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NOTES

A NOTE ON INFLATION PERSISTENCE IN A FAIR WAGE MODEL OF THE BUSINESS CYCLE

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We generalize existing fair wage models to allow effort to vary over the business cycle. When effort is variable, wage fluctuations are partially compensated for by endogenous effort fluctuations, so that the sensitivity of marginal cost to output and employment variations is decreased. This new mechanism decreases the need for sluggishness to explain the observed high inflation persistence.

Keywords: Efficiency Wage, Effort, Inflation Persistence

1. INTRODUCTION

Assuming that workers' effort are affected by the wages paid by the firm, efficiency wage theories have been judged to be very promising given the goal of understanding labor market characteristics [see Akerlof (1982) for the first—static—approach and Bewley (1998) for empirical support]. Danthine and Donaldson (1990) were the first to introduce efficiency wages into a dynamic Real Business Cycles (RBC) model in order to assess whether this mechanism can help explain the wage-employment puzzle. Their finding is that the structural unemployment generated in this way does not help to reduce the procyclicity of wages: as unemployment falls, effort tends to decrease and firms have to pay higher wages to maintain it. Going further, Collard and de la Croix (2000) show that this negative relationship between unemployment and wages can be attenuated by introducing a reference to past wages. In their setup, effort depends not only on wage comparison with

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contemporaneous outside wage opportunities but also on comparison with the workers' own lagged wages. Danthine and Kurmann (2004) embed this idea in a New Keynesian general equilibrium model to analyze labor market and inflation dynamics. Their show that the real rigidities implied by efficiency wages interact with nominal rigidities in such a way that the effect of monetary shocks on output is amplified and more persistent than in other monetary business cycle models.

On the whole, this literature indicates that the fair wage approach constitutes a promising platform for a more complete New Keynesian synthesis. In this paper, we pursue this line by adopting an effort function that is more general and allowing effort to vary over the business cycle. This specification contrasts with the previous studies, which selected a logarithmic effort function, so that the Solow condition, characterizing the optimal firm behavior, implied a constant effort level.

Accordingly, we model efficiency wages within an otherwise standard dynamic model with price staggering à la Calvo. We follow closely the method developed by Bénassy (2005), who obtains closed form solutions within a dynamic model under the following assumptions: logarithmic utility, no capital stock, multiplicative monetary shock. We assess how each parameter of the effort function affects the elasticity of real wages with respect to aggregate unemployment and how this elasticity modifies inflation persistence. In particular, we show that there exist parameterizations of the effort function for which the fair wage model generates more inflation persistence. This results contradicts the criticism of the efficiency wage assumption formulated by Kiley (1997).

Following Angeloni et al. (2006), we define inflation persistence as the extent to which inflation tends to approach slowly, rather than instantly, its long-run level after shocks. We accordingly measure persistence by the coefficient of autocorrelation of inflation. Because the only shock considered here is a permanent shock in the level of the money supply, this autocorrelation measures the tendency of inflation to remain away from its long-run value for a long time after such a policy change.

In any dynamic model with Calvo price staggering, inflation persistence is directly related to the persistence of the marginal cost of producing an additional unit of good. Hence, in these models, output persistence and inflation persistence are tied together by the New Keynesian Phillips curve. Keeping the Calvo parameter (i.e., the probability that a firm will reoptimize its price) constant, inflation persistence cannot be enhanced without increasing output persistence.

2. HOUSEHOLDS

Effort at work has consequences in terms of utility. In fair wage models, utility is negatively related to the distance between the effort provided by household j, denoted $e_t(j)$, and the effort judged fair by the household $e_t^\star(j)$: $[e_t(j) - e_t^\star(j)]^2$. In its simple form, the fair effort is a function of the real wage of the household

$w_t(j)$, of labor market tightness, and of the aggregate wage in the economy w_t:

$$e_t^\star(j) = \phi_1 \frac{w_t(j)^\psi - \phi_2 \left(\frac{1}{1-N_t}\right)^\psi - \phi_3 w_t^\psi - (\phi_0 - \phi_2 - \phi_3)}{\psi},$$

with the following parameter restrictions: $\phi_0 \in \mathbb{R}$, $\phi_1 > 0$, $\phi_2 > 0$, $\phi_3 \in [0, 1)$, and $\psi \in [0, 1)$. ϕ_0 and ϕ_1 are scale parameters. N_t is the aggregate employment rate, i.e., the average fraction of the household's members having a job. The parameter ϕ_2 measures the effect of the tightness of the labor market on individual effort. (We have preferred a formulation with $(1/(1-N_t))^\psi$ to one with N_t^ψ to guarantee that the equilibrium N_t is always below 1.) The parameter ϕ_3 describes the extent to which workers are sensitive to the alternative wage, i.e., the wage they could earn on average in the rest of the economy. Finally, the parameter ψ describes the substitutability between the different elements in the effort function. This effort function is a generalization of the logarithmic function found in the existing literature:

LEMMA 1. *For $\psi \to 0$ and $\phi_0 = 1$, effort is given by*

$$e_t^\star(j) = \phi_1 (\ln w_t(j) - \phi_2 \ln N_t - \phi_3 \ln w_t) .$$

Proof. Compute the limit of $e_t^\star(j)$ when $\psi \to 0$ using l'Hospital's rule. ■

Introducing effort into an otherwise standard money-in-the-utility function, the problem of the household is to maximize

$$\sum \beta^t (\log c_t(j) + \sigma \log(m_t(j)/P_t) - n_t(j)[e_t(j) - e_t^\star(j)]^2)$$

subject to the constraint $P_t c_t(j) + m_t(j) = P_t w_t(j) n_t(j) + \Pi_t + \mu_t m_{t-1}(j)$. The fraction of family members working at date t is $n_t(j)$; Π_t denotes nominal distributed profits, and μ_t is a multiplicative shock affecting all existing money balances. We assume that μ_t is a white noise. This assumption will simplify the computation of inflation persistence, but other assumptions on μ_t could also be used as well.

The above formulation differs from the standard RBC model in one important point: leisure does not enter into the utility function. This implies that the main mechanism at work will not be the standard intertemporal labor substitution effect usually driving RBC models. In this class of models the household supplies inelastically one unit of time, and only a fraction of time is employed by the firm. We call this fraction of time $n_t(j)$. One important point is that the utility drawn from the job itself is separable from the utility drawn from consumption, so that effort is independent of wealth.

The first necessary conditions for a maximum are

$$
\begin{aligned}
e_t(j) &= \phi_1 \frac{w_t(j)^{\psi} - \phi_2\left(\frac{1}{1-N_t}\right)^{\psi} - \phi_3 w_t^{\psi} - (\phi_0 - \phi_2 - \phi_3)}{\psi} \qquad (1)\\
1/c_t(j) &= \lambda_t(j) P_t\\
\sigma/m_t(j) &= \lambda_t(j) - \beta \mathrm{E}_t\left[\lambda_{t+1}(j)\mu_{t+1}\right]
\end{aligned}
$$

and the transversality condition$\lim_{t\to\infty}\beta^t m_t(j)/(P_t c_t(j)) = 0$. The first equation gives optimal effort as a function of real wages and employment rate. The second and third equations can be combined into

$$
\frac{m_t(j)}{P_t c_t(j)} = \sigma + \beta \mathrm{E}_t\left[\frac{m_{t+1}(j)}{P_{t+1}c_{t+1}(j)}\right].
$$

The only solution to this difference equation that satisfies the transversality condition is the constant solution

$$
\frac{m_t(j)}{P_t c_t(j)} = \frac{\sigma}{1-\beta}. \qquad (2)
$$

3. FIRMS

There are two types of firms: final output firms and intermediate good firms. Final output is produced with a combination of intermediate inputs y_i by competitive firms. Their production function is

$$
Y_t = \left[\int_0^1 (y_t(i))^{\theta}\, di\right]^{1/\theta}. \qquad (3)
$$

The elasticity of substitution between intermediate goods is $1/(1-\theta)$ with $\theta \in (0, 1)$. The parameter θ can be seen as an index of competitiveness. Each competitive firm maximizes profits $P_t Y_t - \int_0^1 p_t(i) y_t(i) di$, which leads to an isoelastic demand for intermediate good i:

$$
y_t(i) = \left(\frac{p_t(i)}{P_t}\right)^{1/(\theta-1)} Y_t. \qquad (4)
$$

The aggregate price P_t is a CES index of the intermediate good prices:

$$
P_t = \left[\int_0^1 (p_t(i))^{\theta/(\theta-1)}\, di\right]^{(\theta-1)/\theta}. \qquad (5)
$$

Intermediate good firms face a demand $y_t(i)$. They hire labor input $n_t(i)$ and set a wage to induce an effort level $e_t(i)$ in order to produce the demanded quantity through the following technology:

$$y_t(i) = (e_t(i)n_t(i))^{\alpha} \, . \tag{6}$$

With marginal decreasing returns ($\alpha < 1$), marginal productivity differs across firms as soon as employment differ across them. The intermediate firm minimizes costs $w_t(i)n_t(i)$ subject to technology (6) and effort (1).

The first-order conditions for the cost minimization problem are

$$w_t(i) = \nu_t(i)\alpha y_t(i)/n_t(i)$$
$$n_t(i) = \nu_t(i)\alpha[y_t(i)/e_t(i)]\,[\phi_1 w_t(i)^{\psi-1}],$$

where $\nu_t(i)$ is the Lagrange multiplier associated with the production constraint. $1/\nu_t(i)$ is also the markup over marginal cost. Combining the two conditions, we obtain

$$e_t(i) = \phi_1 w_t(i)^{\psi} . \tag{7}$$

We deduce from this equation the following result.

PROPOSITION 1 (Effort and Wages). *Optimal effort set by firms is given by equation (7). It is constant if $\psi = 0$. Otherwise, there is a positive relation in equilibrium between effort and wages.*

The intuition behind this proposition goes as follows. Firms increase wages up to the point where any marginal gain in effort is offset by an increase in the wage bill. This is translated into the condition that the elasticity of effort to wages should be equal to 1 in equilibrium [which is called in the literature the Solow (1979) condition]. In the case $\psi = 0$; i.e., when the effort function is logarithmic, this elasticity condition is equivalent to imposing constant effort [$e_t(i) = \phi_1$]. Any negative shock to effort, such as a rise in aggregate employment, is met by a rise in the firm's wage to keep effort constant. When $\psi > 0$, i.e., when wages and employment are highly substitutable in the effort function, the elasticity condition is no longer equivalent to keeping effort constant. Any rise in aggregate employment is also met by a rise in the firm's wage; if the wage is raised to the point where effort stays constant, the elasticity of effort to wages will stay above 1, giving an incentive to firms to raise wages above that point. This arises because the derivative of effort with respect to wages decreases more slowly when $\psi > 0$. In some sense, the returns to wages in terms of effort are less decreasing. This underscores that assuming logarithmic utility imposes a very strong restriction on effort. Our generalization of the effort function allows for cases where effort varies positively with wages.

We can now compute the aggregate wage. Equations (1) and (7) imply that $w_t(i) = w_t \; \forall i$, and the optimal aggregate wage is

$$w_t = w_t(i) = \left[\frac{\phi_2}{1-\psi-\phi_3} \left(\frac{1}{1-N_t} \right)^{\psi} + \frac{\phi_0 - \phi_2 - \phi_3}{1-\psi-\phi_3} \right]^{1/\psi} . \tag{8}$$

For w_t to be defined, we need to make one of the following assumptions:

Assumption 1. $1 - \psi - \phi_3 > 0$.

Assumption 2. $1 - \psi - \phi_3 < 0$ and $\phi_0 - \phi_3 < 0$.

Under Assumption 1, the real wage is defined for any $N_t \in (\bar{N}, 1)$ with $\bar{N} = 1 - (-\phi_2/(\phi_0 - \phi_2 - \phi_3))^{1/\psi}$ if $\phi_0 - \phi_2 - \phi_3 < 0$ and $\bar{N} = 0$ otherwise. The real wage is an increasing function of the employment rate. Under Assumption 2, it is defined for any $N_t \in (0, \bar{N}) \subset (0, 1)$. In that case, the real wage is a decreasing function of the employment level. If neither Assumption 1 nor Assumption 2 holds, then the real wage is not defined. It is interesting at this stage to remark the role played by the parameter ψ. When $\psi = 0$, i.e., the effort function is logarithmic, Assumption 1 is not very restrictive. Indeed, ϕ_3 is always below one, reflecting that the wage externality alone cannot overwhelm the direct effect of the firm's wage on effort. When ψ is positive, the story is different. The joint forces of the externality (ϕ_3) and the high substitution in the effort function (ψ) may in fact reverse the positive relationship between wages and employment. In the rest of the paper we suppose that Assumption 1 holds.

We now derive the optimal price setting by the intermediate firm. At each time a fraction $1 - \xi$ of firms sets a new price $p_t^\star(i)$. This price still prevails in period s with probability ξ^{s-t}. Nominal profits at time s are

$$\Pi_s(i) = p_t^\star(i) y_s(i) - w_s P_s n_s(i) = p_t^\star(i) y_s(i) - w_s P_s [y_s(i)]^{1/\alpha}[1/e_s(i)]$$

with $n_s(i) = [y_s(i)]^{1/\alpha}[1/e_s(i)]$. The firm maximizes the discounted flow of expected real profits, multiplied by the marginal utility of consumption $1/C_s$. We use the equilibrium conditions on the final good market $Y_s = C_s$ to write the objective of the firm:

$$\mathrm{E}_t \sum_{s=t}^{\infty} \frac{\Pi_s(i)}{P_s Y_s} = \mathrm{E}_t \sum_{s=t}^{\infty} (\beta\xi)^{s-t} \left(\frac{p_t^\star(i) y_s(i)}{P_s Y_s} - \frac{w_s}{Y_s} [y_s(i)]^{1/\alpha}[1/e_s(i)] \right).$$

Replacing $y_s(i)$ and $e_s(i)$ by their values from equations (4) and (7), and computing the first-order condition for a maximum in $p_t^\star(i)$, we obtain

$$(p_t^\star(i))^{\frac{1-\alpha\theta}{\alpha(1-\theta)}} \mathrm{E}_t \sum_{s=t}^{\infty} (\beta\xi)^{s-t} P_s^{\theta/(1-\theta)} = \frac{1}{\alpha\theta\phi_1} \mathrm{E}_t \sum_{s=t}^{\infty} (\beta\xi)^{s-t} \frac{w_s^{1-\psi}}{Y_s} Y_s^{1/\alpha} P_s^{1/(\alpha(1-\theta))}. \quad \textbf{(9)}$$

The optimal price $p_t^\star(i)$ determined by this equation does not depend on i. All firms that set an optimal price at time t choose the same price $p_t^\star(i) = p_t^\star$.

Given that a fraction $1 - \xi$ of firms set a new price each year, the average price level given in (5) follows:

$$P_t^{\theta/(\theta-1)} = (1-\xi)(p_t^\star)^{\theta/(\theta-1)} + \xi P_{t-1}^{\theta/(\theta-1)}. \quad \textbf{(10)}$$

Following Yun (1996), aggregate output can be written as a function of aggregate inputs by

$$Y_t = \left(\frac{X_t}{P_t}\right)^{1/(1-\theta)} \left(\phi_1 w_t^{\psi} N_t\right)^{\alpha} \text{ with } N_t = \int_0^1 n_t(i)\, di \tag{11}$$

and

$$X_t^{1/(\theta-1)} = (1-\xi)(p_t^{\star})^{1/(\theta-1)} + \xi X_{t-1}^{1/(\theta-1)}. \tag{12}$$

4. LONG-RUN UNEMPLOYMENT

At steady state, all prices are equal, and output is given [from (11)] by $Y = \left(\phi_1 w^{\psi} N\right)^{\alpha}$. All firms are now alike, so that $w = \nu\alpha Y/N$. The optimal price-setting rule (9) leads to $\nu = \theta$. Using these results together with equation (8), the steady state employment rate N satisfies

$$(\theta\alpha)^{\frac{1}{1-\alpha\psi}} \phi_1^{\frac{\alpha}{1-\alpha\psi}} N^{\frac{-(1-\alpha)}{1-\alpha\psi}} = \left[\frac{\phi_0-\phi_2-\phi_3}{1-\psi-\phi_3} + \frac{\phi_2\left(\frac{1}{1-N}\right)^{\psi}}{1-\psi-\phi_3}\right]^{1/\psi}. \tag{13}$$

The left-hand side decreases monotonically from $+\infty$ to $(\theta\alpha)^{1/1-\alpha\psi}\phi_1^{\alpha/1-\alpha\psi}$ as N goes from 0 to one (Assumption 1). The right hand side increases monotonically from $[\frac{\phi_0-\phi_3}{1-\psi-\phi_3}]^{1/\psi}$ to $+\infty$ as N goes from 0 to one. From these properties we can deduce that there is always a unique solution to equation (13).

PROPOSITION 2. *Under Assumption 1, there is a unique steady state employment rate N that satisfies equation (13). N is a positive function of competitiveness θ. It is a negative function of effort sensitivity to employment ϕ_2. If $\phi_0 \geq 1-\psi$, it is a negative function of the strength of the wage externality ϕ_3.*

Under Assumption 1, equation (13) can be interpreted within the usual textbook WS–PS framework. The left-hand side represents the PS curve (price-determined real wage) and is a decreasing function of N. The right-hand side represents the WS curve (wage-setting curve); it is increasing in N and represents the real wage underlying the efficiency wage setup. The PS curve depends on competitiveness on the product market θ and on ϕ_1, which directly influences labor productivity through the level of effort.

A rise in competitiveness reduces the markup of firms, shifts the PS curve to the right, increases the level of employment, and reduces unemployment. A rise in effort sensitivity to employment shifts the WS curve to the left, which lowers employment. When externalities are strong (ϕ_3), the WS curve is higher and unemployment is higher too.

5. REAL WAGE RIGIDITY

We can now define a concept of real rigidity as being the inverse of the sensitivity of wages to employment. Log-linearizing the wage equation (8) around a steady state (w, N), we find

$$\hat{w}_t = \frac{\phi_2}{1-\psi-\phi_3}\left(\frac{N(1-N)^{-1-\psi}}{w^{\psi}}\right)\hat{N}_t = \underbrace{\frac{\phi_2 N(1-N)^{-1-\psi}}{(\phi_0-\phi_2-\phi_3)+\phi_2\left(\frac{1}{1-N}\right)^{\psi}}}_{\equiv\Omega}\hat{N}_t, \tag{14}$$

where hatted variables denote deviations from steady state. Then Ω is the sensitivity to employment and $1/\Omega$ is real wage rigidity.

PROPOSITION 3 (Real Wage Rigidity). *Under Assumption 1, at a given employment rate, real wage rigidity* $(1/\Omega)$ *decreases with the relative sensitivity of effort to employment* ϕ_2*. It decreases with the relative importance of the externality* ϕ_3.

Real rigidity decreases with ϕ_2: if ϕ_2 is small, unemployment affects effort very slightly, and wages do not need to be changed much to respond to changes in market tightness. Real rigidity decreases with ϕ_3: if the externality is large, spillover effects between firms are important, which act as a multiplier for the effect on the aggregate wage of small changes in employment.

6. INFLATION PERSISTENCE

To study inflation persistence, we log-linearize the model (see the Appendix) and study how monetary shocks persist in the price system.

PROPOSITION 4. *After log-linearization around the steady state, the solution to equation (9) is*

$$\hat{P}_t - \hat{P}_{t-1} = \rho(\hat{P}_{t-1} - \hat{P}_{t-2}) + (1-\rho)(\hat{M}_t - \hat{M}_{t-1}). \tag{15}$$

Inflation persistence ρ *increases with the Calvo probability* ξ *and increases with the degree of real wage rigidity* $1/\Omega$*. At a given rigidity* $1/\Omega$*, it also increases with* ψ*, the degree of substitution between wage and employment in the effort function.*

Proof. See Appendix.

Proposition 4 says that when wages and employment are highly substitutible in the effort function, effort co-moves with wages (equation (7)), the influence of the wage on the marginal cost is compensated for by changes in effort, and inflation is more persistent.

Table 1. Numerical illustration

ψ	ρ given $1/\Omega$	N	$1/\Omega$	ρ	σ_l
0.00	0.690	0.95	0.158	0.690	5.67
0.10	0.755	0.94	0.157	0.755	2.84
0.20	0.795	0.93	0.162	0.796	1.55
0.30	0.822	0.91	0.182	0.826	0.79
0.40	0.844	0.88	0.252	0.853	0.26
0.50	0.862	0.73	0.740	0.886	−0.26

Illustration

Assume that one period is one quarter. Take the following values for the parameters: $\phi_0 = 1$ (so that Lemma 1 can be applied), $\alpha = 2/3$, $\phi_1 = 114.25$ (to obtain $N = 0.95$ for $\psi = 0$), $\phi_2 = 1/5$, $\phi_3 = 4/10$, $\theta = 9/10$, $\beta = 99/100$, $\xi = 0.75$. Assumption 1 holds.

We first compute inflation persistence ρ at a given wage rigidity ($1/\Omega = 0.158$) for different values of ψ. We observe from the second column of Table 1 that inflation persistence increases from 0.690 to 0.862 as ψ goes from 0 to 0.5, which illustrates Proposition 4. Second, we let the wage rigidity $1/\Omega$ adapt to variations in ψ. The next three columns report the values of long-run employment, real wage rigidity, and inflation persistence. For this calibration, the endogenous changes in long-run employment and wage rigidity reinforce the effect of the parameter ψ on inflation persistence.

The rise in inflation persistence obtained after an increase in ψ from 0 to 0.3 is far from negligible: it corresponds to an increase in the half-life of inflation after a permanent money shock from 1.87 to 3.63 quarters. In the extreme case where ψ goes to 0.5, the half-life goes to 5.73 quarters.

Finally, we may compare the expression computed for ρ to the expression computed by Bénassy (2005) for this parameter under the assumption of a Walrasian labor market. From this exercise we conclude that the fair wage model generates as much inflation persistence as the traditional competitive labor market model if

$$\frac{(1-\psi)\Omega}{\psi\Omega + 1} = \sigma_l + \alpha,$$

where σ_l is the inverse of the elasticity of intertemporal substitution of labor in the model with Walrasian labor market. We report in the last column of Table 1 the values of σ_l required to produce the same first-order autocorrelation parameter ρ for inflation with the competitive labor market model as obtained with the fair wage model. We observe that σ_l drops rapidly below 2, which is often viewed as the lower extremum in the range of estimates for this parameter from microeconomic studies [for a survey, see Card (1994)]. This proves that the result obtained by Kiley (1997) that the efficiency wage assumption can never produce more endogenous price rigidity than the competitive labor market assumption can be circumvented by considering a more general effort function, with high substitution between wages and unemployment (ψ) and/or large enough wage externalities (ϕ_3).

7. CONCLUDING REMARKS

In this paper, we considered efficiency wages along the gift-exchange argument by allowing effort of the workers to depend on the workers' own wage, the average alternative wage, and the employment rate in the economy. Contrary to the previous studies, which selected an effort function implying constant effort over the business cycle, our effort specification is sufficiently general to allow effort to vary over the business cycle. We showed that, when effort is variable, wage fluctuations are partially compensated for by endogenous effort fluctuations, so that the sensitivity of the marginal cost to output and employment variations is decreased. This mechanism decreases the need for nominal price rigidity to explain the observed low elasticity of inflation to output variations.

We have shown that inflation persistence can be increased in fair wage models without introducing past wages into the effort function as Danthine and Kurmann (2004) did. The question of whether the two mechanisms, past wages and CES effort function, should be viewed as substitutes or complements in future work using the fair wage approach could be settled by the estimation of a fully-fledged DSGE model incorporating these two alternatives.

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APPENDIX

A.1. PROOF OF PROPOSITION 4

Log-linearizing price equations around the steady state equations (10) and (12) yields

$$\hat{P}_t = (1-\xi)\hat{p}^{\star}_t + \xi\hat{P}_{t-1} \text{ and } \hat{X}_t = (1-\xi)\hat{p}^{\star}_t + \xi\hat{X}_{t-1}, \tag{A.1}$$

which implies that $\hat{P}_t = \hat{X}_t$ because their initial conditions are the same. Log-linearizing the optimal price equation (9) yields

$$\frac{(1-\alpha\theta)}{(\alpha(1-\theta))}\hat{p}^{\star}_t + (1-\beta\xi)\sum_{s=t}^{\infty}(\beta\xi)^{s-t}\mathrm{E}_t\frac{\theta}{1-\theta}\hat{P}_s$$

$$= (1-\beta\xi)\sum_{s=t}^{\infty}(\beta\xi)^{s-t}\mathrm{E}_t\left((1-\psi)\hat{w}_s - \hat{Y}_s + \frac{1}{\alpha}\hat{Y}_s + \frac{1}{\alpha(1-\theta)}\hat{P}_s\right). \tag{A.2}$$

Wages follow (14):

$$\hat{w}_t = \Omega\hat{N}_t. \tag{A.3}$$

Effort follows $\hat{e} = \psi\hat{w}$. The output equation leads to

$$\alpha(\hat{e}_t + \hat{N}_t) = \alpha\,(1+\psi\Omega)\,\hat{N}_t = \hat{Y}_t - \frac{1}{1-\theta}(\hat{X}_t - \hat{P}_t) = \hat{Y}_t. \tag{A.4}$$

Hence, the output changes linked to the difference between $\hat{X}_t$ and $\hat{P}_t$ disappear in the linearized version of the model, reflecting that this discrepancy has only a second-order effect. Finally, the equilibrium on the goods market $Y_t = C_t$ together with the first-order condition (2) leads to

$$\hat{Y}_t = \hat{M}_t - \hat{P}_t. \tag{A.5}$$

Starting from (A.2) and replacing $\hat{w}_s$ with its value from (A.3), $\hat{N}_t$ with its value from (A.4), and $\hat{Y}_t$ with its value from (A.5), we get

$$\hat{p}^{\star}_t = \frac{1-\beta\xi}{1-\alpha\theta}\sum_{s=t}^{\infty}(\beta\xi)^{s-t}\mathrm{E}_t\left(h(\hat{M}_s - \hat{P}_s) + (1-\alpha\theta)\,\hat{P}_s\right),$$

where $h = (1-\theta)(1-\alpha+\frac{(1-\psi)\Omega}{\psi\Omega+1})$. The equation for $\hat{p}^{\star}_t$ is equivalent to

$$\hat{p}^{\star}_t = \beta\xi\mathrm{E}_t\hat{p}^{\star}_{t+1} + \frac{1-\beta\xi}{1-\alpha\theta}\left(h(\hat{M}_t - \hat{P}_t) + (1-\alpha\theta)\,\hat{P}_t\right).$$

We now replace $\hat{p}^{\star}_t$ and $\hat{p}^{\star}_{t+1}$ with their values from (A.1), $\hat{p}^{\star}_t = (\hat{P}_t - \xi\hat{P}_{t-1})/(1-\xi)$ and $\beta\xi\mathrm{E}_t\hat{p}^{\star}_{t+1} = \beta\xi(\mathrm{E}_t\hat{P}_{t+1} - \xi\hat{P}_t)/(1-\xi)$:

$$(\hat{P}_t - \xi\hat{P}_{t-1}) = \beta\xi(\mathrm{E}_t\hat{P}_{t+1} - \xi\hat{P}_t) + (1-\xi)\frac{1-\beta\xi}{1-\alpha\theta}\left(h(\hat{M}_t - \hat{P}_t) + (1-\alpha\theta)\,\hat{P}_t\right),$$

which simplifies into $a_1(\hat{P}_t - \hat{M}_t) + a_2(\hat{P}_t - \hat{P}_{t-1}) + a_3(\hat{P}_t - \mathrm{E}_t\hat{P}_{t+1}) = 0$ with

$$a_1 = (1-\xi)\frac{1-\beta\xi}{1-\alpha\theta}h, \quad a_2 = \xi, \quad a_3 = \beta\xi.$$

Using the method of undetermined coefficients, the solution is of the form

$$\hat{P}_t = \rho \hat{P}_{t-1} + b \sum_{j=0}^{\infty} \eta^j \mathrm{E}_t \hat{M}_{t+j} \tag{A.6}$$

with

$$b = \frac{a_1}{a_1 + a_2 + a_3(1+\rho)}, \quad \eta = \frac{a_3}{a_1 + a_2 + a_3(1+\rho)}.$$

ρ is the stable root of $R(\rho) = a_3\rho^2 - (a_1 + a_2 + a_3)\rho + a_2 = 0$. Because the money supply shock is permanent (μ_t is white noise), $\hat{M}_{t+j} = \hat{M}_t$ for all $j \geq 0$, we can write (A.6) as

$$\hat{P}_t - \hat{P}_{t-1} = \rho(\hat{P}_{t-1} - \hat{P}_{t-2}) + \frac{b}{1-\eta}(\hat{M}_t - \hat{M}_{t-1}).$$

Using the definitions of b and η and the fact that $R(\rho) = 0$, we obtain

$$\hat{P}_t - \hat{P}_{t-1} = \rho(\hat{P}_{t-1} - \hat{P}_{t-2}) + (1-\rho)(\hat{M}_t - \hat{M}_{t-1}),$$

which is equation (15) of the main text. If ψ increases at a given Ω, or if Ω decreases, a_1 decreases, which raises inflation persistence ρ through $R(\rho) = 0$.