The natalist bias of pollution control

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ABSTRACT

For a given technology, two ways are available to achieve low polluting emissions: reducing production per capita or reducing population size. This paper insists on the tension between the former and the latter. Controlling pollution either through Pigovian taxes or through tradable quotas schemes encourages agents to shift away from production to tax free activities such as procreation and leisure. This natalist bias will deteriorate the environment further, entailing the need to impose ever more stringent pollution rights per person. However, this will in turn gradually impoverish the successive generations: population will tend to increase further and production per capita to decrease as the generations pass. One possible solution consists in capping population too.

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1. Introduction

Pollution control can be justified on both efficiency and fairness grounds. Under a Kyoto type of regime, a key motivation for capping greenhouse gas emissions arises from a concern for future generations. The aim is to make sure that the climatic conditions they will experience either be not worse than ours or, at the very least, do not prevent them from leading a decent life. Article 2 of the 1997 Kyoto Protocol states that it aims at the

"stabilization of greenhouse gas concentrations in the atmosphere at a level that would prevent dangerous anthropogenic interference with the climate system. Such a level should be achieved within a time-frame sufficient to allow ecosystems to adapt naturally to climate change, to ensure that food production is not threatened and to enable economic development to proceed in a sustainable manner."

The assumption is thus that people in the future will be better off if we cap emissions from now on, than under a business as usual scenario. This is a very plausible claim. And yet, it generates concerns.

One worry has to do with the opportunity cost for today’s poor of such pollution control. The worry we are going to deal with here is the reverse one. It is not so much that pollution control may “impoverish” today’s poor if certain conditions are met. It is rather that it may “impoverish” future generations too, including their poorest members. The mechanism through which future generations might become poorer as a result of pollution control rests on the demographic impact of the latter. In general, the literature in climate economics tends to operate under the assumption that demographics is exogenous (adopting for example global population size projections made by major demographic institutions). One
exception consists in the attempt to connect demographics with the choice of allocation rule of tradable quotas. Consider the following two quotes as illustrations:

“The major objections to [per capita entitlements] are based partly on ethical and practical ‘comparable-burden’ type arguments (since it would imply a huge adjustment burden on industrialized countries, to which they are unlikely to agree), and partly on grounds of concern that such allocation might ‘reward’ population and population growth. Proponents tend to argue that any such effect is negligible compared to other factors influencing population; but to avoid any inducement to population growth, Grubb suggests that the population measure should be restricted to population above a certain age. (...) Grubb et al. note a wider range of possibilities for avoiding any incentive to population growth, including ‘lagged’ allocation (related to population a fixed period earlier); apportionment to a fixed historical date; or the inclusion of an explicit term related inversely to population growth rate” (Grubb [1, pp. 485–486]).

“If we agree that emissions allocations should be based on numbers of people, we effectively encourage something which compounds our problems on Earth: population growth. Solutions have been suggested; in particular, we might tie allocations to population figures for a specific time. Singer, for example, argues that per capita allocations should be based on estimates of a country’s population in the future, to avoid penalizing countries with young populations.” (Garvey [2, p. 218]).

Although neither of these quotes is entirely explicit about the mechanism through which per capita allocation incentivizes population growth, one may assume that what these two authors have in mind is the following idea: under a fully or partly population-based allocation rule of pollution rights, countries may encourage population growth because this will positively impact their relative share in the quota allocation at the next period. This “share preservation/increase” motive can be one incentive for population growth. Grubb is probably right: if this is what actually drives the natalist effect, it is likely to remain “negligible”, as population growth may entail costs likely to more than compensate the value of getting extra emission entitlements.

The mechanism we have in mind differs from the “share reservation/increase” motive. Moreover, its impact is likely to be much more significant. Our starting point is that, when there is only one production sector, capping emissions entails capping production. We will show that this generates a shift from production to other activities, especially procreation. It is this shift from production to reproduction that will generate the demographic impact of capping emissions. Note that in this case, what drives the natalist effect does not directly have to do with the allocation formula. It rather has to do more directly with the very existence of a cap.

As mentioned above, endogenous responses of population have been neglected so far in environmental economics. The main contribution of our paper is accordingly to provide a framework where demography reacts to pollution control. In order to properly focus on that aspect, we simplify matters with regard to technology, and more specifically, its degree of eco-efficiency. We keep it exogenous. This does not need to imply that technology is constant, but rather that technological progress does not depend on the specific policy.

One may object that assuming exogenous technology is far fetched. Admittedly, a portion of technical change is endogenous. However, the empirical literature suggests that this portion is limited. Moreover, there is a second way in which our assumption is realistic: in cases such as climate change, the scale of environment-saving improvements required to stabilize pollution is daunting, at least in the medium-run. Even fully endogenous and highly responsive technology may therefore not suffice.

The essential ingredients of the model are as follows. Individuals allocate their time across three activities: production, leisure, and procreation. Each of these concepts has a specific meaning. Production refers to the time spent on manufacturing consumption and investment goods with an exogenous technology. Leisure involves non-market and emission-free activities, such as chatting with friends, sleeping, sweeping the floor, ... but could also be extended to include time spent on eco-friendly production activities. Procreation refers to the time spent on child rearing by parents.

Substitution of procreation for production is at the heart of this paper. It occurs as soon as rearing children takes time and is sensitive to the relative return of spending time on this or other activities. Various factors can affect this relative return: parents’ income [5], child mortality [6,7], the absence of formal old age support schemes [8], cultural norms (Princeton European Fertility Project), the importance of increasing the relative power of one’s community (see [9,10]), etc. In this paper we concentrate on the first of these factors. Becker and Lewis [5] stress that the wage of the parents is part of the opportunity cost of having children. A rise in parents wage leads to a substitution of production for procreation, and to drop in fertility. Such a mechanism is seen by many economists as a key explanation for the demographic transition (for a

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1 The impact of environmental prices and policy on technological choice by firms has been studied by many. It seems that while some innovation was responsive to energy price changes, a sizable portion of efficiency improvements were still autonomous (see [3]). In addition, although environmental taxes reduce pollution by encouraging the development of new technologies, simply relying on technological change as a panacea for environmental problems is not enough (see [4]).

2 The inclusion of leisure ensures the robustness of the results to the existence of a diversity of activities allowing at every moment in time to shift from eco-efficient activities to less eco-intensive ones.

3 More precisely, the key assumption is that time costs are much more significant than goods costs in the case of child rearing. In the model, we abstract from goods costs, which does not impact on our results as long as such costs remain comparatively smaller.
represents a constant pollution level in the plane population.  

2.1. Production and pollution

aggregate dynamics. describe how pollution is generated. Then we consider the household maximization problem and, finally, the implied allowing production of the same amount with a cleaner technology and lower emissions (for example through higher iso-pollution curve to another with lower level of emissions, one may of course lower production per capita and/or lower right, income per person decreases and population increases, pollution remaining constant. In order to move from one 

The function $C$ per dollar produced. The stock of pollution 

This equation is known in the literature as the Kaya identity. If 

where $\alpha$ represents the pollution coefficient, i.e., the degree to which production generates polluting emissions. Total output is itself the product of adult population size $N_t$ and production per person $y_t$:  

This equation is known in the literature as the Kaya identity. If $E_t$ is measured in tons of CO₂, then $\alpha$ would be tons of CO₂ per dollar produced. The stock of pollution $S_t$ accumulates according to:  

The function $\Psi(\cdot)$ takes different forms in the literature. Fig. 1 displays a map of iso-pollution curves. Each curve represents a constant pollution level in the plane population $\times$ income per person. As we follow the curve towards the right, income per person decreases and population increases, pollution remaining constant. In order to move from one iso-pollution curve to another with lower level of emissions, one may of course lower production per capita and/or lower population size; however, one could also increase either of the two if the other gets reduced strongly enough.

Over time, the map can change. Two factors affect the position of the curves. On the one hand, technical progress allowing production of the same amount with a cleaner technology and lower emissions (for example through higher


5 See for example John and Pecchenino [19] where pollution is the inverse of an "environment" variable which accumulates like capital, assuming some positive degradation rate, i.e., the share of past pollution $S_{t-1}$ that has been absorbed by the environment. In [20], a "world temperature" variable is like pollution here and depends on the past emissions.
energy efficiency) shifts the iso-pollution curves to the Northeast. Environmental efficiency decreases the pollution coefficient \( a \). On the other hand, if there is more pollution accumulated in the past \( (S_{t-1}) \), the iso-pollution curves of today will shift to the Southwest. Fig. 2 represents these two possible shifts.

2.2. Households

At each date \( t \), there is a new adult generation of size \( N_t \) deriving utility from consumption \( c_t \), leisure \( \ell_t \), number of children \( n_t \), and quality of the children \( k_{t+1} \), as measured by their future human capital. Households are homogeneous. We assume a logarithmic utility function:

\[
u(c_t, \ell_t, n_t, k_{t+1}) = \ln c_t + \delta \ln \ell_t + \gamma \ln (n_t k_{t+1}),\]

with \( \delta, \gamma \in \mathbb{R}_+ \). The parameter \( \delta \) is the taste for leisure and \( \gamma \) is the altruism factor. Parents care both about child quantity \( n_t \) and quality \( k_{t+1} \), as measured by the human capital provided to them. Notice that parents do not care about their children’s utility, as would be the case with dynastic altruism. Our formulation of altruism is referred to in the literature as “joy-of-giving” (or warm glove), because parents have a taste for giving (see e.g. [21]). As our aim here is not to assess how agents should behave, impure altruism seems an acceptable assumption as a means to obtain clearcut analytical results.

The choice of a logarithmic utility function is defended by [22] on the grounds that leisure shows no trend despite growing wages. This can only be accounted for when the elasticity of substitution between leisure and consumption is close to one.

We do not introduce pollution into utility, because our objective is not to derive the best policy, but rather to show the side effects of a given type of policy. Adding a disutility term such as \(-\nu(S)\) to the utility would not change any of our results as long as utility is additively separable.
Future human capital is obtained through spending on education, \( e_t \). The human capital production function is

\[
k_{t+1} = \tau e_t^v k_t^\eta
\]

with the following parametric restrictions: \( \tau \in \mathbb{R}_+, \nu, \eta \in (0,1) \). We assume moreover that \( \eta + \nu < 1 \), which will imply that human capital and output person will converge in the long-run to a constant level.\(^6\) The parameter \( \tau \) is a measure of productivity of education technology and \( \eta \) is the elasticity of human capital \( k_{t+1} \) with respect to investment \( e_t \). The parameter \( \nu \) captures the strength of an externality from parents human capital to children human capital. It represents the usual parental influence on child outcome. This production function is standard in the literature on education, starting with Glomm and Ravikumar\([23]\).

Let us provide a few additional details on the parameters \( \eta \) and \( \nu \). \( \eta \) measures the elasticity of earnings with respect to schooling. An idea of its magnitude can be obtained from the survey by Krueger and Lindahl\([24]\) which reports estimates of the return to schooling in developed countries of \( 8\text{–}10\% \), with higher estimates for developing countries and low levels of schooling. Assuming that an additional year of schooling raises education expenditure by \( 20\% \), these returns translate into an earnings elasticity of schooling between \( 0.4 \) and \( 0.8 \). Replacing \( k_{t+1} \) in the utility function by its expression from (1), allows us to stress the importance of the parameter \( \eta \):

\[
\ln(c_t) + \delta \ln \ell_t + \gamma (\ln(n_t) + \eta \ln(e_t)) + \nu \ln k_t + \gamma \tau
\]

The last two terms of the sum are constant. We see that \( \eta \) is not only the elasticity of human capital to education spending in (1), but also the relative weight of quality in the utility function. It has to be smaller than \( 1 \) because the parents’ optimization problem would otherwise not have a solution. More specifically, utility would approach infinity as parents choose arbitrarily high levels of quality spending \( e_t \) and arbitrarily low levels of fertility (a similar condition can be found in Moxå\([25]\) and de la Croix and Doepke\([26]\)).

The parameter \( \nu \) captures the intergenerational transmission of ability, as well as human capital formation within the family that does not work through formal schooling \( (e_t) \). Empirical studies detect such effects, but they are relatively small.\(^7\)

The time needed to produce \( n_t \) children is decreasing in space \( L/N_t \) (land per household) and is given by

\[
\frac{1}{\mu} \left( \frac{N_t}{L} \right)^2 n_t = \phi N_t^2 n_t
\]

with

\[
\phi = \frac{1}{\mu} \left( \frac{1}{L} \right)^2
\]

Compared to the models developed in the recent literature, we introduce land per person as an input in the child production technology. The aim is to take into account that, when households have small dwellings, child production is more costly and people have fewer children (this is known since\([28,29]\)). It also implies that population will be stationary in the long-run. Indeed, as population increase, it becomes more and more costly to have children, lowering progressively the fertility rate to its replacement level. The parameter \( \mu \) measures total factor productivity of the procreation activity and \( z \in (0,1) \) captures the importance of space to produce children.

Households face a budget constraint stating that consumption plus education spending cannot exceed income \( y_t \):

\[
c_t + n_t e_t \leq y_t.
\]

Households have a total time endowment equal to \( 1 \). They face a time constraint that time spent working \( h_t \), rearing children and having leisure cannot exceed \( 1 \):

\[
h_t + \phi N_t^2 n_t + \ell_t \leq 1
\]

Households are self-employed. The productivity of each hour of work is given by the quality of the worker, i.e., his/her human capital \( k_t \). Total production is therefore the product of hours of work \( h_t \) and \( k_t \):

\[
y_t = h_t k_t
\]

Notice that, in this production function, we assume constant returns with respect to input of efficiency units \( h_t k_t \). We also consider hours of work and efficiency units a perfect substitutes: doubling efficiency together with halving hours of work would leave production unchanged. Departing from one of these two assumptions is not expected to modify the results. However, it would make the analysis more complex, requiring a numerical analysis in most cases.

Substituting saturated constraints (2)–(4) into the objective, the household maximization problem can be written as:

\[
\max_{e_t, n_t, h_t} \ln(1-(\ell_t - \phi N_t^2 n_t)k_t-n_t e_t) + \delta \ln \ell_t + \gamma (\ln(n_t) + \eta \ln(e_t)) + \text{constant terms}
\]

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\(^6\) The alternative case \( \eta + \nu = 1 \) would generate endogenous growth, which could be analyzed as well with the tools developed here.

\(^7\) For example, Leibowitz\([27]\) finds that, after controlling for education, parental income has a significant effect on a child’s earnings. The elasticity is of the order of \( 0.1 \).
The first-order conditions are
\[
\frac{-k_t}{(1-\ell_t-\phi N_t^2 n_t)k_t-n_t e_t} + \frac{\delta}{\ell_t} = 0
\]
\[
\frac{-\phi N_t^2 k_t - e_t}{(1-\ell_t-\phi N_t^2 n_t)k_t-n_t e_t} + \frac{\gamma}{n_t} = 0
\]
\[
\frac{-n_t}{(1-\ell_t-\phi N_t^2 n_t)k_t-n_t e_t} + \frac{\gamma n_t}{e_t} = 0
\]
As the maximization problem is convex, the first-order conditions are necessary and sufficient for a maximum. Solving the set of first-order conditions and saturated constraints (2)–(4) for \(c_t, \ell_t, n_t, e_t\), and \(y_t\) yields closed form solutions:
\[
c_t = \frac{k_t}{1+\delta+\gamma}
\]
\[
\ell_t = \frac{\delta}{1+\delta+\gamma}
\]
\[
n_t = \frac{\gamma(1-\eta)}{(1+\delta+\gamma)\phi N_t^2}
\]
\[
e_t = \frac{\eta \phi N_t^2 k_t}{1-\eta}
\]
\[
y_t = \frac{\gamma n_t}{1+\delta+\gamma} k_t
\]

2.3. Aggregate dynamics

Adult population dynamics are given by
\[
N_{t+1} = N_t n_t
\]
Substituting the expressions for \(e_t\) (8) and \(n_t\) (7) into the equations describing the dynamics of human capital (1) and population (10) leads to:
\[
k_{t+1} = \tau \left( \frac{\eta \delta}{1-\eta} \right)^{\frac{\eta}{1-\eta}} N_t^{\eta} k_t^{\eta
+\eta}
\]
\[
N_{t+1} = \frac{\gamma(1-\eta)}{(1+\delta+\gamma)\phi} N_t^{1-\gamma}
\]
This system is recursive as the second equation can be solved independently of the first one. The second equation shows that \(N_{t+1}\) is an increasing and concave function of \(N_t\) which does not depend on \(k_t\). It has a unique nontrivial steady state:
\[
N = \left( \frac{\gamma(1-\eta)}{(1+\delta+\gamma)\phi} \right)^{\frac{1}{1-\gamma}}
\]
which is globally stable. Dynamics of population are monotonic. For a given \(N_t\), the first equation also describes an increasing and concave relation between \(k_{t+1}\) and \(k_t\). When \(N_t\) is close enough to \(N\), the dynamics of \(k_t\) are also monotonic and converge to:
\[
k = \left( \frac{\gamma^{1/\gamma} \eta}{1+\delta+\gamma} \right)^{\frac{\eta(1-\gamma-\eta)}{1-\gamma+\gamma}} \text{ for } \alpha > 0
\]
Income per capita converges to
\[
y = \tau^{1/(1-\eta)} \left( \frac{\gamma n_t}{1+\delta+\gamma} \right)^{(1-\eta+\gamma)/(1-\eta)}
\]
A larger country (higher \(L\), lower \(\phi\)) will have a larger population size. A more productive country (higher \(\tau\)) will have higher income per capita.
If \( x = 0 \), space is not useful to produce children. Population grows unboundedly at rate \( \gamma(1-\eta)/((1+\delta+\gamma)\phi) \) and human capital converges to:
\[
\bar{K} = \left( \frac{\tau^{1/\eta} \delta}{1-\eta} \right)^{\eta/(1-\eta)} \text{ for } x = 0
\]

3. Regulation: pollution cap and tradable rights

At each date, past pollution is given. A given pollution target \( S_t \) can be achieved by imposing an emission target \( E_t \) such that:
\[
S_t = \Psi(S_{t-1}, E_t), \quad \forall t \in \mathbb{N}
\]

Since we do not provide, in this paper, a utility based justification for a given pollution target, the latter is taken to be exogenous. As a result, the path of emission targets \( \{E_t\}_{t=0}^{\infty} \) is exogenous too.

Remember that, for simplification purposes, we have assumed that output is produced by self-employed households. To meet the sequence of emission targets, two policy schemes are available and interchangeable: a Pigovian tax on emissions, which shows clearly that the price of pollution permits affects the first-order conditions for the system formed by the first-order conditions and the constraints; and a tradable pollution rights system with a free initial allocation of rights to households. In a world where agents behave competitively and information on production, with revenue transferred back to households in a lump-sum way; and a tradable pollution rights system with a free initial allocation of rights to households. In a world where agents behave competitively and information is perfect about both the objective that is being pursued and the deep parameters of the model, tradable quotas schemes and price-oriented schemes are fully equivalent. This implies that, despite our focus on tradable right schemes, the results will be of direct relevance for those willing to implement a Pigovian tax.

3.1. Households

A pollution rights system requires each household buys pollution rights in proportion to the output that would exceed their initial endowment. Let us denote the price of the pollution right by \( p_t \) and the initial endowment of rights by \( q_t \). The budget constraint of the household is now:
\[
y_t \geq c_t + n_t e_t + p_t (a_t y_t - q_t)
\]

The constraint can be rewritten:
\[
(1-a_t p_t) y_t + p_t q_t \geq c_t + n_t e_t
\]

which shows clearly that the price of pollution permits \( p_t \) weighted by the pollution coefficient \( a_t \) acts like an income tax, and \( p_t q_t \) as a lump-sum transfer.

Substituting saturated constraints (1), (14), (3) and (4) into the objective, the households maximization problem can be written as:
\[
\max_{c_t, n_t, e_t} L_t = \ln((1-a_t p_t)(1-\ell_t-\phi N_t^2 n_t)k_t - n_t e_t + p_t q_t) + \delta \ln \ell_t + \gamma (\ln(\ell_t) + \eta \ln(e_t)) + \text{constant terms}
\]

The first-order conditions can be written under the form “marginal cost = marginal benefit”:
\[
\begin{align*}
\frac{\partial L_t}{\partial c_t} & = 0 \Rightarrow \frac{(1-a_t p_t) k_t}{c_t} = \frac{\delta}{\ell_t} \\
\frac{\partial L_t}{\partial n_t} & = 0 \Rightarrow \frac{(1-a_t p_t) \phi N_t^2 k_t - e_t}{c_t} = \frac{\gamma}{n_t} \\
\frac{\partial L_t}{\partial e_t} & = 0 \Rightarrow \frac{n_t}{c_t} = \frac{\gamma \eta}{e_t}
\end{align*}
\]

The price \( p_t \) affects the first-order conditions for \( \ell_t \) and \( n_t \) by lowering their marginal cost.

As the maximization problem is convex, the first-order conditions are necessary and sufficient for a maximum. Solving the system formed by the first-order conditions and the constraints leads to:
\[
\begin{align*}
c_t & = \frac{1}{1+\delta+\gamma}((1-a_t p_t) k_t + p_t q_t) \\
\ell_t & = \frac{\delta}{1+\delta+\gamma} \frac{(1-a_t p_t) k_t + p_t q_t}{(1-a_t p_t) k_t} \\
n_t & = \frac{\gamma (1-\eta)}{(1+\delta+\gamma)\phi N_t^2} \frac{(1-a_t p_t) k_t + p_t q_t}{(1-a_t p_t) k_t}
\end{align*}
\]

\(^8\) Uncertainty [30] or strategic interactions [31] draw a wedge between these two instruments.
\[
et_t = \frac{\eta \phi N_t^\phi k_t}{1 - \eta} (1 - a_t p_t) \\
y_t = \frac{(1 - a_t p_t) k_t (1 + \gamma \eta) - (\delta + \gamma - \gamma \eta) p_t q_t}{(1 - a_t p_t)(1 + \delta + \gamma)}
\]

3.2. A small open economy

Before considering our economy as a model of the world in general equilibrium, we analyze the case of a small open economy in which the price \( p_t \) and the quota \( q_t \) are imposed from outside and exogenous.

We first observe that the time spent on emission-free activities, i.e., leisure and procreation, increases with the price of pollution permits \( p_t \). Indeed, \( p_t \) acts as a tax on the time spent on production. Hence, increases in \( p_t \) lower the opportunity cost of leisure and procreation, and so \( p_t \) is similar to a subsidy to procreation:

\[
\frac{\partial \ell_t}{\partial p_t} > 0, \quad \frac{\partial n_t}{\partial p_t} > 0
\]

Leisure and procreation also increase with the endowment of pollution permits. This is because they are both normal goods:

\[
\frac{\partial \ell_t}{\partial q_t} > 0, \quad \frac{\partial n_t}{\partial q_t} > 0
\]

Human capital accumulation (education) is reduced by the price of pollution permits, because of a substitution of quantity \((n_t)\) for quality \((k_t+1)\) of children:

\[
\frac{\partial \ell_t}{\partial p_t} < 0
\]

Finally, net individual income and production are decreasing in \( p_t \):

\[
\frac{\partial y_t}{\partial p_t} = - \frac{(\delta + \gamma (1 - \eta)) q_t}{(1 - p_t)(1 + \delta + \gamma)} < 0.
\]

Let us now analyze how the presence of tradable pollution rights affects the steady state and the dynamics of a small open economy. For this, we assume that exogenous variables are constant, i.e., \( a_t = a, p_t = p \) and \( q_t = q \). The dynamics are represented as follows:

\[
k_{t+1} = \tau \left( \frac{\eta \phi}{1 - \eta} \right)^\eta N_t^{\phi \eta} k_t^{1 + \eta} (1 - ap)^\phi \\
N_{t+1} = \frac{\gamma (1 - \eta)}{(1 + \delta + \gamma)^\gamma} N_t^{1 - \gamma} (1 - ap) k_t + pq \\
\]

This dynamical system is no longer block recursive, i.e., the two different equations need to be solved simultaneously. To analyze its properties, we use the phase diagram in Fig. 3. A first phase line is given by

\[
\Delta k_{t+1} = 0 \iff k_{t+1} - k_t = \tau \left( \frac{\eta \phi}{1 - \eta} \right)^\eta N_t^{\phi \eta} k_t^{1 + \eta} (1 - ap)^\phi - k_t = 0
\]
Solving for \( N_t \) gives

\[
N_t = \tau^{-1/\eta} \left( \frac{\eta \delta}{1-\eta} \right)^{-1/\eta} (1-\alpha p)^{-1/\eta} k_t^{(1-\gamma-\eta)/\eta}
\]  

The right hand side is an increasing function of \( k_t \). We draw this function in the space \( \{k_t, N_t\} \). Considering a point located above that line, i.e., a point with a larger \( N_t \) than the one given by (18), it appears from (17) that it corresponds to a situation where \( \Delta k_{t+1} > 0 \). Accordingly, when located above this phaseline, we draw a horizontal arrow oriented to the right to indicate the direction of motion. Another arrow oriented to the left is drawn when below the phaseline.

The second phase line is given by

\[
\Delta N_{t+1} = 0 \Leftrightarrow N_{t+1} - N_t = \frac{\gamma (1-\eta)}{(1+\delta+\gamma)\phi} N_t^{1-\eta} \frac{(1-\alpha p)k_t + pq}{(1-\alpha p)k_t} - N_t = 0
\]  

Solving for \( N_t \) gives

\[
N_t = \left[ \frac{\gamma (1-\eta)}{(1+\delta+\gamma)\phi} \frac{(1-\alpha p)k_t + pq}{(1-\alpha p)k_t} \right]^{1/\eta}
\]  

which is a negatively sloped function going from +\( \infty \) when \( k_t = 0 \) to 0 when \( k_t = +\infty \). Let us decrease \( k_t \) to consider a point to the left of this curve. It increases the function (19) and hence \( \Delta N_{t+1} > 0 \) in this zone. Hence, to the left (resp. right) of this curve, we can draw arrows pointing upward (resp. downward).

The phase diagram in the left panel of Fig. 3 shows that there is inevitably a unique steady state with oscillatory dynamics. Appendix A linearizes the dynamic system around the steady state and shows that the steady state is locally stable.

Let us now suppose that there is an exogenous increase in the price of pollution permits. Differentiating the two phase lines (18) and (20) leads us to conclude that they both shift upward. As a consequence, the new steady state has a higher population level. We have seen above that pollution control increases fertility. This translates at the steady state level into a larger population.

### 3.3. General equilibrium

We now turn our attention to the most difficult case: the one in which the price of pollution rights, instead of being exogenous, adjusts as a function of market forces. The equilibrium in the market for tradable pollution rights implies that total pollution \( N_t a_t y_t \) equals the total number of quotas \( N_t q_t \), unless the price \( p_t \) is zero:

\[
p_t (N_t a_t y_t - N_t q_t) = 0
\]

Two cases may arise depending on whether the cap is binding or not. A cap is binding if it is set lower than the otherwise desired total amount of pollution. This occurs when

\[
q_t < a_t \frac{1+\gamma \eta}{1+\delta+\gamma} k_t
\]

where \( y_t \) is computed in the business as usual scenario. Replacing \( y_t \) by its value from Eq. (9) leads to:

\[
q_t < a_t \frac{1+\gamma \eta}{1+\delta+\gamma} k_t
\]

**Proposition 1.** At time \( t \), the equilibrium satisfies: if (21) holds then

\[
p_t = \frac{k_t(1+\gamma \eta)-q_t(1+\delta+\gamma)}{(k_t-q_t)(1+\gamma \eta)}
\]

\[
y_t = \frac{q_t}{a_t}
\]

If (21) does not hold then

\[
p_t = 0
\]

\[
y_t = \frac{\gamma \eta}{1+\delta+\gamma} \frac{k_t}{q_t}
\]

If the pollution endowment is sufficiently restrictive, there will be a positive price of pollution permits and production will match the target. If the pollution quota is large, the policy is non-binding. The price of permits then falls to zero, and the output corresponds with the one of the business as usual scenario.
3.4. Dynamics

Let us consider a constant emission cap $E^*$: As a consequence, the pollution endowment per household will be:

$$q_t = \frac{E^*}{N_t}$$

We now analyze how different levels of the emission cap $E^*$ affect the dynamics of population $N_t$. To simplify, we keep technical progress constant $\alpha_t = 1$ (but of course we will not use this simplification when we let $\alpha_t$ increase in one of our scenarios).

The dynamics of human capital $k_t$ and population $N_t$ are obtained by substituting $e_t$, $n_t$, and $p_t$ from (16), (15) and (22) into (1) and (10):

$$k_{t+1} = \tau k_t \left( \frac{\eta \phi N_t^2 k_t}{1-\eta} \left( 1 - \frac{k_t (1+\gamma \eta) - q_t (1+\delta+\gamma)}{(k_t-q_t)(1+\gamma \eta)} \right) \right)^\eta$$

$$N_{t+1} = N_t \left( \frac{\gamma (1-\eta)}{(1+\delta+\gamma)\phi N_t^2} \left( 1 + \frac{k_t (1+\gamma \eta) - q_t (1+\delta+\gamma)}{(k_t-q_t)(1+\gamma \eta)} \right) \right)$$

Using $q_t = E^*/N_t$ and simplifying leads to:

$$k_{t+1} = \tau k_t^{\eta} \left( \frac{\eta \phi N_t^2 E^* (\delta+\gamma-\eta)}{(N_t k_t - E^*)(1-\eta)(1+\gamma \eta)} \right)^\eta$$

$$N_{t+1} = \frac{\gamma (1-\eta)(N_t k_t - E^*)}{\phi N_t^2 k_t (\delta+\gamma-\eta)}$$

The analysis of these dynamics is detailed in Appendix B. The main result is the following.

**Proposition 2** (Population and the pollution cap). For a sufficiently stringent pollution cap $E^*$, there is a locally stable steady state population, decreasing in $E^*$.

If $E^*$ is restrictive enough, the long-run population $N$ is higher if the pollution cap is set at a more stringent level. As a consequence, income per capita will unambiguously be lower, as $y = E^*/N$.

From the dynamic point of view, the pro-population tilt of pollution caps is worrying. For a given $E^*$, emission endowments per person inevitably become more and more stringent as generations pass. Because of this pro-natalist effect, capping emissions impoverishes the successive generations more than in a conventional set-up with exogenous fertility. It is worth spelling out why capping emissions tends to reduce production rather than procreation. This is the case because production generates emissions from the moment it takes place onwards, whereas procreation generates delayed emissions. This rests on two assumptions. First, only physical good production generates pollution. Second, children do not consume physical goods. This implies that the emissions of a person take place at adulthood. In a more general set-up, it would be sufficient to assume that procreation and leisure are simply less emission-intensive activities than production. This is why capping emissions at period $t$ puts less pressure on procreation than on production. In a way, if procreation only generates emissions through future production (i.e., when children will themselves become producers), the capping scheme generates a specific form of externality. Current adults willing to procreate at a rate higher than the replacement rate do not internalize the fact that tomorrow’s pollution cap will have to be divided into smaller pollution endowments.

4. Numerical experiment

In order to provide a meaningful example of the mechanisms studied analytically above, we calibrate the parameters of the model and we simulate the effect of introducing pollution caps on the dynamics of income and population.

4.1. Calibration

Assume that each period lasts 25 years. We will use the year 1983 as representing $t=0$ (initial conditions), and the year 2008 as $t=1$. 2008 is the last year for which we have observations.

We first identify $\gamma$, $\delta$, $\eta$, $\nu$ and $\alpha$ with the following five restrictions:

1. The share of consumption in GDP is 80% (corresponds to public and private consumptions of the national accounts). Using Eqs. (5) and (9), we find that

$$\frac{c_t}{y_t} = \frac{1}{1+\eta} = 0.8$$
2. The time spent on leisure (\( \ell_t \)) and procreation (\( \phi N^x \)) amounts to 2/3 of total available time (this has become a standard value in the literature since [32] found that households allocate approximately one-third of their time to market activities). Using (13):

\[
\phi N^x = \frac{\gamma(1-\eta)}{1+\delta+\gamma}
\]

From (6):

\[
\ell_t + \phi N^x = \frac{\delta}{1+\delta+\gamma} + \frac{\gamma(1-\eta)}{1+\delta+\gamma} = \frac{\delta + \gamma(1-\eta)}{1+\delta+\gamma} = \frac{2}{3}
\]

3. At steady state, the time spent rearing children is equal to 15% (see [33]) of the time remaining after leisure has been accounted for:

\[
\frac{\phi N^x}{1-\ell_t} = 0.15
\]

This implies

\[
\frac{\gamma(1-\eta)}{1+\gamma} = 0.15
\]

4. Following the literature on conditional convergence (see [34] for a survey), the convergence speed of income per capita is 2% per year. For the dynamic equation (11) we get

\[
k_{t+1} = \left(\frac{k_t}{k_{t-1}}\right)^{v+\eta} \left(\frac{N_t}{N_{t-1}}\right)^{\eta}
\]

The required convergence speed is obtained with \( v + \eta = 0.98^{25} \).

5. The dynamics of population are calibrated to match the forecasted evolution of world population between 2008 \((t-1)\), 2033 \((t)\) and 2058 \((t+1)\). From the dynamic equation (12) we get

\[
\frac{N_{t+1}}{N_t} = \left(\frac{N_t}{N_{t-1}}\right)^{1-x}
\]

and we have \( N_{t-1} = 6.67 \), \( N_t = 8.18 \) and \( N_{t+1} = 8.88 \) from the 2007 IIASA World Population Projection.

Solving this system gives \( \gamma = 0.470588 \), \( \delta = 2.279411 \), \( \eta = 0.53125 \), \( v = 0.0722147 \), and \( x = 0.5976 \). Notice that \( \eta \) is in line with estimates of the return from education (see the discussion in [33]). Moreover, this \( \eta \) is almost enough to obtain the required speed of convergence of income per capita, as the additional parameter \( v \) is small. Notice finally that the parameter \( x \) implies an annual convergence speed for population of 3.56% per year.

The two productivity levels, \( \tau \) and \( \phi \), are parameters that determine the size of population and income per capita. Imposing initial conditions so as to start in 1983 requires \( N_0 = 4.68 \) and \( y_0 = 4.541 \). Inverting (9) gives us \( k_0 = 16.0271 \). In order to obtain the right levels \( N_t = 6.67 \) and \( y_t = 7.614 \) in 2008, we need to have \( \phi = 0.0164 \) and \( \tau = 24.0417 \).

4.2. Simulation

Table 1 provides the simulation from 1983 (initial conditions) to 2208 when no pollution cap is imposed. It illustrates the properties of the benchmark model: monotonic convergence of population, which tends to 8.47 billions, and income per capita (38155 dollars per capita per year in 2208). Fertility declines rapidly to its replacement level. Leisure is constant.

Let us now impose a constant pollution cap: \( E^* = 100 \), starting to bind in 2033. The chosen level of \( E^* \) is arbitrary and for illustration purposes only. Table 2 provides the results. There is now one new column: the pollution price \( p_c \). The price in 2033 is 0.24, corresponding to an implicit tax of 24% on production. Following the tax, total output \( Y_t \) is indeed limited to 100. As a consequence of this tax, the households retreat from market activities to devote more time to leisure (66.1% instead of 60.8% in the benchmark) and to procreation (1.151 child per person instead of 1.059 in the benchmark, to be multiplied by 2 to compare to fertility rates per women). The rise in procreation does not look big, but it is large enough to have immense consequences for the future, through its cumulative effect over time. Population in 2058 is now 8.85 billions instead of 8.15 billions in the benchmark and converges in the long-run to more than 12 billions instead of 8.5 billions in the benchmark.

Another way to look at the same data is to plot fertility over time. Fig. 4 represents children born per person over time, for the benchmark (black line) and the constant cap (grey line) scenarios. The drop in fertility is delayed by two periods when the pollution cap is imposed. Notice that delaying the demographic transition does not entail reversing the general trend towards fertility drop. This matters for the following reason. Data tend to show that countries having gone through their demographic transition do not experience later on a rise in fertility in episodes of impoverishment (see e.g. Moldova).
This could suggest that the demographic transition is irreversible. Even if we were to accept this idea, this would not conflict with the delaying effect we identified.

Suppose now that there is some technical progress making production more and more clean over time. Precisely, we assume that

\[ a_t = (1.01)^{-25(t - 2)} \]

which reflects a technical progress of 1% per year. Time \( t \) is equal to 2 in 2033, this formulation is the same as previous for the year 2033, but output is becoming less and less polluting as time passes. Hence we can allow increasing caps: \( E_{2033} = 100, E_{2058} = 128.243 \), etc. Table 3 provides the results. In the long-run, the cap is not binding thanks to
technical progress, and the economy converges to the benchmark steady state. As population has risen fast in the beginning, it actually overshoot its long-run level, and converges from above to its steady state. The cost of this policy in terms of income are still very large. For example, income per person would be 17,058 dollars per year in 2083 with the cap, and 27,334 in the benchmark.

Fig. 5 summarizes the result, comparing the benchmark, the constant pollution cap, and the increasing cap simulations on the figure used in the introduction to present the iso-pollution curve. The benchmark follows a convex path in this plane, and crosses the iso-pollution line $E^* = 100$ early on. The constant cap path, on the contrary, moves Southeast as soon as the cap is binding. It will converge to a situation with a large population and an income per capita only slightly above the 1983 level. The increasing cap path is an intermediate case. In the short-run (which means here a few generations), it follows the constant cap path, with lower income per person and higher population. In the long-run though, the path converges to the benchmark steady state.

In future research, it would be interesting to consider a policy under which we cap population rather than emissions, for example along the lines proposed by [35]. Tradable procreation quotas schemes are of course not the only available option. Policies aimed at addressing population issues—both in terms of absolute level and of heterogeneity—are notoriously difficult to design. If they aim at keeping population below a certain level, they should remain as freedom-friendly as possible while being simultaneously concerned with not increasing poverty and inequality. Women’s education is a policy that can be justified independently (e.g. on gender equality grounds) while being effective at reducing birth rate.

Table 3

<table>
<thead>
<tr>
<th>$t$</th>
<th>$N_t$</th>
<th>$P_t$</th>
<th>$n_t$</th>
<th>$t_t$</th>
<th>$y_t$</th>
<th>$Y_t$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1983</td>
<td>4.680</td>
<td>0.000</td>
<td>1.425</td>
<td>0.608</td>
<td>5.342</td>
<td>25.002</td>
</tr>
<tr>
<td>2008</td>
<td>6.670</td>
<td>0.000</td>
<td>1.153</td>
<td>0.608</td>
<td>9.769</td>
<td>65.164</td>
</tr>
<tr>
<td>2033</td>
<td>7.692</td>
<td>0.240</td>
<td>1.151</td>
<td>0.661</td>
<td>13.000</td>
<td>100.000</td>
</tr>
<tr>
<td>2058</td>
<td>8.855</td>
<td>0.318</td>
<td>1.089</td>
<td>0.680</td>
<td>14.483</td>
<td>128.243</td>
</tr>
<tr>
<td>2083</td>
<td>9.641</td>
<td>0.256</td>
<td>1.012</td>
<td>0.665</td>
<td>17.058</td>
<td>164.463</td>
</tr>
<tr>
<td>2108</td>
<td>9.755</td>
<td>0.140</td>
<td>0.964</td>
<td>0.638</td>
<td>21.622</td>
<td>210.913</td>
</tr>
<tr>
<td>2133</td>
<td>9.402</td>
<td>0.000</td>
<td>0.939</td>
<td>0.608</td>
<td>28.167</td>
<td>264.830</td>
</tr>
<tr>
<td>2158</td>
<td>8.831</td>
<td>0.000</td>
<td>0.975</td>
<td>0.608</td>
<td>33.249</td>
<td>293.627</td>
</tr>
<tr>
<td>2183</td>
<td>8.611</td>
<td>0.000</td>
<td>0.990</td>
<td>0.608</td>
<td>36.025</td>
<td>310.229</td>
</tr>
<tr>
<td>2208</td>
<td>8.525</td>
<td>0.000</td>
<td>0.996</td>
<td>0.608</td>
<td>37.511</td>
<td>319.761</td>
</tr>
</tbody>
</table>

Fig. 5. Income and population dynamics in the examples.
without increasing poverty nor infringing too much on procreative freedom. Other tools have been discussed in the literature, such as taxing skilled people to subsidize unskilled ones ready to limit themselves to a single child [17].

While it is beyond the scope of this paper to explore the respective merits of such population control policies in detail, it would of course be crucial to consider which one to adopt in conjunction with measures of pollution control. Alternatively, a population control policy could also work as a substitute to a directly environmental one. A key question is the following: is there a population cap $N^*$ such that the desired emission level $E^*$ could be met? If yes, does $N^*$ allow for higher income per capita than under the model capping emissions directly? If the answer to these two questions is positive, the next question will become: under which conditions does it follow that we should cap population rather than emissions?

5. Conclusion

Pollution control, and especially greenhouse gas emission reduction, is matters of great importance. Most of the literature looks at environmental policy assuming that demography is exogenous (see e.g. the two influential papers by [20,36] using OLG models). However, we have shown that such policies unexpectedly impact on the population dynamics through a production–procreation substitution effect. Capping pollution subsidies de facto procreation, and may therefore delay the demographic transition in developing countries and the drop in global fertility. Such an increase in population, compared with a business as usual scenario, may in turn be damaging either in environmental terms if the pollution scheme is ineffective, or in terms of the average standard of living—both independently and through the operation of the pollution cap at the next period.

Admittedly, the effect of pollution on utility and/or on productivity has not been modeled. If pollution affects productivity negatively,9 or has a negative effect on the health of workers, the strength of our substitution effect would be weakened. Assuming that consumption and environment quality are complements in the utility function would also weaken our results. Refining the model in that direction would definitely be of interest for a welfare assessment of environmental policies. However, it would not affect the specific conclusion of this paper, as these extensions are unlikely to reverse the direction of the substitution effect we highlighted. Moreover, we assumed that households do not care about future generations beyond their own children. This is not an unusual assumption as some degree of diminishing altruistic behavior seems realistic. Finally, we have considered technological progress to be exogenous. This does not put into question the fact that capping pollution has an impact on population, and even a significant one as we have shown, even if technological progress were endogenous.

We need to make sure as much as possible that pollution control does not take place at the costs of the current least well off or at the cost of those in the future. The natalist bias we identified is worrying in the latter respect. One may then want to address it in two main ways. As was suggested in the introduction, one could adopt an allocation rule of pollution endowments relying on some form of emission grandfathering. It would be such that those deciding to increase their population would not receive extra emission quotas at the next period. Besides the fairness concerns that this would raise, it may, however, not be enough to mitigate the substitution effect to a significant degree. Alternatively, population could be capped directly through a separate scheme, be it in the absence of or as a complement to the pollution capping scheme. Further research is needed to assess the impact of using population and pollution capping schemes either alternatively or complementarily.10

Acknowledgments

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Appendix A. Dynamics in the small open economy

The dynamics to characterize are given by:

$$k_{t+1} = \tau \left( \frac{\eta \delta}{1 - \eta} \right)^{\frac{\eta}{\eta - 1}} N^* \sigma k_t^{1-\eta} (1-ap)^{\eta}$$

9 One possibility would be for example to model the impact of emissions on agricultural productivity through average temperature increase.
10 Another possible extension of the model would consist in adding income heterogeneity among households within generations. This would allow us to study the distributive impact of the substitution effect.
\[ N_{t+1} = \frac{\gamma(1-\eta)}{(1+\delta+\gamma)p} N_t^{1-\eta} \frac{(1-\alpha p)k_t + pq}{(1-\alpha p)k_t} \]

Linearizing the dynamic system around the steady state \((\bar{K}, \bar{N})\) and using the steady state relationships leads to the following Jacobian matrix:

\[
\begin{bmatrix}
\eta + \nu & -p^2 \alpha^2 N^{-2} \frac{1}{(1+\gamma+\delta)p} \\
\bar{N}^{-1} \bar{N}^{-2} \frac{1}{(1+\gamma+\delta)p} & \frac{-p^2 \alpha^2 N^{-2} \frac{1}{(1+\gamma+\delta)p}}{1-\alpha}
\end{bmatrix}
\]

The determinant of this matrix is

\[-\frac{\alpha^2 N^{-2}}{(1+\gamma+\delta)p} + \eta - \nu + \nu \]

It is smaller than one and increasing in \(\bar{N}\). We need to show that it is larger than \(-1\) to establish local stability. For a steady state not too far from the one in the benchmark model

\[ \left( \frac{\gamma(1-\eta)}{(1+\delta+\gamma)p} \right)^{1/\alpha} \]

(Eq. (13)), the determinant is

\[-\frac{\alpha^2 N^{-2}}{(1+\gamma+\delta)p} + \eta - \nu + \nu = (\eta + \nu)(1-\alpha) \in (0,1)\]

Hence, for a larger value of steady state population, the determinant is also \(\in (0,1)\).

The trace of the Jacobian matrix is

\[ 1-\alpha + \eta + \nu \in (0,2) \]

Hence, the two eigenvalues are positive and smaller than one, and the steady state is locally stable.

### Appendix B. Dynamics in the global economy

The dynamics to characterize are given by

\[ k_{t+1} = \tau k_t^{1-\eta} \left( \frac{\eta \phi \bar{N} E^* \gamma}{(N_t k_t - E^*)(1-\eta)(1+\gamma \eta)} \right)^{\gamma} \]

\[ N_{t+1} = \frac{\gamma(1-\eta)N_t k_t - E^*)}{\phi \bar{N}^2 k_t (\gamma + \delta - \gamma \eta)} \]

To analyze these dynamics let us first look for steady states. Solving the last equation for \(k\) at steady state leads to:

\[ \bar{K} = \frac{\gamma(1-\eta)E^*/N}{\gamma(1-\eta) - \phi \bar{N}^2 (\gamma + \delta - \gamma \eta)} \]

Replacing \(k_{t+1}\) and \(k_t\) by this value in the first dynamic equation, we find:

\[ \tau \left( \frac{\bar{N}}{E^*} \right)^{1-\eta} \left( \frac{\eta \phi \bar{N} E^* \gamma}{(\gamma + \delta - \gamma \eta)} \right)^{\gamma} = \left( \frac{\gamma(1-\eta)}{\gamma(1-\eta) - \phi \bar{N}^2 (\gamma + \delta - \gamma \eta)} \right)^{1-x} \] (23)

This equation cannot be solved explicitly for \(\bar{N}\). Let us rewrite this equality as

\[ \Psi_1(E^*, \bar{N}) = \Psi_2(\bar{N}) \]

Fig. 6 represents these two functions. The left hand side \(\Psi_1\) is an increasing and concave function of \(\bar{N}\), starting from 0 when \(\bar{N} = 0\) and going to infinity as \(\bar{N} \to \infty\). The right hand side \(\Psi_2\) is an increasing and convex function of \(\bar{N}\), starting from 1 when \(\bar{N} = 0\) and going to infinity as \(\bar{N} \to \infty\) (vertical asymptote), with

\[ \bar{N} = \left( \frac{\gamma(1-\eta)}{\phi \gamma (\gamma + \delta - \gamma \eta)} \right)^{1/\alpha} \]

Hence, given the characteristics of the two functions, there are either two, one or no steady state, depending on the stringency of the cap \(E^*\).

We can show that, when the cap \(E^*\) is set at its most stringent and yet non-binding level, i.e., such that \(p=0\) and \(q=q\), the steady state is unique. Indeed, in that case,

\[ E = Nk \frac{1+\gamma \eta}{1+\delta + \gamma} \]
Eq. (23) would be, in that case,
\[
(1 + \delta + \gamma) \left( \frac{1 + \delta + \gamma}{1 + \gamma} \right) \left( \frac{1 + \delta + \gamma}{1 + \delta + \gamma} \right)^{-\eta/(1-\gamma)} \left( \frac{1 + \gamma}{1 + \gamma} \right)^{-\eta} = \frac{1}{1 + \gamma} \left( \frac{1 + \gamma}{1 + \delta + \gamma} \right)^{1-\eta}
\]
which simplifies into
\[
\frac{1 + \delta + \gamma}{1 + \gamma} = \frac{\gamma(1-\eta)}{\gamma(1-\eta) - \phi \bar{N}^2 (\gamma - \delta - \gamma \eta)}
\]
and
\[
\bar{N} = \left( \frac{\gamma(1-\eta)}{1 + \delta + \gamma \eta} \right)^{1/2} = \bar{N}
\]
is the only solution to this equality. \(\bar{N}\) and \(\bar{F}\) take their value as in the benchmark model without pollution cap.

Making the pollution cap \(E^*\) marginally more stringent shifts the \(\Psi_1\) function upward. As a result, for any binding pollution cap, we end up with two possible steady state equilibria, respectively one with a larger population than \(\bar{N}\) and one with a smaller. A further step is needed to identify a stable steady state and demonstrate the pro-natalist effect of lowering \(E^*\).

Linearizing the dynamic system around the steady state leads to the following Jacobian matrix:
\[
\begin{bmatrix}
\eta + v - \frac{\gamma(1-\eta)}{(1 + \delta + \gamma \eta) \phi \bar{N}} \\
\frac{\gamma(1-\eta)}{(1 + \delta + \gamma \eta) \phi \bar{N}} \\
\frac{\gamma(1-\eta)}{(1 + \delta + \gamma \eta) \phi \bar{N}} \\
\end{bmatrix}
\begin{bmatrix}
E^* \gamma(1-\eta) \left( \frac{1 + \eta}{1 + \delta + \gamma \eta} \phi \bar{N} \right) \\
\frac{E^* \gamma(1-\eta)}{(1 + \delta + \gamma \eta) \phi \bar{N}} \\
\frac{E^* \gamma(1-\eta)}{(1 + \delta + \gamma \eta) \phi \bar{N}}
\end{bmatrix} = \bar{N}
\]
The determinant of this matrix is
\[
\frac{\gamma(1-\eta)^2}{(\delta + \gamma - \eta \gamma) \phi \bar{N}} - 2v
\]
It is decreasing in \(\bar{N}\). Its trace is
\[
\frac{\gamma(1-\eta)^2}{(\delta + \gamma - \eta \gamma) \phi \bar{N}} - 2 + \eta + v
\]
also decreasing in \(\bar{N}\).

For the steady state close to \(\bar{N}\), the determinant has a value close to \(\nu(1-\nu)\) and a trace close to \(1-\nu+\nu\). It is therefore locally stable.
If $E^*$ is restrictive enough, the low population steady state has a population close to zero, and the high population steady state has a population close to the value of the vertical asymptote $N$. The low population steady state is increasing in $E^*$, the high population steady state, which is locally stable, is decreasing in $E^*$. The latter result proves Proposition 2.

References