
RAOUF BOUCEKKINE
DAVID DE LA CROIX
DOMINIQUE PEETERS

Economic growth, understood as an increase in the gross domestic product over a long period of time, is a contemporaneous phenomenon. As clearly explained in Maddison (2001), humanity was caught in a long-lasting trap of economic stagnation until the nineteenth century. This premodern period was accompanied by even more dire demographic conditions: according to Maddison’s estimates, the size of the world population remained almost constant in the first millennium CE, and life expectancy at birth was below 50 years until the beginning of the twentieth century.

For European countries, premodern times are traditionally associated with successive mortality crises related to wars, famines, and epidemics. Essays by Platt (1996) on England and Herlihy (1997) on Western Europe have stressed the structural changes induced by the bubonic plague in the late medieval ages. However, since Wrigley and Schofield (1989), the view that a unitary premodern demographic regime preceded the industrial revolution has been seriously undermined, especially in the case of England. Births, marriages, and mortality in England sharply fluctuated in the two centuries before the industrial revolution, and the variations cannot be fully attributed to mortality crises. While mortality decline contributed significantly to population growth in England, its effect was clearly overshadowed until 1820 by rising fertility driven notably by increased nuptiality. Another major conclusion to draw from Wrigley and Schofield’s careful empirical work on England is that historical mortality crises show a low correlation with food scarcity and, thus, with the standard of living of the population.
Yet several issues remain unresolved. Even if we agree with the demographic scenario above, the ultimate challenge is to incorporate it into a more global analysis of the transition to modern growth and to identify the economic and demographic mechanisms involved. The demographic determinants have been increasingly emphasized in the development literature (see Lee 1979 and McNicoll 2003). Unified growth theory, a recent stream of economic growth literature, surveyed by Galor (2005), emphasizes the role of demographic change in the transition to the modern economic growth regime. This chapter highlights the role of population density in the economic development of England during the period 1530–1860. Because industrial revolutions rely on innovation and the adoption of new technologies requires a certain density of educated people, population density and literacy are likely to be key variables in the development process. Indeed, the rise in literacy and education in the pre-industrial era may have initiated the process leading to the English industrial revolution. Cipolla (1969) argues that improvements in literacy favored the industrial revolution in more than one way. Literacy increased the ranks of skilled workers in those fields in which such workers were specifically required, and, more generally, it made more people adaptable to new circumstances and receptive to change. In times of rapid technological progress, literate workers assimilate new ideas more readily.

Higher educational achievement might have been triggered by several economic and demographic factors. We distinguish three of them. First, technological progress increased labor productivity and wage rates in the modern sector and thereby increased the return to investment in education. Facing better income prospects in this sector, households would invest in education to benefit from the higher returns. This view is defended, among others, by Hansen and Prescott (2002) and Doepke (2004). Second, improvement in adult longevity is another explanation for the rise in literacy. Although, according to Wrigley and Schofield (1989), mortality decline is not the main engine of population growth in pre-industrial England, increased longevity is potentially an important determinant of literacy: longer lives increase the returns to investment in education, inducing longer schooling, according to the well-known Ben-Porath (1967) mechanism. Recent papers by Boucekkine, de la Croix, and Licandro (2003) and Nicolini (2004) argue that lower mortality induced higher investment in human capital and physical capital at the time of the industrial revolution, paving the way to future growth. Clark (2005a), however, remarks that, if demographic transition and industrial revolution are the two great forces that lead to modern growth, the latter did not lead to fertility decline until over 100 years after its onset. He explores the difficulties in trying to uncover the underlying connection between them.

A third possibility is drawn from various authors who stress that the rising density of population may have played a role in fostering the rise in literacy and education. Higher density can lower the cost of education.
through facilitating the creation of schools. Fujita and Thisse (2002) provide a textbook treatment of this effect. A representative empirical study by Ladd (1992) shows that a small increase in density lowers the costs of providing services, at least at very low levels of population density. Externalities can also be generated by denser population. For Kremer (1993), high population density spurs technological change. Galor and Weil (2000) and Lagerloef (2003) argue for “population-induced” technological progress. Population needs to reach a threshold for productivity to accelerate.

In this chapter we propose a new framework to disentangle the effects of three factors on literacy and economic growth in England: technological progress, mortality decline, and population density. We look at the linkages between literacy, school establishment, and income growth and evaluate the role of each. In our model, the length of schooling is chosen by individuals who maximize lifetime income, which depends on future wages, longevity, and the distance to the nearest school. Then, the number and location of educational facilities is determined, either chosen by the optimizing state or following a free entry process (market solution). Higher population density makes it optimal to increase school density, opening the possibility to attain higher educational levels.

In our model, two sectors coexist: a traditional sector with constant productivity, and a modern sector with exogenously rising productivity. The remuneration of workers in the modern sector depends on their human capital level. Therefore the transition to this modern sector depends on both technological evolution and education. A denser population induces a higher educational level, which promotes the transition to the modern sector.

Wrigley (1988) provides a more elaborate narrative of the industrial revolution. A traditional sector, called “organic economy,” based on agricultural goods, eventually evolves into a more productive sector, an “advanced organic economy,” thanks to animal power. Such a regime is not sustainable, however, because of a fixed land supply and decreasing marginal returns. England’s good fortune, argued Wrigley, was to have abundant coal resources, which made possible the transition to a “mineral-based economy,” in which industries (producing iron, pottery, or glass) could be operated without significant pressure on land, permitting an escape from decreasing returns. The mineral-based economy opened the door to a series of innovations (notably in energy and power production) that elevated productivity and real wages far above the levels allowed by the agricultural economy. Wrigley provides empirical support to his story, especially based on investment data over the period studied.

Our modeling of the traditional and modern sectors is much more stylized than Wrigley’s description. It captures, however, a central message in the latter: the transition to a modern economy represents primarily an escape from decreasing returns. While in Wrigley this transition is made possible by
the much broader set of technological opportunities allowed by the mineral-based economy, in our model it is additionally favored by human capital accumulation, itself boosted by increasing population density.

The next section sets forth the demographic, economic, geographic, and institutional structures of our theoretical model. The third section describes the data and the experimental methodology we use to disentangle the effects of the three aforementioned factors—technological progress, mortality decline, and population density—on literacy and growth. The fourth section displays the findings, and the final section offers concluding comments.

Theory

To assess the development mechanisms just outlined, we first build a theoretical model with the relevant demographic, economic, institutional, and geographic ingredients. The mathematical details, including rigorous proofs of the claims in this section, can be found in Boucekkine, de la Croix, and Peeters (2007).

The demographic structure

We consider an economy populated by overlapping birth cohorts. Individuals belonging to cohort \( t \), that is individuals born at date \( t \), have an uncertain life span: their probability of reaching age \( a \) is given by the survival function:

\[
m_t(a) = \frac{e^{\beta_t a} - \alpha_t}{1 - \alpha_t}.
\]

where \( \alpha_t \) and \( \beta_t \) are two numbers (for fixed \( t \)). This survival function was introduced by Boucekkine, de la Croix, and Licandro (2002). If \( \alpha_t \) and \( \beta_t \) satisfy \( \alpha_t > 1 \) and \( \beta_t > 0 \), then the survival function is concave, that is, the probability of death increases with age and there is a maximum age \( A_t \) that an individual can reach. This parameter configuration allows the function \( m_t(a) \) to accurately represent the empirical adult survival laws and has the advantage of being analytically tractable. The maximum age is obtained by solving \( m_t(A_t) = 0 \) and is equal to

\[
A_t = \frac{\log(\alpha_t)}{\beta_t}.
\]

Note that a higher longevity of individuals belonging to cohort \( t \) corresponds to larger \( \alpha_t \) and/or lower \( \beta_t \). Finally, for the sake of simplicity, we do not explicitly model fertility and instead assume the initial size of each cohort to be given exogenously. If that size, for the cohort born at \( t \), is \( \xi_t \) then its size at any subsequent time \( z \in [t, t + A_t] \) is given by \( \xi_t m_t(z - t) \), reflecting attrition through time. The demographic processes \( \alpha_t, \beta_t, \) and \( \xi_t \) for varying \( t \), are estimated using English data for the period 1530–1860.
The economic structure

To account for the role of technological advances in the transition to modern growth, we postulate two distinct production sectors in the economy, a traditional and a modern sector. The latter is subject to technological progress, inducing rising productivity over time (at a positive rate, say, $\gamma_t$ at time $t$), while the former has a constant productivity level. If workers are paid at their productivity level, as we postulate in our model, then the modern sector will become more attractive over time, eventually yielding a full transition to the modern sector. This way of modeling the transition follows Hansen and Prescott (2002).

Such a sharp transition is not realistic, however; the process is much more gradual and much less mechanistic than outlined above. To generate a more realistic picture, we need to account for human capital formation. Historically, human capital accumulation and its associated literacy improvements have taken place gradually, and this pace is likely to be crucial in determining the actual shape of the transition to modern growth. To incorporate this feature, we model both the supply and demand sides of human capital. The supply side, developed in the next section, builds on the idea that school creation depends on attendance rates, which in turn are determined by population density.

The demand side mechanism originates in a further difference between the two production sectors. Individuals working in the traditional sector have a productivity level, and thus a remuneration level, that are independent of their level of human capital. In contrast, the remuneration of workers in the modern sector is not only determined by their (exogenously) rising productivity, it is also determined by their level of human capital. Thus, there is a complementarity between human capital and technological progress in the development process: for technological innovations to be exploited to their full potential, skilled workers are imperative.

In particular, we take the view that technological progress and human capital interact in a multiplicative way, so that the remuneration of a given worker at time $t$ is the product of his or her human capital and technological progress, say $\exp(\gamma_t)$, and the stock of human capital available in the economy, say $H_t$:

$$Y_t = \exp(\xi_t) H_t.$$

This makes the development process much more complex, since human capital formation is costly. We assume that going to school involves a transportation cost, which is proportional to the distance to the nearest school, and the payment of tuition fees. From the English Schools Inquiry Commission (1868a) we learn that boys could attend a city school from distances up to
20 miles, and with travel times of more than one hour in the morning and in the evening (on foot, ponies, or donkeys). Concerning tuition fees, we know from historical surveys (see again the Schools Inquiry Commission) that schools were funded through income from an endowment and through fees paid by the students' parents. Fees were imposed in order to supplement the endowment, and parents were willing to pay fees, provided they were not excessive and the education provided was of acceptable quality.

Within such a framework, an individual may not find it optimal to go to school. The cost incurred during schooling time can only be paid back (via wages from the modern sector) after this time. There is no guarantee that lifetime earnings allowed by schooling net of school costs are superior to the lifetime revenue that could be directly extracted from the traditional sector. An individual in our framework might choose not go to school for many reasons. The reasons could be demographic. Where life expectancy is markedly low for cohort $t$ (which corresponds to a low parameter $\alpha_t$ and/or a high parameter $\beta_t$ in the model), the returns to schooling are likely to be discouraging, given the expected very short remuneration period. The reasons could be technological. The expected pace of technological progress, denoted by $\gamma_t$ in our model, might be too slow, which would also induce low returns to schooling. Finally, institutional reasons related to the organization and location of schools might yield the same outcome: the absence of schools in the neighborhood and/or prohibitively high tuition fees are very strong barriers to schooling.

As a consequence, the decision to attend school and the resulting schooling time depend on demographic, technological, geographic, and institutional conditions. These conditions change over time, and schooling decisions are therefore likely to vary from one cohort to another. Moreover, there is no reason to believe that all individuals in a cohort will make the same schooling decisions. To allow for within-cohort variation, we postulate that individuals in the same cohort may differ in their location and in their innate abilities. For simplicity, however, a given individual stays at his or her location permanently. In the pre-industrial era, the main reason for households to move was to reach regions with better employment opportunities or higher wages. In our theoretical model, the same technologies are available everywhere, hence the main migration engine is shut down. We postulate that innate abilities are distributed according to a unimodal distribution. We use the log-normal distribution in our experimental studies.

This completes the demand side of human capital. Within the same cohort, other things equal, only the most gifted and those located closest to schools will attend school. In other words, there exists a threshold value for innate ability, such that individuals with an ability above (or below) the threshold will go to school (or remain uneducated). Naturally, this threshold value of ability increases when tuition fees, the distance to the nearest
school, or the alternative remuneration in the traditional sector goes up. The threshold ability value is also sensitive to demographic and technological conditions: a higher life expectancy or faster technological progress should lower the threshold. For individuals above the threshold, the duration of schooling time can be longer or shorter depending on the same technological, demographic, and institutional conditions, for the same plausible reasons. Longer life expectancy, faster technological progress, or closer schools induce a longer duration of schooling and therefore a higher human capital level. And of course, other things equal, more-gifted individuals go to school for a longer time.

We now turn to the supply side of human capital, the school creation part of our theory.

The geographic and institutional structure

Location theory is a field of research that draws on economic geography and operations research. Its purpose is to model, formulate, and solve problems of siting facilities in order to supply goods and services to a spatially dispersed population. The recent survey by ReVelle and Eiselt (2005) gives a bird’s-eye view of the topic and its abundant literature, while the reader can refer to Daskin (1995), among others, for a more in-depth presentation. One of the core models of location theory is the Simple Plant Location Problem (in short SPLP), which can be formulated as follows. Assume a geographically dispersed population with known demands for a certain commodity that is made available at facilities to be created. Opening a facility involves incurring a fixed cost, while distributing the commodity entails transportation costs. The problem is to determine the number, locations, and respective market areas of the facilities in order to minimize total cost, defined as the sum of the transportation costs to the clients and the fixed opening costs. The SPLP captures one of the essential features of economic geography: the tradeoff between transportation costs and economies of scale. The former favor the multiplication of facilities; the latter, expressed by the fixed costs, tend to restrict their number.

In this section, we use an extension of the SPLP to build a theory connecting the creation of schools to population density. We choose a simple geographical setting: a circle of unit circumference. We assume that, at every point of time, the cohort of the newborn generation is uniformly spread along the circle and has the same distribution of abilities at every location. Clearly, such a representation is inconsistent with actual population patterns, since there are strong disparities in density between urban and rural areas, between cities of different sizes, and even within cities. Nonetheless, we argue this is a minor point in our setting: rural population accounted for more than 80 percent of England’s total population by the end of the period we consider. We suppose that every point on the circle can accommodate a school and
that schools are identical in their characteristics (same services, same quality, same reputation, etc.). It follows that a pupil will attend the closest school. Moreover, the results of the preceding section allow us to determine the demand for schooling arising from each point on the circle as a function of the distance to the nearest school. Given the hypothesis on the dispersion of the population, it is obvious that schools will be optimally located if they are evenly spaced. Hence, for a given number of facilities, we can determine the literacy rate of the population, the total amount of fees paid by the pupils, and the total transportation costs. Accordingly, the school location problem is reduced to the single question: how many schools (or classrooms) will be founded at every date \( t \) to educate the newborn cohort?\(^4\) But this entails the formulation of an objective function.

To model the school creation process, one must examine the relevant institutional arrangements at work in the period considered. And in particular, one needs to clarify the objectives pursued by school founders at that time. According to the Schools Inquiry Commission (1868a), the picture is far from uniform. Three types of schools can be distinguished: endowed schools, private schools, and proprietary schools.

Endowed schools usually have some income from funds permanently appropriated to the school. Even in this category, schools vary widely in character and history. Some are part of large charitable foundations, others are run by the Church. Private schools are typically the property of their (head) teacher. They “owe their origin to the operation of the ordinary commercial principle of supply and demand,” according to the Schools Inquiry Commission (1868a). They provide more individual care and teaching, but the Schools Inquiry Commission finds much fault in the quality of these schools. Commissioners noted that “A really large and flourishing school is of course a marketable commodity, and sometimes sells well. But it is always a dangerous purchase for a stranger. ... when the school declines the house is let for a shop or a private residence, and the master betakes himself elsewhere.” And also “Considered commercially, few descriptions of business seem to require less capital than the keeping of a private day school of the second order. A house is taken, a cane and a map of England bought, an advertisement inserted, and the master has nothing more to do but teach. It is not likely that schools established at so slight a cost should have buildings well adapted to purposes of education.” These two quotes stress the commercial nature of private schools. The third type of school is the proprietary school. It too is private property: it belongs to a body of shareholders. This type of school is more recent, dating from the 1820s.

Because we have no information on the composition of English schools by type, we posit two different forms of institutional arrangement. In the first, denoted CP for central planning, the optimal number of classrooms is determined by a central authority every year, by maximization of aggregate profits of the education sector, reflecting that “the purpose of schools was never to
save those from paying who could afford to pay," as noted by the Schools Inquiry Commission (1868a). The return yielded from building a school in a given area is roughly the difference between the tuition fees paid by the individuals in the catchment area of the school who decide to educate themselves or their children, and the cost of building and/or operating a classroom. The link between school creation and population density is therefore clear. Since the profitability of a school mainly depends upon tuition revenues, the size of the population in the catchment area of the school should be a major determinant of school creation.

Our second institutional arrangement is market-based, denoted MA, in which we assume that the density of schools results from a free entry process: schools are created as long as they earn a net profit. This models the functioning of private schools described above. It can readily be shown that MA is equivalent to a model where a central authority maximizes aggregate attendance (for example, for religious reasons). In that case, it would create as many schools as possible, subject to a non-negative profit condition.

Population size is a major determinant of school creation because the main source of a school’s revenues, tuition fees, depends on this demographic variable. This is true for both institutional arrangements CP and MA. No school is viable below a certain threshold of population size (or of cohort size, $\zeta$, in our model). When the newborn population is low, the school creation or set-up costs are unlikely to be covered, hence no schools are created. Once the population reaches a threshold value, many schools may be created at once. The process by which illiteracy is eliminated is thus initiated by a jump. After this initial jump, the process takes place much more smoothly over time depending on the evolution of population density and of the attendance rate at schools of the successive cohorts, which in turn depends on the demographic, technological, and geographic factors outlined above.

We now apply our theory to England over the years 1530–1860 in order to disentangle the salient characteristics and determinants of the English development process.

**Data and methodology**

We first describe our sources with some key descriptive statistics over the period of interest. Then, we give an overview of the chosen experimental setting and an outline of how the data have been brought into the theoretical framework detailed in the previous section (the so-called calibration step).

**The data**

*Literacy.* Figure 1 shows the evolution over time of literacy rates (average of men and women) for England as estimated by Cressy (1980). It suggests that
improvements in literacy started as early as the sixteenth century. The steady rise from 1580 to 1760 is notable.

Technological progress. Technological progress, captured in our model by the productivity growth rate $\gamma_t$, increases the attractiveness of the modern sector and should therefore stimulate schooling. We derive the data on productivity growth from Clark (2001). As illustrated by Figure 2, productivity gains in England started to accelerate in the beginning of the nineteenth century. Consequently, the technological factor cannot account for the higher literacy rates achieved two centuries before any significant gain in productivity. The search for alternative demographic and institutional explanatory factors continues.

Demography. The demographic trends in England over the period considered are taken from the detailed historical studies of Wrigley and Schofield (1989) and Wrigley et al. (1997). We need detailed demographic information to identify the time series of parameters $\alpha_t$ and $\beta_t$ of the survival function postulated earlier and to estimate the size of the successive birth cohorts, $\zeta_t$. These time series are crucial in determining the schooling decisions taken by the individuals in our theoretical set-up.

Survival rates and changes in population size can be extracted from the studies cited above. Figure 3 presents the survival rate to age 40 among individuals surviving to age 5 years. It ignores infant mortality swings in order to concentrate on mortality during the active years of life. Adult longevity was first stagnant and then declined over the period 1600–1700, probably because of the urban penalty associated with the rapid growth of cities.
During this period of high mortality, literacy rose continuously, as seen in Figure 1.

We consider the population aged 5 years and older, because it coincides with the concept of population in our model. Figure 4 shows that population rose rapidly in the sixteenth and nineteenth centuries, while the seventeenth century was a period of demographic stagnation. The corresponding swings in crude birth rates are plotted in Figure 5. Rises in population size in the sixteenth century, together with high (but declining) birth rates, correspond to the first wave of improvement in literacy.
School creation. We collected data on school creation from the appendix to the reports of the Schools Inquiry Commission (1868b). Two lists of schools are provided, together with their dates of establishment. The “endowed grammar schools” taught a mixture of Latin and practical skills to sons of the middle class and the lesser elite (list in the Schools Inquiry Commission). The “endowed non-classical schools” were products of the Charity School Movement, offering Protestant socialization and basic skills to the worthy poor. According to Cressy (1980), although short-lived private schools are omitted from the list, a check against other sources proves the Commission’s work to
be reliable. We use these lists to compute the number of schools created per decade. These data are presented in Table 1.

**Methodology**

To assess the relative importance of the demographic, technological, and institutional factors in the English transition to modern growth, we take the following steps.

*Step 1, calibration of a benchmark model.* We first enter the data into our theoretical model. To this end, we need to specify the institutional arrangements in the education sector. We start with scenario CP, with a central authority determining the optimal number of classrooms as well as the level of the tuition fee so as to maximize profits. This is our benchmark case. We will study in further steps how the results are altered if we switch to maximizing aggregate attendance. Calibration of the benchmark model requires estimation of the three demographic processes, $\alpha_t$, $\beta_t$, $\zeta_t$, and the process of productivity growth, $\gamma_t$. It also requires setting the values of some parameters for which we lack accurate information, including productivity in the tradi-

<table>
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<th>Year</th>
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<th>Endowed non-classical schools</th>
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SOURCE: Schools Inquiry Commission 1868b.
tional sector, transport costs, and the discount factor. An extensive analysis of robustness will be conducted later on these parameters.

Step 2, counterfactual experiments. Of the four exogenous processes in our economy, $\alpha_t$, $\beta_t$, $\xi_t$, and $\gamma_t$, the first two represent mortality factors, the third is a measure of cohort size (determined mainly by fertility net of infant mortality), and the fourth is a measure of technological progress. To evaluate the importance of each of these factors in accounting for literacy and economic growth, we run counterfactual experiments: for example, if we seek to evaluate the extent to which technological progress can explain the observed historical evolution of literacy and growth, we let this factor play in isolation, which amounts to solving our calibrated model with constant $\alpha_t$, $\beta_t$, and $\xi_t$, and with the estimated technological progress, $\gamma_t$, as the unique active force. Repeating the same exercise for each factor permits an assessment of the relative importance of each in explaining the observed transition to modern growth.

Step 3, robustness analysis. The results obtained from the counterfactual experiments are conditional upon the calibrated model. A minimal requirement to test the scientific validity of the results is to conduct extensive robustness analysis, which in our case means not only performing a sensitivity analysis with respect to the values of some parameters but also checking how the results are altered if we move to the alternative institutional arrangement (MA), maximizing aggregate attendance.

Before presenting our main findings from the counterfactual experiments and robustness analysis, we describe the calibration procedure of the benchmark model.

Calibration

The four exogenous processes, $\alpha_t$, $\beta_t$, $\xi_t$, and $\gamma_t$, should be made explicit. We assume that all four processes follow a polynomial function of time. Polynomials of order 3 are sufficient to capture the main trends in the data.

For the survival function processes, $\alpha_t$ and $\beta_t$, the parameters of the polynomial are chosen by minimizing the deviation from the survival functions estimated by Wrigley et al. (1997). These survival functions apply to ages 5–85 years, and accordingly have been normalized to 1 at age 5 (hence excluding the effect of early child mortality). The parameters of the process for $\xi_t$ are chosen so as to minimize the difference between the total population implied by our model and the observed level of population aged 5 and older. Finally, the parameter of exogenous technological progress $\gamma_t$ is set to follow the estimated level of total factor productivity in Figure 2.

In a second step, we select a log-normal distribution for abilities, say the function $g(\mu)$ where $\mu$ stands for ability, which is commonly used to ap-
proximate the actual distribution of innate characteristics. We next choose jointly four parameters in order to satisfy four conditions on endogenous variables, implied by the data. Since we have little information to calibrate these parameters, we choose values that give a reasonable benchmark scenario. The four parameters are: the variance of \( g(\mu) \); the transportation cost (in this benchmark calibration, we assume that the transportation cost is indexed on technological progress); the set-up cost; and the productivity or remuneration in the traditional sector. The four conditions are: ten schools in 1820 (there are 3,000 schools in our database in 1820, so the scale of the model is 1/25); the level of literacy in 1820 (55 percent); the change in literacy over the period 1540–1820; and a skill premium of 60 percent on average over the period for seven years of education (according to van Zanden 2004 this was the premium received by skilled craftsmen after seven years of apprenticeship).

Findings

We first summarize the properties of the benchmark model and then present the results of the counterfactual experiments and sensitivity exercises.

Benchmark simulation

The two first bars in Figure 6 report measured (“own estimation”) and simulated (“baseline”) school density. Each bar represents the change since 1530. The baseline change in the density of schools results from the decision process of the central school authority in the institutional arrangement CP, or central planning. Both the measured and the simulated density of schools increase monotonically. The simulation underestimates school creation in the eighteenth century and overestimates it in the nineteenth century but manages to capture well the overall trend. The literacy rate, presented in Figure 7, follows closely the creation of schools. Estimated literacy rises steadily over the period, while for the baseline simulation there is an initial increase prior to 1600, reflecting the creation of the first schools; this is followed by a period of slower growth and, after 1700, by a second period of rapid growth.

The density of schools and the level of literacy are fully consistent with the estimated data. A precise mapping is not obtained, but this is unsurprising in that literacy data cover the ability to sign a marriage register, not school attendance. Notice also the role of expectations: the sharp acceleration at the end of the period is related to the anticipation by households of strong productivity gains in the modern sector in the nineteenth century.

Figure 8 displays gross domestic product per capita. The height of the bar is proportional to the change in GDP per capita since 1530. Recall that output in the modern sector is postulated to be the product of the level of technology (or productivity) and aggregate human capital. The former in-
put can be immediately extracted from the already-estimated productivity growth process $\gamma_t$. The latter input corresponds to the total stock of human capital of all generations that are currently at work in the modern sector: this implies an exact accounting of all individuals in all co-existing cohorts who

FIGURE 6  Cumulative changes in school density since 1530: Baseline and counterfactuals isolating the roles of mortality, cohort size, and productivity in turn

SOURCE: Simulation of our model.

FIGURE 7  Cumulative changes in literacy rate since 1530: Baseline and counterfactuals isolating the roles of mortality, cohort size, and productivity in turn

SOURCE: Simulation of our model.
attend school. Finally we can compute total GDP as the sum of production in the traditional and modern sectors minus the transportation cost minus the set-up cost of schools. According to the baseline there is a period of no growth after 1530, because the economy has to pay the transportation costs of students and the set-up cost of schools but does not yet benefit from better-educated workers. The seventeenth century is characterized by very low growth—too low compared to Maddison’s data. After this stagnation period, growth starts accelerating after 1700 to reach 0.7 percent per annum at the end of the eighteenth century.

Our GDP numbers should be interpreted as the income generated by the accumulation of human capital and by productivity growth, without any effect from the accumulation of physical capital. The difference between Maddison’s estimate and the baseline simulation can be attributed to physical capital accumulation, which is absent from our model.

Counterfactual experiments

In a first experiment we hold cohort size and technological progress constant over the period; this allows us to isolate the role of mortality. Since mortality declines very late in England (see comparable data on Geneva and Venice in Boucekkine, de la Croix, and Licandro 2003), it does not exert a positive influence before the eighteenth century. The bar “mortality” in the figures represents the hypothetical change in school density, literacy, and GDP per capita if mortality was the only factor in play. If mortality improvements
were the only driving force of the industrial revolution, no school would have been created before 1700 and the literacy rate would have increased by only 6.8 percent by 1850. Compared to the baseline simulation, mortality improvements explain 6.5 percent of total school creations over the period 1530–1850, 12.8 percent of improvements in literacy, and 7.5 percent of growth of income per capita.

Next we run a simulation holding both mortality and technological progress constant. Only cohort size $\xi_t$ varies, reflecting all changes in population that are not due to mortality. In this simulation we observe that the rise in population can explain both school creation in the sixteenth century and the early rise in literacy. In the seventeenth century, however, population stagnates and school creation stops. In the end, the rise in cohort size explains a majority of school creations over the period 1500–1850, 27.5 percent of improvements in literacy, and 7.8 percent of income growth per capita.

In a third simulation both mortality and cohort size are held constant, and only technological progress is variable. In this simulation we observe that technological progress cannot explain the timing of school creation and literacy improvements, but it explains a major part of the changes at the end of the period.

These results display a neat picture of the English transition to modern economic growth. First, the counterfactual analysis highlights the fact that neither increases in productivity nor mortality decline can explain the establishment of schools in the sixteenth century at the high rate documented in Table 1. Only the rise in cohort size can account for it. Second, technological progress is the predominant engine of the growth rate of GDP, while increases in longevity play a small role. These results need to be corroborated by sensitivity tests, which we conduct next.

Robustness analysis

We provide a robustness analysis of changes in some of our key hypotheses. For each experiment, we recalibrate the parameters such that the model matches the four conditions on endogenous variables described earlier.

In the benchmark calibration, transportation costs are indexed on technological progress. This assumption is probably too pessimistic because transportation costs no doubt declined relative to other costs in the eighteenth century. To evaluate the importance of this assumption, we ran a simulation in which the transportation cost was not indexed on productivity; as a consequence, the relative importance of this cost diminished in the eighteenth century and the rise in literacy became more important. We recalibrated the model under this assumption. The new baseline with non-indexed transportation costs yields very similar results, showing that the previous analysis remains valid whether or not transportation costs are indexed on productivity.
Another assumption we test concerns the growth of productivity after 1860. In the baseline, we assumed that households anticipate correctly the growth of future productivity (1 percent per year). This creates an incentive to accumulate more human capital. To assess the importance of this mechanism, we ran a simulation where households suppose that productivity will stay at a constant level beyond 1860 (i.e., they consider that the industrial revolution is a temporary phenomenon). This change in assumption does not require any modification in the calibration. Results show that the effect of lower expectations is quite small.

In another robustness test, we set a lower value of the risk-free interest rate, assuming a rate of 3 percent per year instead of 5 percent. The other parameters need to be adjusted. A lower interest rate gives an incentive for households to invest in more education, so we need higher transportation costs to match the observed education investment. The number of schools is very close to the baseline, while literacy increases faster in the beginning of the period. Using 3 percent as an interest rate would bring our simulated literacy closer to the estimate by Cressy for the beginning of the period.

The robustness analysis indicates that the results on literacy and economic growth are little affected by changes in the parameters. This conclusion, however, does not extend to the parameter measuring productivity in the traditional sector, $w^h$. If for example we index $w^h$ on productivity in the modern sector, $A_t$, there is no way to choose the parameters so as to fulfill our four conditions relating endogenous variables to data, and in particular the rise in literacy over the period. In fact, the non-indexation of $w^h$ is the main mechanism through which technological progress plays a role in the model. If we shut down this channel by indexing $w^h$, we reduce drastically the role of technological progress, and we are left with the two other factors, mortality and cohort size, which together explain about 40 percent of the observed rise in literacy.

We also investigate the robustness of the results to the assumption about the institutional arrangement. We ran simulations for the MA arrangement in which schools are created in a decentralized way as long as profit opportunities exist. Comparing CP to MA, we reach two conclusions. First, the timing of the take-off for school creation does not vary across models; in both cases it starts as early as 1540. Second, the density of schools increases much faster with the market solution than with a central authority. This very rapid rise entails important fixed costs for the economy, slowing down growth compared to the central authority case. The model with the market solution therefore does not perform as well as the model with a central authority in reproducing the acceleration in growth during the early nineteenth century because the former would imply the creation of too many schools.

Finally, one issue that is potentially important but very complex involves the modeling of spatial structure. In our formulation, space is modeled as a circle, with schools spread evenly along its circumference. This is a one-
dimensional model of location. In real life, of course, the English countryside is two dimensional. To see whether the predictions of the one-dimensional space can be transposed to a more realistic set-up, let us consider the infinite plane. There are basically two ways of covering the plane with regular shapes of the same size: squares and hexagons. It is well known from the literature on central place theory (see, e.g., Beckmann 1968) that the latter is more efficient than the former, hence we will consider an infinite covering of the plane with hexagons of the same size. The relevant descriptor for our problem is the density of centers, that is, the number of centers in a unit area. An equivalent descriptor is the edge length of the hexagon. Consider the simple case where all children have to attend school and there is free entry for school creation. Then it can be shown that the density of schools is a linear function of the density of population, exactly as it is in the one-dimensional case. The relationship between the number of schools and population density is linear whatever the dimensionality of the space.

This result prompts two additional comments. First, in the one-dimensional world, the average travel distance is linearly related to the number of schools, implying that the average travel distance falls in proportion to the inverse of population density. However, in the two-dimensional world, the average travel time is proportional to the reciprocal of the square root of population density. Whether this affects our estimation of the importance of changing population density in explaining rising schooling is an open question. Recent papers in the optimal location literature (see, e.g., Morgan and Bolton 2002) have provided estimates in some special cases. Using their results in a simple example, we find that if the number of schools in the two-dimensional world doubles, then the average distance decreases by a factor of 0.267, while, in the one-dimensional world, doubling the number of schools decreases the average distance by a factor of 0.25. Hence, the difference between 1/density and 1/(square root of density) is compensated by a scaling factor that makes the difference between the two worlds acceptable. In the more complex model, the discrepancy will depend, among other things, on whether tuition fees are set lower to attract students from more distant places in the two-dimensional space.

Second, while space is generically two dimensional, most of human activities at any time have been organized along certain principal routes. Our circular representation of space could well fit such an organization, and that is precisely why it is so frequently adopted in economic geography.

Conclusion

In this chapter, we developed a theoretical model with the main demographic, economic, and institutional factors traditionally considered to be crucial in the transition to modern economic growth. We provided a formal link between population density and the provision of schools, that is, given economies of
scale, higher density allows one to reduce the cost of education per capita and to increase the level of human capital. This is in agreement with the literature on agglomeration economies (see, e.g., Duranton and Puga 2004 and Henderson 2005).

We applied our theory to England over the period 1530–1860. Using a calibrated version of our model, we measured the impact of mortality, cohort size, and technological progress on school density, literacy, and economic growth through a set of counterfactual experiments. We found that one-third of the rise in literacy over the period 1530–1850 can be directly related to the effect of cohort size, while one-sixth is linked to higher longevity, and one-half to exogenous total factor productivity growth.

Some concluding remarks are in order. First, one has to mention the reduced role of mortality decline relative to other factors in explaining England’s development over the period studied. This is at odds with studies on other countries (see, e.g., Boucekkine, de la Croix, and Licandro 2003) but is not so surprising if we have in mind Wrigley and Schofield’s study described in the introduction. Because we rely on this study to calibrate the demographic components of the model, it is a fortunate outcome of our simulations that mortality declines do not play the major role. Second, while the model used is properly calibrated to capture the main observed demographic and technological characteristics of the English transition, it is built on several simplifying assumptions that can, we hope, be relaxed in future work to bring the model closer to reality. Including physical capital accumulation and human capital externalities should be the next steps. Working with a two-dimensional representation of space and determining whether it really matters compared to the one-dimensional space used here would also be a desirable extension.

Notes

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1 England is not an exception, though: Ireland and Holland, for example, share some historical demographic trends with England, notably the preeminent role of fertility compared to mortality in the acceleration of population growth in the mid-eighteenth century. Sweden and France exhibit a completely different picture.

2 Note that we are invoking this complementarity argument at the implementation stage of innovations; it is even more obvious if one has in mind the prior research and development stage giving rise to innovations.

3 The Schools Inquiry Commission of 1867–68, appointed by the British Parliament, was in charge of a survey on the state of education. The appendix to the 24-volume report
contains a list of endowed schools in England and Wales with their formation dates ranging from the twelfth century to 1860.

4 Here we take the view that classrooms are specific to cohorts. In particular, they are assumed to be closed when the last person of cohort $t$ graduates.

5 This set-up and operating cost can be seen as being net of the possible endowment.

6 Culp and Smith (1989) mention that in *The Wealth of Nations*, Adam Smith reviewed eighteenth-century public attitudes toward two new forms of wealth creation: “forestalling” and “engrossing.” Both activities had become possible only as transportation costs dropped.

References


