EFFICIENT OVER-MANNING

By DAVID DE LA CROIX* and ERIC TOULEMONDE†

* National Fund for Scientific Research (Belgium) and University of Louvain, IRES, Place Montesquieu 3, B-1348 Louvain-la-Neuve, Belgium
† University of Namur, Faculty of Economics, Rempart de la Vierge 8, B-5000 Namur, Belgium

1. Introduction

Over the last 15 years, much work has been done on the analysis of non-competitive wage formation and, in particular, on models in which unions and firms bargain over wages (right-to-manage models) or over both wages and employment (‘efficient’ bargaining, see Leontief 1946; and McDonald and Solow 1981). In these traditional union models, firms are generally assumed to be price-takers in the goods market. Recently, the models of bargaining have been extended by Svejnar (1986) and Dowrick (1989) to the case where the firm has some market power in its goods market. The interest in coupling bargaining models with monopolistic price formation is to analyse how two different sources of non-competitive behaviour interact. One of the most important results is the following: in the ‘efficient’ bargaining framework, in which efficient contracts covering both wages and employment are signed, it is shown that the price-cost margin is influenced by union preferences and by union power; in the face of a more powerful union, the firm has to lower its price in order to honour its commitment regarding the employment level. This framework provides theoretical foundations to the empirical literature of price-cost margin determination under unionism (see Cowling and Waterson 1976; Cowling and Mohlo 1982; Veugelers 1989; Conyon 1992; and Bughin 1993) and can also be extended to study open economy topics as in Mezzetti and Dinopoulos (1991).

The result of Svejnar (1986) and Dowrick (1989) is derived under the constraint that the firm produces on its production frontier. Let us call this case ‘constrained efficient bargaining’. In this paper, we show that, if the union is risk averse, the constrained efficient bargaining solution may lead to a negative marginal revenue. In Fig. 1, we have represented this kind of outcome for the case of a linear production function (to be able to represent the labour market and the goods market on the same graph). Isoprofit curves ($\pi = \tilde{\pi}$) attain their maximum on the marginal revenue curve ($MR$). The constrained efficient outcome must lie on the contract curve, i.e., at a tangency point between isoprofit curves and union’s indifference curves. In the presence of a risk averse union, the contract curve passing through A and B is positively sloped. The outcome A implies a positive marginal revenue, while at outcome B the marginal revenue is negative.

In this paper, we find the conditions under which constrained efficient
bargaining leads to outcome $B$ and we show that it can be optimal for both agents to produce less than in $B$ with more workers, therefore, producing below the production frontier. In this case, the so-called constrained efficient solution is not efficient in the sense that it is Pareto-dominated by an unconstrained efficient solution. We will show that this unconstrained efficient bargaining implies over-manning.\footnote{The terms 'featherbedding' and/or 'over-manning' are used in the literature to describe a situation where the marginal revenue product of labour falls short of wages. Here, we use 'over-manning' to describe a situation where the marginal revenue of employment is zero.}

In the second section, we derive the conditions under which the constrained efficient solution implies negative marginal revenue. In the third section, we demonstrate that this constrained outcome is false in the sense that it is not Pareto-efficient. Section 4 concludes.

2. The constrained efficient bargaining solution

Let us first introduce the following assumptions:

(i) The utility of the union is taken from Veugelers (1989) and Pencavel (1991) and is defined over wages and employment; $u(l, w) = l(w - \bar{o})^{\nu} / \nu$ for $w \geq \bar{o}$. It is a simplified Stone-Geary utility function, where $\nu$
measures the concavity of the utility function with respect to net income and where $\bar{\omega}$ is a minimum acceptable wage. For $v < 1$, the union is risk averse ($u_{ww} < 0$).

Our results are also derived for the utilitarian-union case in Appendix 1.

(ii) The firm uses labour $l$ to produce $q$ with a production function $f(l)$ with decreasing marginal returns ($f_l > 0$, $f_{ll} < 0$).

(iii) The firm faces a downward-sloping demand function. $p(q)$ is the inverse of this function with $p_q < 0$.

(iv) Firm’s profit is $\pi = pq - wl$.

The constrained efficient bargaining solution is characterised by a vector $w^*$, $l^*$, $p^*$, $q^*$ such that

$$w^*, l^* = \arg\max_{w, l} \left[ \frac{l(w - \bar{\omega})^v}{v} \right]^\beta \left[ p(q)q - wl \right]^{1 - \beta}$$

s.t. $w \geq \bar{\omega}$ and $l \geq 0$

$$q^* = f(l^*)$$

$$p^* = p(q^*)$$

Wages and employment are determined by the maximisation of the asymmetric Nash product between the utility function of the union and the profit of the firm. $\beta$ represents the weight of the union in the bargaining ($0 < \beta < 1$). For simplicity, both fall-back levels are assumed to be zero.

Denoting the elasticity of $x$ to $y$ by $\eta_{x,y}$, let us now derive an important property of the constrained efficient solution:

**Lemma 1** With the constrained efficient bargaining

$$\eta_{p^*, q^*} < -1 \iff (v < 1) \quad \text{and} \quad \left( w^* > \frac{\bar{\omega}}{1 - v} \right)$$

**Proof** The first-order condition for wages gives

$$\beta \frac{v}{w^* - \bar{\omega}} + (1 - \beta) \frac{-l^*}{p^*q^* - w^*l^*} = 0$$

The first-order condition for labour gives

$$\beta \frac{1}{l^*} + (1 - \beta) \frac{p_q f_l q^* + p f_{lq} - w^*}{p^*q^* - w^*l^*} = 0$$

using both conditions leads to

$$\left( \frac{w^* - \bar{\omega}}{v w^*} - 1 \right) + (\eta_{p^*, q^*} + 1) \eta_{f_{l^*}, l^*} \frac{p^*q^*}{w^*l^*} = 0$$

(1)
which implies that
\[ \eta_{p^*, q^*} < -1 \iff (v < 1) \quad \text{and} \quad \left( w^* > \frac{\bar{\omega}}{1 - v} \right) \]

\( \eta_{p^*, q^*} < -1 \) is equivalent to negative marginal revenue.\(^2\) Lemma 1 says that, with constrained efficient bargaining, the marginal revenue is negative if and only if the union is risk averse and the wage is larger than \( \bar{\omega}/(1 - v) \). This case may arise for instance when the union is so powerful and so interested in employment that the negotiated employment is high enough to provoke a negative marginal revenue (as in point B of Fig. 1). If the union is risk neutral or a risk lover, the contract curve is vertical or negatively sloped, and the marginal revenue is always positive.

The case with negative marginal revenue is ignored in the literature. For instance, in eq. (7) of Svejnar (1986), a strongly risk averse union (\( \delta \) small) may imply negative marginal revenue, in which case, as we will show, Svejnar's bargaining solution is Pareto-dominated by an unconstrained solution.

3. Unconstrained efficient bargaining

Let us now select the subset of constrained efficient solutions in which \( w^* > \bar{\omega}/(1 - v) \) and \( v < 1 \) and try to improve the welfare of the agents.

**Proposition 1** If the constrained efficient bargaining solution is such that \( v < 1 \) and \( w^* > \bar{\omega}/(1 - v) \), its outcome is Pareto-dominated by a situation where the firm produces less with more workers, being therefore at a point in the interior of its production possibility set.

**Proof** By Lemma 1, if \( v < 1 \) and \( w^* > \bar{\omega}/(1 - v) \) we know that marginal revenue is negative at this equilibrium. In this case profit can be increased by reducing the output of the firm at given level of wages and employment (the union’s utility level does not change). Moreover, the benefit from reducing the output of the firm may be transferred to the union through an increase in employment; let us decrease the output by a small amount (increasing the price) and let us engage more workers. Let us assume that wages are set in order to keep profits constant and see if the utility of the union is higher. Total differentiation of profits \( (d\pi = q^* \, dp + p^* \, dq - l^* \, dw - w^* \, dl = 0) \) implies
\[ (\eta_{p^*, q^*} + 1 \, p^* \, dq - l^* \, dw - w^* \, dl = 0) \]

Since the constrained efficient bargaining solution we consider \( \eta_{p^*, q^*} < -1 \), the expression \( (\eta_{p^*, q^*} + 1 \, p^* \, dq \) is positive if \( dq < 0 \). The implies that
\[ l^* \, dw + w^* \, dl > 0 \quad \text{if} \quad dq < 0 \]

\( \quad \text{(2)} \)

\(^2\) If the demand curve is such that \( \eta_{p, q} \) is constant and below \(-1\) it is a common result that the firm does not have any incentive to produce. If the elasticity is variable, the firm never has interest to determine \( q^* \) such that \( \eta_{p, q} < -1 \) (even if union has interest) because reducing \( q^* \) would improve its profit.
From (2), \( dw > -(w^*/l^*) \, dl \). The variation of the utility of the union (\( du = u_{w^*} \, dw + u_{l^*} \, dl \)) is

\[
du > -\frac{w^*}{l^*} u_{w^*} \, dl + u_{l^*} \, dl
\]

Or equivalently

\[
du > \left( -\frac{vw^*}{w^* - \bar{w}} + 1 \right) u_{l^*} \, dl \tag{3}
\]

Which implies that \( du > 0 \) if \( dl > 0 \) since \( w^* > \bar{w}/(1 - \nu) \).

Therefore, under the conditions of Proposition 1, it is possible to improve the firm's profit and/or the utility of the union by relaxing the requirement of being on the production frontier. An easy way of finding one of the Pareto-superior solutions starting from the constrained efficient outcome is to maximize the utility of the union subject to the fact that profit remains the same. This solution is represented in Fig. 2.\(^3\) Firstly, it can be shown that the isoprofit line changes when we relax the requirement of producing on the production frontier. When marginal revenue is negative, the unconstrained isoprofit line is computed at given output and price, is higher than in the constrained case, and is no

---

\( ^3 \) In Fig. 2, there is no longer a one-to-one relationship between \( q \) and \( l \), implying that the definition of the axes is different from the one in Fig. 1.
longer concave. Secondly, production takes place at a point $C$, where marginal revenue is zero. Wages and employment are located at $C'$. The above solution amounts to assuming that all the benefits from relaxing the constraint of being on the production frontier go to the union. This is only one example of a situation that Pareto-dominates the constrained efficient solution. More work should be done to understand how such a solution may take place and how the surplus can be shared among the players. An alternative and probably a more intuitive way of treating the problem could be to impose the constraint $q \leq f(l)$ on the Nash product and to maximize the corresponding Lagrangean with respect to wages, employment and output. In this case, two problems arise: (i) we need to calculate the Lagrange multiplier of the inequality constraint as a function of the parameters only, in order to know if the unconstrained case applies or not; and (ii) this unconstrained solution does not necessarily Pareto-dominate the constrained solution. This could cause a dilemma if the agents were able to enforce a solution on the production frontier. Another way of presenting the problem could be to recalculate an unconstrained Nash product taking the constrained efficient outcome as the fall-back. A third way is to let the firm alone choose its output level.

4. Conclusion

In this paper it has been shown that, under certain conditions, the standard ‘efficient’ bargaining outcome between a union and a monopolistic firm is not efficient and is Pareto-dominated by an unconstrained solution where the firm engages more workers than necessary for the production. One interesting point is that our unconstrained framework could give an explanation for productive inefficiencies or persistent over-manning in unionised firms. Contrary to Osano (1990), this result does not depend on the existence of asymmetric information and uncertainty. Moreover, if our framework is also applicable to public firms, the policy conclusions linked with the reduction of over-manning (see e.g. Avila 1986) could be altered, because over-manning is mutually optimal in our model.

Efficient over-manning appears when unions are strong and risk averse. These union characteristics fit casual observation and what we know about the data (see Pencavel 1991). Moreover, over-manning seems to be a frequent practice in the real-life: conditions of work including an inordinately high number of workers per machine are often contracted (for example, the struggle of unions to maintain three pilots in the cockpit of the new airbus). The current negotiations in France, which aim at moderating wage increases and at hiring the unskilled, long-term unemployed, may also lead to over-manning.

It is also worth noting that the properties of the unconstrained efficient solution are quite different from those of constrained ‘efficient’ bargaining and do merit further analysis. In particular, the result of this paper might be of importance for some of the many tests that have been undertaken on whether there is efficient bargaining or the firm unilaterally determines the employment level. Our framework implies a regime switch at the point where marginal
revenue is zero. At this point, the slope of the contract curve changes, involving strong non-linearities. This makes inconsistent the estimation of the parameters of the contract curve in the majority of the previous studies (summarised in Pencavel 1991) if the condition of Proposition 1 is satisfied. Moreover, these tests are often bedeviled by the observational equivalence of the estimating equation. The presence of the regime switch stressed by our work may suggest a way out of this.

ACKNOWLEDGEMENTS

We are grateful to P. Dehez, R. Deschamps, L. Gevers, F. Maniquet, and D. Weiserbs for their comments on an earlier draft of this paper. We also thank two anonymous referees for their constructive suggestions for sharpening the presentation.

The financial support of the ‘Pôles d’attraction interuniversitaires’ (Belgian State, Prime Minister’s Office) is gratefully acknowledged.

REFERENCES


APPENDIX

The utilitarian-union case

The utility of the union takes the following form

\[ u(l, w) = lh(w) + (m - l)h(r) \]

where \( h(w) \) is the utility of the individual member \( (h_w > 0, h_{ww} < 0) \), \( m \) is the union membership, and \( r \) is the outside income.
Lemma 1 With the constrained efficient bargaining

\[ \eta_{p^*, q^*} < -1 \iff \frac{h_w(w^*)w^*}{h(w^*) - h(r)} < 1 \]

Proof Using the first order conditions of the constrained bargaining problem, one gets

\[ \left( \frac{h(w^*) - h(r)}{h_w(w^*)w^*} - 1 \right) + (\eta_{p^*, q^*} + 1) \eta_{p^*, q^*} \frac{p^*}{w^*} q^* w^* = 0 \]  \hspace{1cm} (1b)

which proves the lemma. \qed

Proposition 1 If the constrained efficient bargaining solution is such that \( h_w(w^*)w < h(w^*) - h(r) \), its outcome is Pareto-dominated by a situation where the firm produces less with more workers, being therefore below its production frontier.

Proof Following the same procedure as in the main text, one ends with

\[ du > \left( - \frac{h_w(w^*)w^*}{h(w^*) - h(r)} + 1 \right) u_r \, dl \]  \hspace{1cm} (3b)

which implies that \( du > 0 \) if \( h_w(w^*)w^* < h(w^*) - h(r) \). \qed