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The natalist bias of pollution control

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ABSTRACT

For a given technology, two ways are available to achieve low polluting emissions: reducing production per capita or reducing population size. This paper insists on the tension between the former and the latter. Controlling pollution either through Pigovian taxes or through tradable quotas schemes encourages agents to shift away from production to tax free activities such as procreation and leisure. This natalist bias will deteriorate the environment further, entailing the need to impose ever more stringent pollution rights per person. However, this will in turn gradually impoverish the successive generations: population will tend to increase further and production per capita to decrease as the generations pass. One possible solution consists in capping population too.

1. Introduction

Pollution control can be justified on both efficiency and fairness grounds. Under a Kyoto type of regime, a key motivation for capping greenhouse gas emissions arises from a concern for future generations. The aim is to make sure that the climatic conditions they will experience either be not worse than ours or, at the very least, do not prevent them from leading a decent life. Article 2 of the 1997 Kyoto Protocol states that it aims at the

> "stabilization of greenhouse gas concentrations in the atmosphere at a level that would prevent dangerous anthropogenic interference with the climate system. Such a level should be achieved within a time-frame sufficient to allow ecosystems to adapt naturally to climate change, to ensure that food production is not threatened and to enable economic development to proceed in a sustainable manner."

The assumption is thus that people in the future will be better off if we cap emissions from now on, than under a business as usual scenario. This is a very plausible claim. And yet, it generates concerns.

One worry has to do with the opportunity cost for today's poor of such pollution control. The worry we are going to deal with here is the reverse one. It is not so much that pollution control may "impoverish" today's poor if certain conditions are met. It is rather that it may "impoverish" future generations too, including their poorest members. The mechanism through which future generations might become poorer as a result of pollution control rests on the demographic impact of the latter. In general, the literature in climate economics tends to operate under the assumption that demographics is exogenous (adopting for example global population size projections made by major demographic institutions). One

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exception consists in the attempt to connect demographics with the choice of allocation rule of tradable quotas. Consider the following two quotes as illustrations:

> "The major objections to [per capita entitlements] are based partly on ethical and practical 'comparable-burden' type arguments (since it would imply a huge adjustment burden on industrialized countries, to which they are unlikely to agree), and partly on grounds of concern that such allocation might 'reward' population and population growth. Proponents tend to argue that any such effect is negligible compared to other factors influencing population; but to avoid any inducement to population growth, Grubb suggests that the population measure should be restricted to population above a certain age. (…) Grubb et al. note a wider range of possibilities for avoiding any incentive to population growth, including 'lagged' allocation (related to population a fixed period earlier); apportionment to a fixed historical date; or the inclusion of an explicit term related inversely to population growth rate" (Grubb [1, pp. 485–486]).

> "If we agree that emissions allocations should be based on numbers of people, we effectively encourage something which compounds our problems on Earth: population growth. Solutions have been suggested; in particular, we might tie allocations to population figures for a specific time. Singer, for example, argues that per capita allocations should be based on estimates of a country's population in the future, to avoid penalizing countries with young populations." (Garvey [2, p. 218]).

Although neither of these quotes is entirely explicit about the mechanism through which per capita allocation incentivizes population growth, one may assume that what these two authors have in mind is the following idea: under a fully or partly population-based allocation rule of pollution rights, countries may encourage population growth because this will positively impact their relative share in the quota allocation at the next period. This "share preservation/increase" motive can be one incentive for population growth. Grubb is probably right: if this is what actually drives the natalist effect, it is likely to remain "negligible", as population growth may entail costs likely to more than compensate the value of getting extra emission entitlements.

The mechanism we have in mind differs from the "share reservation/increase" motive. Moreover, its impact is likely to be much more significant. Our starting point is that, when there is only one production sector, capping emissions entails capping production. We will show that this generates a shift from production to other activities, especially procreation. It is this shift from production to reproduction that will generate the demographic impact of capping emissions. Note that in this case, what drives the natalist effect does not directly have to do with the allocation formula. It rather has to do more directly with the very existence of a cap.

As mentioned above, endogenous responses of population have been neglected so far in environmental economics. The main contribution of our paper is accordingly to provide a framework where demography reacts to pollution control. In order to properly focus on that aspect, we simplify matters with regard to technology, and more specifically, its degree of eco-efficiency. We keep it exogenous. This does not need to imply that technology is constant, but rather that technological progress does not depend on the specific policy.

One may object that assuming exogenous technology is far fetched. Admittedly, a portion of technical change is endogenous. However, the empirical literature suggests that this portion is limited.[1] Moreover, there is a second way in which our assumption is realistic: in cases such as climate change, the scale of environment-saving improvements required to stabilize pollution is daunting, at least in the medium-run. Even fully endogenous and highly responsive technology may therefore not suffice.

The essential ingredients of the model are as follows. Individuals allocate their time across three activities: production, leisure, and procreation. Each of these concepts has a specific meaning. Production refers to the time spent on manufacturing consumption and investment goods with an exogenous technology. Leisure involves non-market and emission-free activities, such as chatting with friends, sleeping, sweeping the floor, … but could also be extended to include time spent on eco-friendly production activities.[2] Procreation refers to the time spent on child rearing by parents.

Substitution of procreation for production is at the heart of this paper. It occurs as soon as rearing children takes time[3] and is sensitive to the relative return of spending time on this or other activities. Various factors can affect this relative return: parents' income [5], child mortality [6,7], the absence of formal old age support schemes [8], cultural norms (Princeton European Fertility Project), the importance of increasing the relative power of one's community (see [9,10]), etc. In this paper we concentrate on the first of these factors. Becker and Lewis [5] stress that the wage of the parents is part of the opportunity cost of having children. A rise in parents wage leads to a substitution of production for procreation, and to drop in fertility. Such a mechanism is seen by many economists as a key explanation for the demographic transition (for a

[1] The impact of environmental prices and policy on technological choice by firms has been studied by many. It seems that while some innovation was responsive to energy price changes, a sizable portion of efficiency improvements were still autonomous (see [3]). In addition, although environmental taxes reduce pollution by encouraging the development of new technologies, simply relying on technological change as a panacea for environmental problems is not enough (see [4]).

[2] The inclusion of leisure ensures the robustness of the results to the existence of a diversity of activities allowing at every moment in time to shift from eco-efficient activities to less eco-intensive ones.

[3] More precisely, the key assumption is that time costs are much more significant than goods costs in the case of child rearing. In the model, we abstract from goods costs, which does not impact on our results as long as such costs remain comparatively smaller.

recent critical survey see [11]). If wages are so important for fertility outcomes, taxes on wages are relevant too. Pollution control policy, either explicitly through a Pigovian tax, or implicitly through a tradable quota scheme, implies a drop in the *net* wage of those working in the final good sector. For households, production becomes relatively more expensive through the conjunction of the tax resulting from the cap and the absence of such a tax on procreation and leisure. Agents then tend to substitute reproduction and leisure for production. As a result, unless substitution is exclusively of a production—leisure type, population will tend to increase and production per worker to decrease.

How strong is this substitution effect? Many empirical studies show that fertility is negatively related to mothers' wages or education.[4] This tends to confirm that opportunity costs such as women's income are essential determinants of fertility. However, even if this mechanism was weak it would not matter much for our purpose. This is because our time horizon is long. The cumulative impact on population of even a weak substitution effect is indeed likely to be significant.

When, in addition to choosing the number of children, parents also choose the quality of these children, capping pollution will have additional effects. Child quality amounts to future productivity and is the result of investment in education, health, etc. which we distinguish from simple child rearing. Many authors rely on the notion of a quantity–quality tradeoff, introduced by Becker [15], to explain the rise in education and the fertility decline during the demographic transition: as the economy grows and wages rise the opportunity cost of child rearing increases. At the same time, rising income implies that investment in education becomes more affordable. Parents therefore substitute child quality for child quantity, and decide to have smaller families with better education (see e.g. [16]). In the case of environmental policy, this mechanism goes in the opposite direction. Taxing production lowers wages, decreases the opportunity cost of raising children, and increases the cost of education relative to the wage. Parents would therefore substitute child quantity for child quality, and decide to have larger and less educated families. In the case of developing countries in the middle of their demographic transition, taxing pollution, and therefore production and income, would thus delay the drop in fertility and the rise in education, going in the opposite direction to what is suggested in the literature, i.e., taxing birth and subsidizing education [17,18].

In Section 2 we introduce pollution dynamics into a standard model of fertility with a quality-quantity tradeoff. In Section 3 we set a pollution cap and analyze the dynamics of population and income under such a policy. A numerical experiment is provided in Section 4 aimed at illustrating and quantifying the main effects derived in the previous sections. The last section concludes.

2. The benchmark model

Time is discrete and goes from 0 to infinity. Each individual only lives for two periods: childhood and (active) adulthood. We consider a closed economy, possibly the whole Earth, endowed with a certain quantity of land L, having an initial level of human capital per person k_0, an adult population size of N_0, and an initial pollution level P_{-1}. We first describe how pollution is generated. Then we consider the household maximization problem and, finally, the implied aggregate dynamics.

2.1. Production and pollution

At a given time, for a given technology, polluting emissions E_t are proportional to total output Y_t:

$$E_t = a_t Y_t$$

where a_t represents the pollution coefficient, i.e., the degree to which production generates polluting emissions. Total output is itself the product of adult population size N_t and production per person y_t:

$$E_t = a_t N_t y_t$$

This equation is known in the literature as the Kaya identity. If E_t is measured in tons of CO_2, then a_t would be tons of CO_2 per dollar produced. The stock of pollution S_t accumulates according to:

$$S_t = \Psi(S_{t-1}, E_t), \quad \forall t \in \mathbb{N}$$

The function $\Psi(.)$ takes different forms in the literature.[5] Fig. 1 displays a map of iso-pollution curves. Each curve represents a constant pollution level in the plane population $\times$ income per person. As we follow the curve towards the right, income per person decreases and population increases, pollution remaining constant. In order to move from one iso-pollution curve to another with lower level of emissions, one may of course lower production per capita and/or lower population size; however, one could also increase either of the two if the other gets reduced strongly enough.

Over time, the map can change. Two factors affect the position of the curves. On the one hand, technical progress allowing production of the same amount with a cleaner technology and lower emissions (for example through higher

[4] For example, Fernández and Fogli [12] for migrants in the US, deb and Rosati [13] for India and Baudin [14] for France.

[5] See for example John and Pecchenino [19] where pollution is the inverse of an "environment" variable which accumulates like capital, assuming some positive degradation rate, i.e., the share of past pollution S_{t-1} that has been absorbed by the environment. In [20], a "world temperature" variable is like pollution here and depends on the past emissions.

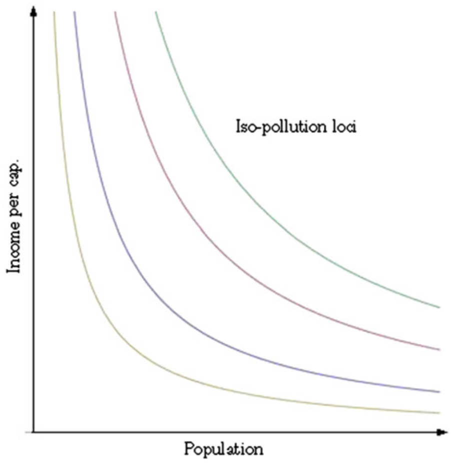

Fig. 1. Map of iso-pollution curves.

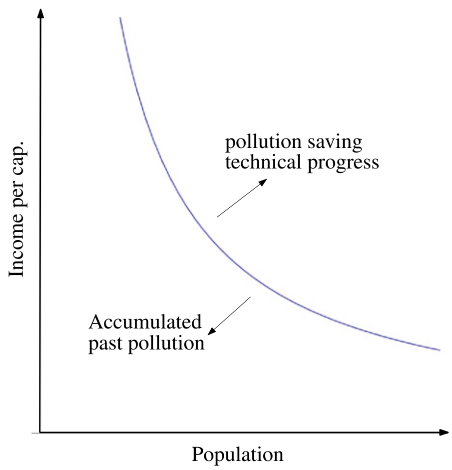

Fig. 2. Shifts over time of one iso-pollution curve.

energy efficiency) shifts the iso-pollution curves to the Northeast. Environmental efficiency decreases the pollution coefficient a_t. On the other hand, if there is more pollution accumulated in the past (S_{t-1}), the iso-pollution curves of today will shift to the Southwest. Fig. 2 represents these two possible shifts.

2.2. Households

At each date t, there is a new adult generation of size N_t deriving utility from consumption c_t, leisure ℓ_t, number of children n_t, and quality of the children k_{t+1}, as measured by their future human capital. Households are homogeneous. We assume a logarithmic utility function:

$$u(c_t,\ell_t,n_t,k_{t+1}) = \ln c_t + \delta \ln \ell_t + \gamma \ln (n_t k_{t+1}),$$

with $\delta,\gamma \in \mathbb{R}_+$. The parameter δ is the taste for leisure and γ is the altruism factor. Parents care both about child quantity n_t and quality k_{t+1}, as measured by the human capital provided to them. Notice that parents do not care about their children's utility, as would be the case with dynastic altruism. Our formulation of altruism is referred to in the literature as "joy-of-giving" (or warm glove), because parents have a taste for giving (see e.g. [21]). As our aim here is not to assess how agents should behave, impure altruism seems an acceptable assumption as a means to obtain clearcut analytical results.

The choice of a logarithmic utility function is defended by [22] on the grounds that leisure shows no trend despite growing wages. This can only be accounted for when the elasticity of substitution between leisure and consumption is close to one.

We do not introduce pollution into utility, because our objective is not to derive the best policy, but rather to show the side effects of a given type of policy. Adding a disutility term such as $-v(S)$ to the utility would not change any of our results as long as utility is additively separable.

Future human capital is obtained through spending on education, e_t. The human capital production function is

$$k_{t+1} = \tau e_t^{\eta} k_t^{\nu} \tag{1}$$

with the following parametric restrictions: $\tau \in \mathbb{R}_+$, $\nu, \eta \in (0,1)$. We assume moreover that $\eta + \nu < 1$, which will imply that human capital and output per person will converge in the long-run to a constant level.[6] The parameter τ is a measure of productivity of education technology and η is the elasticity of human capital k_{t+1} with respect to investment e_t. The parameter ν captures the strength of an externality from parents human capital to children human capital. It represents the usual parental influence on child outcome. This production function is standard in the literature on education, starting with Glomm and Ravikumar [23].

Let us provide a few additional details on the parameters η and ν. η measures the elasticity of earnings with respect to schooling. An idea of its magnitude can be obtained from the survey by Krueger and Lindahl [24] which reports estimates of the return to schooling in developed countries of 8–10%, with higher estimates for developing countries and low levels of schooling. Assuming that an additional year of schooling raises education expenditure by 20%, these returns translate into an earnings elasticity of schooling between 0.4 and 0.8. Replacing k_{t+1} in the utility function by its expression from (1), allows us to stress the importance of the parameter η:

$$\ln(c_t) + \delta \ln \ell_t + \gamma(\ln(n_t) + \eta \ln(e_t)) + \gamma\nu \ln k_t + \gamma\tau$$

The last two terms of the sum are constant. We see that η is not only the elasticity of human capital to education spending in (1), but also the relative weight of quality in the utility function. It has to be smaller than 1 because the parents' optimization problem would otherwise not have a solution. More specifically, utility would approach infinity as parents choose arbitrarily high levels of quality spending e_t and arbitrarily low levels of fertility (a similar condition can be found in Moav [25] and de la Croix and Doepke [26]).

The parameter ν captures the intergenerational transmission of ability, as well as human capital formation within the family that does not work through formal schooling (e_t). Empirical studies detect such effects, but they are relatively small.[7]

The time needed to produce n_t children is decreasing in space L/N_t (land per household) and is given by

$$\frac{1}{\mu}\left(\frac{N_t}{L}\right)^{\alpha} n_t = \phi N_t^{\alpha} n_t$$

with

$$\phi = \frac{1}{\mu}\left(\frac{1}{L}\right)^{\alpha}$$

Compared to the models developed in the recent literature, we introduce land per person as an input in the child production technology. The aim is to take into account that, when households have small dwellings, child production is more costly and people have fewer children (this is known since [28,29]). It also implies that population will be stationary in the long-run. Indeed, as population increase, it becomes more and more costly to have children, lowering progressively the fertility rate to its replacement level. The parameter μ measures total factor productivity of the procreation activity and $\alpha \in (0,1)$ captures the importance of space to produce children.

Households face a budget constraint stating that consumption plus education spending cannot exceed income y_t:

$$c_t + n_t e_t \le y_t. \tag{2}$$

Households have a total time endowment equal to 1. They face a time constraint that time spent working h_t, rearing children and having leisure cannot exceed 1:

$$h_t + \phi N_t^{\alpha} n_t + \ell_t \le 1 \tag{3}$$

Households are self-employed. The productivity of each hour of work is given by the quality of the worker, i.e., his/her human capital k_t. Total production is therefore the product of hours of work h_t and k_t:

$$y_t \le h_t k_t \tag{4}$$

Notice that, in this production function, we assume constant returns with respect to input of efficiency units $h_t k_t$. We also consider hours of work and efficiency units a perfect substitutes: doubling efficiency together with halving hours of work would leave production unchanged. Departing from one of these two assumptions is not expected to modify the results. However, it would make the analysis more complex, requiring a numerical analysis in most cases.

Substituting saturated constraints (2)–(4) into the objective, the household maximization problem can be written as:

$$\max_{\ell_t, n_t, e_t} \ln((1 - \ell_t - \phi N_t^{\alpha} n_t)k_t - n_t e_t) + \delta \ln \ell_t + \gamma(\ln(n_t) + \eta \ln(e_t)) + \text{constantterms}$$

[6] The alternative case $\eta + \nu = 1$ would generate endogenous growth, which could be analyzed as well with the tools developed here.

[7] For example, Leibowitz [27] finds that, after controlling for education, parental income has a significant effect on a child's earnings. The elasticity is of the order of 0.1.

The first-order conditions are

$$\frac{-k_t}{(1-\ell_t-\phi N_t^{\alpha} n_t)k_t-n_t e_t}+\frac{\delta}{\ell_t}=0$$

$$\frac{-\phi N_t^{\alpha} k_t-e_t}{(1-\ell_t-\phi N_t^{\alpha} n_t)k_t-n_t e_t}+\frac{\gamma}{n_t}=0$$

$$\frac{-n_t}{(1-\ell_t-\phi N_t^{\alpha} n_t)k_t-n_t e_t}+\frac{\gamma\eta}{e_t}=0$$

As the maximization problem is convex, the first-order conditions are necessary and sufficient for a maximum. Solving the set of first-order conditions and saturated constraints (2)–(4) for c_t, ℓ_t, n_t, e_t, and y_t yields closed form solutions:

$$c_t=\frac{k_t}{1+\delta+\gamma} \tag{5}$$

$$\ell_t=\frac{\delta}{1+\delta+\gamma} \tag{6}$$

$$n_t=\frac{\gamma(1-\eta)}{(1+\delta+\gamma)\phi N_t^{\alpha}} \tag{7}$$

$$e_t=\frac{\eta\phi N_t^{\alpha} k_t}{1-\eta} \tag{8}$$

$$y_t=\frac{\gamma\eta}{1+\delta+\gamma}k_t \tag{9}$$

2.3. Aggregate dynamics

Adult population dynamics are given by

$$N_{t+1}=N_t n_t \tag{10}$$

Substituting the expressions for e_t (8) and n_t (7) into the equations describing the dynamics of human capital (1) and population (10) leads to:

$$k_{t+1}=\tau\left(\frac{\eta\delta}{1-\eta}\right)^{\eta} N_t^{\eta\alpha} k_t^{\nu+\eta} \tag{11}$$

$$N_{t+1}=\frac{\gamma(1-\eta)}{(1+\delta+\gamma)\phi}N_t^{1-\alpha} \tag{12}$$

This system is recursive as the second equation can be solved independently of the first one. The second equation shows that N_{t+1} is an increasing and concave function of N_t which does not depend on k_t. It has a unique nontrivial steady state:

$$\overline{N}=\left(\frac{\gamma(1-\eta)}{(1+\delta+\gamma)\phi}\right)^{1/\alpha} \tag{13}$$

which is globally stable. Dynamics of population are monotonic. For a given N_t, the first equation also describes an increasing and concave relation between k_{t+1} and k_t. When N_t is close enough to $\overline{N}$, the dynamics of k_t are also monotonic and converge to:

$$\overline{k}=\left(\frac{\tau^{1/\eta}\gamma\eta}{1+\delta+\gamma}\right)^{\eta/(1-\nu-\eta)} \quad \text{for } \alpha>0$$

Income per capita converges to

$$\overline{y}=\tau^{1/(1-\nu\eta)}\left(\frac{\gamma\eta}{1+\delta+\gamma}\right)^{(1-\nu\eta+\eta)/(1-\nu\eta)}$$

A larger country (higher L, lower ϕ) will have a larger population size. A more productive country (higher τ) will have higher income per capita.

If $\alpha=0$, space is not useful to produce children. Population grows unboundedly at rate $\gamma(1-\eta)/((1+\delta+\gamma)\phi)$ and human capital converges to:

$$\overline{k}=\left(\frac{\tau^{1/\eta}\eta\delta}{1-\eta}\right)^{\eta/(1-\nu\eta)} \quad \text{for } \alpha=0$$

3. Regulation: pollution cap and tradable rights

At each date, past pollution is given. A given pollution target $S_t^\star$ can be achieved by imposing an emission target $E_t^\star$ such that:

$$S_t^\star=\Psi(S_{t-1}^\star,E_t^\star), \quad \forall t\in\mathbb{N}$$

Since we do not provide, in this paper, a utility based justification for a given pollution target, the latter is taken to be exogenous. As a result, the path of emission targets $\{E_t^\star\}_{t=0..+\infty}$ is exogenous too.

Remember that, for simplification purposes, we have assumed that output is produced by self-employed households. To meet the sequence of emission targets, two policy schemes are available and interchangeable: a Pigovian tax on emissions, hence on production, with revenue transferred back to households in a lump-sum way; and a tradable pollution rights system with a free initial allocation of rights to households. In a world where agents behave competitively and information is perfect about both the objective that is being pursued and the deep parameters of the model, tradable quotas schemes and price-oriented schemes are fully equivalent.[8] This implies that, despite our focus on tradable right schemes, the results will be of direct relevance for those willing to implement a Pigovian tax.

3.1. Households

A pollution rights system requires each household buys pollution rights in proportion to the output that would exceed their initial endowment. Let us denote the price of the pollution right by p_t and the initial endowment of rights by q_t. The budget constraint of the household is now:

$$y_t\ge c_t+n_te_t+p_t(a_ty_t-q_t) \tag{14}$$

The constraint can be rewritten:

$$(1-a_tp_t)y_t+p_tq_t\ge c_t+n_te_t$$

which shows clearly that the price of pollution permits p_t weighted by the pollution coefficient a_t acts like an income tax, and p_tq_t as a lump-sum transfer.

Substituting saturated constraints (1), (14), (3) and (4) into the objective, the households maximization problem can be written as:

$$\max_{\ell_t,n_t,e_t}\mathcal{L}_t=\ln((1-a_tp_t)(1-\ell_t-\phi N_t^\alpha n_t)k_t-n_te_t+p_tq_t)+\delta\ln\ell_t+\gamma(\ln(n_t)+\eta\ln(e_t))+\text{constant terms}$$

The first-order conditions can be written under the form "marginal cost = marginal benefit":

$$\frac{\partial\mathcal{L}_t}{\partial\ell_t}=0\Rightarrow\frac{(1-a_tp_t)k_t}{c_t}=\frac{\delta}{\ell_t}$$

$$\frac{\partial\mathcal{L}_t}{\partial n_t}=0\Rightarrow\frac{(1-a_tp_t)\phi N_t^\alpha k_t-e_t}{c_t}=\frac{\gamma}{n_t}$$

$$\frac{\partial\mathcal{L}_t}{\partial e_t}=0\Rightarrow\frac{n_t}{c_t}=\frac{\gamma\eta}{e_t}$$

The price p_t affects the first-order conditions for ℓ_t and n_t by lowering their marginal cost.

As the maximization problem is convex, the first-order conditions are necessary and sufficient for a maximum. Solving the system formed by the first-order conditions and the constraints leads to:

$$c_t=\frac{1}{1+\delta+\gamma}((1-a_tp_t)k_t+p_tq_t)$$

$$\ell_t=\frac{\delta}{1+\delta+\gamma}\frac{(1-a_tp_t)k_t+p_tq_t}{(1-a_tp_t)k_t}$$

$$n_t=\frac{\gamma(1-\eta)}{(1+\delta+\gamma)\phi N_t^\alpha}\frac{(1-a_tp_t)k_t+p_tq_t}{(1-a_tp_t)k_t} \tag{15}$$

[8] Uncertainty [30] or strategic interactions [31] draw a wedge between these two instruments.

$$e_t = \frac{\eta\phi N_t^{\alpha} k_t}{1-\eta}(1-a_t p_t) \tag{16}$$

$$y_t = \frac{(1-a_t p_t)k_t(1+\gamma\eta)-(\delta+\gamma-\gamma\eta)p_t q_t}{(1-a_t p_t)(1+\delta+\gamma)}$$

3.2. A small open economy

Before considering our economy as a model of the world in general equilibrium, we analyze the case of a small open economy in which the price p_t and the quota q_t are imposed from outside and exogenous.

We first observe that the time spent on emission-free activities, i.e., leisure and procreation, increases with the price of pollution permits p_t. Indeed, p_t acts as a tax on the time spent on production. Hence, increases in p_t lower the opportunity cost of leisure and procreation, and so p_t is similar to a subsidy to procreation:

$$\frac{\partial \ell_t}{\partial p_t} > 0, \quad \frac{\partial n_t}{\partial p_t} > 0$$

Leisure and procreation also increase with the endowment of pollution permits. This is because they are both normal goods:

$$\frac{\partial \ell_t}{\partial q_t} > 0, \quad \frac{\partial n_t}{\partial q_t} > 0$$

Human capital accumulation (education) is reduced by the price of pollution permits, because of a substitution of quantity (n_t) for quality (k_{t+1}) of children:

$$\frac{\partial e_t}{\partial p_t} < 0$$

Finally, net individual income and production are decreasing in p_t:

$$\frac{\partial y_t}{\partial p_t} = -\frac{(\delta+\gamma(1-\eta))q_t}{(1-p_t)^2(1+\delta+\gamma)} < 0.$$

Let us now analyze how the presence of tradable pollution rights affects the steady state and the dynamics of a small open economy. For this, we assume that exogenous variables are constant, i.e., $a_t = a$, $p_t = p$ and $q_t = q$. The dynamics are represented as follows:

$$k_{t+1} = \tau\left(\frac{\eta\delta}{1-\eta}\right)^{\eta} N_t^{\eta\alpha} k_t^{\nu+\eta}(1-ap)^{\eta}$$

$$N_{t+1} = \frac{\gamma(1-\eta)}{(1+\delta+\gamma)\phi} N_t^{1-\alpha} \frac{(1-ap)k_t + pq}{(1-ap)k_t}$$

This dynamical system is no longer block recursive, i.e., the two different equations need to be solved simultaneously. To analyze its properties, we use the phase diagram in Fig. 3. A first phase line is given by

$$\Delta k_{t+1} = 0 \Leftrightarrow k_{t+1} - k_t = \tau\left(\frac{\eta\delta}{1-\eta}\right)^{\eta} N_t^{\eta\alpha} k_t^{\nu+\eta}(1-ap)^{\eta} - k_t = 0 \tag{17}$$

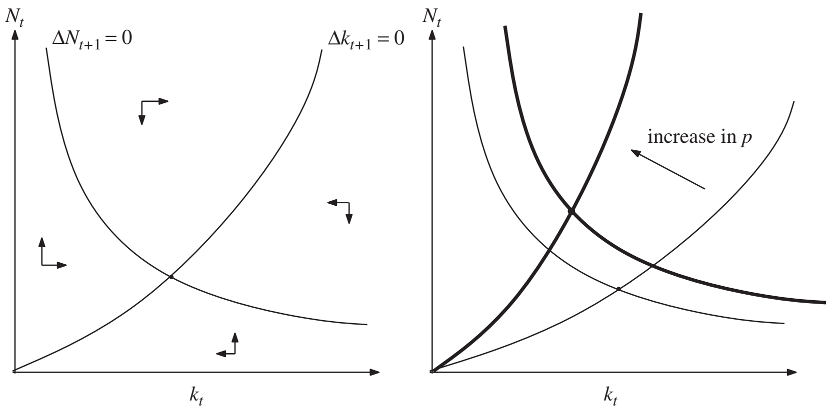

Fig. 3. Phase diagram for a small open economy.

Solving for N_t gives

$$N_t = \tau^{-1/\eta\alpha}\left(\frac{\eta\delta}{1-\eta}\right)^{-1/\alpha}(1-ap)^{-1/\alpha}k_t^{(1-\nu-\eta)/\eta\alpha} \tag{18}$$

The right hand side is an increasing function of k_t. We draw this function in the space $\{k_t,N_t\}$. Considering a point located above that line, i.e., a point with a larger N_t than the one given by (18), it appears from (17) that it corresponds to a situation where $\Delta k_{t+1} > 0$. Accordingly, when located above this phaseline, we draw a horizontal arrow oriented to the right to indicate the direction of motion. Another arrow oriented to the left is drawn when below the phaseline.

The second phase line is given by

$$\Delta N_{t+1} = 0 \Leftrightarrow N_{t+1} - N_t = \frac{\gamma(1-\eta)}{(1+\delta+\gamma)\phi}N_t^{1-\alpha}\frac{(1-ap)k_t+pq}{(1-ap)k_t} - N_t = 0 \tag{19}$$

Solving for N_t gives

$$N_t = \left[\frac{\gamma(1-\eta)}{(1+\delta+\gamma)\phi}\frac{(1-ap)k_t+pq}{(1-ap)k_t}\right]^{1/\alpha} \tag{20}$$

which is a negatively sloped function going from $+\infty$ when k_t=0 to 0 when $k_t = +\infty$. Let us decrease k_t to consider a point to the left of this curve. It increases the function (19) and hence $\Delta N_{t+1} > 0$ in this zone. Hence, to the left (resp. right) of this curve, we can draw arrows pointing upward (resp. downward).

The phase diagram in the left panel of Fig. 3 shows that there is inevitably a unique steady state with oscillatory dynamics. Appendix A linearizes the dynamic system around the steady state and shows that the steady state is locally stable.

Let us now suppose that there is an exogenous increase in the price of pollution permits. Differentiating the two phase lines (18) and (20) leads us to conclude that they both shift upward. As a consequence, the new steady state has a higher population level. We have seen above that pollution control increases fertility. This translates at the steady state level into a larger population.

3.3. General equilibrium

We now turn our attention to the most difficult case: the one in which the price of pollution rights, instead of being exogenous, adjusts as a function of market forces. The equilibrium in the market for tradable pollution rights implies that total pollution $N_t a_t y_t$ equals the total number of quotas $N_t q_t$, unless the price p_t is zero:

$$p_t(N_t a_t y_t - N_t q_t) = 0$$

Two cases may arise depending on whether the cap is binding or not. A cap is binding if it is set lower than the otherwise desired total amount of pollution. This occurs when

$$q_t < a_t y_t$$

where y_t is computed in the business as usual scenario. Replacing y_t by its value from Eq. (9) leads to:

$$q_t < a_t\frac{1+\gamma\eta}{1+\delta+\gamma}k_t \tag{21}$$

Proposition 1. *At time t, the equilibrium satisfies: If (21) holds then*

$$p_t = \frac{k_t(1+\gamma\eta) - q_t(1+\delta+\gamma)}{(k_t-q_t)(1+\gamma\eta)} \tag{22}$$

$$y_t = \frac{q_t}{a_t}$$

If (21) does not hold then

$$p_t = 0$$

$$y_t = \frac{\gamma\eta}{1+\delta+\gamma}\frac{k_t}{q_t}$$

If the pollution endowment is sufficiently restrictive, there will be a positive price of pollution permits and production will match the target. If the pollution quota is large, the policy is non-binding. The price of permits then falls to zero, and the output corresponds with the one of the business as usual scenario.

3.4. Dynamics

Let us consider a constant emission cap $E^\star$. As a consequence, the pollution endowment per household will be:

$$q_t = \frac{E^\star}{N_t}$$

We now analyze how different levels of the emission cap $E^\star$ affect the dynamics of population N_t. To simplify, we keep technical progress constant $a_t=1$ (but of course we will not use this simplification when we will let a_t increase in one of our scenarios).

The dynamics of human capital k_t and population N_t are obtained by substituting e_t, n_t, and p_t from (16), (15) and (22) into (1) and (10):

$$k_{t+1} = \tau k_t^\nu \left(\frac{\eta \phi N_t^\alpha k_t}{1-\eta} \left(1 - \frac{k_t(1+\gamma\eta) - q_t(1+\delta+\gamma)}{(k_t - q_t)(1+\gamma\eta)} \right) \right)^\eta$$

$$N_{t+1} = N_t \left(\frac{\gamma(1-\eta)}{(1+\delta+\gamma)\phi N_t^\alpha} \left(1 + \frac{\frac{k_t(1+\gamma\eta) - q_t(1+\delta+\gamma)}{(k_t - q_t)(1+\gamma\eta)}}{1 - \frac{k_t(1+\gamma\eta) - q_t(1+\delta+\gamma)}{(k_t - q_t)(1+\gamma\eta)}} \frac{q_t}{k_t} \right) \right)$$

Using $q_t = E^\star / N_t$ and simplifying leads to:

$$k_{t+1} = \tau k_t^{\nu+\eta} \left(\frac{\eta \phi N_t^\alpha E^\star (\delta + \gamma - \gamma\eta)}{(N_t k_t - E^\star)(1-\eta)(1+\gamma\eta)} \right)^\eta$$

$$N_{t+1} = \frac{\gamma(1-\eta)(N_t k_t - E^\star)}{\phi N_t^\alpha k_t (\delta + \gamma - \gamma\eta)}$$

The analysis of these dynamics is detailed in Appendix B. The main result is the following.

Proposition 2 (*Population and the pollution cap*). *For a sufficiently stringent pollution cap $E^\star$, there is a locally stable steady state population, decreasing in $E^\star$.*

If $E^\star$ is restrictive enough, the long-run population $\overline{N}$ is higher if the pollution cap is set at a more stringent level. As a consequence, income per capita will unambiguously be lower, as $y = E^\star / N$.

From the dynamic point of view, the pro-population tilt of pollution caps is worrying. For a given $E^\star$, emission endowments per person inevitably become more and more stringent as generations pass. Because of this pro-natalist effect, capping emissions impoverishes the successive generations more than in a conventional set-up with exogenous fertility. It is worth spelling out why capping emissions tends to reduce production rather than procreation. This is the case because production generates emissions from the moment it takes place onwards, whereas procreation generates delayed emissions. This rests on two assumptions. First, only physical good production generates pollution. Second, children do not consume physical goods. This implies that the emissions of a person take place at adulthood. In a more general set-up, it would be sufficient to assume that procreation and leisure are simply less emission-intensive activities than production. This is why capping emissions at period t puts less pressure on procreation than on production. In a way, if procreation only generates emissions through future production (i.e., when children will themselves become producers), the capping scheme generates a specific form of externality. Current adults willing to procreate at a rate higher than the replacement rate do not internalize the fact that tomorrow's pollution cap will have to be divided into smaller pollution endowments.

4. Numerical experiment

In order to provide a meaningful example of the mechanisms studied analytically above, we calibrate the parameters of the model and we simulate the effect of introducing pollution caps on the dynamics of income and population.

4.1. Calibration

Assume that each period lasts 25 years. We will use the year 1983 as representing $t=0$ (initial conditions), and the year 2008 as $t=1$. 2008 is the last year for which we have observations.

We first identify γ, δ, η, ν and α with the following five restrictions:

1. The share of consumption in GDP is 80% (corresponds to public and private consumptions of the national accounts). Using Eqs. (5) and (9), we find that

$$\frac{c_t}{y_t} = \frac{1}{1+\gamma\eta} = 0.8$$

2. The time spent on leisure (ℓ_t) and procreation ($\phi\overline{N}^{\alpha}$) amounts to 2/3 of total available time (this has become a standard value in the literature since [32] found that households allocate approximately one-third of their time to market activities). Using (13):

$$\phi\overline{N}^{\alpha} = \frac{\gamma(1-\eta)}{1+\delta+\gamma}$$

From (6):

$$\ell_t + \phi\overline{N}^{\alpha} = \frac{\delta}{1+\delta+\gamma} + \frac{\gamma(1-\eta)}{1+\delta+\gamma} = \frac{\delta+\gamma(1-\eta)}{1+\delta+\gamma} = \frac{2}{3}$$

3. At steady state, the time spent rearing children is equal to 15% (see [33]) of the time remaining after leisure has been accounted for:

$$\frac{\phi N^{\alpha}}{1-\ell_t} = 0.15$$

This implies

$$\frac{\gamma(1-\eta)}{1+\gamma} = 0.15$$

4. Following the literature on conditional convergence (see [34] for a survey), the convergence speed of income per capita is 2% per year. For the dynamic equation (11) we get

$$\frac{k_{t+1}}{k_t} = \left(\frac{k_t}{k_{t-1}}\right)^{\nu+\eta}\left(\frac{N_t}{N_{t-1}}\right)^{\alpha\eta}$$

The required convergence speed is obtained with $\nu+\eta = 0.98^{25}$.

5. The dynamics of population are calibrated to match the forecasted evolution of world population between 2008 ($t-1$), 2033 (t) and 2058 ($t+1$). From the dynamic equation (12) we get

$$\frac{N_{t+1}}{N_t} = \left(\frac{N_t}{N_{t-1}}\right)^{1-\alpha}$$

and we have $N_{t-1} = 6.67$, $N_t = 8.18$ and $N_{t+1} = 8.88$ from the 2007 IIASA World Population Projection.

Solving this system gives $\gamma = 0.470588$, $\delta = 2.27941$, $\eta = 0.53125$, $\nu = 0.0722147$, and $\alpha = 0.5976$. Notice that η is in line with estimates of the return from education (see the discussion in [33]). Moreover, this η is almost enough to obtain the required speed of convergence of income per capita, as the additional parameter ν is small. Notice finally that the parameter α implies an annual convergence speed for population of 3.56% per year.

The two productivity levels, τ and ϕ, are parameters that determine the size of population and income per capita. Imposing initial conditions so as to start in 1983 requires $N_0 = 4.68$ and $y_0 = 4.541$. Inverting (9) gives us $k_0 = 16.0271$. In order to obtain the right levels $N_1 = 6.67$ and $y_1 = 7.614$ in 2008, we need to have $\phi = 0.0164$ and $\tau = 24.0417$.

4.2. Simulation

Table 1 provides the simulation from 1983 (initial conditions) to 2208 when no pollution cap is imposed. It illustrates the properties of the benchmark model: monotonic convergence of population, which tends to 8.47 billions, and income per capita (38155 dollars per capita per year in 2208). Fertility declines rapidly to its replacement level. Leisure is constant.

Let us now impose a constant pollution cap: $E^{\star} = 100$, starting to bind in 2033. The chosen level of $E^{\star}$ is arbitrary and for illustration purposes only. Table 2 provides the results. There is now one new column: the pollution price p_t. The price in 2033 is 0.24, corresponding to an implicit tax of 24% on production. Following the tax, total output Y_t is indeed limited to 100. As a consequence of this tax, the households retreat from market activities to devote more time to leisure (66.1% instead of 60.8% in the benchmark) and to procreation (1.151 child per person instead of 1.059 in the benchmark, to be multiplied by 2 to compare to fertility rates per women). The rise in procreation does not look big, but it is large enough to have immense consequences for the future, through its cumulative effect over time. Population in 2058 is now 8.85 billions instead of 8.15 billions in the benchmark and converges in the long-run to more than 12 billions instead of 8.5 billions in the benchmark.

Another way to look at the same data is to plot fertility over time. Fig. 4 represents children born per person over time, for the benchmark (black line) and the constant cap (grey line) scenarios. The drop in fertility is delayed by two periods when the pollution cap is imposed. Notice that delaying the demographic transition does not entail reversing the general trend towards fertility drop. This matters for the following reason. Data tend to show that countries having gone through their demographic transition do not experience later on a rise in fertility in episodes of impoverishment (see e.g. Moldova).

Table 1
Benchmark simulation—World Economy 1983–2208.

t	N_t	n_t	ℓ_t	y_t	Y_t
1983	4.680	1.425	0.608	5.342	25.002
2008	6.670	1.153	0.608	9.769	65.164
2033	7.692	1.059	0.608	15.737	121.052
2058	8.146	1.023	0.608	21.954	178.847
2083	8.336	1.009	0.608	27.334	227.862
2108	8.414	1.004	0.608	31.428	264.432
2133	8.445	1.002	0.608	34.290	289.596
2158	8.458	1.001	0.608	36.185	306.058
2183	8.463	1.000	0.608	37.397	316.498
2208	8.465	1.000	0.608	38.155	322.992

Table 2
Simulation with a constant pollution cap—1983–2208.

t	N_t	p_t	n_t	ℓ_t	y_t	Y_t
1983	4.680	0.000	1.425	0.608	5.342	25.002
2008	6.670	0.000	1.153	0.608	9.769	65.164
2033	7.692	0.240	1.151	0.661	13.000	100.000
2058	8.855	0.505	1.171	0.731	11.294	100.000
2083	10.366	0.556	1.087	0.746	9.647	100.000
2108	11.272	0.574	1.042	0.752	8.871	100.000
2133	11.746	0.582	1.020	0.754	8.513	100.000
2158	11.982	0.586	1.010	0.755	8.346	100.000
2183	12.097	0.587	1.005	0.756	8.266	100.000
2208	12.153	0.588	1.002	0.756	8.229	100.000

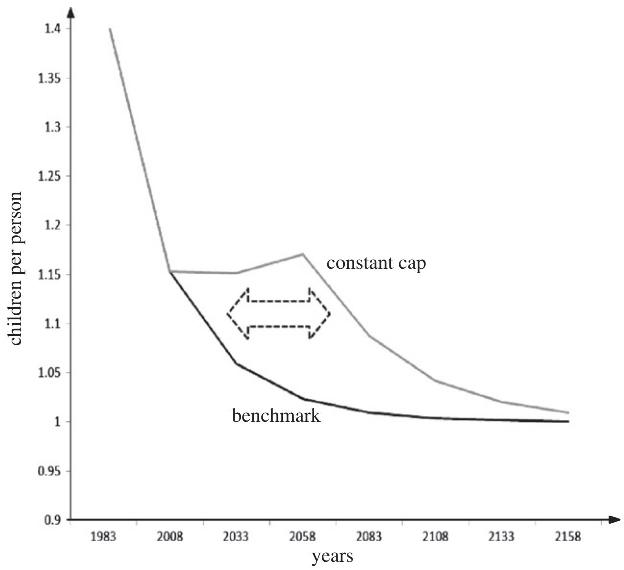

Fig. 4. Delay in the demographic transition.

This could suggest that the demographic transition is irreversible. Even if we were to accept this idea, this would not conflict with the delaying effect we identified.

Suppose now that there is some technical progress making production more and more clean over time. Precisely, we assume that

$$a_t = (1.01)^{-25(t-2)}$$

which reflects a technical progress of 1% per year. Time t is equal to 2 in 2033, this formulation is the same as previous for the year 2033, but output is becoming less and less polluting as time passes. Hence we can allow increasing caps: $E^{\star}_{2033} = 100$, $E^{\star}_{2058} = 128.243$, etc. Table 3 provides the results. In the long-run, the cap is not binding thanks to

Table 3
Simulation with technical progress—1983–2208.

t	N_t	p_t	n_t	ℓ_t	y_t	Y_t
1983	4.680	0.000	1.425	0.608	5.342	25.002
2008	6.670	0.000	1.153	0.608	9.769	65.164
2033	7.692	0.240	1.151	0.661	13.000	100.000
2058	8.855	0.318	1.089	0.680	14.483	128.243
2083	9.641	0.256	1.012	0.665	17.058	164.463
2108	9.755	0.140	0.964	0.638	21.622	210.913
2133	9.402	0.000	0.939	0.608	28.167	264.830
2158	8.831	0.000	0.975	0.608	33.249	293.627
2183	8.611	0.000	0.990	0.608	36.025	310.229
2208	8.525	0.000	0.996	0.608	37.511	319.761

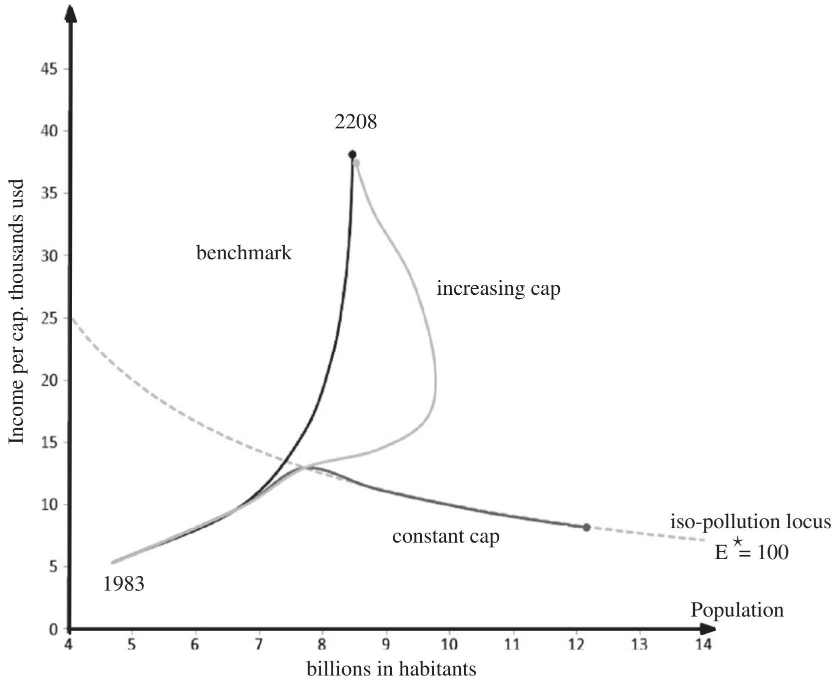

Fig. 5. Income and population dynamics in the examples.

technical progress, and the economy converges to the benchmark steady state. As population has risen fast in the beginning, it actually overshoot its long-run level, and converges from above to its steady state. The cost of this policy in terms of income are still very large. For example, income per person would be 17 058 dollars per year in 2083 with the cap, and 27 334 in the benchmark.

Fig. 5 summarizes the result, comparing the benchmark, the constant pollution cap, and the increasing cap simulations on the figure used in the introduction to present the iso-pollution curve. The benchmark follows a convex path in this plane, and crosses the iso-pollution line $E^{\star} = 100$ early on. The constant cap path, on the contrary, moves Southeast as soon as the cap is binding. It will converge to a situation with a large population and an income per capita only slightly above the 1983 level. The increasing cap path is an intermediate case. In the short-run (which means here a few generations), it follows the constant cap path, with lower income per person and higher population. In the long-run though, the path converges to the benchmark steady state.

In future research, it would be interesting to consider a policy under which we cap population rather than emissions, for example along the lines proposed by [35]. Tradable procreation quotas schemes are of course not the only available option. Policies aimed at addressing population issues—both in terms of absolute level and of heterogeneity—are notoriously difficult to design. If they aim at keeping population below a certain level, they should remain as freedom-friendly as possible while being simultaneously concerned with not increasing poverty and inequality. Women's education is a policy that can be justified independently (e.g. on gender equality grounds) while being effective at reducing birth rate without increasing poverty nor infringing too much on procreative freedom. Other tools have been discussed in the literature, such as taxing skilled people to subsidize unskilled ones ready to limit themselves to a single child [17].

While it is beyond the scope of this paper to explore the respective merits of such population control policies in detail, it would of course be crucial to consider which one to adopt in conjunction with measures of pollution control. Alternatively, a population control policy could also work as a substitute to a directly environmental one. A key question is the following: is there a population cap $N^\star$ such that the desired emission level $E^\star$ could be met? If yes, does $N^\star$ allow for higher income per capita than under the model capping emissions directly? If the answer to these two questions is positive, the next question will become: under which conditions does it follow that we should cap population rather than emissions?

5. Conclusion

Pollution control, and especially greenhouse gas emission reduction, is matters of great importance. Most of the literature looks at environmental policy assuming that demography is exogenous (see e.g. the two influential papers by [20,36] using OLG models). However, we have shown that such policies unexpectedly impact on the population dynamics through a production–procreation substitution effect. Capping pollution subsidies de facto procreation, and may therefore delay the demographic transition in developing countries and the drop in global fertility. Such an increase in population, compared with a business as usual scenario, may in turn be damaging either in environmental terms if the pollution scheme is ineffective, or in terms of the average standard of living—both independently and through the operation of the pollution cap at the next period.

Admittedly, the effect of pollution on utility and/or on productivity has not been modeled. If pollution affects productivity negatively,[9] or has a negative effect on the health of workers, the strength of our substitution effect would be weakened. Assuming that consumption and environment quality are complements in the utility function would also weaken our results. Refining the model in that direction would definitely be of interest for a welfare assessment of environmental policies. However, it would not affect the specific conclusion of this paper, as these extensions are unlikely to reverse the direction of the substitution effect we highlighted. Moreover, we assumed that households do not care about future generations beyond their own children. This is not an unusual assumption as some degree of diminishing altruistic behavior seems realistic. Finally, we have considered technological progress to be exogenous. This does not put into question the fact that capping pollution has an impact on population, and even a significant one as we have shown, even if technological progress were endogenous.

We need to make sure as much as possible that pollution control does not take place at the costs of the current least well off or at the cost of those in the future. The natalist bias we identified is worrying in the latter respect. One may then want to address it in two main ways. As was suggested in the introduction, one could adopt an allocation rule of pollution endowments relying on some form of emission grandfathering. It would be such that those deciding to increase their population would not receive extra emission quotas at the next period. Besides the fairness concerns that this would raise, it may, however, not be enough to mitigate the substitution effect to a significant degree. Alternatively, population could be capped directly through a separate scheme, be it in the absence of or as a complement to the pollution capping scheme. Further research is needed to assess the impact of using population and pollution capping schemes either alternatively or complementarily.[10]

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Appendix A. Dynamics in the small open economy

The dynamics to characterize are given by:

$$k_{t+1} = \tau \left(\frac{\eta\delta}{1-\eta} \right)^{\eta} N_t^{\eta\alpha} k_t^{\nu+\eta} (1-ap)^{\eta}$$

[9] One possibility would be for example to model the impact of emissions on agricultural productivity through average temperature increase.

[10] Another possible extension of the model would consist in adding income heterogeneity among households within generations. This would allow us to study the distributive impact of the substitution effect.

$$N_{t+1} = \frac{\gamma(1-\eta)}{(1+\delta+\gamma)\phi} N_t^{1-\alpha} \frac{(1-ap)k_t+pq}{(1-ap)k_t}$$

Linearizing the dynamic system around the steady state $(\overline{k},\overline{N})$ and using the steady state relationships leads to the following Jacobian matrix:

$$\begin{bmatrix} \eta+v & \frac{pq\alpha\gamma\eta(1-\eta)}{\overline{N}(p-1)(\gamma(1-\eta)-\overline{N}^{\alpha}(\gamma+\delta+1)\phi)} \\ \frac{\overline{N}^{1-\alpha}(p-1)(\gamma(1-\eta)-\overline{N}^{\alpha}(\gamma+\delta+1)\phi)^2}{pq\gamma(\gamma+\delta+1)(1-\eta)\phi} & 1-\alpha \end{bmatrix}$$

The determinant of this matrix is

$$-\frac{\alpha\gamma\eta(1-\eta)\overline{N}^{-\alpha}}{(1+\gamma+\delta)\phi} + \eta - \alpha v + v$$

It is smaller than one and increasing in $\overline{N}$. We need to show that it is larger than -1 to establish local stability. For a steady state not too far from the one in the benchmark model

$$\left(\frac{\gamma(1-\eta)}{(1+\delta+\gamma)\phi}\right)^{1/\alpha}$$

(Eq. (13)), the determinant is

$$-\alpha\eta+\eta-\alpha v+v = (\eta+v)(1-\alpha) \in (0,1)$$

Hence, for a larger value of steady state population, the determinant is also $\in (0,1)$.

The trace of the Jacobian matrix is

$$1-\alpha+\eta+v \in (0,2)$$

Hence, the two eigenvalues are positive and smaller than one, and the steady state is locally stable.

Appendix B. Dynamics in the global economy

The dynamics to characterize are given by

$$k_{t+1} = \tau k_t^{v+\eta} \left(\frac{\eta\phi N_t^{\alpha} E^{\star}(\delta+\gamma-\gamma\eta)}{(N_t k_t - E^{\star})(1-\eta)(1+\gamma\eta)} \right)^{\eta}$$

$$N_{t+1} = \frac{\gamma(1-\eta)(N_t k_t - E^{\star})}{\phi N_t^{\alpha} k_t (\delta+\gamma-\gamma\eta)}$$

To analyze these dynamics let us first look for steady states. Solving the last equation for k at steady state leads to:

$$\overline{k} = \frac{\gamma(1-\eta)E^{\star}/N}{\gamma(1-\eta)-\phi N^{\alpha}(\delta+\gamma-\gamma\eta)}$$

Replacing k_{t+1} and k_t by this value in the first dynamic equation, we find:

$$\tau\left(\frac{\overline{N}}{E^{\star}}\right)^{1-v-\eta}\left(\frac{\gamma\eta}{1+\gamma\eta}\right)^{\eta} = \left(\frac{\gamma(1-\eta)}{\gamma(1-\eta)-\phi\overline{N}^{\alpha}(\gamma+\delta-\gamma\eta)}\right)^{1-v} \tag{23}$$

This equation cannot be solved explicitly for $\overline{N}$. Let us rewrite this equality as

$$\Psi_1(E^{\star},\overline{N}) = \Psi_2(\overline{N})$$

Fig. 6 represents these two functions. The left hand side Ψ_1 is an increasing and concave function of $\overline{N}$, starting from 0 when $\overline{N}=0$ and going to infinity as $\overline{N}\to\infty$. The right hand side Ψ_2 is an increasing and convex function of $\overline{N}$, starting from 1 when $\overline{N}=0$ and going to infinity as $\overline{N}\to\hat{N}$ (vertical asymptote), with

$$\hat{N} = \left(\frac{\gamma(1-\eta)}{\phi(\gamma+\delta-\gamma\eta)}\right)^{1/\alpha}$$

Hence, given the characteristics of the two functions, there are either two, one or no steady state, depending on the stringency of the cap $E^{\star}$.

We can show that, when the cap $E^{\star}$ is set at its most stringent and yet non-binding level, i.e., such that $p=0$ and $y=q$, the steady state is unique. Indeed, in that case,

$$E = Nk\frac{1+\gamma\eta}{1+\delta+\gamma}$$

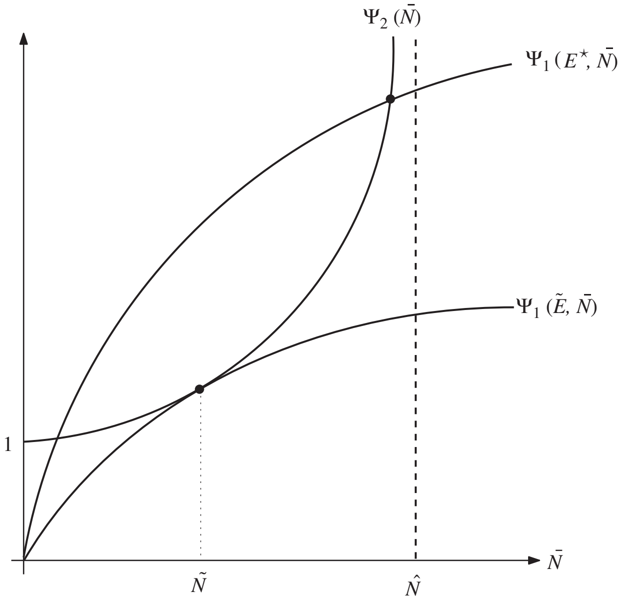

Fig. 6. Steady state population with pollution cap.

Eq. (23) would be, in that case,

$$\tau\left(\frac{1+\delta+\gamma}{1+\gamma\eta}\left(\frac{\tau^{1/\eta}\,\gamma\eta}{1+\delta+\gamma}\right)^{-\eta/(1-v-\eta)}\right)^{1-v-\eta}\left(\frac{\gamma\eta}{1+\gamma\eta}\right)^{\eta}=\left(\frac{\gamma(1-\eta)}{\gamma(1-\eta)-\phi\overline{N}^{\alpha}(\gamma+\delta-\gamma\eta)}\right)^{1-v}$$

which simplifies into

$$\frac{1+\delta+\gamma}{1+\gamma\eta}=\frac{\gamma(1-\eta)}{\gamma(1-\eta)-\phi\overline{N}^{\alpha}(\gamma+\delta-\gamma\eta)}$$

and

$$\overline{N}=\left(\frac{\gamma(1-\eta)}{(1+\delta+\gamma)\phi}\right)^{1/\alpha}\equiv\tilde{N}$$

is the only solution to this equality. $\overline{N}$ and $\overline{k}$ take their value as in the benchmark model without pollution cap.

Making the pollution cap $E^\star$ marginally more stringent shifts the Ψ_1 function upward. As a result, for any binding pollution cap, we end up with two possible steady state equilibria, respectively one with a larger population than $\tilde{N}$ and one with a smaller. A further step is needed to identify a stable steady state and demonstrate the pro-natalist effect of lowering $E^\star$.

Linearizing the dynamic system around the steady state leads to the following Jacobian matrix:

$$\begin{bmatrix} \eta+v-\frac{\gamma\eta(1-\eta)}{(\delta+\gamma-\eta\gamma)\phi\overline{N}^{\alpha}} & -\frac{E^\star\gamma\eta(1-\eta)\left(\gamma(1-\eta)-\alpha(\delta+\gamma-\eta\gamma)\phi\overline{N}^{\alpha}\right)}{\overline{N}^{2}(\delta+\gamma-\eta\gamma)\phi\overline{N}^{\alpha}\left((\delta+\gamma-\eta\gamma)\phi\overline{N}^{\alpha}+\gamma(1-\eta)\right)} \\ \frac{\overline{N}^{2}\left(\gamma(1-\eta)-(\delta+\gamma-\eta\gamma)\phi\overline{N}^{\alpha}\right)^{2}}{E^\star\gamma(\delta+\gamma-\eta\gamma)(1-\eta)\phi\overline{N}^{\alpha}} & \frac{\gamma(1-\eta)}{(\delta+\gamma-\eta\gamma)\phi\overline{N}^{\alpha}}-\alpha \end{bmatrix}$$

The determinant of this matrix is

$$\frac{\gamma(1-\eta)v}{(\delta+\gamma-\eta\gamma)\phi\overline{N}}^{\alpha}-\alpha v$$

It is decreasing in $\overline{N}$. Its trace is

$$\frac{\gamma(1-\eta)^{2}}{(\delta+\gamma-\eta\gamma)\phi\overline{N}^{\alpha}}-\alpha+\eta+v$$

also decreasing in $\overline{N}$.

For the steady state close to $\hat{N}$, the determinant has a value close to $v(1-\alpha)$ and a trace close to $1-\alpha+v$. It is therefore locally stable.

If $E^{\star}$ is restrictive enough, the low population steady state has a population close to zero, and the high population steady state has a population close to the value of the vertical asymptote $\hat{N}$. The low population steady state is increasing in $E^{\star}$, the high population steady state, which is locally stable, is decreasing in $E^{\star}$. The latter result proves Proposition 2.

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