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Easter Island's collapse: a tale of a population race

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Abstract The Easter Island tragedy has become an allegory for ecological catastrophe and a warning for the future. In the economic literature the collapse is usually attributed to irrational or myopic behavior in the context of a fragile ecosystem. In this paper we propose an alternative story involving non-cooperative bargaining between clans to share the crop. Each clan's bargaining power depends on its threat level when fighting a war. The biggest group has the highest probability of winning. A clan's fertility is determined *ex ante* by each group. In the quest for greater bargaining power, each clan's optimal size depends on that of the other clan, and a population race follows. This race may exhaust the natural resources and lead to the ultimate collapse of the society. In addition to well-known natural factors, the likelihood of a collapse turns out to be greater when the cost of war is low, the probability of succeeding in war is highly responsive to the number of fighters, and the marginal return to labor is high. We analyze whether these factors can account for the difference between Easter and Tikopia Islands. The paper also makes a methodological contribution in that it is the first fertility model to include strategic complementarities between groups' fertility decisions.

Keywords Fertility · War · Bargaining power · Collapse · Natural resources

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1 Introduction

Easter Island had a rising population and a prosperous civilization until about the XVth century, after which it declined sharply. The most popular sign of that past glory is *moai*, the enormous statues carved in stone and erected all over the island by the inhabitants. By the XVIIIth century, when the island was discovered by European explorers, the population had been decimated. Easter Island is therefore an example of how a closed system can collapse, but what exactly happened is still an unresolved matter.

Tikopia is another small island in the Pacific Ocean that provides an interesting contrast. It is similar to Easter Island in many ways, but no collapse occurred and a long-lasting society with a relatively stable population evolved. The population reached 1200 in the XIth century and stayed approximately constant from then on; the environment remains well preserved. Tikopians managed to control population growth (unlike Easter Islanders), thus avoiding the over-exploitation of resources.

Why did Tikopia and Easter Island differ so much, and experience such diverging patterns? In order to identify the central factors which may have led Easter Islanders along the path to collapse, we explore a model where there is interaction among island clans. The expected payoffs turn out to be affected by the relative size of the groups, thus making fertility decision a strategic choice. In the absence of strong property rights, the clans have to agree how to share the total crop, and this involves non-cooperative bargaining. Bargaining power is affected by the threat of war. We assume that the probability of winning a war depends on the relative size of the group. This is quite a plausible assumption since at that time war-technology was very human-intensive, and we have no reason to believe that the groups differed substantially in their war-technology or any input other than the number of people able to fight. Under certain conditions (which we will identify) there will be a population race, where each group tries to increase its size in order to maintain its bargaining power. Such a population race is likely to have brought about over-population and an over-exploitation of resources, even though it emerged as the equilibrium of a 'rational game'.

So far, the economic literature on Easter Island has usually assumed that the dynamics of population growth is mechanistic, following a Malthusian approach according to which fertility is regulated by nutrition. In such a world, the birth rate is increasing with material living standards, while material living standards and natural resources decline as population increase.[1] Highly-myopic behavior is often postulated to explain what, from an external viewpoint, looks like irrational over-exploitation of resources leading to the collapse of the society. Here, however, we look at endogenous population dynamics when fertility is a strategic choice for groups. Hence this paper also has a methodological contribution in providing the first fertility model with strategic complementarity between fertility decisions across groups.[2]

The paper is organized as follows: in Sect. 2 the main archeological evidence and the literature on the collapse of small societies are reviewed. In Sect. 3 the model is described and studied analytically. Sect. 4 provides numerical simulations of the model, both in a static and in a dynamic framework. In Sect. 5 we discuss the role of different factors on Easter Island and Tikopia. Sect. 6 concludes with some final remarks.

[1] Clark (2007)'s first chapters contain a rich description of how this Malthusian logic applies to all pre-1800 societies.

[2] In his conclusion, Lagerloef (2006) evokes a mechanism close to ours, suggesting that if population size is an input in land acquisition, clans have an extra motive for high fertility. Good and Reuveny (2007) mention that in some episodes population growth was enhanced by the need for a large army.

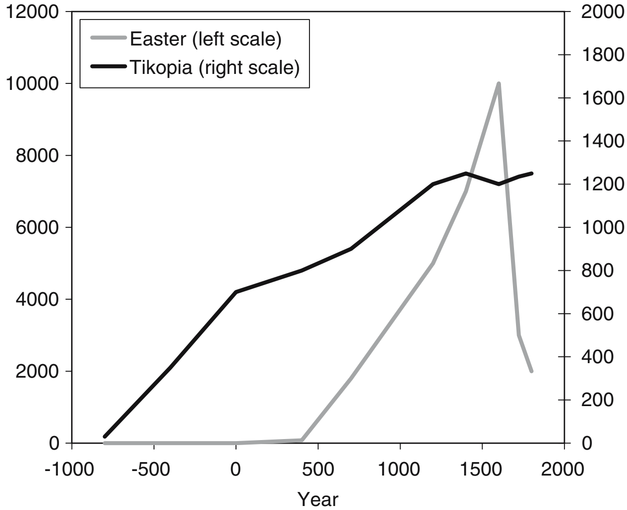

Fig. 1 Population of Easter Island and Tikopia

2 Historical evidence and literature

2.1 Data

Data on Easter Island are available from archaeological studies,[3] but they generally lack precision. The timing of events is also far from certain, and sources disagree to a considerable extent. The best-accredited theory is that less than 100 people arrived on Easter Island from the Marquesas Islands around CE 400. Thereafter the population began to increase; however according to Cohen (1995) the population remained relatively low until CE 1100, when the increase accelerated. The peak was probably reached around CE 1400–1600 at over 10,000 (perhaps as high as 20,000, see Reuveny and Maxwell (2001), Brander and Taylor (1998)). Figure 1 displays the likely evolution of population over time.

In the XVIth century there was a decline of food consumption with likely episodes of cannibalism around CE 1600 and a big population crash during the XVIIth century. A new religion and a new political order settled down after mid 1600. When Easter Island was discovered by Europeans in CE 1722, estimated population amounted to 3000. At the end of the XVIIIth century a reliable estimate is of about 2000. Since the XIXth many exogenous shocks happened (kidnapping for slave market, epidemics due to foreign germs, etc.) and the island can no longer be considered as a closed system.

As far as natural resources are concerned, island forest was at its carrying capacity when the first settlers arrived. Cutting trees would have begun almost immediately in order to have firewood, land for agriculture, and to make canoes. The process of deforestation was reinforced during the *moai*-construction period since trees were cut to facilitate the transportation of the statues. The rate of deforestation reached its peak around CE 1400, and the forest clearance was likely completed by CE 1600. In CE 1722 when the Island was discovered by Europeans there was basically no tree. The XVth century is rated to be the period of maximum deforestation. Figure 2 illustrates the deforestation process by

[3] For more details see Flenley and Bahn (2003), Keegan (1993), Ponting (1991).

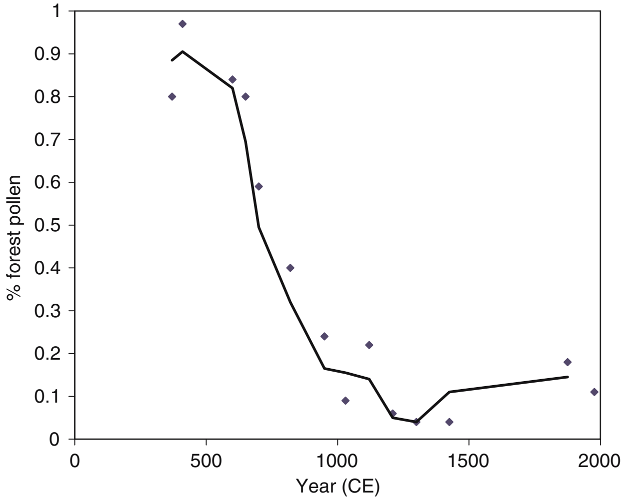

Fig. 2 Forest coverage on Easter Island

reporting the occurrence of forest pollen as a percentage of total pollen over time (from Flenley et al. 1991). The deforestation had a clear impact on human life, having "led to leaching and soil erosion, more wind damage, increased soil evaporation and a reduction in crop yields." (Flenley and Bahn 2003, pp. 191–192).

We have also some data on *moai*-carving from Van Tilburg and Mach (1995). There were 887 statues on Easter Island weighting on average 13.78 tons (that is the number of known statues; there are certainly more buried in sediments). The largest *moai* once erected weights 82 tons. They were constructed between the XIIth and the XVIth century. The fact that *moai*-carving ceased around CE 1600 is confirmed by Dalton and Coats, when "the tools of the artisan were replaced with tools of war, such as the *mataa* (apparently a dagger or spearhead)".[4]

Tikopia is located in the Pacific Ocean, in the far east of the Solomon Islands; culturally and linguistically its people are of Polynesian stock.[5] A small group of Polynesians likely from the East settled on Tikopia about BCE 900 and lived by slash and burn agriculture. Around BCE 100 the economy began to change: "initially high yields from the wild resources of land and sea were seriously reduced by the pressure of harvesting"[6] and the people began to rear pigs intensively to compensate for the drastic decline in birds and seafood. After a second wave of immigration from West Polynesia, the population reached 1200 and then remained roughly constant (see Fig. 1). An important decision was taken around the late XVIIth century: to eliminate pigs, which were becoming too costly to feed in terms of resources.

Over-population was avoided by somewhat conscious mechanisms (Kirch 1997). Several practices can be related to such a purpose: celibacy, prevention of conception, abortion, infanticide, sea voyaging by young males, expulsion of some segment of population, etc. Firth (1967) reports the existence of a traditional ritual cycle ("the work of the gods") which

4 Dalton and Coats (2000) p. 491.

5 For an in-depth study of Tikopian society see Firth (1936), Kirch (1986) and Kirch and Yen (1982).

6 Kirch and Yen (1982) p. 354.

encoded the ideology of zero-population growth.[7] Kirch and Yen (1982) refers to "a range of social and ritual controls [that] served to maintain population levels" (p. 355).

2.2 Literature

The collapse of Easter Island's civilization has fascinated historians, sociologists, anthropologists, biologists and—since the seminal article by Brander and Taylor (1998)—economists. The literature can be broadly divided into two groups: one is based on the dynamic properties of interactions between the laws of motion of renewable resources and exploiters, an idea that goes back to the Lotka-Volterra predator/prey model. The other explicitly assesses the possibility of conflict among groups over resources.

In the first group, the article by Brander and Taylor (1998) explains Easter Island's collapse in a Malthusian-Ricardian fashion: the fertility rate increases with the use of renewable resource (i.e. harvesting), while the increased exploitation of resources necessary to feed people encounters decreasing returns. In this set-up damped cyclical behavior may occur, as well as monotonic convergence. Calibrating parameters in Easter Island's case, Brander and Taylor simulate a bell-shape for the changes in the population.[8]

However historical and archaeological studies indicate a sharper decline in population than that predicted by Brander and Taylor. Moreover archaeological studies provide evidence of episodes of cannibalism, movements of islanders into caves and fortified dwellings, fights, the felling of statues, construction of weapons, etc., all of which suggests a sharp and dramatic decline in the civilization. Anderies (2000) and Pezzey and Anderies (2003) introduce a minimum subsistence requirement into Brander and Taylor's model. This allows the implausible assumption of a constant division of time between manufacturing and harvesting whatever the availability of resources to be avoided; in the revised model people stop manufacturing and devote themselves to harvesting when resources become scarce enough, although this means that the remaining resources are depleted faster. Some possible institutional adaptations to prevent collapse, such as an *ad-valorem* tax and quotas on harvesting are discussed. Erickson and Gowdy (2000) obtain a similar asymmetric decline by introducing capital accumulation of manufactured goods and assuming some substitution between natural and artificial stocks. In this way fertility reacts more slowly initially, because people could use manufactured capital as a partial substitute for natural resources. However at some point the poverty of resources becomes such that this substitution is no longer possible and the population starts to decline abruptly. Tikopia Island is presented as a differently parameterized example, where different institutional habits (i.e. more careful attention to population growth) reduced the natural fertility rate and the rate at which stocks depreciated.

D'Alessandro (2007) assumes there exists a certain threshold for the stock of renewable resources below which there is an inexorable tendency to exhaustion that implies an irreversible collapse. Basener and Ross (2005) adopt a logistic growth rate for population dynamics, as well as for resource dynamics. This produces a sudden decrease in population when the total population comes close to the carrying capacity of the eco-system (which depends on the current stock of resources). The role of fertility is highlighted by Reuveny and Decker (2000), who study the effects of different fertility rates on the feast-famine cycle, finding that higher fertility is associated with sharper rises and falls in population. They also examine the effects of technological progress: if technology affects harvesting it tends to exacerbate

7 Kirch (1997) rates the text of this ritual as "a remarkable document, unambiguously reflecting the Tikopia ideology of zero-population growth" (p. 40).

8 A discrete version of Brander and Taylor's model is provided by Matsumoto (2002) who shows how rich dynamics may arise also in such a framework.

the feast-famine cycle whereas progress affecting the resource stock tends to moderate the cycle.[9]

Finally, Harford (2000) introduces endogenous fertility in the context of renewable resource management, treating the number of children as an optimizing decision. He shows that, in this setting, individuals decide to have children on the basis of a trade off between increasing their own dynasty's utility and devoting part of their income to child rearing, but the cost for the whole society of a greater pressure on the natural resource stock is not internalized. The focus of Harford's paper is in fact on mechanisms to contrast 'the tragedy of the commons' (e.g. Pigouvian tax or individual transferable quotas) and not on societal collapses such as that on Easter Island. The fertility choice is driven by bequest motives, and is not strategic.

The other stream of literature deals with conflicting groups. It stems from the evidence of conflicting and competing groups on Easter Island. Being organized in chiefdoms was a common feature of basically all Polynesian societies (Kirch 1997). As Flenley and Bahn (2003) report, Easter Island clans were governed by religious-military chiefs who preserved their power through prestige and awe gained by erecting *moai*.[10] Property rights were not enforced so that crop sharing was ruled by other institutions, basically bargaining between the chiefs of the various groups.

Reuveny and Maxwell (2001) suggest that two clans may conflict in order to appropriate the total harvest. Each group has to choose its strategy for allocating the group's members between harvesting and conflict, so as to maximize the group's share of the yield, taking into account the choice of the other group. A greater effort in conflict will increase the group's relative share, but the total crop will be negatively affected by the conflict. Population growth in both groups is assumed to increase with nutrition and decrease with conflict, but its laws of motion are not internalized. Since the only objective is to maximize current income and the clans are myopic with respect to their growth in population, there is no strategy involving fertility choice. Reuveny and Maxwell argue that this shortsightedness might be appropriate for a primitive society, but there is evidence that Easter Islanders were neither primitive nor completely short-sighted. Think, for example, about building enormous statues: it involves constructing something to be impressive and intimidating for a long time; in the short term it is costly and useless. Rainbird (2002) suggests that Easter Islanders "were not mere unthinking pawns in an environment where they had little choice, but, instead, decision making with the ability and knowledge to manipulate extreme environments for their own ends." Moreover, Tikopians, who were unlikely to have been more developed than Easter Islanders, had a long-term perspective. Even if Easter Island clans were myopic in the long run, they could well have had a multi-period view and the perception that their relative size would have provided them with an advantage in bargaining power.

In a more recent paper Maxwell and Reuveny (2005) compare a non-cooperative outcome with a cooperative one, where groups are not involved in any conflict. They find that the interior steady state is characterized by higher per capita income and higher resource stocks with conflict than without. The intuitive reasoning behind this finding is that when there is no

9 Dalton et al. (2005) also study how technological change could have altered the results if Easter Island is used as a metaphor for managing resources in a closed system. The fertility rate still increases with nutrition, but now the feast and famine cycle can be attenuated by technological improvements, encouraged by demographic pressure.

10 When resources became scarcer and social discontent started to spread, the chiefs reacted by trying to increase their prestige in order to keep power. Bigger and bigger *moai* were built until misery and poverty gave rise to social rebelion and new military leaders ousted the original chiefs. A new religion was established, confirming that great upheavals accompanied the collapse.

conflict more resources are devoted to harvesting, thus depleting the global stock of resources faster. If conflict comes at a price (for instance, by reducing the amount of natural resources) or if agents behave less short-sightedly, the cooperative setting could bring about a Pareto improvement.

Prskawetz et al. (2003) use a framework similar to Maxwell and Reuveny (2005), but they also account for the possible harmful effects of conflict on death rates and the growth rate of resources and perform a local bifurcation analysis, focused in particular on the parameters related to such feedback effects. They find that sustainability requires stabilizing feedback mechanisms to become active at an early stage. In their model conflict is postulated to arise when the natural resource stock per capita falls below some threshold, and so it does not explicitly result in a strategic equilibrium. A model of rational conflict is provided by Lasserre and Souberyan (2003) who consider the conditions under which groups decide how to allocate time between production and conflict in a tragedy of the commons context and whether to specialize in one or other activity. They find that multiple equilibria may arise, both interior and at the extremes. In their model, however, population growth is not analyzed.

3 The model

In this section we will develop a stylized model to show how the main mechanism we put forward can explain an environmental collapse. We consider an overlapping generations framework where agents live for two periods. Every agent belongs to a clan. In each period, the timing of decisions is as follows. Each clan chooses its fertility rate, taking the other's fertility rate as given, and having perfect foresight. A Nash(-Cournot) equilibrium in fertility rates follows, characterized by a match between the strategy actually played and the strategy expected by the other group. The crop is then shared, following a non-cooperative bargaining process between clans.

3.1 Preferences and technology

Clans were organized along the lines of a genealogical trees. Clan members have a common ancestor, often a son or grandson of the traditional original settler of the island, see Sahlins (1955). For the sake of simplicity we assume that there are only two clans, and that all individuals belong either to one or the other clan, with no possibility of changing clan membership.

Group i at time t consists of $N_{i,t}$ adults. In the first period the adults are young, they work, they support their parents' consumption and they make fertility choices; in the second part of their life they consume what their children provide for them. A representative adult belonging to group i receives utility from both periods of consumption and bears a disutility cost from child rearing. We assume that households are risk neutral, i.e. preferences are represented by a linear utility function:[11]

$$U_{i,t} = c_{i,t} + \beta d_{i,t+1} - \lambda n_{i,t} \tag{1}$$

[11] Introducing small risk aversion turns out to benefit the clan with higher military power, thus enhancing the strategic motive for fertility. For higher degree of risk aversion numerical analysis shows a non-monotonic effect on fertility, since at some point clans are both so averse to the uncertain outcome of war that a greater military power is no more effective as a threat. For moderate risk aversion (as in the log-utility case) the effects on fertility are close to those predicted by the model. All results with positive risk aversion are in appendix which is available from the authors upon request.

where $c_{i,t}$, and $d_{i,t+1}$ are the first and second period consumption respectively, $n_{i,t}$ is the number of children per adult in Group i at time t, $\beta > 0$ is the psychological discount factor, and $\lambda > 0$ is the marginal disutility of child rearing.

In order to ensure marginal decreasing benefits to procreation, we assume that supporting parents costs a fraction $\tau/(1 + n_{i,t-1})$ of young adults' total income $y_{i,t}$. To simplify we assume that $\tau \in (0, 1)$ results from an exogenous social norm.[12] Hence the parent receives a total of

$$\tau \frac{n_{i,t-1}}{(1 + n_{i,t-1})} y_{i,t}$$

This formulation introduces the idea that each additional child will bring some additional resources to the parents, but the returns are not constant, since the fraction each child pays decreases with the number of siblings. The chosen formulation makes the support from children a concave function of the number of children. The assumption of decreasing returns in support from children is important to avoid the possibility of offsetting decreasing returns in production just by increasing fertility.[13] Therefore the budget constraint in the first period is:

$$c_{i,t} = \left(1 - \frac{\tau}{1 + n_{i,t-1}}\right) y_{i,t} \tag{2}$$

In the second part of their life, agents are economically supported by their children. Hence the budget constraint is

$$d_{i,t+1} = n_{i,t} \frac{\tau}{1 + n_{i,t}} y_{i,t+1} \tag{3}$$

The population evolves for each group according to the law of motion

$$N_{i,t+1} = n_{i,t} N_{i,t} \tag{4}$$

In each period, crop Y_t is obtained according to the production function

$$Y_t = A(R_t) L^{\alpha} (N_{1,t} + N_{2,t})^{1-\alpha}$$

where L is the fixed amount of land,[14] and $A(R_t)$ is the TFP which depends on the stock of natural resources R_t available at time t. The function $A()$ may reflect exogenous conditions determining differences between productivity on Tikopia and Easter Island.

The dynamics of the stock of resources is taken from Matsumoto (2002) as:

$$R_{t+1} = (1 + \delta - \delta R_t/K - b(N_{1,t} + N_{2,t}))R_t, \quad R_0 > 0 \text{ given} \tag{5}$$

where $K > 0$ is the carrying capacity, i.e. the maximum possible size of the stock of resources, $\delta > 0$ is the intrinsic growth rate of natural resources, and $b > 0$ is a coefficient weighting the effect of human absorption of resources. Equation 5 results from the difference between a logistic 'natural' growth rate and a negative human impact on resources.[15] It has the following

[12] In the literature on old-age support (Ehrlich and Lui 1991), the compensation rate τ is determined through self-enforcing implicit contracts. At equilibrium, the value of τ typically depends on longevity and the availability of capital markets.

[13] The functional form in the text is chosen for analytical tractability.

[14] Since the amount of land is fixed we can set its value at 1 without loss of generality.

[15] An analogous specification in continuous time is common to almost the whole literature reviewed in Sect. 2 (see, for instance, Brander and Taylor (1998), Pezzey and Anderies (2003), Dalton and Coats (2000) and Reuveny and Maxwell (2001)).

property:

$$\lim_{t\to\infty} N_{1,t} + N_{2,t} = 0 \quad \Rightarrow \quad \lim_{t\to\infty} R_t = K.$$

Let us denote by θ Group 1's share of the total crop. The representative adult's income can be written as a function of his or her clan's share as:

$$\begin{aligned} y_{1,t} &= \theta_t Y_t / N_{1,t}; \\ y_{2,t} &= (1-\theta_t) Y_t / N_{2,t}. \end{aligned}$$

In the absence of strong property rights and input markets, groups bargain to reach an agreement on crop-sharing: if they did not a war could take place. The bargaining power of each group is affected by its relative size. Therefore under perfect foresight each group has an incentive to increase its fertility in order to increase its threat level in the future. Before studying the bargaining problem analytically let us discuss the two main assumptions on which the mechanism we propose is built.

First, fertility is fixed before the bargaining begins, as a social norm. It may be questioned whether it is not too extreme to assume that clans control their members' fertility. In our framework there is no division between the subjects who are involved in bargaining (the clans) and those who actually have children (all agents are perfectly equal within the clans). In reality the number of children could in fact have been to some extent a social norm: Erickson and Gowdy (2000) refer to archaeological evidence for Tikopia in support of "the adoption of cultural beliefs that incorporated the ethic of zero population growth";[16] Diamond (2005) reports the highly hierarchical and centralized structure of clans on Easter Island.[17]

Second, the probability of success in war is ultimately determined by the relative number of individuals able to fight, i.e. young people: when two groups conflict, using basically the same labor-intensive war technology, it is plausible to expect that the bigger group is more likely to win.[18] There is good evidence for episodes of conflict on Easter Island (Ponting 1991; Keegan 1993), but Diamond (2005) also reports clashes on Tikopia.

3.2 The bargaining problem

The outcome of non-cooperative bargaining can be modeled by maximizing a Nash product (Binmore et al. 1986):

$$(U_{1,t} - \bar{U}_{1,t})^{\gamma} (U_{2,t} - \bar{U}_{2,t})^{1-\gamma}$$

where $U_{i,t}$ is the pay-off of Group i, $\bar{U}_{i,t}$ is the fall-back pay-off for Group i if no agreement is reached, and $\gamma \in (0, 1)$ is a parameter for the possible residual asymmetries in bargaining power. The parameter γ may represent exogenous asymmetries in bargaining positions, for

16 See Erickson and Gowdy (2000) pp. 350, 351. The 'fono', an annual address by Tikopian chiefs, is described as a proclamation encoding the idea of zero population growth. Section 2 mentions several indications of conscious control over population growth. For further evidence of control over population growth in Tikopia see Borrie et al. (1957).

17 The chief was a religious and military leader with great power, who needed to retain his prestige through successful competition with other clans.

18 Reuveny and Maxwell (2001), Lasserre and Souberyan (2003), Maxwell and Reuveny (2005) and Prskawetz et al. (2003) all consider a labor-intensive war technology.

instance the quality of the land occupied by each group,[19] or the religious authority of the clan's leader.

Using Eqs. 1–3, the indirect lifetime utilities of a representative member of the current young generation at time t are respectively:

$$U_{1,t} = \left(1 - \frac{\tau}{1+n_{1,t-1}}\right)\frac{\theta_t Y_t}{N_{1,t}} + \beta\frac{n_{1,t}\tau}{1+n_{1,t}}\frac{\theta_{t+1}Y_{t+1}}{N_{1,t+1}} - \lambda n_{1,t};$$
$$U_{2,t} = \left(1 - \frac{\tau}{1+n_{2,t-1}}\right)\frac{(1-\theta_t)Y_t}{N_{2,t}} + \beta\frac{n_{2,t}\tau}{1+n_{2,t}}\frac{(1-\theta_{t+1})Y_{t+1}}{N_{2,t+1}} - \lambda n_{2,t}.$$

The fall back pay-off $\bar{U}_{i,t}$ is represented by the utility of a failed agreement. If the parties do not come to an arrangement they fight. In other words we can see the emergence of war as a threat to induce compliance. When a war happens the income is an expected value, since it depends on the probability of the group of winning the war. We allow for the possibility that, during the war, a fraction $\omega \in [0, 1)$ of the total crop is wasted. This is likely to occur because fighting often brings about a depletion of resources, or missed production; at the very least the resources are consumed in fighting and so do not provide the usual utility. Moreover, we assume that when a clan wins a war, it appropriates the total crop and that war entails no human loss.[20]

We denote the probability of Group 1 winning the war by π_t. We assume that the form of the function $\pi_t = p(N_{1,t}, N_{2,t})$ relating the winning probability to group sizes satisfies the properties specified in an axiomatic approach by Skaperdas (1996) for contest success functions:

- being positive, between zero and one, and the sum of all groups' probabilities adding up to one (conditions for a probability distribution function);
- being increasing with the groups' own contest effort, and decreasing with the other groups' efforts;
- being anonymous, in the sense that the outcome only depends on the relative efforts and if every group makes the same effort in the conflict then their probability of winning is equal;
- contests among smaller numbers of groups are qualitatively similar to contests among a large number of groups;[21]
- homogeneity, in the sense that multiplying each effort by the same amount does not affect the probability of winning.

Skaperdas (1996) shows that

$$p(N_{1,t}, N_{2,t}) = \frac{N_{1,t}^{\mu}}{N_{1,t}^{\mu} + N_{2,t}^{\mu}} \in (0, 1) \tag{6}$$

[19] Easter Island land was divided among clans in a radial way, but the quality of land was not uniform. Nevertheless there is no reason to believe that these differences alone enabled one group to achieve a clear and definitive success in taking possession of the crop.

[20] This assumption allows the extreme possibility of the extermination of a group (which was probably not the intention of the clans and was also unrealistic in such societies, considering the available weapons and war technology) to be avoided. There is evidence (see Owsley et al. (1994)) that struggles very seldom entailed mortal wounds, given the kind of weapons used, and few fatalities were attributable directly to violence. Hence we assume, like Maxwell and Reuveny (2005), that war does not imply a cost in terms of human life. Since war plays the role of a fall-back outcome that never occurs in the bargaining equilibrium, permitting human loss would simply alter the bargaining power quantitatively without adding substantive insights.

[21] With only two groups this property is trivial, but it can be proved to hold if a larger number of groups is involved.

is the only continuous function to satisfy all these properties. The parameter $\mu \geq 1$ is related to the sensitivity of the probability of winning a war to the size of the clan. It has to be greater than or equal to 1 to guarantee the convexity of the maximization problem. The higher μ, the greater the influence of the size of the group, so that μ captures the decisiveness of groups' relative size. The main implication of this is that the ratio of winning probabilities of any two players depends on the ratio of their efforts.[22] An increase in μ has a positive impact on π_t if and only if $N_{1,t} > N_{2,t}$. Here we have:

$$\frac{\partial p}{\partial \mu} = \frac{\mu N_{1,t}^{\mu} N_{2,t}^{\mu}}{(N_{1,t}^{\mu} + N_{2,t}^{\mu})^2} \ln\left(\frac{N_{1,t}}{N_{2,t}}\right).$$

The fall back utility of Group 1 can be written as:

$$\bar{U}_{1,t} = \pi_t \left[\left(1 - \frac{\tau}{1+n_{1,t-1}}\right) \frac{(1-\omega)Y_t}{N_{1,t}}\right] + \beta \frac{n_{1,t}\tau}{1+n_{1,t}} \frac{\theta_{t+1}Y_{t+1}}{N_{1,t+1}} - \lambda n_{1,t}.$$

Notice that fertility has already been chosen when groups bargain, therefore the disutility of child-rearing is not affected by the outcome of the bargaining. Second period consumption depends on expectations about the next period agreement (θ_{t+1}) and output (Y_{t+1}). Perfect foresight is assumed. Notice that the outcome of the bargaining today has no influence on the bargaining tomorrow. The utility net of fall-back is:

$$\begin{aligned} U_{1,t} - \bar{U}_{1,t} &= \left(1 - \frac{\tau}{1+n_{1,t-1}}\right)\left(\theta_t - \pi_t(1-\omega)\right)\frac{Y_t}{N_{1,t}}; \\ U_{2,t} - \bar{U}_{2,t} &= \left(1 - \frac{\tau}{1+n_{2,t-1}}\right)\left(1 - \theta_t - (1-\pi_t)(1-\omega)\right)\frac{Y_t}{N_{2,t}}. \end{aligned}$$

Abstracting from multiplicative constant terms, the net utilities become

$$\begin{aligned} U_{1,t} - \bar{U}_{1,t} &= \theta_t - \pi_t(1-\omega); \\ U_{2,t} - \bar{U}_{2,t} &= 1 - \theta_t - (1-\pi_t)(1-\omega). \end{aligned}$$

Using these values the bargaining problem simplifies to:

$$\theta_t \equiv \arg\max \left[\theta_t - \pi_t(1-\omega)\right]^{\gamma} \left[1 - \theta_t - (1-\pi_t)(1-\omega)\right]^{1-\gamma}. \tag{7}$$

Proposition 1 [Bargaining Outcome] *The bargaining share of the crop for Clan* 1 *is:*

$$\theta_t = \gamma\omega + \frac{N_{1,t}^{\mu}}{N_{1,t}^{\mu} + N_{2,t}^{\mu}}(1-\omega). \tag{8}$$

[22] An alternative functional form that is frequently used is the logistic: $\pi = 1/\exp[\mu(F_2 - F_1)]$ where F_i denotes the effort made by Group i. This formulation implies that it is the difference between efforts that determines the probability of success. Notice that if we take a logarithmic effort function ($F(N_i) = \ln(N_i)$) we still end up with Eq. 6. The issue of the linearity of effort to the number of combatants is not central to our model: taking a linear effort function and probability of success, or taking a logarithmic effort function and a logistic probability of success both yield Eq. 6, which turns out to be suitable for analytical study since it allows useful simplifications. Such simplifications are not possible if we consider a linear effort function and a logistic probability. Although Hirshleifer (1991) argues that such a form matches some empirical data on battles better, we prefer to retain the formulation in Eq. 6 for several reasons: as Skaperdas (1996) points out, such evidence is far from being conclusive; the homogeneity axiom is not satisfied; some outcomes may be counter-intuitive; and, last but not least, the possibility of in-depth analytical study would be complicated without providing additional insights.

The following comparative statics results are derived:

$$\frac{\partial \theta_t}{\partial \gamma} = \omega \geq 0;$$
$$\frac{\partial \theta_t}{\partial \omega} = \gamma - \pi_t \geq 0 \quad \text{iff } \pi_t \leq \gamma;$$
$$\frac{\partial \theta_t}{\partial \mu} = \frac{N_{1,t}^{\mu} N_{2,t}^{\mu} (1-\omega) \ln\left(\frac{N_{1,t}}{N_{2,t}}\right)}{(N_{1,t}^{\mu} + N_{2,t}^{\mu})^2} \geq 0 \quad \text{iff } N_{1,t} \geq N_{2,t}.$$

Proof The first order condition for Problem (7) yields Eq. 8. The second order condition for a maximum, $[-(1-\gamma)\gamma\omega^2]^{-1} < 0$, is satisfied. □

The effect of γ on θ_t is unambiguously positive: when it increases the asymmetries in bargaining change in favor of Clan 1 thus strengthening its bargaining position. An increase in the dead-weight loss ω reduces the fall back of both groups, but in an asymmetric way depending on the groups' relative size. As Eq. 8 shows, θ_t turns out to be a weighted sum of the exogenous contractual power γ and the endogenous force, due to the likelihood of winning the war, π_t; therefore an increase in the relative weight of the former increases θ_t if and only if $\gamma > \pi_t$. Finally an increase in the sensitivity of the probability of victory to the numbers of fighters involved determines the advantage of the group with the bigger size.

3.3 The fertility choice

Let us now turn to the choice of fertility made by adults at time t. The variable part of the utility, for Clans 1 and 2 respectively, is given by:

$$\frac{\beta\tau n_{1,t}}{1+n_{1,t}} \left[\gamma\omega + \frac{(N_{1,t}n_{1,t})^{\mu}(1-\omega)}{(N_{1,t}n_{1,t})^{\mu} + (N_{2,t}n_{2,t})^{\mu}}\right] \frac{A(R_{t+1})(N_{1,t}n_{1,t} + N_{2,t}n_{2,t})^{1-\alpha}}{N_{1,t}n_{1,t}} - \lambda n_{1,t}$$

and

$$\frac{\beta\tau n_{2,t}}{1+n_{2,t}} \left[1 - \gamma\omega - \frac{(N_{1,t}n_{1,t})^{\mu}(1-\omega)}{(N_{1,t}n_{1,t})^{\mu} + (N_{2,t}n_{2,t})^{\mu}}\right] \frac{A(R_{t+1})(N_{1,t}n_{1,t} + N_{2,t}n_{2,t})^{1-\alpha}}{N_{2,t}n_{2,t}} - \lambda n_{2,t}$$

At this point it is worth noting the channels through which endogenous fertility affects utility. There are two kinds of benefits from having more children: a higher old age support and a strategic motive which operates through an increase in the probability of succeeding in war, and hence through the clan's threat-value in bargaining. On the other hand there are two costs from having children: a direct disutility cost from rearing them, and greater pressure on crops in the next period, as the crops have to be divided among more people.

The first order conditions for Clans 1 and 2 are given by:

$$\frac{1-\omega}{1+n_{1,t}} \frac{\partial \pi_{t+1}}{\partial n_{1,t}} + \frac{(1-\alpha)N_{1,t}\theta_{t+1}}{(1+n_{1,t})(N_{1,t}n_{1,t} + N_{2,t}n_{2,t})} = \frac{\theta_{t+1}}{(1+n_{1,t})^2} + \frac{\lambda}{\beta\tau}\frac{N_{1,t}}{Y_{t+1}}; \quad (9)$$

$$-\frac{1-\omega}{1+n_{2,t}} \frac{\partial \pi_{t+1}}{\partial n_{2,t}} + \frac{(1-\alpha)N_{2,t}(1-\theta_{t+1})}{(1+n_{2,t})(N_{1,t}n_{1,t} + N_{2,t}n_{2,t})} = \frac{1-\theta_{t+1}}{(1+n_{2,t})^2} + \frac{\lambda}{\beta\tau}\frac{N_{2,t}}{Y_{t+1}} \quad (10)$$

where θ_{t+1} is obtained from Eq. 8, while from Eqs. 4 and 6 we have

$$\pi_{t+1} = \frac{(N_{1,t}n_{1,t})^{\mu}}{(N_{1,t}n_{1,t})^{\mu} + (N_{2,t}n_{2,t})^{\mu}}.$$

At optimum the marginal benefit from increasing fertility (the l.h.s. of Eqs. 9 and 10) equals the marginal cost (the r.h.s.). The benefit is represented by the increase in consumption in the second period, due to the larger clan's share and the larger total contribution of their offspring; the cost is represented by the reduction in crop per capita (due to decreasing marginal returns) and the cost of child rearing.

Each group's optimal fertility rate turns out to depend on expectations about the other group's fertility: in this sense we can talk of fertility reaction functions. We assume that when clans choose their own fertility rate they take the other group's one as given, i.e. they behave á la Cournot. A necessary condition for a population race to occur is that fertility reaction functions must have a positive slope, so that it is best for each group to respond to increases in the other group's fertility rate by having more children itself.

Although there is no explicit solution to Eqs. 9 and 10, we can characterize the solution for a particular parameter configuration as satisfying

Assumption 1 Parameters satisfy $\mu = 1$, $\omega = 0$, $\lambda = 0$, and $\alpha = 1$.

Assumption 1 tells us that π_t is equal to the proportion of fighters who belong to Group 1, there is no dead-weight loss with war and no disutility from child-rearing. Finally $\alpha = 1$ means that at every period the island yields a given amount of food depending solely on the stock of natural resources, e.g. coconuts. In this scenario Proposition 2 holds:

Proposition 2 *Under Assumption 1:*

- *the fertility reaction functions have positive slopes;*
- *the Nash equilibrium is*

$$n^*_{1,t} = \sqrt[3]{\frac{N_{2,t}}{N_{1,t}}}, \quad n^*_{2,t} = \sqrt[3]{\frac{N_{1,t}}{N_{2,t}}}\,; \tag{11}$$

- *the Nash equilibrium is stable.*[23]

Proof Setting $\omega = 0$, $\mu = 1$, $\lambda = 0$ $\alpha = 1$ in Eqs. 9 and 10 we obtain the reaction functions

$$n_{1,t} = \sqrt{\frac{N_{2,t}}{N_{1,t}} n_{2,t}} \quad \text{and} \quad n_{2,t} = \sqrt{\frac{N_{1,t}}{N_{2,t}} n_{1,t}} \tag{12}$$

which show that the best fertility rate for each group is to increase as the other group's fertility increases. At the Nash equilibrium, there is no incentive for either group to change its fertility rate, given the other group's fertility rate; this amounts to solving the system in Eq. 12, which yields the results in Eq. 11. This solution represents a maximum as

$$U''_{i,t} = -\frac{2AN_{j,t}\left(2N_{i,t} + \sqrt[3]{\frac{N^4_{i,t}}{N_{j,t}}} + \sqrt[3]{N^2_{i,t}N_{j,t}}\right)R_{t+1}\beta\tau}{\left(N_{j,t} + \sqrt[3]{N^2_{i,t}N_{j,t}} + 2\sqrt[3]{N_{i,t}N^2_{j,t}}\right)^3} < 0.$$

At equilibrium the slopes of the reaction functions are

$$\frac{\partial n_{i,t}}{\partial n_{j,t}} = \frac{1}{2}\left(\frac{N_{j,t}}{N_{i,t}}\right)^{2/3}.$$

[23] The stability of the Nash equilibrium should not be confused with the concept of (dynamic) stability which is used in Sect. 3.4. The Nash equilibrium is stable when small changes in the strategy of one clan lead to a situation where the clan which did not change has no better strategy than its original one (i.e. the original reaction function) and the clan which did change is now using a strategy which is worse. Intuitively, we can see that, in the neighborhood of an equilibrium, adjustments along the reaction functions converge toward the original equilibrium (see Hahn 1962).

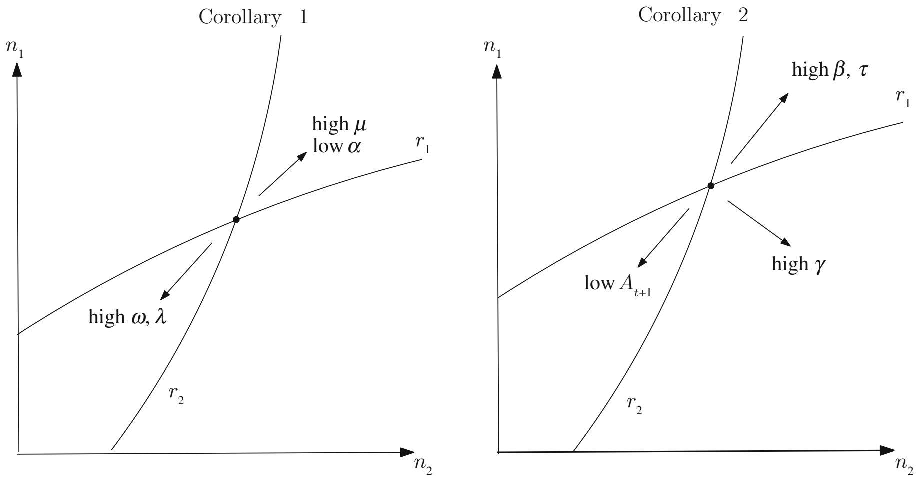

Fig. 3 Reaction functions (r_1 r_2) and comparative statics

To demonstrate the stability of the Nash equilibrium, we compute the inverse reaction function of Group 2 as $n_{1,t} = n_{2,t}^2(N_{2,t}/N_{1,t})$. The slope of this curve at equilibrium is $(N_{1,t}/N_{2,t})^{2/3}$, which is always greater that for Group 1. This ensures that in the $\{n2, n1\}$ space Group 1's reaction function intersects Group 2's from above and the equilibrium is stable. The equilibrium can be reached through adjustments along the reaction functions. □

The finding that fertility reaction functions have positive slopes means that there is a strategic complementarity between group sizes, and therefore between the groups' fertility rates as implied by the population race mechanism. It is interesting to observe how the fertility changes if we allow the parameters to be perturbed. The results are summarized in Corollary 1 and are illustrated graphically in Fig. 3.

Corollary 1 *In the neighborhood of the Nash equilibrium obtained under Assumption 1, the following comparative statics hold:*

$$\frac{\partial n_{i,t}}{\partial \omega} < 0,$$
$$\frac{\partial n_{i,t}}{\partial \mu} > 0,$$
$$\frac{\partial n_{i,t}}{\partial \lambda} < 0,$$
$$\frac{\partial n_{i,t}}{\partial \alpha} < 0.$$

Proof See Appendix A.1. □

We observe that equilibrium fertility decreases with ω. Indeed, when a war has a high potential cost the importance of military power in determining the outcome of the bargaining decreases, and this reduces the strength of the strategic motive for fertility. If μ increases, the fertility rates at equilibrium also increase: when the bargained share is equal to the probability of success, then, if this probability becomes more sensitive to the relative size of the clans, it is best to increase the size of each clan. As expected, an increase in the disutility of child rearing decreases fertility because having children entails a (greater) disutility cost. Finally, when returns to labor are greater (α is lower) it is optimal to increase fertility because the negative impact on crop per capita is less.

Notice that, when evaluating the marginal effects under Assumption 1, β τ, and γ have no effect. If we evaluate their effects in different configurations, further insights can be gained as long as the reaction functions continue to have positive slopes. In particular we have Corollary 2, below.

Corollary 2 *If the fertility reaction functions have positive slopes the following comparative statics hold:*

$$\begin{aligned} & \frac{\partial n_{i,t}}{\partial \beta} \geq 0, \\ & \frac{\partial n_{i,t}}{\partial \tau} \geq 0, \\ \textit{if} \quad \{\alpha = 1\} \quad & \frac{\partial n_{i,t}}{\partial \gamma} \leq [\geq]\, 0 \quad \textit{with} \quad i = 1\,[2], \\ \textit{if} \ \{\alpha = 1, \mu = 1\} \quad & \frac{\partial n_{i,t}}{\partial A_{t+1}} \geq 0. \end{aligned}$$

Proof See Appendix A.2. □

Both τ and β are associated with greater importance of the role of children as old-age support. Therefore they have the common effect of making the cost of child rearing, and hence of enhancing fertility, relatively more bearable.[24] The parameter γ for the exogenous contractual asymmetries in favor of Group 1 has a negative impact on the optimal fertility of Group 1 and a positive impact on the fertility of Group 2. In other words, when the exogenous contractual strength of one group increases, it prefers to reduce its own fertility, whereas the fertility of the other group increases. The reason why this happens can be seen as a substitution effect: *ceteris paribus*, the same bargained share can be achieved with a lower military threat, and since the military threat comes at a cost, the more powerful group chooses a lower fertility rate, whereas the other group has to compensate by investing in the other source of bargaining power (i.e. the threat of war).[25] Total factor productivity A_{t+1} is not a parameter as it depends on the current stock of resources; nevertheless it is exogenous to clans' fertility choices and can be dealt with in a comparative statics set-up. It has a positive effect that can be seen as an income effect, making more children affordable for a given disutility of child rearing.[26]

Figure 3 summarizes Corollaries 1 and 2. It is worth remarking that Corollary 1 evaluates the parameters at the Nash equilibrium under Assumption 1, whereas the parameters in Corollary 2 have zero effect at that configuration, but have the effects already discussed when evaluated elsewhere (as long as the fertility reaction functions have positive slopes and the specific restrictions given in Corollary 2 are satisfied).

[24] They only have zero effect when such a cost is not present.

[25] Notice that when $\omega = 0$, the bargained share is unaffected by γ and accordingly this parameter has no effect on fertility rates.

[26] When $\lambda = 0$ we have a zero effect.

3.4 Dynamics

We can now assess the dynamics of the model under Assumption 1. The dynamics of the population is obtained by inserting Eq. 11 into Eq. 4 to give

$$N_{i,t+1} = N_{i,t}\sqrt[3]{\frac{N_{j,t}}{N_{i,t}}}.$$

Denoting $\ln x$ by $\hat{x}$, the population dynamics is determined by a system of two linear difference equations:

$$\hat{N}_{i,t+1} = \hat{N}_{i,t}\ 2/3 + \hat{N}_{j,t}/3.$$

The solution is given by

$$\hat{N}_{i,t} = \frac{\hat{N}_{1,0} + \hat{N}_{2,0}}{2} + \frac{3^{-t}}{2}(\hat{N}_{i,0} - \hat{N}_{j,0})$$

and at steady state, the population of the two clans is equal and given by the geometric average of the two initial population levels:

$$\bar{N}_i = \bar{N}_j = \sqrt{N_{1,0}}\sqrt{N_{2,0}}. \tag{13}$$

The dynamics of the stock of resources is provided by Eq. 5, which has a steady state

$$\bar{R} = K\left(1 - \frac{b(\bar{N}_i + \bar{N}_j)}{\delta}\right). \tag{14}$$

A positive steady state exists if and only if $\bar{N}_i + \bar{N}_j < \delta/b$. We can write this as a condition on the initial populations to give the following proposition:

Proposition 3

- *If a strictly positive steady state for resources exists, then it is stable.*
- *Under Assumption 1:*
  - *a positive stable steady state exists if and only if the initial populations are not too high:*

$$2b\sqrt{N_{1,0}}\sqrt{N_{2,0}} < \delta.$$

  - *at steady state* $\bar{\pi} = 1/2$ *and* $\bar{\theta} = 1/2$.

Proof The steady state for Eq. 14 is strictly positive if and only if

$$\frac{b(N_i + N_j)}{\delta} < 1. \tag{15}$$

The first derivative of Eq. 5 evaluated at the steady state reads $1 - \delta + b(N_i + N_j)$. This is less than 1 if Condition (15) holds. Hence the positive steady state is asymptotically stable. This proves the first point.

Inserting Eq. 13 into Eq. 14 we can reduce the condition for the convergence of resources to a strictly positive long-run level to a condition for the initial population (see above). This proves the second point.

Finally, when $\omega = 0$, $\theta_t = \pi_t$. At steady state the two groups have the same population, and so $\bar{\pi} = 1/2$. Therefore we also have $\bar{\theta} = \bar{\pi} = 1/2$. □

With Assumption 1, the ecosystem can sustain a long run equilibrium providing the initial population is not too high; in other circumstances an environmental trap occurs. The dynamics for more general cases, and the role of factors affecting the occurrence of an environmental collapse, are studied through numerical simulations in Sect. 4.2. At this stage it is possible to see from Eq. 5 that a higher population uses more resources, and from Eq. 14 that the occurrence of an environmental trap depends on the long run dynamics of the total population. Therefore, any change in parameters which has the effect of increasing equilibrium fertility rates also has the effect of increasing the likelihood of an environmental trap. This implies that two similar societies with different μ's may end up in very different situations: the one where μ is low may experience a moderate population growth and so achieve a long run stable equilibrium, whereas the society where μ is high may be doomed to collapse.

4 Numerical simulations and robustness analysis

In the previous section we derived some theoretical results for a specific configuration of the parameters. In this section we investigate whether these results still hold when the parameters take different values. In Sect. 4.1 we study the fertility problem for one generation: we numerically compare the benchmark model studied analytically in Sect. 3.3 with that resulting from a different setting of the parameters. Sect. 4.2 and Sect. 4.3 deal with the dynamics, focusing in particular on the role of μ and ω; in the former section, we identify in the $\{\mu, \omega\}$ space regions where dynamics converge to a feasible long run equilibria, and we analyze how these regions are affected by perturbation of other parameters; in the latter section, we report some examples of transition paths in order to show that the model is able to reproduce qualitatively plausible patterns of population and resources.

4.1 The Nash equilibrium

Let us begin by setting some values for the parameters, which are summarized in Table 1. A few words on these values: $\gamma = 1/2$ means that there is no exogenous asymmetry in the bargaining process; setting $\tau = 0.8$ implies that, for instance, young adults with two siblings support their parents with 20% of their income, and the parent receives a total of 60% of the income of a young adult in this group. For the sake of simplicity the discount factor β is set to 1. The returns parameter α and the coefficient on child disutility λ are set to 1 and 0 respectively, as in Assumption 1. From Sect. 3.3 we know that fertility rates are increased (decreased) by a lower α (a greater λ). We take initial population levels of 90 and 100 respectively, the fact that Group 2 is bigger than Group 1 implying that it is more likely to win if there is a conflict. The total factor productivity $A()$ depends on the stock of resources. For the sake of simplicity we assume $A(R_t) = AR_t$ and set $A = 10$.

When we consider the dynamics, then the environmental parameters δ, b and K also play a role as shown in the law of motion in Eq. 5. Assuming that one period lasts for 20 years, the intrinsic regeneration growth rate δ can be set at 0.08 (in accord with Matsumoto (2002) and Dalton and Coats (2000) that suggest a 4% growth rate per decade). Using the same

Table 1 Benchmark parametrization

γ	λ	α	τ	β	A	$N_{1,t}$	$N_{2,t}$	δ	b	K
.5	0	1	.8	1	10	90	100	.08	$1.2 \cdot 10^{-4}$	12000

Table 2 Outcome for generation born at t, Cases I and II

Case	$n_{1,t}$	$n_{2,t}$	θ_t	θ_{t+1}
I	1.036	0.965	.474	.491
II	1.465	1.356	.469	.492

assumptions as Matsumoto, the coefficient for the human impact on resources b turns out to have the order of magnitude of 10^{-4} on our scale. The carrying capacity K, which acts as a scaling parameter, is set at 12000 as elsewhere in the literature.[27]

Here we will focus on the parameters μ and ω in two cases. In Case I they take the same values as in Sect. 3.2: $\{\omega = 0; \mu = 1\}$, so that Case I is a numerical example satisfying Assumption 1. Case II is characterized by a larger value for both parameters: $\{\omega = 0.1; \mu = 1.3\}$. For both cases we consider the fertility choice problem at given t.[28] The resulting equilibrium fertility choices and bargained share are shown in Table 2. In both cases optimal fertility is higher for Group 1 than for Group 2 because its size is initially lower. Moreover, in Case II, there is a higher equilibrium level of fertility for both groups. This arises because the probability of success in war is more sensitive to the groups' relative sizes, giving a stronger incentive to have children. This is also reflected in the fact that, for given population size, Group 1's share (θ_t) is smaller in Case II than in Case I. Finally, we verified that the slopes of the reaction functions, which were shown to be positive in the neighborhood of the equilibrium under Assumption 1 (see Proposition 2), are still positive when the values of μ and ω are somewhat different.

4.2 Resources and population dynamics

In Sect. 3.4 we studied the dynamics of the particular configuration which allows analytical solutions. Here we will use numerical simulations to extend the analysis to the general model, so as to see which factors make the occurrence of a collapse more likely. The aim is to discover under which configuration of parameters the dynamics converge to a steady state.

Non-convergence can reflect two different situations. The first—which we label 'population collapse'—arises when the population converges asymptotically to zero while resources achieve their carrying capacity K. This represents a world where fertility is below replacement level in the long-run. The second situation—labeled 'environmental collapse'—arises when resources are exhausted following a population boom. This also causes the extinction of the population, as no crops are produced. Contrary to the 'population collapse' case, the 'environmental collapse' leads to the extinction of population in finite time.

For the benchmark parametrization we focus on the role of the parameters ω and μ, which have not been considered so far in the literature. The meaning of these parameters will be discussed more extensively in Sect. 5, as will the historical evidence for differences between Easter and Tikopia Islands. The parameters for the benchmark specification are as shown in Table 1 but for λ which is set to 1. If $\lambda = 0$ fertility does not depend on resources and on actual clans' populations but just on their ratio: if this is the case, then fertility remains generally different from 1 (except in the case of Assumption 1–Proposition 3), leading either

[27] See Matsumoto (2002), Brander and Taylor (1998), Pezzey and Anderies (2003), Erickson and Gowdy (2000), Dalton and Coats (2000), Flenley et al. (1991). The meaning of the parameters will be discussed in more detail in Sect. 5, with respect to the cases of Tikopia and Easter Island.

[28] Notice that in the fertility problem R_{t+1} is exogenous. Moreover given that $\lambda = 0$, the resource stock is just a multiplicative term that does not affect the fertility choice.

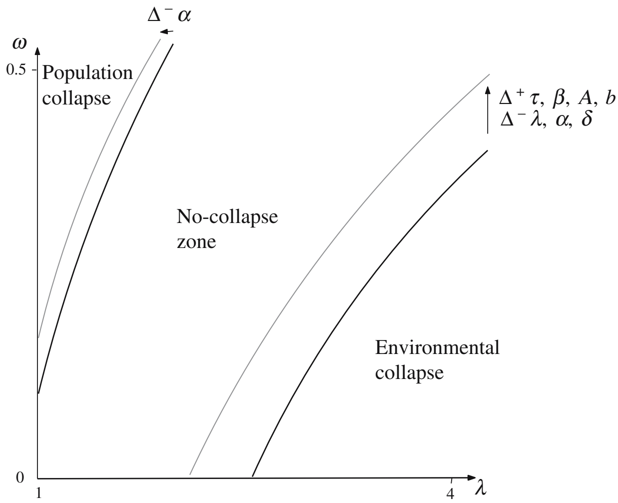

Fig. 4 Collapse zones as a function of parameters

to population collapse or to environmental collapse. Instead with a strictly positive λ, fertility is related to the actual stock of resources and populations and a there exists a range of parameters μ and ω such that collapse can be avoided. The setting $\lambda = 1$ is chosen for the sake of simplicity. The values for $N_{1,t}$ and $N_{2,t}$ are used as initial values, and the value for K is taken as the initial value for R_t. Figure 4 shows the three regions in the $\{\mu; \omega\}$ space separated by bold curves.

The top-left region represents the zone of population collapse, the central area is the no-collapse zone where a long run stable equilibrium of population and resources can be reached, and the bottom right represents environmental collapse. We can see that the frontiers all have positive slopes: a collapse is avoided if neither of these two parameters is 'too large' compared to the other. In particular, an environmental collapse becomes more likely if, for a given ω, μ is high, or if for a given μ, ω is low.

By performing a *ceteris paribus* perturbation of the parameters we can assess the effects of other model parameters on the collapse zones.[29] Figure 4 shows how the borderlines move as the parameters change. Both borderlines are affected by every parameter, but the effect on the left frontier is so small in some cases that we do not report it systematically. This shows that:

- a drop in the disutility of children λ reduces the stable population zone and increases the environmental collapse zone;
- higher old age transfers τ enlarge the zone of environmental collapse and shrink the zone of no-collapse;
- less severe decreasing returns to labor ($\Delta^- \alpha$) reduce the scope for population collapse, enlarge the zone of environmental collapse, and move the zone of no-collapse;
- lower discounting with time (higher β) enlarges the zone of environmental collapse and reduces the zone of no-collapse;
- higher productivity A reduces the stability zone and enlarges the zone of environmental collapse.

[29] All the simulations are available from the authors upon request.

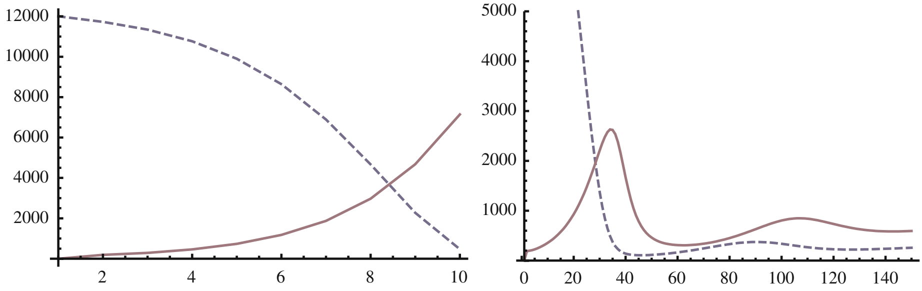

Fig. 5 Simulation for environmental collapse (left panel) and no collapse (right panel)
Young Population $N_{1,t} + N_{2,t}$ (solid); Resources (dashed). Left panel: $\{\omega = .4; \mu = 1.22\}$; Right panel: $\{\omega = .5; \mu = 1.12\}$. Other parameters as in Table 1 but for $\lambda = 1$ and $\alpha = .5$

- lower δ or higher b, both implying a more fragile ecosystem, reduces the zone of no-collapse and enlarges the zone of environmental collapse.

Basically, the parameters which have been proved in Corollary 1 and 2 to have a positive impact on fertility enlarges the scope for environmental collapse by increasing the pressure on resources.

Additional non-reported simulations show that, when exogenous asymmetries are very important (γ close to 0 or to 1 and ω high), the population collapse zone is enlarged and the environmental collapse zone is reduced. With large exogenous asymmetries, the strategic incentive to compete on size is weaker. The weaker group knows that it is not worth increasing fertility for strategic reasons.

4.3 Simulation of transition paths

As an example of dynamics we report in Fig. 5 a simulation for the case of an environmental collapse (Panel A) and one for the case of no collapse (Panel B). In our model an "Environmental Collapse" occurs when the law of motion of resources (Eq. 5) given the current stocks of R, N_1, and N_2 would imply a negative stock of resources in the next period. Should this happen, in the simulation resources are set to zero since they cannot be negative, with the eventual extinction of population following from the lack of crop. In this sense we model the process that can lead to a collapse but not what happens thereafter. In the collapse scenario, the basic structure of our model no longer makes sense and one must turn to a different model.[30]

Panel A shows an economy with a relatively high μ and low ω, high fertility and high population growth. At some point (after 10 periods in the example), resources are depleted and we reach a state with $R = 0$. In Panel B, a lower μ and higher ω allow to avoid the collapse. The strategic motive toward fertility is still at work and for high initial levels of resources, population initially increases, but as resources are more and more depleted, the decrease in A_t lowers fertility offsetting the strategic incentive. The reduction in fertility turns out to be enough in the setting of Panel B to allow the resource stock to recover; then, when resources are again high enough, population increases again but more slowly than before and one can observe that the model is able to display oscillatory behavior in the long run as in Brander and Taylor (1998).

[30] This seems in line with what happened on Easter Island, since it seems that cultural practices changed drastically once the resources were exhausted.

In Proposition 3 the long run population turned out to be highly dependent on initial values of N_1 and N_2. This result was obtained under the specific parametrization of Assumption 1. Away from that setting the dynamics lose such a serious dependence on initial values of population. By repeating simulation for different $N_{1,0}$ and $N_{2,0}$, we observe that whether a collapse occurs and the rough pattern of the dynamics are not substantially affected: in the case of Panel A a collapse occurs also for small initial levels of population, whereas in the case of Panel B also for relatively high initial values,[31] population exhibits an initial hump-shape and then it oscillates with resources in a similar fashion as in the benchmark case of Panel B. What is affected is the timing of the peak, that is achieved earlier for high initial values.

5 Discussion

In this model, fertility is not determined through mechanistic Malthusian dynamics but is the outcome of a rational choice by decision makers, with a finite life-span, who are aware of some trade-off in choosing how many children to raise. In particular one of the motives related to fertility choice has a strategic nature, since it is related to bargaining over crop sharing in the next period.

A new way to look at fertility is proposed, suggesting that strategic complementarities can characterize fertility behavior across groups. When the relative size provides advantages, as in this model in relation to military power, each group's fertility decision is affected by expectations about the other group's decision. We studied under what conditions this can result in over-population. Up to now the number of children has usually been studied either as automatically determined by nutrition (in a Malthusian fashion), as a consumption good (assuming altruistic motives), or as an investment good (for old age support). Our framework has elements of all these traditions, but differs substantially from them since the relative size also matters, thus determining fertility complementarity.

Moreover, groups are neither completely myopic, nor very-long-term optimizers. It is assumed that they were not interested in the impact of their fertility behavior in the very long run, but also that they cared about the effects of their actions for at least the whole of their lifetime. The assumption of perfect foresight is the simplest way to model agents who are aware that their consumption tomorrow will be affected by their groups' relative size and thus by their fertility decision today. It is noteworthy that the lack of internalization of long term harmful effects is not due to myopic behavior but to the crucial features of agents as finite-lived and non altruistic in the sense of Barro (1974). If altruism is modeled as 'joy of giving', it can be handled within the model as a shift on τ (e.g. more altruism toward children implies a lower τ). Instead if altruism involves taking into account the utility of all features generations (Barro 1974), the clan's problem becomes analogous to the case of infinite-lived agents: in such a framework perfect foresight would preclude any collapse (understood as the extinction of population) in finite time as it cannot be an optimal path.

As Kirch (1997) remarks the "diversification of the Oceanic societies and cultures resulted in part from the process of island settlement and inevitable isolation, in part from adaptation to contrastive island environments, *and in no small measure from internal social dynamics*".[32] In this respect, the model adds new factors (e.g. ω, μ) that play a role influencing human decisions to the elements already considered in the literature on fragile ecosystems (δ, b).

[31] Clearly $N_{1,0}$ and $N_{2,0}$ cannot be "too high" otherwise a collapse follows almost immediately.

[32] Kirch (1997), p. 31, italic is ours.

Even if two societies are similar with respect to the second group of parameters they may experience very different patterns if they differ substantially with respect to the first group. Though simple, the model highlights how these factors can affect fertility decisions when crop sharing is decided by bargaining. It is worth remarking that fertility reaction functions with positive slopes is a necessary but not sufficient condition for a population race to occur. In order for a population race to happen the strategic complementarity must force fertility high enough. This turns out to depend also on parameters related to geography and climate.

Previous economic literature has stressed the climatic differences between Easter and Tikopia Islands. Easter Island is located at a latitude of 27° South, while Tikopia's latitude is 12.17° South: this determines a first important difference since the climate is colder on Easter Island. Moreover while Tikopia has relatively high rainfall (4000 mm per year according to Taylor (1973)), with an annual dry season (the *tonga* period) in May–June, Easter Island exhibits drier weather (rainfall: 1270 mm per year according to Sahlins (1955)). Rolett and Diamond (2004) find that both cold and dryness are important in determining the likelihood of deforestation in a sample of small Pacific Islands. Other important factors are the exposure to aerial volcanic fallout and Asian dust plumes, and the presence of a rock called *makatea*. Tikopia is ranked much higher than Easter Island on these factors. Brander and Taylor (1998) remark that the indigenous palm has a very low inherent growth rate due to the southern latitude of the island, the temperature, the amount of annual rainfall, and the quality of the soil (see also Ladefoged et al. (2005)). All those features makes Easter Island an outlier in the Polynesian context.[33] Applying all this evidence to the model suggests that the intrinsic resource growth rate δ was lower for Easter Island.[34] As Eq. 14 shows, a lower δ increases the likelihood of an environmental trap that determines the collapse of a society. Moreover the remoteness of Easter Island[35] might have spurred the deforestation by increasing the opportunity for doing things which have an impact on the environment (as suggested by Diamond 2005, p. 108). This can be seen in a higher b, the coefficient of human impact on resource growth, for Easter Island, which *ceteris paribus* makes it easier for the environmental trap to occur.

The role that differences in parameters such as b and δ have played in favoring the Tikopian ecosystem has already been considered by Erickson and Gowdy (2000). But there are also geographical features against Tikopia. For example the smaller area of Tikopia[36] means that the perimeter/area ratio is larger, and the initial percentage of forest area is smaller. Moreover Tikopia was subject to periodic cyclones, *afa*, (approximately 20 per decade) which prevented almost all human activity and compelled the inhabitants to stockpile food in advance. Diamond (2005) reports that "fields in Tikopia require almost constant labor input for weeding, plus mulching with grass and brushwood to prevent crop plants from drying out." and Kirch (1997) remarks that in Tikopia "a near-continuous rotation of field crops (yams, taro, and today manioc) is maintained through labor-intensive mulching" (p. 35). However in Easter Island the "upland plantation did not require year-round effort: the peasants just had to march up and plant taro and other root crops in the spring then return later in the year for the harvest" (Diamond 2005, p. 93). All this can be interpreted as a higher

33 Despite that was known to archaeologists, Brander and Taylor (1998) have been the first to stress how such factors could have created an overshoot and a collapse on Easter but not on other islands in Polynesia.

34 A lower δ could also be caused by a higher presence of rats on Easter Island which ate palm fruits and prevented regeneration (Flenley and Bahn, 2003).

35 Easter Island is more than 3200 km west of the nearest continent (South America) and 2250 km from the nearest populated island (Pitcairn). Tikopia's closest island, Anuta, is 136.8 km away, while the closest large island is 225 km away.

36 Tikopia covers 4.6 km^2 of land area whereas Easter Island's area is 166 km^2.

value of ω in the Tikopian economy: ω really captures the cost of waging war, in terms of missed/wasted/not-enjoyed crop consumption. When agents have less time available, and they have to use part of this time in stockpiling resources, devoting their time to fighting is likely to involve a greater cost. Our model suggests that a greater ω reduces fertility, so that even if being bigger provides greater military power, this is unlikely to trigger a population race. Notice that Page (2005) in his review of Diamond's book also relates a lower extraction in Tikopia to the presence of cyclones, as we do; he has in mind some institutional adaptation to climatic variability, whereas in our model the channel works through the credibility of a military threat.

The parameter μ also plays an important role in our model. Such a parameter is inherent to warfare and captures the sensitivity of the probability of success in war to relative numbers of fighters. We have seen that, with a high sensitivity, a population race can be triggered. Münster and Staal (2005) suggest that such a parameter should be considered as determined both by technological factors (for example, war technology) and institutional factors (social security and cohesion). It is reasonable to believe that war technology at the time we are interested in was very labor intensive, so that differences in the number of fighters could play a substantial role. On the other hand we have no clear evidence that the clans differed very much with respect to war technology or weapons. However Hirshleifer (1995) claims that a low μ "in military struggles [...] corresponds to the defense having the upper hand" (p. 32).[37] Possible advantages for the defensive position, with respect to the attacking one, can depend on features of the terrain. Easter Island's topography is mostly gentle without the deep valleys which are common elsewhere in the Pacific Islands.[38] Kirch and Yen (1982) instead describe the territory of some of the main clans living in Tikopia as "an apron of narrow, steep, gravely soils" (p. 365). Moreover the *makatea* terrain—which is present on Tikopia but not on Easter Island—is extremely difficult to walk over as it is razor-sharp and cutting. Such terrain is likely favor defenders rather than attackers in a fight, and can so be interpreted as evidence for a lower μ on Tikopia.

Hirshleifer (1995) suggests several historical examples where a high decisiveness of conflict is compatible with the establishment of a hierarchical society, whereas when μ is small a less stratified society without strong hierarchies can be sustainable. All that is consistent with the differences between Easter Island and Tikopian societies: Easter Island was characterized by a highly hierarchic structure whereas Tikopia was one of the least stratified societies in Polynesia. We do not claim that the institutional setting was the determinant of μ (as Münster and Staal (2005) do); on the contrary, it is possible that the institutional setting has been the outcome of the differential decisiveness of conflict. At the very least, what occurred historically is consistent with a higher value of μ for Easter Island.

Despite we focused on Easter Island the model could be applied to other less well-known Pacific societies. For instance, Kirch (1997) compares the successful case of Tikopia with the collapse of Mangaia civilization claiming that the human behavior played a critical role in the different outcomes of the societies.[39] Also in Mangaia groups competed in order to control the vital irrigation systems and hence the bulk of the crop, with the weakest groups

[37] This view is not shared by Grossman and Kim (1995) who model disparities between attack and defense through another parameter.

[38] "Except at the step sided craters and cinder crones, [it is] possible almost anywhere in Easter to walk safely in a straight line to anywhere else nearby." (Diamond 2005, p. 83). Easter Island's gentle terrain may also help to explain why it was much more integrated than the Marquesas Islands, where people living in adjacent valleys communicated each other mainly by sea rather than overland.

[39] Mangaia is a geologically old island, part of the Cook Islands, which ranks somewhat in between Easter Island and Tikopia with respect to magnitude (52km^2) and rainfall (1967 mm).

relegated to seek for their subsistence consumption from vastly inferior yields. What occurred then is qualitatively very similar to Easter Island. As the population increased up to its peak of approximately 4500 people around XVIIth century and the ecosystem became more and more damaged,[40] the social terror increased accordingly, as witnessed by Mangaia myths and traditions, and by archaeological records of fortified refuge caves and more and more frequent episodes of warfare. When Europeans arrived in the XVIIIth century native population was estimated at 2000.

6 Conclusion

In this paper we suggested that a population race could help to explain the different patterns experienced in the past by two small societies in the Pacific Ocean: Easter Island where the population grew considerably over time and then decreased sharply, and Tikopia where the population remained roughly steady over centuries. In the absence of strong property rights, income distribution is governed by bargaining between clans. Bargaining power is influenced by the threat of waging war. Since the probability of winning a war is positively affected by the number of fighters available, relative size matters: a bigger clan is more powerful and can achieve a greater share of the crop.

The resulting strategic interaction among clans can result in over-population, rapid depletion of renewable resources, and ultimately the collapse of the whole society. We discussed factors making the occurrence of a collapse more likely, using historical and archaeological evidence to assess possible differences in them between Easter Island and Tikopia. On top of the factors already highlighted in the literature, we considered the role played by the cost of conflict (ω), and the decisiveness of having more people in determining the outcome of a war (μ). Without neglecting the role of other elements, we suggest that these factors, through their influence on human behavior, may also have played an important role in the intensity of a population race and the path to collapse. In particular, the agriculture type and the presence of cyclones makes war more costly in Tikopia, deterring clans to compete in size to gain military power. Evaluation by researchers in other fields (e.g. experimental archeology) of μ and ω in Tikopia and Easter would be highly interesting to assess the strength of the mechanism we put forward. Along this line one might also want to know more about the role the *moai* played—perhaps they were a way of keeping track of the bargaining shares of the respective clans; at least this paper has a better chance of explaining what the *moai* were for than other models of population dynamics on Easter Island.

One interesting feature of our approach, compared to that elsewhere in the literature, is that the assumption of a mechanistic dynamics for population is relaxed. It is often assumed that, in the least developed societies, fertility simply follows Malthusian dynamics. However the different demographic patterns followed by similar non-developed societies, plus evidence of an awareness of fertility behavior (supported by historical indications of fertility as a social norm) suggest that a deeper analysis of fertility may be profitable. In our model fertility turns out to be a utility-maximizing choice, whose micro-determinants can be investigated.

The commonly made assumption of highly myopic behavior is also relaxed by introducing foresight and inter-temporal choice (as recommended by Reuveny and Maxwell (2001)). This does not mean that decision makers are aware of long-run resource dynamics, but simply that they are aware that they have to take decisions which will affect them during the whole of

40 First settlers arrived in Mangaia around BCE 500. Forest resources were reduced by about 60% in 500 years, and consequently also the faunal resources began to decline sharply. See Kirch (1997) for further details.

their lifetime: perhaps the clearest example of this is the decision about how many children to rear.

Another contribution of this paper is that it provides a new way of looking at fertility, since it features strategic complementarity between groups' sizes. The mechanism through which being larger yields an advantage provides a further motive for fertility choices, in addition to those generally highlighted (old age support, family altruism, etc.). In our model this motive can be traced back to the absence of property rights over output, but the principle can easily be extended to any situation where the relative size of a group influences its expected payoffs. Consider, for instance, tensions between two groups where one feels much weaker in war or weapon technology: enlarging its population can be seen as a means to increase its power and partly bridge the gap. In a bargaining situations the bigger group can take advantage of its size by making its voice louder (for example, in subsidy seeking); or in contexts where creating a relative advantage through size can be seen as means of defense and evaluated positively.[41]

Easter Island is often cited as an example of how a closed system can collapse in an endogenous way. Although the focus of this paper is on an episode of the past, it can be also seen as a metaphor for the management of resources within a society. If important features of modern development (technological progress, international trade) are disregarded, the model highlights the dangers involved in a race for a positional rent. In this respect the population race can become an arms race, reminiscent of episodes in the Cold War as well as more recent international tensions.

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Appendix

A.1 Proof of Corollary 1

Total differentiation of Eqs. 9 and 10 at the Nash equilibrium under Assumption 1 yields:

$$-\frac{2N_{1,t}^{2/3}}{\left(N_{1,t}^{1/3}+N_{2,t}^{1/3}\right)^4 N_{2,t}^{1/3}}dn_{1,t}+\frac{N_{2,t}^{1/3}}{\left(N_{1,t}^{1/3}+N_{2,t}^{1/3}\right)^4}dn_{2,t}-\frac{\gamma}{N_{1,t}^{1/3}\left(N_{1,t}^{1/3}+N_{2,t}^{1/3}\right)^2}d\omega$$
$$+\frac{N_{1,t}^{1/3}+N_{2,t}^{1/3}+\frac{1}{3}N_{1,t}^{1/3}\ln\left(\frac{N_{2,t}}{N_{1,t}}\right)}{\left(N_{1,t}^{1/3}+N_{2,t}^{1/3}\right)^4}d\mu-\frac{\left(\frac{N_{1,t}}{N_{2,t}}\right)^{1/3}}{\left(N_{1,t}^{1/3}+N_{2,t}^{1/3}\right)^3}d\alpha-\frac{1}{A_{t+1}\beta\tau}d\lambda=0$$

and

[41] An example of a population race for strategic motives, is, according to many, the episode of *la revanche des berceaux* (the revenge of the cradles) when Quebec's population more than doubled in 40 years in the mid XIXth century.

$$\frac{N_{1,t}^{1/3}}{\left(N_{1,t}^{1/3}+N_{2,t}^{1/3}\right)^4}dn_{1,t}-\frac{2N_{2,t}^{2/3}}{N_{1,t}^{1/3}\left(N_{1,t}^{1/3}+N_{2,t}^{1/3}\right)^4}dn_{2,t}-\frac{1-\gamma}{N_{2,t}^{1/3}\left(N_{1,t}^{1/3}+N_{2,t}^{1/3}\right)^2}d\omega$$
$$+\frac{N_{1,t}^{1/3}+N_{2,t}^{1/3}+\frac{1}{3}N_{2,t}^{1/3}\ln\left(\frac{N_{1,t}}{N_{2,t}}\right)}{\left(N_{1,t}^{1/3}+N_{2,t}^{1/3}\right)^4}d\mu-\frac{\left(\frac{N_{2,t}}{N_{1,t}}\right)^{1/3}}{\left(N_{1,t}^{1/3}+N_{2,t}^{1/3}\right)^3}d\alpha-\frac{1}{A_{t+1}\beta\tau}d\lambda=0.$$

These expressions can be written in matrix form as:

$$\begin{pmatrix} 1 & -\frac{1}{2}\left(\frac{N_{2,t}}{N_{1,t}}\right)^{2/3} \\ -\frac{1}{2}\left(\frac{N_{1,t}}{N_{2,t}}\right)^{2/3} & 1 \end{pmatrix}\begin{bmatrix} dn_{1,t} \\ dn_{2,t} \end{bmatrix}=$$
$$\begin{pmatrix} \frac{N_{2,t}^{1/3}}{2N_{1,t}}\left[\left(\frac{N_{2,t}}{N_{1,t}}\right)^{1/3}+1+\frac{1}{3}\ln\left(\frac{N_{2,t}}{N_{1,t}}\right)\right] & -\gamma\frac{\left(N_{1,t}^{1/3}+N_{2,t}^{1/3}\right)^2N_{2,t}^{1/3}}{2N_{1,t}} \\ \frac{N_{1,t}^{1/3}}{2N_{2,t}}\left[\left(\frac{N_{1,t}}{N_{2,t}}\right)^{1/3}+1+\frac{1}{3}\ln\left(\frac{N_{1,t}}{N_{2,t}}\right)\right] & -(1-\gamma)\frac{\left(N_{1,t}^{1/3}+N_{2,t}^{1/3}\right)^2N_{1,t}^{1/3}}{2N_{2,t}} \end{pmatrix.$$
$$\left.\begin{matrix} -\frac{\left(N_{1,t}^{1/3}+N_{2,t}^{1/3}\right)^4N_{2,t}^{1/3}}{2A_{t+1}N_{1,t}^{2/3}\beta\tau} & -\frac{1}{2}\left[1+\left(\frac{N_{2,t}}{N_{1,t}}\right)^{1/3}\right] \\ -\frac{\left(N_{1,t}^{1/3}+N_{2,t}^{1/3}\right)^4N_{1,t}^{1/3}}{2A_{t+1}N_{2,t}^{2/3}\beta\tau} & -\frac{1}{2}\left[1+\left(\frac{N_{1,t}}{N_{2,t}}\right)^{1/3}\right] \end{matrix}\right)\begin{bmatrix} d\mu \\ d\omega \\ d\lambda \\ d\alpha \end{bmatrix}. \tag{16}$$

The left coefficient matrix is positive definite and can be inverted to give

$$\begin{pmatrix} \frac{4}{3} & \frac{2}{3}\left(\frac{N_{2,t}}{N_{1,t}}\right)^{2/3} \\ \frac{2}{3}\left(\frac{N_{1,t}}{N_{2,t}}\right)^{2/3} & \frac{4}{3} \end{pmatrix}.$$

Pre-multiplying both sides of Eq. 16 by this matrix gives:

$$\begin{bmatrix} dn_{1,t} \\ dn_{2,t} \end{bmatrix}=\left(\begin{matrix} \frac{1}{3}+\left(\frac{N_{2,t}}{N_{1,t}}\right)^{\frac{1}{3}}\left[1+\frac{1}{9}\ln\left(\frac{N_{2,t}}{N_{1,t}}\right)\right]+\frac{2}{3}\left(\frac{N_{2,t}}{N_{1,t}}\right)^{\frac{2}{3}} & -\frac{\left(N_{1,t}^{\frac{1}{3}}+N_{2,t}^{\frac{1}{3}}\right)^2\left[(1-\gamma)\left(N_{1,t}^2N_{2,t}\right)^{\frac{1}{3}}+2\gamma N_{2,t}\right]}{3N_{1,t}N_{2,t}^{\frac{2}{3}}} \\ \frac{1}{3}+\left(\frac{N_{1,t}}{N_{2,t}}\right)^{\frac{1}{3}}\left[1+\frac{1}{9}\ln\left(\frac{N_{1,t}}{N_{2,t}}\right)\right]+\frac{2}{3}\left(\frac{N_{1,t}}{N_{2,t}}\right)^{\frac{2}{3}} & -\frac{\left(N_{1,t}^{\frac{1}{3}}+N_{2,t}^{\frac{1}{3}}\right)^2\left[(1-\gamma)2N_{1,t}+\gamma\left(N_{1,t}N_{2,t}^2\right)^{\frac{1}{3}}\right]}{3N_{1,t}^{\frac{2}{3}}N_{2,t}} \end{matrix}\right.$$
$$\left.\begin{matrix} -\frac{\left(N_{1,t}^{1/3}+N_{2,t}^{1/3}\right)^4\left(N_{1,t}^{1/3}+2N_{2,t}^{1/3}\right)}{3A_{t+1}N_{1,t}^{2/3}\beta\tau} & -\frac{1}{3}\left[1+\left(\frac{N_{2,t}}{N_{1,t}}\right)^{1/3}\right]\left[2+\left(\frac{N_{2,t}}{N_{1,t}}\right)^{1/3}\right] \\ -\frac{\left(N_{1,t}^{1/3}+N_{2,t}^{1/3}\right)^4\left(2N_{1,t}^{1/3}+N_{2,t}^{1/3}\right)}{3A_{t+1}N_{2,t}^{2/3}\beta\tau} & -\frac{1}{3}\left[1+\left(\frac{N_{1,t}}{N_{2,t}}\right)^{1/3}\right]\left[2+\left(\frac{N_{1,t}}{N_{2,t}}\right)^{1/3}\right] \end{matrix}\right)\begin{bmatrix} d\mu \\ d\omega \\ d\lambda \\ d\alpha \end{bmatrix}. \tag{17}$$

The effects are all unambiguous: the effects of ω, λ, and α are negative whereas the effect of μ is positive.

A.2 Proof of Corollary 2

Let us denote Eqs. 9 and 10 by Φ and Ψ, respectively. If we differentiate with respect to β and τ we obtain the following equations whose sign is unambiguous:

$$\frac{\partial \Phi}{\partial \beta} = \frac{\partial \Psi}{\partial \beta} = \frac{\lambda}{\beta^2 \tau};$$
$$\frac{\partial \Phi}{\partial \tau} = \frac{\partial \Psi}{\partial \tau} = \frac{\lambda}{\beta \tau^2}.$$

When the reaction functions both have positive slopes the effect of a change in parameters on the equilibrium fertility can be assessed by looking directly at the shift in Φ and Ψ.

When γ is involved we obtain an unambiguous effect if we keep $\alpha = 1$:

$$\left.\frac{\partial \Phi}{\partial \gamma}\right|_{\alpha=1} = -\frac{A\omega}{(1+n_{1,t})^2 N_{1,t}};$$
$$\left.\frac{\partial \Psi}{\partial \gamma}\right|_{\alpha=1} = \frac{A\omega}{(1+n_{2,t})^2 N_{2,t}}$$

as long as $\omega \neq 0$ the effect of an increase in γ is negative for Group 1's fertility, and positive for Group 2's fertility.

Keeping $\alpha = 1$ and setting $\mu = 1$, it is possible to compute each reaction function explicitly, obtaining long expressions. Plugging these expressions into the derivatives of Φ and Ψ with respect to A_{t+1} evaluated at $\alpha = 1$ and $\mu = 1$ the following equations hold after simplification:

$$\left.\frac{\partial \Phi}{\partial A_{t+1}}\right|_{\alpha=1,\mu=1} = \frac{\lambda}{A_{t+1}\beta\tau};$$
$$\left.\frac{\partial \Psi}{\partial A_{t+1}}\right|_{\alpha=1,\mu=1} = \frac{\lambda}{A_{t+1}\beta\tau}.$$

Again, provided the reactions functions have positive slopes, we can see that the effect of A_{t+1} on fertility rates is positive in the neighborhood of $\mu = 1$ and $\alpha = 1$.

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