

Unemployment and Capacity Under-Utilisation in a Tobin's q Model

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Introduction

The ability of an economy to ensure full-employment is partly determined by its ability to provide sufficient production capacities (places of work). Some studies have shown that the lack of capacities plays an important role in explaining hysteresis in European unemployment (See *e.g.* Modigliani *et al.* [1987] and Sneessens and Drèze [1986]). More precisely, the relation between capital and labour utilisation in a large number of European economies can be characterised as follows:

- Both capital and labour tend to remain under-utilised: the degree of capital utilisation (d) and the unemployment rate (u) remain at levels which are incompatible with full utilisation of resources without any tendency to converge to natural levels close to full utilisation.
- In Chart 1 (from Drèze and Bean [1990]), the dynamics of the u, d pair between 1970 and 1991 shows a gradual rise in unemployment and a cyclical behaviour of d . There is a negative correlation between the two variables in the beginning of each business cycle and no correlation in the long run. Chart 1 suggests that the gap between labour supply and production capacities has widened.

The major part of the literature concerned with equilibrium unemployment does not consider the role of capital accumulation. In this

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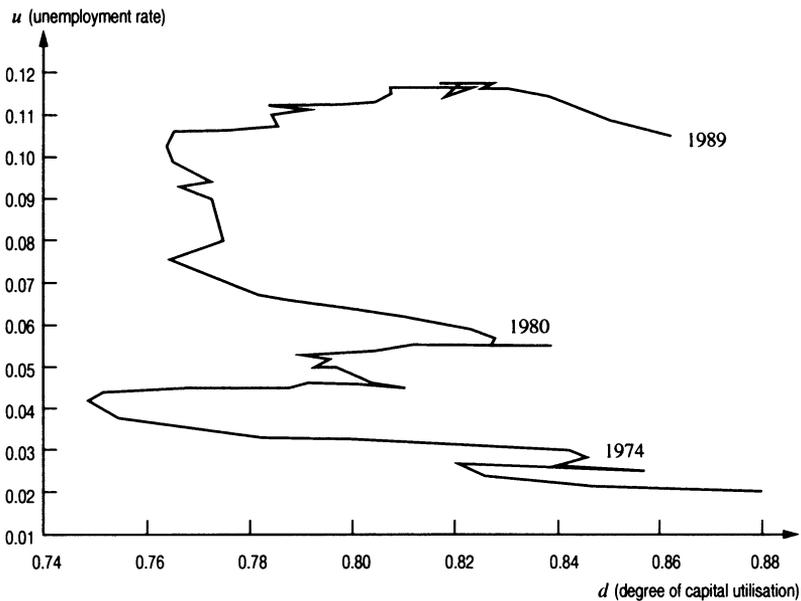


Chart 1

Unemployment rate (u) and degree of capital utilisation (d) in the EEC

paper, we want to analyse the link between equilibrium unemployment and the equilibrium degree of capacity utilisation and study their joint dynamics. Stated from another point of view, we introduce underutilisation of labour and capital in an investment model in the Tobin's q tradition. Besides the standard phase diagram showing the relation between profitability (marginal q) and the capital stock (see *e.g.* Blanchard and Fisher [1989], p. 64) we introduce a diagram depicting the relation between unemployment and investment. The effect of unexpected changes in the environment on the dynamic paths of unemployment and capacity utilisation will be studied with these diagrams and have real novelty value, as far as we know.

A model proposed by Sneessens [1987] makes a first step towards this direction. It is based on a monopolistic competition framework where firms face uncertainty with a Leontief technology. Uncertainty and technological uncertainty imply that, even in the long run, capital is under-utilised. In Sneessens' model, investment takes place through entry of new firms as long as profits are positive. We try to extend this approach by giving better foundations to the dynamics of the system by considering an infinite horizon problem. We also introduce a simple type of wage formation through the introduction of a bargaining set-

up at the firm level⁽¹⁾ We use an efficient bargaining framework where firms and unions bargain over wages and expected employment; similar problems have been shown to be time-consistent if the discount factor is high enough (Espinoza and Rhee [1989] and Strand [1989]) and presents the best solution for both players.

With respect to previous work in bargaining models with endogenous investment (Grout [1984], Anderson and Devereux [1988] and Manning [1994]), our framework is closer to the usual models of investment (infinite horizon), thus allowing us to analyse the effect of unions on the relationship between average and marginal Tobin's q . With respect to the scarce literature on dynamic bargaining with adjustment cost (Card [1986] and Lockwood and Manning [1989]) we endogenise the capital stock. The outcome of the model will be a joint determination of the equilibrium (steady-state) degree of utilisation of capital and unemployment rate and a dynamic pattern for these two variables. Moreover, our model gives some insights into the foundations of unemployment.

The limitations of this paper are numerous; in this sense, it can be considered as a preliminary attempt to build a (dynamic) non-Walrasian general equilibrium model explaining the persistence of unemployment. These limitations are the following: the household maximisation problem is not treated; the utility of the union is not derived from household utility; the discount rate is exogenous; technology is Leontief (no substitutability of production factors); Investment as a component of aggregate demand is not treated; The labour market is strongly segmented.

The structure of the paper is the following: The main assumptions are presented in section 1. Section 2 discusses wage and price formation as well as the determination of short-run unemployment (*i.e.*, at given capital stock). Section 3 studies the investment decision and its link with union power. The steady state is analysed in section 4. The dynamics of the system is described in section 5 as well as the responses of the model in the face of permanent unexpected shocks.

1 Assumptions

1.1 Households and Union

The utility function of the representative household is defined over a basket of imperfectly substitutable goods. The elasticity of substitution between the different goods, ε , is assumed constant and larger than one. From the utility function of the representative household we derive

⁽¹⁾ Wage bargaining in a Sneessens type of model with exogenous capital stock is analysed in Arnsperger and de la Croix [1993] and de la Croix [1993].

demand functions for goods: The demand for each good at time t , y_t^d , is a share of total real demand \bar{C}_t/P_t depending upon the ratio of the good's price p_t to the general price index P_t and upon a good-specific random shock μ_t :

$$y_t^d = \left(\frac{p_t}{P_t} \right)^{-\varepsilon} \frac{\bar{C}_t}{P_t} \mu_t, \quad \varepsilon > 1. \quad (1)$$

\bar{C}_t depends on various elements (income, wealth, interest rate, spill-over etc.) which are exogenous at the firm level. It gives a nominal anchorage to the model and makes aggregate demand a negative function of aggregate price. The labour supply is firm-specific and each firm-specific union is composed of the l_t^s households which supply their work force to the representative firm. The union's utility function is defined over employment l_t and over the difference between the real wage w_t/P_t and the disutility of work ω_t :

$$U_t = l_t \frac{1}{\nu} \left(\frac{w_t}{P_t} - \omega_t \right)^\nu, \quad \nu < 1. \quad (2)$$

The parameter ν can be interpreted as a relative preference for wages with respect to employment. It measures the concavity of the utility function with respect to income; when we allow for uncertainty, something we shall do later, $1-\nu$ is also the coefficient of relative risk aversion of the workers, with $\nu = 1$ implying risk neutrality.

1.2 Firms

The representative firm has a Leontief production function⁽²⁾ which combines labour l_t and capital k_t in a given proportion (which may however vary exogenously over time through technical progress). Assuming that labour supply l_t^s is firm-specific and exogenous⁽³⁾ (as long as the wage is larger than the disutility of work), the full-employment output y_t^s is:

$$y_t^s = a_t l_t^s \eta_t \quad (3)$$

where a_t is the exogenous technical coefficient associated with labour in the production function and η_t is a firm-specific random shock affecting labour availability (*e.g.* modelling absenteeism etc.).

⁽²⁾ A generalisation of the model to a putty-clay production function is proposed in de la Croix and Fagnart [1995]. The resolution of the equilibrium trajectories requires in that case the use of numerical simulations.

⁽³⁾ But not necessarily constant through time.

Assuming a one period time-to-build (*i.e.* investment only becomes productive after one period), the full-capacity output y_t^p is:

$$y_t^p = b_t k_{t-1} \lambda_t \tag{4}$$

where b_t is the technical coefficient associated with capital and where λ_t is a firm-specific random shock on capital availability (*e.g.* reflecting technical breakdowns etc.). The capital stock at s is equal to the previous capital stock k_{t-1} times $(1 + i_t)$ where i_t is the investment rate. For simplicity, we assume a zero depreciation rate:

$$k_t = k_{t-1}(1 + i_t). \tag{5}$$

The cost of investment $P_t k_{t-1} \Psi(i_t)$ includes installation costs

$$P_t k_{t-1} (\Psi(i_t) - i_t),$$

where $\Psi(i_t)$ is the installation function with $\Psi'(i_t) > 0$, $\Psi''(i_t) < 0$, $\Psi(0) = 0$ and $\Psi'(0) = 1$. Real profits V_t are:

$$V_t = \frac{p_t y_t - w_t l_t - P_t \Psi(i_t) k_{t-1}}{P_t}. \tag{6}$$

We assume that the price level has to be announced by the firm before it knows the current realisation of the random shocks μ_t, η_t, λ_t and given its technical rigidities (Leontief production function, fixed labour supply). The timing of the decisions is described by Chart 2.

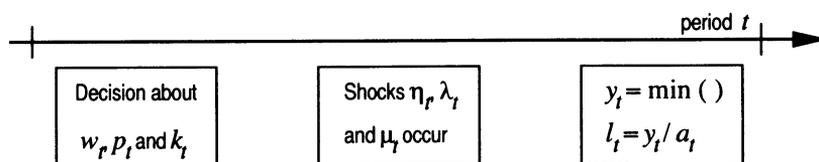


Chart 2: Timing of the Firms' decisions on price level

After the realisation of the shocks, the firm can be constrained either by labour availability, capital availability or demand. In this case, output is the minimum of the three constraints:

$$y_t = \min(y_t^d, y_t^s, y_t^p), \tag{7}$$

and the employment level which is decided after the realisation of the shock is simply:

$$l_t = y_t / a_t. \tag{8}$$

Furthermore, if the shocks η_t, λ_t, μ_t are i.i.d.⁽⁴⁾ and lognormally distributed with a variance-covariance matrix with identical variances and identical covariances, the conditional expectation of output taken at time s before the realisation of the shocks can be written as a CES function of the three expected constraints (Lambert's [1988] theorem adapted by Sneessens (1983) to a three-constraint case):

$$E_s(y_t) = [E_s(y_t^d)^{-\rho} + E_s(y_t^s)^{-\rho} + E_s(y_t^p)^{-\rho}]^{-1/\rho} \quad \rho > 0 \quad (9)$$

where ρ is a function of the variances and covariances of η_t, λ_t, μ_t . The higher ρ , the lower is the loss of production due to heterogeneous situations: Intuitively, if the variance of the shocks is higher, the allocation of excess supply and demand across markets is more heterogeneous and the resulting aggregate loss of production due to the impossibility of transferring the disequilibrium from one market to another is higher. Note that these shocks have no dynamic implications by themselves since they are time-uncorrelated. However, the magnitude of their variance will turn to have an effect on the dynamics of the model through the parameter ρ .

The interest of this formulation is twofold; first, the elasticities of expected output to each expected constraint (which are also equal to the probability of facing that constraint) have a simple formulation:

$$\eta_{E_s(y_t).E_s(y_t^d)} = \Pr(y_t^d = y_t) = \pi_t^d = \left[\frac{E_s(y_t)}{E_s(y_t^d)} \right]^\rho \quad (10)$$

$$\eta_{E_s(y_t).E_s(y_t^p)} = \Pr(y_t^p = y_t) = \pi_t^p = \left[\frac{E_s(y_t)}{E_s(y_t^p)} \right]^\rho \quad (11)$$

$$\eta_{E_s(y_t).E_s(y_t^s)} = \Pr(y_t^s = y_t) = \pi_t^l = 1 - \pi_t^d - \pi_t^p = \left[\frac{E_s(y_t)}{E_s(y_t^s)} \right]^\rho \quad (12)$$

Secondly, if the number of firms is sufficiently large, aggregation gives interesting properties, to which we now turn.

1.3 Aggregation

If the number of firms is large, aggregate output (resp. employment) is equal to each firm's expected output (resp. employment) times the number of firms. In that case, the ex post proportions of firms in

⁽⁴⁾ If the shocks exhibit time dependence, each individual firm's output would depend on the firm's history, and the aggregation problem would become untractable.

each regime are equal to the above ex ante elasticities and can be related to two business cycle indicators, namely the unemployment rate, $u_t = 1 - l_t/l_t^s$ and the degree of capacity utilisation $d_t = y_t/y_t^p$:

$$d_t = (\pi_t^p)^{1/\rho} \tag{13}$$

$$u_t = 1 - (\pi_t^l)^{1/\rho} \tag{14}$$

1.4 Efficient bargaining

Each union-firm couple sign a contract on wages and expected employment. This agreement amounts to determine actual values for prices, wages and investment and contingent plans for those variables in the future. This decision is taken before the realisation of the shocks, whereas output and employment are decided by the firm after the realisation. This explains why output and employment are expressed in expected terms at the time of bargaining. In this set-up, bargaining directly over prices and investment amounts to bargain over an *expected* level of employment.

The mechanism through which such a cooperative outcome could emerge is detailed in Eberwein and Kollintzas [1995]. Their argument applied to our framework is the following: At the cooperative solution, the union sets a wage along the contract curve. It does so, however, expecting the firm to cooperate and set its price and investment compatible with the expected employment level along the contract curve. If the firm cooperates by setting a low enough price and a high enough investment, the union continues to cooperate in the future. If the firm does not cooperate, the union starts a punishment strategy by moving towards the non-cooperative equilibrium as in Espinosa and Rhee [1989]. If the agents give enough weight to the future (*i.e.* if their discount factor is high enough) it is optimal for them to cooperate. In the sequel, we assume θ to be high enough, preventing the firm to renege on the efficient bargaining agreement.

Given the imposed timing of the decisions, a cooperative agreement on p_t , w_t and k_t is the best solution for the agents. This decision timing (prices - shocks - quantities) is consistent with the fact that a large number of firms do report demand shortages in business surveys. The objective function is the weighted sum of all future utilities and profits:

$$\max_{\{k_t\}_{t=s}^{\infty}, \{p_t\}_{t=s}^{\infty}, \{w_t\}_{t=s}^{\infty}} \sum_{t=s}^{\infty} E_s(U_t) \theta^{t-s} + \lambda \sum_{t=s}^{\infty} E_s(V_t) \theta^{t-s}$$

such that(1) – (9).

with $\infty > \lambda > 0$. The parameter λ which weights the agents' objective function is called "firm power"; it represents relative power: If $\lambda > 1$ the firm has more power than the union⁽⁵⁾. θ is equal to $1/(1+r)$ where r is the union's time-preference parameter and the firm's discount rate. Allowing r to differ across agents would not change the nature of our results. Note that there is a lower bound on λ in order to guarantee non-negative profits at the steady state; from the computation of the steady state value of V (see further), this condition writes: $\lambda \geq (a - \omega)^{\nu-1}$.

Note finally that, in the absence of unexpected changes in the exogenous variables, it is equivalent to negotiate at each point in time or to determine once for all the complete path of these variables.

2 Wage and price formation and unemployment

The first-order conditions of the maximisation programme determine wage, price and investment. In this section, we focus on the two first components, which are closely related to the short run equilibrium unemployment (at given capital stock). This equilibrium unemployment rate makes compatible the claims of the union (wage claims) and the claims of the firm (price claims). At this rate, wage and price are constant.

The first-order conditions for wage and price lead to (see Appendix B):

$$\begin{cases} w_t &= (\tau + \omega_t) P_t \\ p_t &= \frac{\varepsilon \pi_t^d}{\varepsilon \pi_t^d - 1} \left[\frac{w_t}{a_t} - \frac{P_t}{\lambda a_t} \frac{1}{\nu} \left(\frac{w_t}{P_t} \right)^\nu \right] \end{cases} \tag{15}$$

where $\tau = \lambda^{1/(\nu-1)}$. Moving to the aggregate level, we use the properties of a symmetric Nash equilibrium: all firms are the same before the realisation of the shocks; they will therefore fix the same wage and the same price. Consequently, $p_t = P_t$. Using this, we may rewrite (15) as:

$$\begin{cases} \frac{w_t}{p_t} &= \tau + \omega_t \\ p_t &= \left[1 + \frac{\tau}{a_t \nu} - \frac{1}{\varepsilon \pi_t^d} \right]^{-1} \frac{w_t}{a_t} \end{cases} \tag{16}$$

Real wages are equal to the disutility of work plus a positive term negatively related to firm power. This result is comparable to the usual

⁽⁵⁾ Assuming that the utility of the union and of the firm can be compared interpersonally.

result of bargaining models where the wage is a mark-up over the fall-back position of the union or over its “outside wage” (e.g. Holmlund [1989]). However, it is clear that wage formation is given a very limited role in the model. An interesting extension should be to endogenise ω_t .

Prices are a mark-up over marginal labour cost. The mark-up rate increases with product differentiation and with firm power. It decreases with the endogenous probability of facing a demand constraint and with workers’ risk aversion.

Product differentiation appears in the mark-up rate because of the monopolistic competition nature of the goods market. As in Sneessens [1987], the probability of a demand constraint reflects the fact that the firm can be quantity rationed. The two other elements reflect the efficient nature of the bargain: If the power of the firm is reduced or if workers are more risk averse, the union will negotiate a lower mark-up rate in order to increase expected employment. This is consistent with Dowrick [1989] and Arnspenger and de la Croix [1990].

System (16) can now be solved for π_t^d :

$$\pi_t^d = \frac{1}{\varepsilon} \left[1 + \frac{\tau}{a_t} \left(\frac{1}{\nu} - 1 \right) - \frac{\omega_t}{a_t} \right]^{-1}. \quad (17)$$

This π_t^d is the only *ex post* proportion of firms constrained by demand (\equiv *ex ante* probability of a demand constraint) which makes compatible the claims of the unions and the mark-up requirement of the firms. It is determined by the value of the parameters and by exogenous variables. An interesting feature of our model is to derive this proportion as a function of a set of well-identified parameters: The proportion of firms constrained by demand is positively related to product differentiation, to the ratio between the disutility of work and labour productivity, and to firm power. It is negatively related to workers’ risk aversion. In other words, the parameters which are at the root of imperfect competition models are the main determinants of π_t^d , because π_t^d depends only on price and wage formation. Using (9), (13) and (14) we derive the short run equilibrium unemployment and the corresponding degree of utilisation of capital:

$$\begin{cases} u_t &= 1 - \left(\frac{1 - \pi_t^d}{1 + (1 - g_t)^{-\rho}} \right)^{(1/\rho)} \\ d_t &= \left(\frac{1 - \pi_t^d}{1 + (1 - g_t)^{-\rho}} \right)^{(1/\rho)} \end{cases} \quad (18)$$

where g_t denotes the “capital gap” $g_t = 1 - b_t k_{t-1}/a_t l_t^s$, i.e. the shortage of production capacities relatively to the capacities needed to obtain full-employment. Due to the one period time-to-build assumption, u_t and d_t

are conditional on the capital stock of the previous period. An important property of this system is the following:

Property 1. *At given capital stock, any shock affecting the proportion of demand-constrained firms affects unemployment and d in opposite directions.*

This property is straightforward in the presence of fixed coefficients in the production function. It implies that we should observe, in the short run, a negative correlation between u_t and d_t when the shock comes from (17). Two other properties of (17)–(18) are worth noting:

Property 2. *If the goods tend to be perfectly substitutable ($\varepsilon \rightarrow \infty$), the proportion of firms constrained by demand tends to zero.*

In that case, unemployment related to π_t^d is zero. Clearly, in our model, the existence of unemployment due to the possibility of a demand constraint is related to goods differentiation. In its absence, firms no longer have any market power on the good market and cannot be constrained by demand.

Property 3. *In the absence of uncertainty, ρ tends to infinity and unemployment tends to zero.*

If there is no uncertainty, the production function exhibits constant returns to scale and constant marginal productivities. For this reason, it is always optimal to choose a full-employment level for prices and wages.

3 Investment and union power

The first-order condition for investment leads to (see Appendix C):

$$k_s \Psi'(i_s) = \sum_{t=s+1}^{\infty} \left[\frac{1}{\lambda} \pi_t^p E_s(U_t) + \pi_t^p E_s(V_t) + (\pi_t^p - 1) k_{t-1} \Psi(i_t) \right] \theta^{t-s}. \quad (19)$$

This equation says that investment will be such that the marginal cost of capital is equal to its marginal value. This marginal value is itself equal to the firm's marginal value

$$\sum_{t=s+1}^{\infty} [\pi_t^p E_s(V_t) + (\pi_t^p - 1) k_{t-1} \Psi(i_t)] \theta^{t-s}$$

plus the marginal value of capital for the union

$$\sum_{t=s+1}^{\infty} [\pi_t^p E_s(U_t)] \theta^{t-s}$$

weighted by $1/\lambda$. The firm's marginal value can be rewritten as

$$\sum_{t=s+1}^{\infty} [\pi_t^p y_t (1 - w_t/a_t/p_t) - k_{t-1} \Psi(i_t)] \theta^{t-s}.$$

This is the gain of increasing the stock of capital (the presence of π_t^p reflects the fact that the elasticity of output to capital is lower than one) minus the cost of investment. The implications of this condition for the relation between marginal q and average q can be drawn easily by using the following definitions:

Definition 1. Let \tilde{Q}_s be the average value of capital for the union:

$$\tilde{Q}_s = \frac{\sum_{t=s+1}^{\infty} E_s(U_t) \theta^{t-s}}{k_s(1+r)}. \quad (20)$$

Capital has a value for the union because it allows to employ workers and to pay wages.

Definition 2. Let Q_s be the private average value of capital:

$$Q_s = \frac{\sum_{t=s+1}^{\infty} E_s(V_t) \theta^{t-s}}{k_s(1+r)}. \quad (21)$$

Q_s is the usual "average q " of Hayashi [1982].

Definition 3. Let Q_s be the social average q :

$$Q_s = \tilde{Q}_s/\lambda + Q_s.$$

Q_s is the social average value of capital as evaluated by the bargaining function.

Definition 4. Let $\tilde{\pi}_s^p$ be the "mean capital output elasticity" from the union's point of view.

$$\tilde{\pi}_s^p = \sum_{t=s+1}^{\infty} \left(\frac{E_s(U_t) \theta^{t-s-1}}{\sum_{t'=s+1}^{\infty} E_s(U_{t'}) \theta^{t'-s}} \right) \pi_t^p. \quad (22)$$

$\hat{\pi}_s^p$ is the weighted sum of all future elasticities of output with respect to capital. The weights are a function of all future expected union utilities.

Definition 5. Let $\hat{\pi}_s^p$ be the “mean capital output elasticity” from the firm’s point of view.

$$\hat{\pi}_s^p = \sum_{t=s+1}^{\infty} \left(\frac{E_s(V_t)\theta^{t-s-1}}{\sum_{t'=s+1}^{\infty} E_s(V_{t'})\theta^{t'-t}} \right) \pi_t^p. \tag{23}$$

$\hat{\pi}_s^p$ is the weighted sum of all future elasticities of output with respect to capital. The weights are a function of all future expected profits. $\hat{\pi}_s^p$ and $\hat{\pi}_s^q$ are two different weighted averages of all future elasticities of production with respect to capital. In both cases, the sum of the weights is equal to 1. Using these definitions, and denoting the marginal value of capital q_s , we may rewrite (19) as:

$$\Psi'(i_s) = q_s = \frac{1}{\lambda} \hat{\pi}_s^p \tilde{Q}_s + \hat{\pi}_s^q Q_s + \frac{1}{k_s} \sum_{t=s+1}^{\infty} [(\pi_t^p - 1)k_{t-1} \Psi(i_t)] \theta^{t-s}. \tag{24}$$

The following property follows:

Property 4. Marginal q and social average q are equal only if the firm and the union expect to fully employ the capacities during all future periods. Otherwise, marginal q is smaller than social average q .

If the firm and the union expect to fully employ the capacities during all future periods, the elasticity of production to capital is always 1, the two “mean capital output elasticities” are also 1 in all periods ($\pi_t^p = 1$, $\hat{\pi}_t^p = 1$ and $\hat{\pi}_t^q = 1$ for all s). This implies $q_s = Q_s$.

Note that, in our model, monopolistic competition is not sufficient to imply a gap between marginal and average q (contrary to Schiantarelli and Georgoutsos [1990] who introduce monopolistic competition in a more standard model with a putty-putty technology) because, in the absence of uncertainty, marginal productivities are constant.

Property 5. Marginal q and private average q differ in general because firms expect to underutilise their future capacities and because unions attach a value to capital.

The first element, which comes from uncertainty, tends to make marginal q smaller than average q , implying a lower level of investment

than in the deterministic case. The second element, which comes from the efficient bargaining framework, tends to make marginal q larger than average q , implying a tendency to over invest in order to increase employment and therefore union utility.

4 The steady state

The steady state is characterised by the absence of net investment ($i_t = 0 \forall s$) implying that marginal q is equal to 1 ($q = \Psi'(0) = 1$). To compute the steady state we use the properties of the installation function: $\Psi(0) = 0$ and $\Psi'(0) = 1$. Using

$$\sum_{s=t+1}^{\infty} \theta^{s-t} = 1/r$$

the value of the agents' objective functions at steady state are (see Appendix E):

$$\begin{cases} U &= y \frac{\tau^\nu}{a\nu} \\ V &= y \left(1 - \frac{\tau}{a} - \frac{\omega}{a} \right). \end{cases} \quad (25)$$

This shows that both the utility of the union and the profit of the firm are linear in output. Let us now define \mathcal{W} as the value of the bargaining function per unit of output divided by λ :

$$c\mathcal{W} = \frac{1}{\lambda} \frac{U}{y} + \frac{V}{y}.$$

Using (25), \mathcal{W} can be rewritten:

$$\mathcal{W} = \frac{\tau}{a} \left(\frac{1}{\nu} - 1 \right) + 1 - \frac{\omega}{a}. \quad (26)$$

The definition of \mathcal{W} will be very useful in the following results. A first result comes from combining investment and wage equations at steady state, which leads to the determination of an equilibrium level for π^p :

$$\pi^p = \left(\frac{r}{b\mathcal{W}} \right)^{\rho/(\rho+1)}. \quad (27)$$

For large value of ρ we have $\pi^p b \mathcal{W} \approx r$. Remembering that π^p is also the elasticity of output to capital, the model implies that:

Property 6. *At steady state, the gross social marginal productivity of capital is equal to the discount rate.*

This equation is to be compared with the standard result of growth theory, keeping in mind that we provide only partial equilibrium results. According to the modified golden rule, the marginal productivity of capital is equal to the growth rate of population plus the discount rate. Our equation differs from this result in two respects: (1) The marginal product of capital is multiplied by the elasticity of output to capacity π^p , which is smaller than 1. (2) Marginal productivity is replaced by a “social marginal productivity” $b\mathcal{W}$.

As in the short run, the combination of price and wage equations leads to the determination of an equilibrium level for π^d :

$$\pi^d = \frac{1}{\varepsilon\mathcal{W}}. \quad (28)$$

Using the definitions of d and u , (27) and (28) lead to the determination of the unemployment rate and the degree of utilisation of capital at steady state:

$$\begin{cases} d &= \left(\frac{r}{b\mathcal{W}}\right)^{(1/(\rho+1))} \\ u &= 1 - \left(1 - \left(\frac{r}{b\mathcal{W}}\right)^{(\rho/(\rho+1))} - \frac{1}{\varepsilon\mathcal{W}}\right)^{(1/\rho)}. \end{cases} \quad (29)$$

This leads to:

Property 7. *At steady state, d can be smaller than 1 and u larger than 0.*

Moreover, unemployment at steady state results from the interaction of four elements:

- r/b : this first element is linked with π^p and refers to unemployment due to the eventuality of a capacity constraint. It is positively affected by the discount rate and negatively affected by capital productivity.
- $1/\varepsilon$: this second element is linked with π^d and refers to unemployment due to the eventuality of a demand constraint. This unemployment is related to the nature of the goods market: it increases with product differentiation. It cannot be cured by Keynesian policy since aggregate demand has no real effect in our model.
- $1/\mathcal{W}$: this third element is linked with both π^p and π^d . This $1/\mathcal{W}$ is determined by the parameters of the bargaining process (workers' risk aversion, firm power and disutility of work). It

influences both types of unemployment through income and profit formation. For instance, an increase in workers' risk aversion will decrease the labour share in value-added, leading to an increase in productive capacities (drop in unemployment due to the eventuality of a capacity constraint), and to a decrease in the mark-up rate, inducing a rise in demand (drop in unemployment due to the eventuality of a demand constraint).

- $1/\rho$: this last element is linked with structural unemployment. If the allocation of demand and supply shocks among markets is less "equal" and ρ decreases. The loss of production due to the fact that workers in low production firms cannot move to high production firms is thus more important.

Using the properties of (9), (10), (11) and (12), the levels of output and capital can be expressed as functions of u and d .

$$\begin{cases} y &= (1-u) a l^s \\ k &= \frac{1-u}{d} \frac{a}{b} l^s. \end{cases} \quad (30)$$

This implies that the source of growth in the steady state is the (deterministic) growth of population and productivities.

Note on Union Power and the Level of Capital

Note that a rise in firm power always leads to increases in u and d and to decreases in output and capital. In general, one can identify the two reasons for which unions have a positive impact on investment:

- Due to the restrictive technology under consideration, an increase in the capital stock always increases employment so that capital has a positive value for the union.
- The union bargain also over investment (efficient bargaining).

This can be compared to the cases in which unions do not bargain over investment: In partial equilibrium analysis, if the union is a simple monopoly union and if the firm commits itself to a capital decision before the wage decision (see Anderson and Devereux [1988] or the non-binding contract case in Grout [1984]), capital decreases with union power. Because an increase in capital motivates the union to ask for a higher wage to capture a part of the new rent, union power is a component of the cost of capital. In general equilibrium analysis (see the overlapping generation model of Devereux and Lockwood [1991]), a rise in union power increases the income of the workers (young generation), leading to a rise in savings and a drop in the interest rate. This goes in the

opposite direction from the partial-equilibrium effect, so that the total effect of a rise in union power on capital is undetermined.

Note on The Perfect Competition Case

An interesting benchmark is the competitive equilibrium corresponding to our economy. If we have perfect competition on the labour market ($\lambda \rightarrow \infty$), the real wage will fall until it reaches the level of the disutility of work; in that case, the unemployed workers are not worse off than the employed workers. If we have perfect competition on the goods market ($\varepsilon \rightarrow \infty$), the firms produce the same good and the price will always adjust in order to ensure that demand is equal to supply. The firm will never be constrained by demand ($\pi^d = 0$) and expected output will be a CES function of potential output and full-employment output. The firm has only to decide about investment; this leads to:

$$\pi^p = \left(\frac{r}{b(1 - \omega/a)} \right)^{\rho/(\rho+1)}$$

which gives the following d and u :

$$\begin{cases} d &= \left(\frac{r}{b(1 - \omega/a)} \right)^{1/(\rho+1)} \\ u &= 1 - \left(1 - \left(\frac{r}{b(1 - \omega/a)} \right)^{\rho/(\rho+1)} \right)^{1/\rho} \end{cases} .$$

Comparing this with (29), we see that the value of d in the perfect competition case is larger than the one in the general model. The value of u is lower or larger than in the general model (the effect is indeterminate since perfect competition in the goods market decreases unemployment but perfect competition in the labour market increases unemployment with respect to the efficient bargaining outcome). Of course, all the unemployed workers are no worse off than the employed workers, so that unemployment is voluntary.

5 The dynamics of u_t and d_t

The dynamics of the system is described by a two-equation system containing one forward-looking variable (q) and one backward variable (k or d) (see Appendix F). The first equation comes from the capital accumulation rule (5) expressed in d terms; the second one comes from

the Euler equation associated with investment and used to obtain (19).

$$\begin{cases} \Delta d_t &= -i_t (d_t - d_t^{\rho+1}) \\ \Delta q_t &= (r - i_t) q_{t-1} - d_t^{\rho+1} \mathcal{W}_t b_t + \Psi(i_t). \end{cases} \quad (31)$$

This system is comparable to the one we obtain in standard investment theory (See Blanchard and Fisher [1989]). Note that the investment rate i_t in (31) depends on q through $\Psi'(i_t) = q_t$. Using (31) we draw the corresponding phase diagram (Figure 1). The horizontal $\Delta d = 0$ locus is given by the first equation of (31) and implies $i = 0$ at steady state, which determines a constant value for $q = \Psi'(0) = 1$. The slope of the $\Delta q = 0$ locus is obtained by differencing the right-hand side of the second equation.

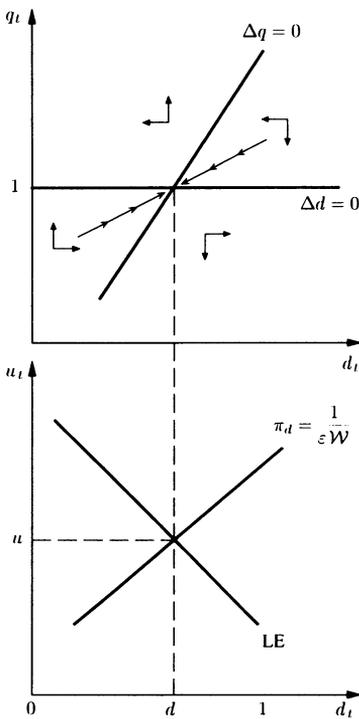


Figure 1: The phase diagram

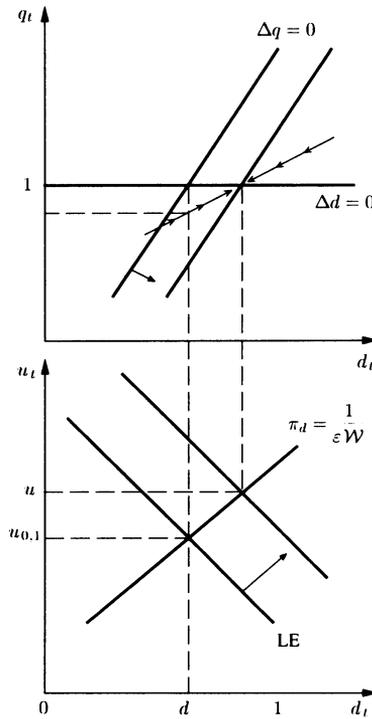


Figure 2: Increase in r

The local stability of the system is analysed by linearising around the equilibrium and by computing the roots of the characteristic equation. The detailed results are given in Appendix F. A first root is smaller

than 1 and larger than -1 for certain value of the parameters (if ρ is not too large). The second root is larger than 1. There is therefore a unique, negatively sloped path converging to the steady state: If marginal q exceeds 1, firms invest so that d decreases. Under rational expectations the economy is always located on the saddle path. Considering the evaluation of the eigenvalues, let us stress the importance of the uncertainty parameter ρ in determining the speed of adjustment of the model: A higher variance of the shocks (a lower ρ) reduces the speed of adjustment of the economy.

In order to obtain a joint determination of d and u , we put a second graph on the bottom of the phase diagram. On this graph, we draw two relations between d and u which have to be satisfied at all times. The positively sloped relation uses (12), (13) and (14) and the value of π_t^d resulting from (17):

$$1 - d_t^\rho - (1 - u_t)^\rho = \pi_t^d = \frac{1}{\varepsilon \mathcal{W}}. \quad (32)$$

This condition says that the wage and price equations which determine π^d imply a positive relation between u and d . The position of this relation in the u, d space is a function of ε and of the parameters λ, ν and a embedded in \mathcal{W} . At steady state, this relation gives the unemployment rate compatible with the d of the above figure.

The negatively sloped relation, called LE, is obtained by using the properties of the Leontief technology; it is not necessary to compute the steady state u but will be useful in analysing the dynamic effect of some shocks:

$$u_t = 1 - d_t \frac{b_t k_{t-1}}{a_t l_t^s}. \quad (33)$$

Note that this relation is conditional on the lagged capital stock so that it shifts as long as the system is not at steady state.

We now analyse the effect of some unexpected changes in exogenous parameters on the $\{u, d\}$ dynamics. We consider in turn changes affecting r, ε and $\{a_t, b_t\}$. It is important to realise that unions and firms operate in a stationary world (the idiosyncratic shocks driving the model are assumed to be i.i.d.). The conclusions would change if some persistence in the shocks would be allowed for.

Let us first consider a negative supply shock through an unexpected increase in the discount rate r (Figure 2). At the moment of the shock, the curves of the bottom chart are not affected (neither technology nor π^d depend on r). However, the dynamic equation which results from the investment decision is affected: It moves to the right, implying that for the same d, q is smaller than one; the increase in the discount rate

reduces the marginal profitability of investment. The union-firm pair begins to reduce the stock of capital. d progressively increases along the new convergence path. The decrease in the capital stock pushes the LE curve eastward: the number of persons that the firm can employ at given d is reduced so that unemployment increases. In this case, we observe a progressive rise in unemployment and in d due to the reduction in the capital stock. Thus,

Property 8. *An unexpected increase in the discount rate generates monotonic rises in both u and d .*

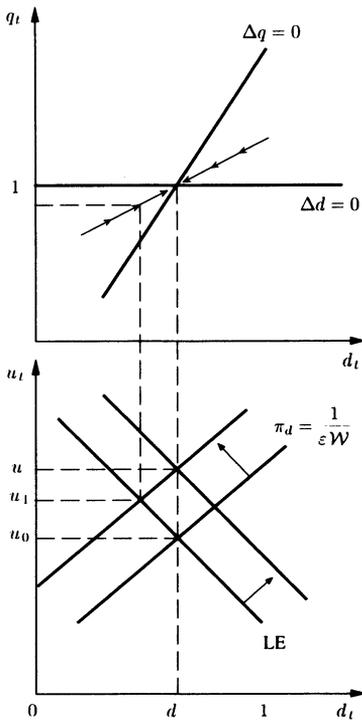


Figure 3: Increase in $1/\varepsilon$

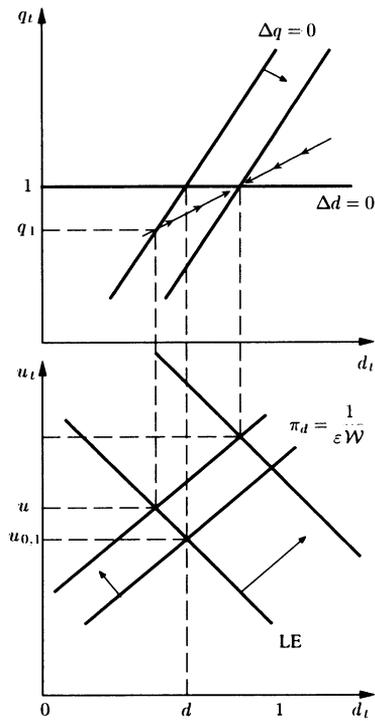


Figure 4: Decrease in a_t and b_t

Let us now consider an unexpected negative demand shock through a decrease in ε ⁽⁶⁾. The goods market becomes less competitive (Figure

⁽⁶⁾This demand shock has nothing to do with an increase in aggregate consumption. In our model, aggregate demand shocks which do not modify the elasticity of demand with respect to prices are neutral.

3). The reasons of a change in ε could be found in Rotemberg and Woodford [1993], although these interpretations are beyond the framework of this model. The immediate effect of such a shock is to shift the $\pi^d = 1/(\varepsilon\mathcal{W})$ curve to the North-West: Given the new structure of the goods market, the price and wage rules require that to the same level of d correspond a higher level of u . The effect of this shift is to increase unemployment and to decrease d . At this time, as shown by the phase diagram, marginal q has fallen below 1 (note that the curves $\Delta q = 0$ and $\Delta d = 0$ are not affected by ε). The union-firm pair gradually decreases the capital stock to bring d back to its initial level. On the bottom chart, the LE curve moves eastward until the steady state is reached. At this point, u is higher and d is unchanged. The proportion of firms constrained by demand has increased while the proportion of firms constrained by capacities has first decreased and then reverted to its initial level. Thus,

Property 9. *An unexpected decrease in price-elasticity of demand generates in a first period a rise in u and a drop in d . It generates in the following periods monotonic rises in u and d . Once the steady state has been reached, d has reverted to its initial level.*

The last shock we consider is a negative productivity shock affecting both a_t and b_t . This could be interpreted as an oil shock if energy is a complementary factor to capital and labour in the production function. In this case, our shock models a drop in the “productivity” of added value. The shock is presented in Figure 4. We observe a shift of the $\pi^d = 1/(\varepsilon\mathcal{W})$ curve to the North-West (if $\omega > \tau(1/\nu - 1)$). Moreover, the dynamic equation $\Delta q = 0$ is affected: At given d , the marginal profitability of investment is reduced. After one period we observe an increase in unemployment and a decrease in d . Then, following the new convergence path to the steady state, investment is reduced so that q increases progressively up to 1. On the bottom chart, the LE curve moves eastward until the steady state is reached. This steady state implies a higher value for d : Contrary to the previous case, the proportion of firms constrained by capacities is increased, because the drop in productivity lowers profits and investment. In this simulation, u and d are negatively correlated in the short run and positively correlated in the long run.

Property 10. *An unexpected decrease in productivity generates in a first period a rise in u and a drop in d . It generates in the following periods monotonic rises in u and d . At the new steady state, d is above its initial level.*

6 Conclusion

Building on previous work in investment theory and in quantity rationing models we have provided a dynamic model with endogenous wages, prices and investment. Its main characteristics are the following. Due to technical rigidities and uncertainty, both capital and labour are under-utilised at steady state. Marginal Tobin's q differs from average q because firms expect to under-utilise their capacities and because unions attach a certain value to capital. At steady state, the gross "social" marginal productivity of capital is equal to the discount rate.

At steady state, unemployment results from the interaction of four elements: The first element refers to the unemployment due to the eventuality of a capacity constraint. It is positively affected by the discount rate and negatively affected by capital productivity. The second element refers to the unemployment due to the eventuality of a demand constraint. This unemployment is related with the nature of the goods market: it increases with product differentiation. The third element is linked with the parameters of the bargaining process (workers' risk aversion, firm power and disutility of work). It influences both types of unemployment through income and profit formation. The last element is linked with structural unemployment. If the allocation of demand and supply shocks among markets is less "equal", the loss of production due to the fact that workers in low production firms cannot move to high production firms is more important.

From the dynamic point of view, we are mainly interested in explaining the joint evolution of unemployment rate and capacity utilisation rate: We find very different results depending upon the type of shock. An unexpected increase in the discount rate generates monotonic rises in both u and d . An unexpected decrease in goods substitutability generates in a first period a rise in u and a drop in d . It generates in the following periods monotonic rises in u and d . At the new steady state, d has reverted to its initial level. An unexpected decrease in productivity generates in a first period a rise in u and a drop in d . It generates in the following periods a rise in u and a rise in d which pushes d above its initial level. In general, d and u are negatively correlated right after the shock. Some time after, they are either positively correlated or not correlated at all. The relevance of the three shocks we have explored should be investigated further by the mean of an econometric and/or simulation-based study⁽⁷⁾.

Some improvements of the model would be necessary to make it more general. A generalisation to a less restrictive technology would be

⁽⁷⁾ A first step in this direction is in de la Croix and Lubrano [1995].

welcome. More importantly, it should be useful to overcome the partial equilibrium spirit of the model. This would allow to introduce in the analysis three important mechanisms: the demand spillovers (if a firm faces a capacity constraint, what happens to its rationed customers? Do they switch to other firms which produce close substitutes? What is the effect on overall employment?), the savings and interest rate determination and, more importantly perhaps, the endogenous treatment of outside opportunities (see Layard and Nickell [1990]).

APPENDIX A The maximisation problem

The maximisation problem is:

$$\max_{\{k_t\}_{t=s}^{\infty}, \{p_t\}_{t=s}^{\infty}, \{w_t\}_{t=s}^{\infty}} \sum_{t=s}^{\infty} E_s(U_t) \theta^{t-s} + \lambda \sum_{t=s}^{\infty} E_s(V_t) \theta^{t-s}$$

s.t.

$$\begin{aligned} E_s(U_t) &= \frac{E_s(y_t)}{a_t} \frac{1}{\nu} \left(\frac{w_t}{P_t} - \omega_t \right)^{\nu} \\ E_s(V_t) &= \frac{E_s(y_t)}{P_t} \left(p_t - \frac{w_t}{a_t} \right) - \Psi(i_t) k_{t-1} \\ k_t &= (1 + i_t) k_{t-1} \\ E_s(y_t) &= [E_s(y_t^d)^{-\rho} + (a_t l_t^s)^{-\rho} + (b_t k_{t-1})^{-\rho}]^{-1/\rho} \\ E_s(y_t^d) &= \left(\frac{p_t}{P_t} \right)^{-\varepsilon} \frac{\bar{C}_t}{P_t} \end{aligned}$$

with $\infty > \lambda > 0, \nu < 1, \rho > 0, \varepsilon > 1$.

APPENDIX B Wage and price determination

The first-order condition for wage implies

$$\frac{E_s(y_t)}{a_t} \frac{1}{P_t} \left(\frac{w_t}{P_t} - \omega_t \right)^{\nu-1} - \lambda \frac{E_s(y_t)}{a_t} \frac{1}{P_t} = 0.$$

Denoting $\tau = \lambda^{1/(\nu-1)}$ we get:

$$\frac{w_t}{P_t} = \tau + \omega_t.$$

The first-order condition for prices implies

$$0 = \frac{1}{a_t} \frac{1}{\nu} \left(\frac{w_t}{P_t} - \omega_t \right)^\nu \frac{\partial E_s(y_t)}{\partial p_t} + \lambda \frac{E_s(y_t)}{P_t} + \lambda \frac{1}{P_t} \left(p_t - \frac{w_t}{a_t} \right) \frac{\partial E_s(y_t)}{\partial p_t}.$$

Knowing that

$$\frac{\partial E_s(y_t)}{\partial p_t} = -\varepsilon \pi_t^d \frac{E_s(y_t)}{p_t}$$

and multiplying both sides by p_t it comes:

$$0 = -\frac{1}{a_t} \frac{1}{\nu} \left(\frac{w_t}{P_t} - \omega_t \right)^\nu \varepsilon \pi_t^d + \lambda \frac{p_t}{P_t} - \lambda \frac{1}{P_t} \left(p_t - \frac{w_t}{a_t} \right) \varepsilon \pi_t^d.$$

Solving for p_t leads to:

$$\lambda (\varepsilon \pi_t^d - 1) \frac{p_t}{P_t} = -\frac{1}{a_t} \frac{1}{\nu} \left(\frac{w_t}{P_t} - \omega_t \right)^\nu \varepsilon \pi_t^d + \lambda \left(\frac{w_t}{P_t a_t} \right) \varepsilon \pi_t^d,$$

which gives the mark-up rule:

$$p_t = \frac{\varepsilon \pi_t^d}{\varepsilon \pi_t^d - 1} \left[\frac{w_t}{a_t} - \frac{P_t}{\lambda a_t} \frac{1}{\nu} \left(\frac{w_t}{P_t} - \omega_t \right)^\nu \right].$$

APPENDIX C

Short-run equilibrium unemployment

Solving

$$\begin{cases} \frac{w_t}{p_t} = \tau + \omega_t \\ p_t = \left[1 + \frac{1}{a_t \nu} \tau - \frac{1}{\varepsilon \pi_t^d} \right]^{-1} \frac{w_t}{a_t} \end{cases}$$

for π_t^d leads to:

$$\pi_t^d = \frac{1}{\varepsilon} \left[1 + \frac{\tau}{a_t} \left(\frac{1}{\nu} - 1 \right) - \frac{\omega_t}{a_t} \right]^{-1}.$$

Using the output expression, we have:

$$y_t^{-\rho} = (a_t l_t^s)^{-\rho} + (b_t k_{t-1})^{-\rho} + \pi_t^d y_t^{-\rho},$$

which gives:

$$y_t = \left(\frac{1 - \pi_t^d}{(a_t l_t^s)^{-\rho} + (b_t k_{t-1})^{-\rho}} \right)^{(1/\rho)}$$

Knowing that $u_t = 1 - y_t/(a_t l_t^s)$ and denoting $g_t = 1 - b_t k_{t-1}/a_t l_t^s$, we find the unemployment rate at time s as:

$$u_t = 1 - \left(\frac{1 - \frac{1}{\varepsilon} [1 + (\tau/a_t) (\frac{1}{\nu} - 1) - (\omega_t/a_t)]^{-1}}{1 + (1 - g_t)^{-\rho}} \right)^{(1/\rho)}$$

Knowing that $d_t = y_t/(b_t k_{t-1})$, we find the degree of utilisation of capital:

$$d_t = \left(\frac{1 - \frac{1}{\varepsilon} [1 + (\tau/a_t) (\frac{1}{\nu} - 1) - (\omega_t/a_t)]^{-1}}{1 + (1 - g_t)^{\rho}} \right)^{(1/\rho)}$$

APPENDIX D Investment determination

The first-order condition for capital for $s > t$ implies

$$\begin{aligned} \lambda \frac{\partial \Psi}{\partial i_{t-1}} &= \frac{E_s(U_t)}{E_s(y_t)} \frac{\partial E_s(y_t)}{\partial k_{t-1}} \theta \\ &+ \lambda \left[\frac{1}{P_t} \frac{\partial E_s(y_t)}{\partial k_{t-1}} \left(p_t - \frac{w_t}{a_t} \right) - \left(\frac{\partial \Psi}{\partial i_t} (-i_t - 1) + \Psi(i_t) \right) \right] \theta. \end{aligned}$$

Knowing that $\partial E_s(y_t)/\partial k_{t-1} = \pi_t^p E_s(y_t)/k_{t-1}$ and multiplying both sides by k_{t-1}/λ , we get:

$$\begin{aligned} k_{t-1} \frac{\partial \Psi}{\partial i_{t-1}} &= \frac{\pi_t^p}{\lambda} E_s(U_t) \theta + \frac{\pi_t^p}{P_t} E_s(y_t) \left(p_t - \frac{w_t}{a_t} \right) \theta \\ &- k_{t-1} \left(\frac{\partial \Psi}{\partial i_t} (-i_t - 1) + \Psi(i_t) \right) \theta. \end{aligned}$$

Using the law of motion of capital we find:

$$k_{t-1} \frac{\partial \Psi}{\partial i_{t-1}} = k_t \frac{\partial \Psi}{\partial i_t} \theta + \frac{\pi_t^p}{\lambda} E_s(U_t) \theta + \left[\frac{\pi_t^p}{P_t} E_s(y_t) \left(p_t - \frac{w_t}{a_t} \right) - k_{t-1} \Psi(i_t) \right] \theta.$$

Summing over s it comes:

$$k_s \frac{\partial \Psi}{\partial i_s} = \sum_{t=s+1}^{\infty} \left[\frac{1}{\lambda} \pi_t^p E_s(U_t) \frac{\pi_t^p}{P_t} E_s(y_t) \left(p_t - \frac{w_t}{a_t} \right) - k_{t-1} \Psi(i_t) \right] \theta^{t-s}.$$

Finally, using the definition of profit we find

$$k_s \frac{\partial \Psi}{\partial i_s} = \frac{1}{\lambda} \sum_{t=s+1}^{\infty} [\pi_t^p E_s(U_t) + \pi_t^p E_s(V_t) + (\pi_t^p - 1)k_{t-1} \Psi(i_t)] \theta^{t-s}.$$

APPENDIX E

Steady state

The steady state ($\Psi(0) = 0$ and $\Psi'(0) = 1$) implies for the investment equation:

$$k_t = \frac{1}{\lambda} \sum_{t=s+1}^{\infty} [\pi_t^p U_t] \theta^{s-t} + \sum_{t=s+1}^{\infty} [\pi_t^p V_t] \theta^{s-t}.$$

Using $\sum_{t=s+1}^{\infty} \theta^{s-t} = 1/r$ we have:

$$rk = \pi^p \left[\frac{1}{\lambda} U + V \right]$$

$$U = \frac{y}{a} \frac{1}{\nu} \left(\frac{w}{p} - \omega \right)^\nu$$

$$V = \frac{y}{p} \left(p - \frac{w}{a} \right)$$

$$\frac{w}{p} = \tau + \omega$$

$$p = \frac{\varepsilon \pi^d}{\varepsilon \pi^d - 1} \left[\frac{w}{a} - \frac{p}{\lambda a} \frac{1}{\nu} \left(\frac{w}{p} - \omega \right)^\nu \right].$$

Computing the value of V and U leads to

$$U = \frac{y \tau^\nu}{a \nu}$$

$$V = y \left(1 - \frac{\tau}{a} - \frac{\omega}{a} \right).$$

Replacing U and V by their value, we find

$$rk/y = \pi^p \left[\frac{\tau}{a \nu} + \left(1 - \frac{\tau}{a} - \frac{\omega}{a} \right) \right]$$

$$\frac{w}{p} = \tau + \omega$$

$$p = \frac{\varepsilon \pi^d}{\varepsilon \pi^d - 1} \left[\frac{w}{a} - \frac{p}{\lambda a} \frac{1}{\nu} \left(\frac{w}{p} - \omega \right)^\nu \right].$$

Replacing w/p by its value gives:

$$\begin{aligned}(r)k/y &= \pi^p \left[\frac{\tau^\nu}{\lambda a \nu} + 1 - \frac{1}{a}(\tau + \omega) \right] \\ 1 &= \frac{\varepsilon \pi^d}{\varepsilon \pi^d - 1} \left[\frac{1}{a}(\tau + \omega) - \frac{1}{\lambda a} \frac{1}{\nu} \tau^\nu \right].\end{aligned}$$

Using $y/k = b(\pi^p)^{1/\rho}$, we may solve the system for π^p and π^d :

$$\begin{aligned}\pi^p &= \left(\frac{r}{b} \left[\frac{\tau}{a} \left(\frac{1}{\nu} - 1 \right) + 1 - \frac{\omega}{a} \right]^{-1} \right)^{\rho/(\rho+1)} \\ \pi^d &= \frac{1}{\varepsilon} \left[1 + \frac{\tau}{a} \left(\frac{1}{\nu} - 1 \right) - \frac{\omega}{a} \right]^{-1}.\end{aligned}$$

Using the definition

$$\mathcal{W} = \left[1 + \frac{\tau}{a} \left(\frac{1}{\nu} - 1 \right) - \frac{\omega}{a} \right]$$

we may now compute the equilibrium unemployment rate and the equilibrium d :

$$\begin{aligned}d &= \left(\frac{r}{b\mathcal{W}} \right)^{1/(\rho+1)} \\ u &= 1 - \left(1 - \left(\frac{r}{b\mathcal{W}} \right)^{\rho/(\rho+1)} - \frac{1}{\varepsilon \mathcal{W}} \right)^{1/\rho}.\end{aligned}$$

APPENDIX F Dynamics

The dynamic behaviour of the system is given by the two following equations:

$$\begin{aligned}k_t &= (1 + i_t)k_{t-1} \\ k_{t-1} \frac{\partial \Psi}{\partial i_{t-1}} &= \frac{\pi_t^p}{\lambda} E_s(U_t) \theta + \frac{\pi_t^p}{P_t} E_s(y_t) \left(p_t - \frac{w_t}{a_t} \right) \theta \\ &\quad - k_{t-1} \left(\frac{\partial \Psi}{\partial i_t} (-i_t - 1) + \Psi(i_t) \right) \theta\end{aligned}$$

which are the accumulation rule and the first-order condition for investment. Using $q_s = \frac{\partial \Psi}{\partial i_s}$, multiplying both sides by $1/\theta k_{t-1}$, using $E_s(y_t)/k_{t-1} =$

$b(\pi_t^p)^{(1/\rho)}$ and the values of $E_s(U_t)$ and $E_s(V_t)$ implied by the price and wage formation, it comes:

$$\begin{aligned}\Delta k_t &= (i_t)k_{t-1} \\ \Delta q_t &= (r - i_t)q_{t-1} - (\pi_t^p)^{1/\rho+1} \mathcal{W} b + \Psi(i_t).\end{aligned}$$

Using the fact that, by definition,

$$\Delta d_t = \frac{d_t^{\rho+1} - d_t}{k_{t-1}} \Delta k_s$$

and that $\pi_t^p = d_t^\rho$, the dynamic system becomes:

$$\begin{aligned}\Delta d_t &= -(i_t)(d_t - d_t^{\rho+1}) \\ \Delta q_t &= (r - i_t)q_{t-1} - d_t^{\rho+1} \mathcal{W} b + \Psi(i_t).\end{aligned}$$

The linearization around the steady state leads to

$$\begin{pmatrix} d_t \\ q_t \end{pmatrix} = \begin{pmatrix} 1 & -\gamma_1 \\ -\gamma_3 & 1 + \gamma_2 \end{pmatrix} \begin{pmatrix} d_{t-1} - \left(\frac{r}{b\mathcal{W}}\right)^{1/(\rho+1)} \\ q_{t-1} - 1 \end{pmatrix} \quad (32)$$

with

$$\begin{aligned}\gamma_1 &= \left[\left(\frac{r}{b\mathcal{W}}\right)^{1/(\rho+1)} - \left(\frac{r}{b\mathcal{W}}\right)^{(\rho+1)/\rho} \right] \Psi''(0)^{-1} \\ \gamma_2 &= r + \frac{1}{\Psi''(0)} \\ \gamma_3 &= r(\rho + 1).\end{aligned}$$

The roots of the characteristic equation are:

$$\lambda_1 = 1 + \frac{\gamma_2 - \sqrt{\gamma_2^2 + 4\gamma_1\gamma_3}}{2} \quad \lambda_2 = 1 + \frac{\gamma_2 + \sqrt{\gamma_2^2 + 4\gamma_1\gamma_3}}{2}.$$

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