lambda, \tag{31}$$

$$\frac{C_{t+1}}{C_t} = (1 + r_{t+1})\rho, \tag{32}$$

$$K_t = (1-\delta)K_{t-1} + e_t\, q_t^\lambda I_t, \tag{33}$$

$$\frac{b_t q_t}{\tau\phi} = (n_{t-T} + (n_t - n_{t-T})\mu^{1-\sigma})^{1/(\sigma-1)}, \tag{34}$$

$$\frac{\tau^{1-\sigma}(\sigma-1)^{1-\sigma}\sigma^\sigma}{\phi^\sigma a}\left(\frac{w_t}{m_t} - \frac{R_t^{t+1} w_{t+1}}{m_{t+1}}\right) = b_t^{1-\sigma} I_t q_t^{1-\sigma} - R_t^{t+T} b_{t+T}^{1-\sigma} I_{t+T} q_{t+T}^{1-\sigma}, \tag{35}$$

$$m_t = (1-\theta)n_{t-T} + \theta n_t. \tag{36}$$

The proof is given in Appendix A.1. Eq. (33) gives the accumulation law of efficient capital at equilibrium. $E_t = e_t q_t^\lambda$ measures the marginal efficiency of the new investment goods in the production process of efficient capital. This is obviously consistent with the Cobb–Douglas production specification in the efficient capital sector seen in the early stage of our description. E_t measures embodied technical progress, in contrast to the variable z_t in the final good sector, which measures neutral or disembodied technical progress. Our apparently peculiar capital accumulation law is at the basis of Greenwood et al. (1997) seminal contribution. However, the embodied technical progress variable

E_t is endogenous in our model in contrast to the latter. In our case, q_t and then E_t depend on the software to hardware ratio, and this ratio is endogenously determined in our model. The first contribution to growth theory with endogenous embodied technical progress is due to Krusell (1998). Nonetheless, the latter author is mainly interested in reproducing a decreasing relative price of capital along the balanced growth path of his economy, a task which can be undertaken within an elementary setup (cf. Krusell, 1998) in contrast to our objectives. This will be much clearer in the next sections.

3.2. The balanced growth path

We assume that labor supplies L and H are constant. The productivity variables e_t and z_t are assumed constant in the long term. Along a balanced growth path, each variable grows at a constant rate. For output we have

$$Y_t = \bar{Y} g_Y^t,$$

where g_Y is the growth factor and $\bar{Y}$ the initial level of output. n_t, C_t, I_t, q_t, b_t, w_t and K_t grows, respectively, with factors g_n, g_C, g_I, g_q, g_b, g_w and g_K. The interest rate r_t is constant.

Proposition 2. *If q_t grows at a rate $g_q > 1$, then all the other variables grow at strictly positive rates with*

$$g_n = g_q^{(\sigma-1)[1-(1-\lambda)\gamma]/(1-\gamma)}, \tag{37}$$

$$g_Y = g_C = g_I = g_w = g_b = g_q^{\lambda\gamma/(1-\gamma)}, \tag{38}$$

$$g_K = g_q^{\lambda/(1-\gamma)}. \tag{39}$$

The proof is given in Appendix A.2. Along a balanced growth path, output, consumption, investment and wages grow at the same rate. The stock of capital grows faster as it includes improvement in the embodied productivity. It is worth pointing out that for given $g_q > 1$, all the growth rates are increasing functions of λ, the softwares share in human capital. This property reflects that the engine of growth in the model is the expanding variety of softwares: The bigger the impact of the latter on efficient capital, the higher the resulting long-run growth rates.

To determine g_q, we need an additional information, which is provided by the restrictions on the long-run levels. Computing these restrictions from the dynamic system (28)–(35) we end with 8 equations for 9 unknowns ($\bar{w}, \bar{b}, \bar{n}, \bar{q}, \bar{I}, \bar{C}, \bar{K}, \bar{r}$ and g_q) since all the other growth rates can be expressed in terms of g_q. The system in terms of levels is therefore undetermined, which is a usual property of endogenous growth models. Fortunately, it is always possible to rewrite this system in such way that we get rid of this indeterminacy. As usual, this is done by “stationarizing” the equations by the means of some auxiliary variables. Indeed, the dynamic system (28)–(35) can be rewritten as a function of eight stationary variables, which are

$$r_t, \quad \hat{w}_t = \frac{w_t}{b_t}, \quad \hat{K}_t = \frac{K_t}{n_t^{\omega_1}}, \quad \hat{C}_t = \frac{C_t}{b_t}, \quad \hat{I}_t = \frac{I_t}{b_t}, \quad \hat{q}_t = \frac{q_t}{b_t^{\omega_2}}, \quad g_t = \frac{n_t}{n_{t-1}},$$

$$\hat{n}_t = \frac{n_t}{b_t^{\omega_3}},$$

and $\hat{m}_t = m_t/n_t$, with $\omega_1 = \lambda/(\sigma-1)(1-\gamma+\lambda\gamma)$, $\omega_2 = (1-\gamma)/\lambda\gamma$ and $\omega_3 = 1/\gamma\omega_1$. The stationarized dynamic system is given in Appendix A.3. Note that as for the original system, we have two pre-determined variables $\hat{K}_t$ and $\hat{g}_t$. Hence our stationarization does not alter the dynamic order of the original system.

The corresponding restrictions on the levels are given in Appendix A.4. Note that since the other growth rates g_q, g_K, g_w and g_Y depend on $g_n = g$ through (37)–(39), the restrictions on the levels determine all the growth rates of the variables of the model, together with seven other ratios, namely $\bar{r}, \hat{w}$, $\hat{K}$, $\hat{C}$, $\hat{I}$, $\hat{q}$ and $\hat{n}$. Our choice of stationarization is indeed the simplest algebraically speaking given the long run relationships described in Proposition 2. Obviously, we can recover any relevant stationary ratio from the seven previous ones.

Given the complexity of the long run steady state described above, it is impossible to derive an analytical solution. However, though the corresponding system of equations is indeed extremely heavy to manipulate, it is possible to bring out some interesting intermediate results which turn out to be crucial to understand the issues related to the existence and uniqueness of steady state growth paths in our model. In particular, the following proposition reveals most useful.

Proposition 3. *At any growth rate value g, there exist explicit functions expressing the long run level $\hat{K}$, $\hat{I}$, $\hat{r}$, $\hat{C}$, $\hat{q}$, $\hat{n}$ and $\hat{w}$ exclusively in terms of g: $\hat{K} = \Psi_K(g)$, $\hat{I} = \Psi_I(g)$, $\hat{r} = \Psi_r(g)$, $\hat{C} = \Psi_C(g)$, $\hat{q} = \Psi_q(g)$, $\hat{n} = \Psi_n(g)$ and $\hat{w} = \Psi_w(g)$.*

It follows the corollary:

Corollary 1. *There exists an explicit function $\Psi(g)$ such that the long run equilibrium growth rate value solves the equation $\Psi(g) = 0$.*

Clearly if Proposition 3 holds, then we can obtain an explicit equation involving only g by using the g-functional expressions of the long run levels in any equation of the steady-state system. Therefore we can reduce our eight-dimensional system to an explicit scalar equation involving the growth rate g. Once this equation is solved, the remaining long run levels can be recovered using the explicit g-functions of Proposition 3. A proof of Proposition 3 can be found in Appendix A.4. The proof of the following useful property can be also found in the same appendix.

Proposition 4. *Assuming that a solution for the steady-state system exists, the long values of z and e only affect the stationary values $\hat{n}$, $\hat{q}$* and *$\hat{K}$. In particular, z and e have no impact on the growth rate g. In contrast, the R&D productivity parameter a affects g.*

The property above is particularly useful in the interpretation of the outcomes of permanent shocks on the disembodied technological progress variable z_t or on the productivity in the efficient capital sector through the parameter e_t. Proposition 4 implies in particular that a permanent change in both variables will not affect the long run growth rate of the economy, in contrast to a permanent shock on R&D productivity. Interestingly it only affects the long run levels of the variables related to embodied technological progress such as n_t and q_t (stationarized by an adequate measure of raw labor cost). This suggests that permanent movements in z_t or e_t will call for optimal responses in the embodied component of technological progress. As for the insensitivity of long-term growth to changes in z, it is worth pointing out that a similar result can be obtained from Romer's model (1990) while the lab-equipment counterpart of the same model (see Rivera-Batiz and Romer, 1991) gives just the contrary. The intuition behind these properties is the following. In both versions of the model, long-term growth relies on horizontal differentiation R&D. However in the lab-equipment version, the production function in the latter sector is implicitly the same as in the final good sector, in sharp contrast to Romer's model (1990) in which R&D is more labor intensive. Therefore, while a shock on the total factor productivity of the final good sector should have an effect on long-term growth in the lab-equipment version, it should not have this effect in a Romer-like model like ours.

More interestingly, the result on the variable e_t can be advantageously connected to Gordon's argument about the long-term viability of the recent IT-based economic expansion in the USA. Recall that Gordon casts a serious doubt about the real scope of the recent recovery by figuring out that TFP growth has been only boosted in the production of hardware (or of efficient capital in our terminology). Our model displays two main lessons regarding this issue. First of all, if the IT revolution relies exclusively on an acceleration (even permanent) of TFP in the production of efficient (physical) capital, this should not have any everlasting effect on growth. In contrast, when the productivity of R&D is boosted, stimulating the creation of softwares (or immaterial capital), long-term growth is expected to rise. Obviously, this does say much about the short term dynamics, and the possible occurrence of transitory IT revolutions. This appealing issue is treated below after the description of our calibration procedure.

3.3. Parameterized example

Although we can find by Corollary 1 an explicit equation $\Psi(g) = 0$ giving the eventual steady-state growth rate(s), this equation is unsurprisingly so complicated—as it summarizes the algebra of eight non-redundant equations—that no exact solution(s) can be found out. So we resort to numerical resolution using a parameterized example.

Consider the following calibration or the model. A first set of parameters is fixed a priori to what we view as reasonable values given the empirical evidence available (see Table 1). The skilled population is 10% of total population (roughly the share of workers with higher education in developed countries). The length of copyrights is set at five years. It means that the profits made on software invented five years ago falls to zero. The total factor productivity parameterized z is normalized to 1. The rate of depreciation of physical capital is 10% and the psychological discount

Table 1
Parameters

Parameter	Symbol	Value
Unskilled labor supply	L	9
Skilled labor supply	H	1
Copyrights length	T	5
Total factor productivity in the final sector	z	1
Rate of depreciation of capital	δ	0.1
Psychological discount factor	ρ	0.97
Elasticity of substitution between varieties of softwares	σ	2
Diffusion parameter	θ	0.5
Unskilled labor share in the final sector	α	0.5
Skilled labor share in the final sector	β	1/6
Total factor productivity in the research sector	a	1.179
Total factor productivity in the efficient capital sector	e	6.15
Unskilled labor productivity in the intermediate sector	τ	0.18
Share of software in the production of efficient capital	λ	0.5

factor is 0.97. We select the elasticity of substitution between varieties of softwares to obtain a mark-up rate of 2. The parameter θ is called the diffusion coefficient in the literature. It is equal to one when knowledge is non excludable despite the existence of copyrights. On the contrary it is equal to zero, as in Judd (1985), when copyrights prevent any positive externality from protected software to public knowledge. We take an intermediate value of $\frac{1}{2}$.

A second set of parameters is fixed in order to match a series of moments of the steady state we consider. The target moments are drawn from the report of Atkinson et al. (1999) on the state of the new economy in the US. The parameters α and β are such that the share of labor in the final sector is $\frac{2}{3}$ and the ratio of the two wages about 3 (the ratio of elite workers wage to unskilled workers wage). The total factor productivity in the research sector a is set in order to obtain a growth rate of the number of patents of 5% a year. Indeed there was 60 000 new patents in 1983 and 110 000 in 1997, which corresponds to an annual growth rate of 5%. The productivity parameter in the production of efficient capital e is fixed at 6.15 to have a ratio capital to output of 2. The two remaining parameters, λ and τ, are used to calibrate the size of the new economy. Jobs in high-tech electronic manufacturing, softwares and computer related services, and telecommunications represents 4.5% of the total employment. Since we do not have data on the composition of these jobs by skills, we calibrate the parameters so that 4.5% of both skilled and unskilled workers work in the immaterial capital and research sectors. This yields $\lambda = \frac{1}{2}$ and $\tau = 0.18$.

To further evaluate the quality of the calibration, let us report a few other moments of the balanced growth path. As said above, we have calibrated the model to have a growth rate of the number of patents of 5%; this leads to a growth rate of output of 0.98% per year, which may be interpreted as the part of actual output growth generated by embodied technical progress. This is reasonable in the light of the empirical accounting

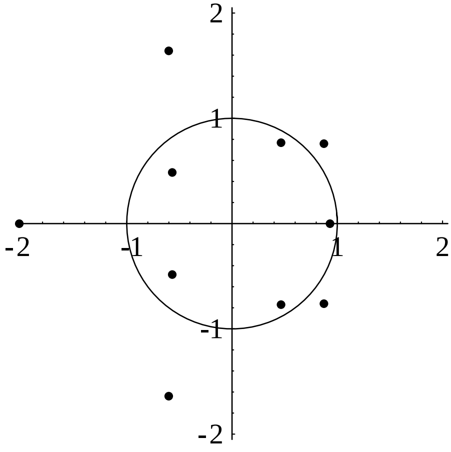


Fig. 1. Eigenvalues.

debate on the measurement of growth rate under embodiment.[2] The interest rate is 4.1%. The share of investment in physical capital over output is 13.8%, which is also close to actual numbers.

For this parameterization, there is a unique balanced growth path with positive growth (negative growth is excluded in the model as it implies a negative number of skilled people in the research sector). Computing the eigenvalues of the linearized model around it, we are able to assess that it is a saddle-point.[3] The eigenvalues of the steady state are depicted in Fig. 1, with their real part on the horizontal axis and their imaginary part on the vertical axis. There are five eigenvalues outside the unit circle, corresponding to the five forward-looking variables. The presence of complex eigenvalues reveals an oscillatory dynamic behavior. This behavior is due to the finite patent specification as one can infer from the solution paths in Section 4 (see also Boucekkine et al., 2002).

An analysis of the sensitivity of the results to the parameters θ and a is presented in Fig. 2. The upward sloping curve labelled $\theta = \frac{1}{2}$ describes how the productivity of R&D a affects the steady-state growth of the number of patents g_N for our calibration. A similar curve is drawn in the case where new software never affect public knowledge ($\theta = 0$); for any given a, the steady state with $\theta = 0$ displays lower growth, but it remains a saddle-point. Note that in these cases, a should be sufficiently high (above 1.075) for a balanced growth path to exist. When new softwares are public knowledge ($\theta = 1$), the threshold below which there is no balanced growth path is 1.05. When a

[2] For example, Greenwood et al. (1997) found that 63% of the US growth rate in the period 1954–1990 is due to embodied technical change. Given that the average annual growth rate over this period is around 1.24%, the contribution of embodiment into this figure amounts to 0.8% approximately.

[3] For dynamic simulation as well as for stability assessment, we use the Dynare package designed by Juillard (1996).

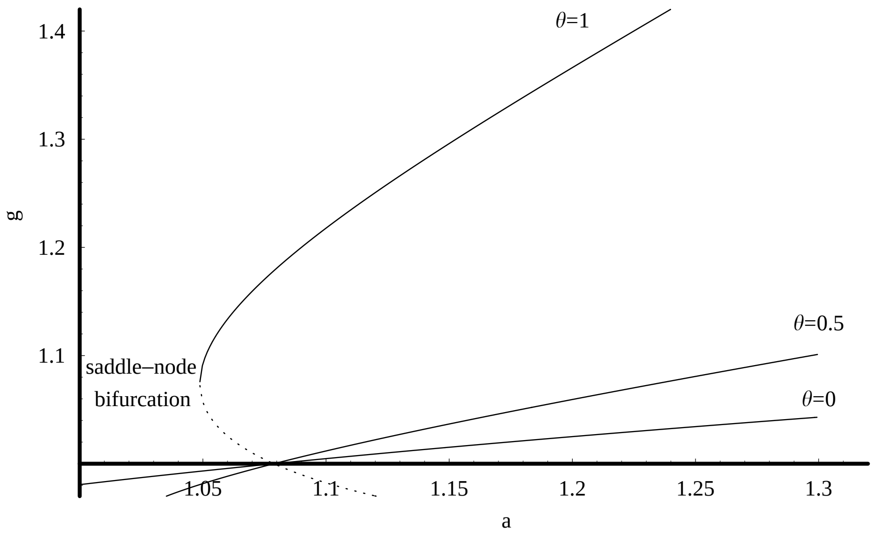


Fig. 2. Bifurcation diagram.

rises above this value, two steady states appear: the high steady state is a saddle-point (solid line) and the low one is unstable (dotted line). Increasing a further, the low steady state enters the region of negative growth, i.e. disappears, and the high steady state is increasing in a. We conclude that the effect of a on g_N is positively related to the value of the diffusion parameter θ.

4. The dynamics of the model

In this section, we will present the results of dynamic simulations conducted on the calibrated model described above. Except in Section 4.3, the magnitude of the shocks is set equal to 1%. One should have in mind this figure in order to value correctly the obtained quantitative results.

4.1. Transitory vs. permanent IT revolutions

We shall perform two experiments in this section. In the first simulation, the R&D productivity parameter a is increased permanently by 1%. In the second experiment, the parameter e, measuring total factor productivity in the capital sector, is increased permanently by 1%. By Proposition 4, we already know that a permanent boom in the IT sector is impossible in the second case. We shall investigate whether there is room for a transitory IT boom in this context. In contrast, the first experiment can give rise to a permanent IT boom.

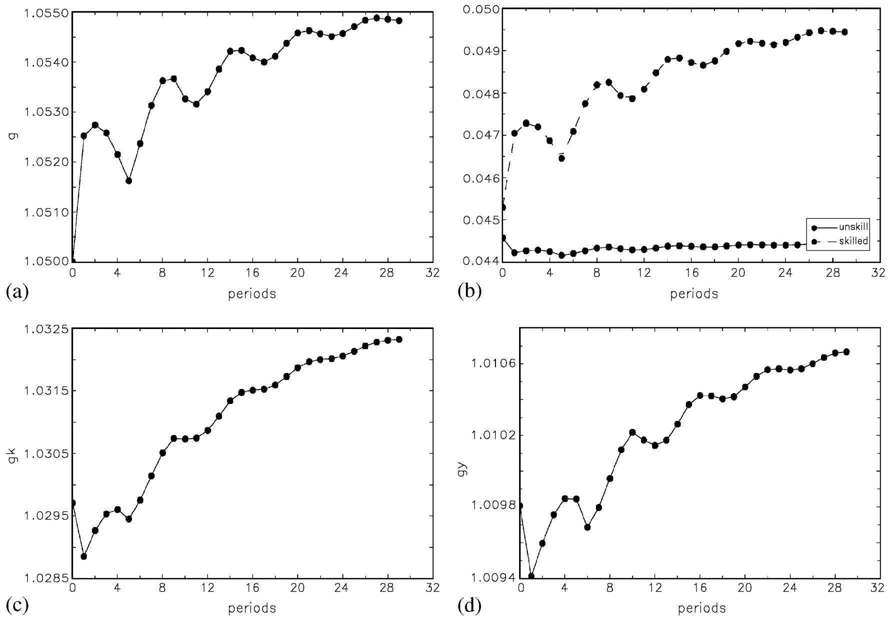


Fig. 3. (a) Growth rate of the number of potents—increase in a. (b) Skilled and unskilled labor in software sector (%)—increase in a. (c) Growth rate of efficient capital—increase in a. (d) Growth rate of production—increase in a.

Figs. 3(a)–(d) present the main results of the first simulation. Figs. 4(a)–(d) are the corresponding solution paths for the second experiment. The first simulation features a permanent IT boom. On the contrary, the productivity shock in the capital sector only gives rise to a temporary IT boom. Another difference is the following: While the dynamics registered under the R&D shock are relatively smooth, the short-run fluctuations obtained under the capital sector shock are larger. This is in particular clear for the solution paths obtained for the growth rate of softwares (Figs. 3(a) and 4(b)) and for the fraction of skilled people working in R&D (Figs. 3(b) and 4(b)). Concerning the growth rate of softwares, it rises by 5% (with respect to the initial steady-state value) at $t = 1$ under the R&D shock while the corresponding magnitude under the capital sector shock is almost the double—9.8%. As for the fraction of skilled people doing research, it rises by 9.5% at $t = 1$ under the latter shock and only by 3.8% when R&D productivity increases. In the long run, both variables return to their initial steady-state values in the case of the capital sector shock while the growth rate of softwares and the fraction of skilled in R&D increase by 10.6% and 10%, respectively, in the long run according to the first experiment.

In both simulations, the crucial aspect is the inter-sectoral reallocation of labor resources. In particular, the inter-sectoral allocation of skilled workers turns out to be the main engine of the obtained IT booms. In contrast, the reassignment of unskilled labor

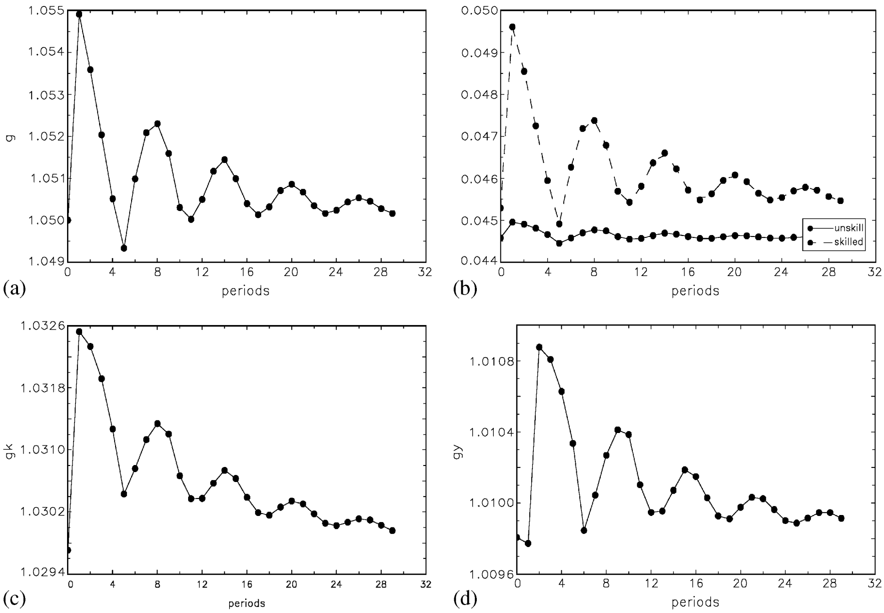
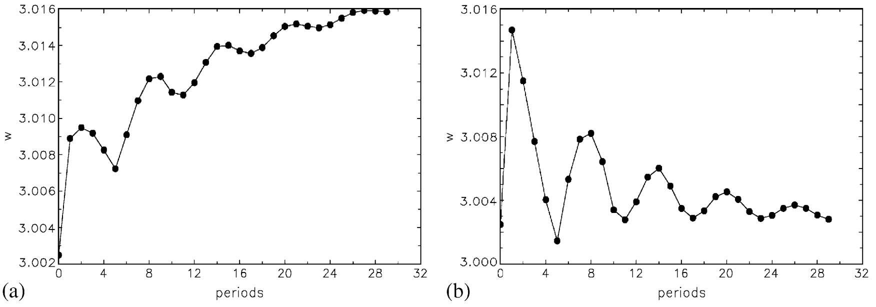


Fig. 4. (a) Growth rate of the number of potents—increase in e. (b) Skilled and unskilled labor in software sector (%)—increase in a. (c) Growth rate of efficient capital—increase in e. (d) Growth rate of production—increase in e.

resources is quantitatively much less significant. This is clear in Figs. 3(b) and 4(b). In the permanent IT experiment, the fraction of unskilled people producing softwares very slightly decreases both in the short run and in the long run. Precisely, it decreases by 0.7% at $t = 1$ and by 0.26% in the long run with respect to the initial steady-state value. The transitory IT revolution experiment yields the reverse result in the short run, +0.8%.

Note that the latter findings are consistent with the empirical literature dealing with skill-biased technological progress (for example Bartel and Lichtenberg, 1987). It is important here to understand the precise mechanisms behind the registered massive inter-sectoral reallocation of skilled labor. To each productivity shock corresponds a specific transmission mechanism. When the productivity of research permanently increases, the cost of this activity is lowered forever, which gives rise to a massive reallocation effect of skilled people at the expense of the final good sector. When total factor productivity in the capital sector is shifted upwards, the mechanisms involved are quite different. At first, the marginal return to both softwares and hardware increase in this sector, which stimulates demand for both inputs. This in turn stimulates the creation and production of more softwares, and so the hiring of more skilled people in the R&D sector. More skilled workers doing research means more created softwares, and this launches the IT boom in both cases.

There are further differences between the two experiments, in particular in the timing of capital deepening, as it can be deduced from Figs. 3(c) and 4(c). The registered



(a) (b)

Fig. 5. (a) Skill premium—increase in a. (b) Skill premium—increase in e.

paths for the growth rate of production in the final good sector (see Figs. 3(d) and 4(d)) result mainly from this difference in timing. Indeed, the growth rate of efficient capital decreases by 2.8% at $t=1$ (but rises by more than 10% in the long run) in the permanent IT boom scenario. Capital deepening starts much earlier in the alternative scenario, +9.8% at $t=1$. The solution paths for the growth rate of production in the final good sector follow basically the same lines. There is an intertemporal arbitrage mechanism behind this. Under perfect foresight, the economic agents know that the economy will experience an expansion in the long run in the first experiment while no long-term growth is possible in the second experiment. This is in particular crucial for capital goods producers. There is no question about the massive capital deepening starting at $t=1$ when total factor productivity increases in the capital sector. In contrast, when the productivity shock does not affect directly this sector, as it does occur when there is a productivity improving shock in the R&D activity, things are much less mechanistic. In such a case, the capital producer may not accumulate capital massively from $t=1$ and wait for a further stage of the economic expansion with a more favorable supply of softwares. This is exactly what happens in our permanent IT revolution experiment.

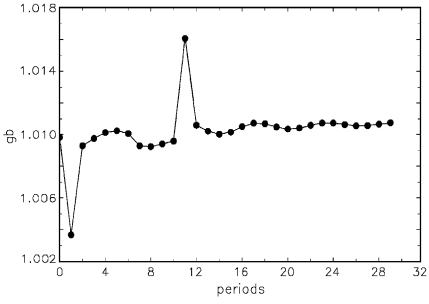
4.2. *The income and wealth effects of IT revolutions*

One of the most interesting debates around the IT revolution concerns the skill premium that presumably increases in the information age. There is a huge literature on this topic starting from the capital-skill complementarity hypothesis advocated by Griliches (1969) to Galor and Moav (2000). Admittedly the skill premium increases in periods of sharp technical change since skilled workers have a comparative advantage in running the new technologies but this premium tends to vanish as the innovations get assimilated by the economy (see Bartel and Lichtenberg, 1987, for empirical evidence). Therefore, the IT revolution should be accompanied by a rise in the skill premium at least in the short run. In our stationarized dynamic model, the skill premium is directly measured by the variable $\hat{w}_t = w_t/b_t$. Figs. 5(a) and (b) display the dynamics of the skill premium under a permanent IT revolution (i.e. when a increases permanently) and under a transitory IT revolution (i.e. when e increases permanently), respectively.

In both cases, the skill premium rises in the short run, which is consistent with the related empirical and theoretical literature. The skill premium rises by 0.4% (resp. 0.2%) at $t = 1$ in the case of the transitory (resp. permanent) IT boom. In the long run, the productivity shock in the efficient capital sector has no effect on the skill premium while the productivity shock in the R&D sector, producing the permanent IT revolution, does. Indeed in the latter case, the skill premium increases by 0.5%. The rational behind these results can be deduced from the dynamics of skilled and unskilled labor inter-sectoral allocation given above. In both cases, the productivity shocks induce an increase in the demand for skilled people. This increase is permanent when the shock improves the productivity of R&D, and it is only transitory when the shock affects the capital sector. Meanwhile, and this is one of the originality of our model, the demand for unskilled people may also be shifted upwards under an IT revolution because raw labor is the unique input in the production of softwares. However, as explained in the previous subsection, the reallocation of labor resources after the shocks relies principally on skilled workers. Despite the wage of unskilled increases at $t = 1$ under the transitory IT boom, it is not enough to compensate the sharp rise in skilled people's wage.

The specifications adopted as for the type of labor input required in each sector are obviously crucial in the results stated above. In particular, since our IT sectors also employ unskilled workers in pure production tasks, which seems to us highly reasonable, things are far from obvious. Undeniably if adoption costs are included in our model as in Greenwood and Yorukoglu (1997), the story would be more complete: Skilled labor is not only needed to create new softwares, it is also needed to adopt the innovations. Though skilled people are employed in the final goods sector in our model, there is no explicit adoption task which entirely lies on the shoulders of the latter. Our approach is indeed complementary to the story told by Greenwood and Yorukoglu: While skilled labor is required for technology adoption, unskilled people are also needed in the production of the intermediate goods associated with the innovations.

The skill premium debate is part of a larger debate on the income distribution effects of the IT revolution. Another fundamental point concerns the consequences on the stock market evolution in line with the discussion opened by Greenwood and Jovanovic (1999) and even more recently by Jovanovic and Rousseau (2000). We could have much more comprehensively studied this question by introducing a stock market and an explicit decision on the distribution of dividends as did Greenwood and Yorukoglu. Our model is too simple on the financial markets side to bring out interesting lessons regarding these issues. The linear technologies adopted for the intermediate and research sector do not help much in this respect. As an example, one can check that the ratio "value of the firms in the research sector" to "skilled labor wage", namely v_t/w_t, is exactly equal at equilibrium to the skilled labor assigned to the research sector. We know from the previous subsection that the latter increases in the short run when the economy is affected by the productivity shocks, which in turn means that the value of the firms creating softwares will sharply increase in the short run too. Though this result is good in itself, it is too mechanically generated to be taken more seriously than an elementary consequence of our simple technological specifications of the IT sectors.



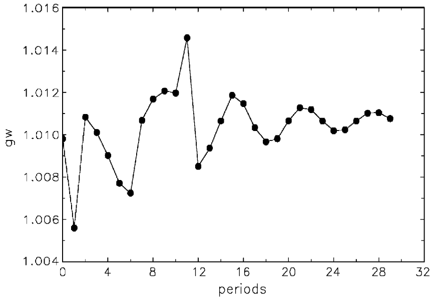


Fig. 6. (a) Growth rate of unskilled labor productivity-productivity slowdown. (b) Growth rate of skilled labor productivity-productivity slowdown.

4.3. IT revolutions and the productivity slowdown

The theoretical studies devoted to the link between embodied technological change and labor productivity slowdown are now numerous. As pointed out in the introduction, the fundamental contributions dealing with this issue treat the former variable as exogenously given (see specially Greenwood and Yorukoglu, 1997). Since embodied technological progress is endogenous in our model, an interesting issue concerns the behavior of labor marginal productivity in the transition dynamics. Obviously it is an issue if we consider the final goods sector, and not the other two sectors which have linear production functions. We propose here an alternative story of the productivity slowdown and a simple way to get through the Solow paradox. Our idea is quite simple. A productivity slowdown may occur even when the economy comes up with a major achievement in research (i.e. the invention of the electronic chip by Intel) if at the same time it is affected by (an eventually small) negative supply shock (i.e. oil shock). Since the research sector is small relative to the final good sector (in terms of labor and production), and provided that technological progress is embodied in the capital goods, labor productivity in the latter sector may perfectly slowdown under the latter configuration. Hence there is no need to adoption costs to generate a productivity slowdown followed by an eventually spectacular recovery.

Figs. 6(a) and (b) present some results of a simulation in which the productivity of R&D, a increases permanently by 1%, while total factor productivity in the final good sector, z, is reduced by 0.5% from $t=1$ to 10, featuring a kind of small oil shock. Figs. 6(a) and (b) display the dynamics of the growth rate of labor productivity for both unskilled and skilled. Both go down during the oil shock but recover around the date at which the negative supply shock ends. The burst in productivity growth is delayed.

Clearly our story can also incorporate adoption costs, which will delay even more the IT revolution. The point we would like to make here is simple. Independently of the adoption costs and of more or less sophisticated technological diffusion theories, the Solow paradox completely disappears within a basic research-based growth model à la Romer with embodied technical progress when a major advance in the research process (permanent shock on R&D productivity) is combined with an oil shock (negative transitory shock on TFP in the final good sector).

5. Concluding remarks

In this paper, we build up a multi-sectoral growth model with endogenous embodied technical progress in order to draw some useful lessons about the consequences and long-term viability of an IT driven economic expansion. Some analytical results have been derived despite the extreme analytical complexity of the model. Two main results have been brought out. First, an IT-based expansion period may not have long-term effects on growth if it exclusively relies on an acceleration (even permanent) of TFP in the production of efficient (physical) capital. In such a case, the IT revolution is just transitory. Second, when the productivity of R&D is boosted, giving rise to a permanent increase in the number of softwares (or immaterial capital), a new everlasting growth regime is likely to set in. A rigorous diagnosis of the viability of an IT based economic regime should take into account these aspects. Further properties of such a regime have been identified (skill premium, capital deepening, etc.) and an alternative story of the productivity slowdown has been presented.

Clearly our model need to be enriched in several directions in order to provide a more complete description of a *digital economy*, it should be therefore seen as a first step in an ongoing research program with numerous ramifications. Two main extensions are currently considered. First of all, it is clear that the consumer behavior under the digital economy requires a much more specific modelling. In particular, a fundamental characteristic of the recent US expansion is the boom of households' purchases of IT equipment, favored by the strong decline of the price of this equipment (see Jorgenson and Stiroh, 1999). It is consequently necessary to incorporate this aspect into the growth models like ours. Second, the R&D policy followed by the IT firms is certainly much less standard and much more complex than the elementary horizontal differentiation story told here. The recent attempt of Segerstrom (2000) to meet this specific behavior is most worthwhile in this respect. It seems to us that one of the most important challenges faced by the theorists nowadays is to understand how the process of innovation in the IT sector operates and why in this context some well known properties (like Moore's law) may or may not hold. One outcome of our contribution is precisely to claim that no safe diagnosis can be drawn concerning the long-term viability of an IT based economic regime if the latter issues are not properly addressed.

Acknowledgements

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Appendix A.

A.1. Proof of Proposition 1

The demand of unskilled labor from the intermediate goods sector is obtained using Eqs. (17), (16), (18) and (19):

$$\int_0^{n_t} \tilde{L}_{i,t}\,\mathrm{d}i = \int_0^{n_t^c} \left(\frac{\phi I_t}{b_t Q_t}\right)^{\sigma} Q_t\,\mathrm{d}i + \mu^{-\sigma} \int_{n_t^c}^{n_t} \left(\frac{\tau\phi I_t}{b_t Q_t}\right)^{\sigma} Q_t\,\mathrm{d}i$$

$$= (n_t^c + \mu^{-\sigma}(n_t - n_t^c)) \left(\frac{\phi I_t}{b_t Q_t}\right)^{\sigma} Q_t.$$

Using this result and Eqs. (8) and (26), the equilibrium on the unskilled labor market leads to Eq. (28) of the proposition. From Eqs. (9), (25), and (23) we have

$$H = \left(\frac{\alpha^\alpha \beta^{1-\alpha} z_t}{b_t^\alpha w_t^{1-\alpha}}\right)^{1/\gamma} K_t + \frac{\Delta n_t}{a m_t}$$

which is Eq. (29) of the proposition. The equilibrium on the final good market, using (8), (9), (27), (11), is given in Eq. (30) of the proposition. Replacing $L_{t,s}$ from (6) and $H_{t,s}$ from (7) in (3), using the definition of $K_{t,s}$ given in (1), and simplifying yields:

$$\gamma \sum_{s=t}^{\infty} R_t^s z_s^{1/\gamma} (1-\delta)^{s-t} \left(\frac{\alpha}{b_s}\right)^{\alpha/\gamma} \left(\frac{\beta}{w_s}\right)^{\beta/\gamma} = d_t.$$

Replacing d_t by its value from (14) yields:

$$\gamma(1-\lambda) e_t \left(\frac{Q_t}{I_t}\right)^{\lambda} \sum_{s=t}^{\infty} R_t^s z_s^{1/\gamma} (1-\delta)^{s-t} \left(\frac{\alpha}{b_s}\right)^{\alpha/\gamma} \left(\frac{\beta}{w_s}\right)^{\beta/\gamma} = 1$$

which gives the law of motion for Q_t in Eq. (31) of the proposition. Consumption dynamics is given by (24) yielding Eq. (32) of the main text. Capital accumulation is given by

$$K_t = \sum_{s=-\infty}^{t} K_{s,t} = \sum_{s=-\infty}^{t} K_{s,s}(1-\delta)^{s-t} = \sum_{s=-\infty}^{t} e_s Q_s^{\lambda} I_s^{1-\lambda} (1-\delta)^{s-t}$$

which is Eq. (33) of the proposition. The variable Q_t can be determined using Eqs. (13), (16), (18) and (19), which yields Eq. (34) of the proposition. Using (21), (22), and (16), the free entry condition becomes

$$\frac{w_t}{a m_t} = \frac{1}{\sigma - 1} \mu^{-\sigma} \sum_{z=t}^{t+T} R_t^z \left(\frac{b_z}{\tau}\right)^{1-\sigma} (\phi I_z)^{\sigma} Q_z^{1-\sigma}$$

which yields Eq. (35) of the proposition. Finally, (36) is obtained from (20).

A.2. Proof of Proposition 2

If a balanced growth path should satisfy the nine Eqs. (28)–(36), then one should have the following eight restrictions among the various growth rates:

$$(g_b)^{-(1-\beta)/\gamma}(g_w)^{-\beta/\gamma}g_K = 1 = \frac{g_N}{g_b^{\sigma}}\, g_q^{1-\sigma}g_I, \tag{A.1}$$

$$g_K = (g_b)^{\alpha/\gamma}(g_w)^{(1-\alpha)/\gamma}, \tag{A.2}$$

$$g_C = g_I = g_Y = g_K g_b^{-\alpha/\gamma}g_w^{-\beta/\gamma}, \tag{A.3}$$

$$g_q^{\lambda}(g_b)^{-\alpha/\gamma}(g_w)^{-\beta/\gamma} = 1, \tag{A.4}$$

$$g_N^{1/(\sigma-1)} = g_b g_q, \tag{A.5}$$

$$g_Y = (1+r)\rho, \tag{A.6}$$

$$g_K = g_q^{\lambda}g_Y, \tag{A.7}$$

$$\frac{g_w}{g_N} = g_b^{1-\sigma}g_q^{1-\sigma}g_Y, \tag{A.8}$$

$$g_m = g_N. \tag{A.9}$$

We use implicitly the condition $g_Y = g_C = g_I$ in (A.1)–(A.8), a condition implied by the good market equilibrium and by the fact that the share of consumption in production cannot tend to zero or to infinity along a balanced growth path. Using (A.1) and (A.2) to eliminate g_K we have $g_b = g_w$. Eq. (A.4) gives

$$g_b = g_w = g_q^{\lambda\gamma/(1-\gamma)}$$

and by Eq. (A.2), $g_K = g_q^{\lambda/(1-\gamma)}$. Eq. (A.3) yields $g_Y = g_q^{\lambda\gamma/(1-\gamma)}$. It turns out that (A.7) is redundant with (A.3). Now, by using (A.5) we get

$$g_N = (g_b g_q)^{\sigma-1} = g_q^{(\sigma-1)[1-(1-\lambda)\gamma]/(1-\gamma)}.$$

This result is the same obtained from Eq. (A.8) or from the restriction $1 = (g_N/g_b^{\sigma})g_q^{1-\sigma}g_I$ listed in (A.1). Hence, the two latter equations are redundant with (A.5). At the end, the seven unknowns of the problem $(g_b, g_w, g_q, g_Y, g_N, g_K, \bar{r})$ are shown to be truly related by a system of six equations (out of the nine initial restrictions since three redundant equations have been identified). For given g_q, all the other unknowns can be found. They are thus parameterized by g_q, including $\bar{r}$ since by (A.6), we have $\bar{r} = g_q^{\lambda\gamma/(1-\gamma)}/\rho - 1$.

A.3. The stationarized dynamic system

The dynamic system (28)–(36) can be rewritten as

$$\left(\frac{\beta^{\beta}\alpha^{1-\beta}z_t}{\hat{w}_t^{\beta}}\right)^{1/\gamma}\hat{K}_t\hat{n}_t^{\omega_1} + (u_t + \mu^{-\sigma}(1-u_t))(\tau\phi)^{\sigma}\hat{q}_t^{1-\sigma}\hat{n}_t\hat{I}_t = L,$$

$$\left(\frac{\alpha^{\alpha}\beta^{1-\alpha}z_t}{\hat{w}_t^{1-\alpha}}\right)^{1/\gamma}\hat{K}_t\hat{n}_t^{\omega_1}+\frac{g_t-1}{ag_t\hat{m}_t}=H,$$

$$z_t^{1/\gamma}\hat{K}_t\hat{n}_t^{\omega_1}\alpha^{\alpha/\gamma}\left(\frac{\beta}{\hat{w}_t}\right)^{\beta/\gamma}=\hat{C}_t+\hat{I}_t,$$

$$\gamma\hat{q}_t^{\lambda}(1-\lambda)e_t z_t^{1/\gamma}\alpha^{\alpha/\gamma}\left(\frac{\beta}{\hat{w}_t}\right)^{\beta/\gamma}+\frac{1-\delta}{1+r_{t+1}}\left(\frac{\hat{n}_t g_{t+1}}{\hat{n}_{t+1}}\right)^{-(1-\gamma)\omega_1}\left(\frac{\hat{q}_t}{\hat{q}_{t+1}}\right)^{\lambda}=1,$$

$$\frac{\hat{C}_{t+1}}{\hat{C}_t}\left(\frac{\hat{n}_t g_{t+1}}{\hat{n}_{t+1}}\right)^{\gamma\omega_1}=(1+r_{t+1})\rho,$$

$$\hat{K}_t-(1-\delta)\hat{K}_{t-1}g_t^{-\omega_1}=e_t\hat{q}_t^{\lambda}\hat{I}_t\hat{n}_t^{-\omega_1},$$

$$u_t+(1-u_t)\mu^{1-\sigma}=\left(\frac{\hat{q}_t}{\tau\phi}\right)^{\sigma-1}\frac{1}{\hat{n}_t},$$

$$\frac{\xi}{a}\left(\frac{\hat{w}_t}{\hat{m}_t}-\frac{\hat{w}_{t+1}\,g_{t+1}^{\gamma\omega_1-1}(\hat{n}_t/\hat{n}_{t+1})^{\gamma\omega_1}}{(1+r_{t+1})\hat{m}_{t+1}}\right)$$

$$+\left(\prod_{i=1}^{T}\frac{1}{1+r_{t+i}}\right)\hat{I}_{t+T}\hat{q}_{t+T}^{1-\sigma}\hat{n}_{t+T}\left(\prod_{i=0}^{T-1}g_{t+i+1}^{\gamma\omega_1-1}\left(\frac{\hat{n}_{t+i}}{\hat{n}_{t+1+i}}\right)^{\gamma\omega_1}\right)=\hat{I}_t\hat{q}_t^{1-\sigma}\hat{n}_t,$$

$$\hat{m}_t=\theta+(1-\theta)u_t$$

with

$$u_t=\prod_{i=0}^{T-1}\frac{1}{g_{t-i}}\quad\text{and}\quad\xi=\frac{\tau^{1-\sigma}(\sigma-1)^{1-\sigma}\sigma^{\sigma}}{\phi^{\sigma}}.$$

A.4. The proofs of Propositions 3 and 4

Denote $g_n=g$. Considering that $\lim z_t=z$ and $\lim e_t=e$, and defining the following stationary variables:

$$\hat{w}=\frac{\bar{w}}{\bar{b}},\quad\hat{K}=\frac{\bar{K}}{\bar{n}^{\omega_1}},\quad\hat{C}=\frac{\hat{C}}{\bar{b}},\quad\hat{I}=\frac{\bar{I}}{\bar{b}},\quad\hat{q}=\frac{\hat{q}}{\bar{b}^{\omega_2}},\quad\hat{n}=\frac{\bar{n}}{\bar{b}^{\omega_3}},$$

the restrictions on the levels can be rewritten as

$$\left(\frac{\beta^{\beta}\alpha^{1-\beta}z}{\hat{w}^{\beta}}\right)^{1/\gamma}\hat{K}\hat{n}^{\omega_1}+(g^{-T}+\mu^{-\sigma}(1-g^{-T}))(\tau\phi)^{\sigma}\hat{q}^{1-\sigma}\hat{I}\hat{n}=L,\tag{A.10}$$

$$\left(\frac{\alpha^{\alpha}\beta^{1-\alpha}z}{\hat{w}^{1-\alpha}}\right)^{1/\gamma}\hat{K}\hat{n}^{\omega_1}+\frac{1-1/g}{a(g^{-T}+\theta(1-g^{-T}))}=H,\tag{A.11}$$

$$z^{1/\gamma}\hat{K}\hat{n}^{\omega_1}\alpha^{\alpha/\gamma}\left(\frac{\beta}{\hat{w}}\right)^{\beta/\gamma}=\hat{C}+\hat{I}, \tag{A.12}$$

$$\gamma(1-\lambda)e\hat{q}^{\lambda}z^{1/\gamma}\alpha^{\alpha/\gamma}\left(\frac{\beta}{\hat{w}}\right)^{\beta/\gamma}+\frac{1-\delta}{(1+r)}g^{-(1-\gamma)\omega_1}=1, \tag{A.13}$$

$$g^{\gamma\omega_1}=(1+r)\rho, \tag{A.14}$$

$$\hat{K}(1-(1-\delta)g^{-\omega_1})\hat{n}^{\omega_1}=e\,\hat{q}^{\lambda}\hat{I}, \tag{A.15}$$

$$\tau\phi(g^{-T}+(1-g^{-T})\mu^{1-\sigma})^{1/(\sigma-1)}(\hat{n})^{1/(\sigma-1)}=\hat{q}, \tag{A.16}$$

$$\frac{\xi\hat{w}(1-(g^{\gamma\omega_1-1})/(1+r))}{a((1-\theta)g^{-T}+\theta)}-\hat{I}\hat{q}^{1-\sigma}\hat{n}\left(1-\left(\frac{g^{\gamma\omega_1-1}}{1+r}\right)^{\mathrm{T}}\right)=0. \tag{A.17}$$

We write $\hat{x}=x$ for any variable x to unburden the notations in this appendix. Using Eq. (A.14), we get directly

$$r=\Psi_r(g)=\frac{g^{\gamma\omega_1}}{\rho}-1.$$

Using Eq. (A.17), we can derive the following important relation:

$$w=\Psi_1(g)Iq^{1-\sigma}n, \tag{A.18}$$

with $\Psi_1(g)$ given by the following expression provided $\Psi_r(g)$

$$\Psi_1(g)=\frac{1-(\rho/g)^{\mathrm{T}}}{A^1(1-\rho/g)}((1-\theta)g^{-T}+\theta)$$

with $A^1=\tau^{1-\sigma}[(\sigma-1)^{1-\sigma}\sigma^{\sigma}]/a\phi^{\sigma}$. On the other hand, Eq. (A.11) yields

$$Kn^{\omega_1}=\Psi_2(g)w^{(1-\alpha)/\gamma} \tag{A.19}$$

with

$$\Psi_2(g)=\frac{H-g-1/[ag((1-\theta)g^{-T}+\theta)]}{A^2z^{1/\gamma}},$$

where $A^2=(\alpha^{\alpha}\beta^{1-\alpha})^{1/\gamma}$. Putting (A.18) and (A.19) into Eq. (A.10) we get the intended functional relation $w=\Psi_w(g)$ with

$$\Psi_w(g)=\frac{L}{(\beta^{\beta}\alpha^{1-\beta}z)^{1/\gamma}\Psi_2(g)+(\phi\tau)^{\sigma}[g^{-T}+\mu^{-\sigma}(1-g^{-T})]/\Psi_1(g)}.$$

Note that by construction of $\Psi_1(g)$ and $\Psi_2(g)$, $\Psi_w(g)$ *does not depend neither on* z *nor on* e. *In contrast*, $\Psi_w(g)$ *depends on* a. Now, we can derive $\Psi_I(g)$. Indeed, using Eq. (A.16), one can find:

$$q^{1-\sigma}n=(\tau\phi)^{1-\sigma}(g^{-T}+(1-g^{-T})\mu^{1-\sigma})^{-1}=\Psi_3(g),$$

from which we can derive immediately a g-function for I given $\Psi_w(g)$ and the relationship (A.18):

$$I = \Psi_I(g) = \frac{\Psi_w(g)}{\Psi_1(g)\Psi_3(g)}.$$

The g-functional expression for C is then computed from Eq. (A.12) and using additionally the relation (A.19):

$$C = \Psi_C(g) = z^{1/\gamma}\Psi_2(g)\alpha^{\alpha/\gamma}\beta^{\beta/\gamma}(\Psi_w(g))^{-\beta/\gamma} - \Psi_I(g).$$

Also note that by construction of $\Psi_2(g)$ and $\Psi_w(g)$, $\Psi_C(g)$ and $\Psi_I(g)$ *do not depend neither on z nor on e but both depend on a*, consistently with Proposition 4. According to the same proposition, z and e should enter the g-expressions of the stationarized variables q, n and K. This is indeed obvious to check. Immediately from Eq. (A.13), on can directly check that q is a decreasing function of z and e:

$$q^{\lambda} = (\Psi_q(g))^{\lambda} = \frac{1-(1-\delta)/(1+r)g^{-(1-\gamma)\omega_1}}{\gamma(1-\lambda)\alpha^{\alpha/\gamma}ez^{1/\gamma}}\left(\frac{\Psi_w(g)}{\beta}\right)^{\beta/\gamma}.$$

A decrease in z or in e increases the value of the stationarized q, or in other terms it rises the value of embodied technological progress with respect to a measure of the raw labor cost by definition of the stationarized q. One also can trivially check that z has the same effect on the stationarized n. Effectively using Eq. (A.16) and then condition (A.19) above, one finds:

$$n^{1/(\sigma-1)} = (\Psi_n(g))^{1/(\sigma-1)} = \frac{\tau\phi}{\Psi_q(g)}[g^{-T} + (1-g^{-T})\mu^{1-\sigma}]^{1/(\sigma-1)},$$

$$K = \Psi_K(g) = \frac{\Psi_2(g)(\Psi_w(g))^{(1-\alpha)/\gamma}}{(\Psi_n(g))^{\omega_1}}.$$

The relation above implies that Kn^{ω_1} depends on z through the term $\Psi_2(g)$. Given the g-functional expression of $\Psi_2(g)$ written above, this implies that Kn^{ω_1} includes a term $z^{-1/\gamma}$. Consider now Eq. (A.13), namely

$$Kn^{\omega_1}(1-(1-\delta)g^{-\omega_1}) = eq^{\lambda}I.$$

Substituting q, I and Kn^{ω_1} by their respective g-functional expressions, one can find an equation involving only g. It is easy to check that this equation does not depend on z neither on e. In effect, the term in z appearing in Kn^{ω_1} is offset by the multiplicative factor $z^{-1/\gamma}$ appearing in the g-expression of q^{λ}. Also the product eq^{λ} appearing in the right side of the equation is independent of e given the latter g-expression. In contrast, the parameter a does not vanish from the equation although we cannot say much about the behavior of the long-term growth rate with respect to this parameter.

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