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Irreversibilities, uncertainty and underemployment equilibria

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Abstract. In a competitive overlapping generations model, technological irreversibilities and idiosyncratic uncertainty generate a misallocation of resources among segments, which takes the form of underemployment and underutilization of capacities at the aggregate level. This affects the qualitative properties of the equilibrium path. Indeed, increases in the variance of the technological shock can be responsible, a.o., for an "inescapable poverty trap," or for periodic orbits generating endogenous fluctuations in underemployment.

JEL classification: D90

Key words: Underemployment, underutilization, irreversibility, poverty trap, endogenous fluctuations

Introduction

The existence of underemployment of both capital and labor is an important stylized fact of actual economies. It is relevant for business cycle analysis but also for growth theory, as shown by the Europe experience of a continuous growth over the last twenty years with a high degree of resources underutilization. Several authors have stressed the importance of structural mismatch in explaining such a situation. The structural mismatch literature covers both the inadequacy of skills and the lack of regional mobility. In both contexts, the bottom line is the high degree of rigidity: the labor force does not seem to move easily across

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skills and regions. Regional mismatch seems highly due to low migration and participation elasticities to local conditions (Jimeno and Bentolila 1998), and skill mismatch is due to an increasing distance between industry needs and the actual composition of the labor force (Sneessens and Shadman-Mehta 1995) and, hence, to the inadequacy of the training system (Padoa-Schioppa 1990).

"Equilibrium unemployment" may exist if the process of matching workers and jobs is not instantaneous as, e.g., in Blanchard and Diamond (1989) and Pissarides (1990). This process is formalised by a matching function, which represents a transaction technology different from a standard production technology. An alternative, but in some ways complementary, approach was proposed by Hansen (1970) and Tobin (1972) but it was not followed by other authors in a neo-classical growth framework. It relies on the aggregation over heterogeneous markets, with the outcome in each separate market being either unemployed workers or unfilled vacancies.[1]

In this paper we go back to Hansen's original idea and we show that the existence of underemployment in an "aggregation over heterogeneous markets" context introduces additional non-linearities that could modify the qualitative properties of equilibrium paths in the Diamond economy. Our main assumption is the existence of technological irreversibilities in both physical and human capital, in an uncertain world. Indeed, in our model, irreversibilities and uncertainty generate a misallocation of resources among segments. These segments should be thought either as industries or as regions. Aggregation over heterogeneous markets with underemployment and underutilized capacities (unfilled vacancies), as in Hansen (1970), generates the coexistence of underutilized capital and labor at the macroeconomic equilibrium. This misallocation affects the equilibrium path and we show that it can be responsible for catastrophes like "inescapable poverty traps" and/or self-driven oscillatory phenomena.[2]

We thus study the implications of underemployment in an otherwise standard equilibrium model, by adding a simple theory of underemployment to an overlapping generations models à la Diamond (1965). In Sect. 1 we present the main characteristics of the model and we solve the agents' problems. Firms choose a technology and commit capital to this technology one period in advance, before shocks on their productivity take place. Workers commit their labor to a specific technology. The aggregate equilibrium conditions are derived in Sect. 2, and equilibrium underutilization of resources appears as agents cannot reallocate: in the segments with high productivity there are no workers enough to make use of machines; in the segments with low productivity, the machines are so inefficient that it is not worth to use all the workers. The dynamic properties of equilibria are analyzed by mean of some examples in Sect. 3; we show that by increasing

[1] The "aggregation over micromarkets in disequilibrium" hypothesis, proposed by Muellbauer (1978), is very related to Hansen's idea and it was fruitfully utilized in the fixed-price literature. It was developed for empirical proposes by Lambert (1988) and Kooiman (1984).

[2] In this paper, we are mainly interested on long run factor underutilization. Under similar assumptions, Fagnart et al. (1999) analyze the business cycle implications of factor underutilization.

the uncertainty faced by firms, it is more likely to find more complex dynamics for the system, and patently pessimistic outcomes. Section 4 concludes.

1 The model

The main assumptions under which this model is built are the following.

First, it is a two period overlapping generations model, as in Diamond (1965), where individuals live two periods and markets are competitive. In each period we have two generations, young and old. Let us assume that there is a continuum of young agents in the interval $[0, N_t]$, with N_t growing at the rate n. Each young individual has one unit of a specific labor endowment, works, consumes and lends savings to firms. Old people only consume. There is only one good in the economy, which can be consumed or accumulated as capital. To produce this good there exist different technologies, each of them depending on specific labor and capital inputs.

Secondly, technological choices are irreversible (a putty-clay technology). As it is standard in OLG models, the capital stock is decided one period ahead. A given technology is always associated to this capital stock, in the sense that machines incorporate, when they are bought, a particular capital-labor ratio.

Third, the factors of production are firm specific, i.e., the labor markets are segmented and investment is irreversible and cannot be valuable elsewhere. There is a continuum of segments in the interval $[0, 1]$, with a large number of firms and workers in each segment. Each segment is denoted by i.[3] The number of workers in each segment is supposed to be equal to N_t.[4] We normalize the number of firms to N_t in order to simplify the notation, which allows us to work with per-capita variables.[5]

Finally, at the time of the decisions on capital and the related technology, there is some idiosyncratic uncertainty concerning the average productivity of capital. Moreover, after their realization, shocks become public information. Consumption, savings, wages, employment and production take place simultaneously under full-information. Since all uncertainty is firm specific (there is no aggregate uncertainty), the bond market portfolio pays the riskless rate of return. This timing, even if it is relatively standard, is relevant in generating underemployment of production factors.

[3] This economy can be seen as a particular case of a more general economy where there is a continuum of goods, each of them being produced with specific capital and labor inputs. In this particular version all goods are perfect substitutes. Alternatively, we can see this economy as one in which firms are geographically located and segments represent a particular location; goods are allowed to move costlessly among places, while inputs are not.

[4] We can see this economy as if individuals live three periods. In each period we have three generations: children, young and olds. Each child does not consume at all (its consumption is implicitly in the utility function of its parents) and chooses costlessly a specific human capital. At the time of the kids' decision, expected labor incomes are the same for all types of human capital, implying that kids are distributed uniformly over the different segments of the labor market at equilibrium.

[5] Because the production function has constant returns to scale, the number of firms is undetermined and irrelevant.

1.1 The consumer problem

All individuals have identical preferences over consumption when young c_{i1t} and consumption when old c_{i2t+1}, represented by a utility function $U(c_{i1t}, c_{i2t+1})$, which is supposed to be homothetic and increasing in its arguments, differentiable and concave in the positive orthant. The representative individual of generation t, with specific labor endowment i, solves the following problem:

$$\max_{c_{i1t}, c_{i2t+1}} U(c_{i1t}, c_{i2t+1})$$

subject to the intertemporal budget constraint

$$c_{i1t} + \frac{c_{i2t+1}}{1 + r_{t+1}} = w_{it} l_{it}.$$

The real wage w_{it} paid in segment i and the real interest rate r_{t+1} are taken as given by the individual. Labor supply is infinitely inelastic, but as it is shown later, an individual could be underemployed at equilibrium, i.e., $l_{it} \leq 1$.[6]

The optimal savings are:

$$s_{it} = \theta(r_{t+1})\, w_{it}\, l_{it}, \tag{1}$$

where s_{it} represents savings. Given that the utility function is supposed to be homothetic, individual savings are a proportion of individual labor income, i.e., the function θ represents the "propensity to save" and depends only on the interest rate. Since human capital is segment specific, individuals from the same generation could have a different labor income. However, they have the same propensity to save, implying that aggregate savings are a proportion θ of aggregate labor income.

1.2 The putty-clay technology

Firms in a particular segment have the same technology and employ segment specific inputs. Technology is different from one segment to another, even if for reason of symmetry the production function is assumed to take the same functional form for all segments.

As it is standard in OLG models, the capital stock in segment i at time t, k_{it}, is assumed to be bought at time $t-1$. Production y_{it} and the labor input l_{it} are chosen at time t. The technology is putty-clay:

$$y_{it} = \min\left\{ f(x_{it}) l_{it}, \frac{f(x_{it})}{x_{it}} \mu_{it} k_{it} \right\}.$$

Labor and capital productivities depend on x_{it}, which can be seen as the ex-ante capital-labor ratio. At $t-1$ when buying capital, the firm chooses its technology

[6] Equivalently, there could be assumed that $l_{it} \in \{0, 1\}$ for each individual. In which case, if employment is strictly smaller than one in segment i, some workers would be unemployed.

by deciding on x_{it} (capital embodies given factor productivities). μ_{it} is a stochastic shock and $f(x)$ is assumed to be increasing and concave.

In this technology, the productivity of capital is affected by the multiplicative stochastic shock $\mu_{it} \in R_+$.[7] We assume that μ_{it} is drawn from the continuous distribution $F(\mu_{it})$, the same for all i and t, such that $E(\mu_{it}) = 1$. There is "heterogeneity" in this economy and it is related to the realizations of the μ_{it} idiosyncratic shocks. The distribution over segments of the realized μ_{it} shocks follows the same distribution $F(\mu_{it})$.

1.3 Labor market equilibrium

In each segment, wages and employment are determined competitively. The labor supply is infinitely inelastic, because workers do not care about leisure. Concerning the labor demand, since k_{it} and x_{it} were decided in period $t-1$, production technology is Leontief at period t, which allows us to define an upper bound on labor demand, i.e., the employment level needed to produce at full-capacity,

$$l_{it}^p \equiv \frac{k_{it}}{x_{it}} \mu_{it}.$$

From the Leontief technology, labor demand is infinitely elastic until full-capacity is reached and then it becomes infinitely inelastic.

Figure 1 shows the two type of equilibrium that could arrive in the labor market, in which l^s and l^d represents labor supply and labor demand respectively: In Fig. 1a, capacities are relatively low, because the productivity shock is low, implying that equilibrium wages are equal to the reservation wage (equal to zero) and firms produce at full-capacity; otherwise, as in Fig. 1b, if the shock is high, workers are fully employed at equilibrium and wages are equal to the productivity of labor.

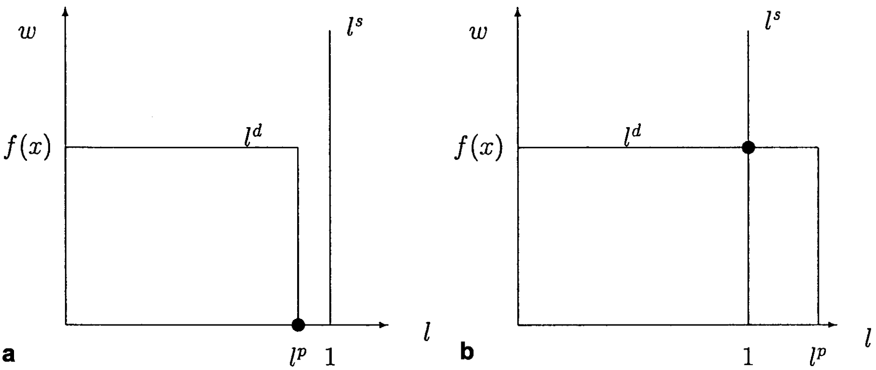

Fig. 1. The labor market equilibrium. **a** Underemployment equilibrium; **b** under-utilisation equilibrium

Let us call $\bar{\mu}_{it}$ the value of μ_{it} such that capacities are just enough to employ all workers, i.e.,

[7] We assume for simplicity that the productivity of labor is non stochastic.

$$\bar{\mu}_{it} = \frac{x_{it}}{k_{it}},$$

which implies that $l^p_{it} = 1$. As all firms in segment i are identical, they have set the same k_{it} and x_{it} and they face the same shock μ_{it}. The definition of $\bar{\mu}$ allows us to express the outcome of the competitive equilibrium as a function of the idiosyncratic shock:[8]

$$w_{it} = \begin{cases} 0 & \text{if} \quad \mu_{it} < \bar{\mu}_{it} \\ f(x_{it}) & \text{if} \quad \mu_{it} > \bar{\mu}_{it} \end{cases} \tag{2}$$

$$l_{it} = \begin{cases} l^p_{it} & \text{if} \quad \mu_{it} < \bar{\mu}_{it} \\ 1 & \text{if} \quad \mu_{it} \geq \bar{\mu}_{it}. \end{cases} \tag{3}$$

In Eq. (3) we assume implicitly that the rationing scheme is uniform, i.e., when firms are rationed total labor supply is allocated proportionally among them.[9] Figure 1 represents the two possible situations. The realized productivity shock μ_{it} could be: (a) "bad" (i.e. $\mu_{it} < \bar{\mu}_{it}$), in which case capacities are so small that firms, even producing at full-capacity, can not hire all workers; since labor do not generate any disutility, the corresponding equilibrium wage is zero; or (b) "good" (i.e. $\mu_{it} > \bar{\mu}_{it}$), in which case the equilibrium wage is given by the average productivity of labor and capacities are underemployed. We call "underemployment equilibrium" the first case (Fig. 1a) and "underutilization (of capital) equilibrium" the second case (Fig. 1b).

1.4 Firm's capital and technological choices

As stated before, the capital stock and the capital-labor ratio for period t are chosen at $t-1$. Since there is uncertainty concerning the productivity of capital, the representative firm of segment i chooses x_{it} and k_{it} in order to maximize expected profits, i.e.,

$$\max_{k_{it}, x_{it}} \mathrm{E}_{t-1} \left[(f(x_{it}) - w_{it}\, l_{it} - (\delta + r_t) k_{it} \right]$$

where

$$l_{it} = \min \left[\frac{k_{it}}{x_{it}} \mu_{it}, \bar{l}_{it} \right].$$

and

$$\bar{l}_{it} = \begin{cases} \infty & \text{if} \quad \mu_{it} \leq \bar{\mu}_{it} \\ 1 & \text{otherwise} \end{cases}$$

[8] The equilibrium wage rate is undetermined at $\mu_{it} = \bar{\mu}_{it}$. However, the aggregate equilibrium wage rate is well determined since the distribution F(.) is atomless.

[9] Notice that, even if rationed firms were interested in increasing labor supply by paying a wage greater than the equilibrium wage, this policy would not be optimal since such a wage would engender negative profits.

The parameter $0 \leq \delta \leq 1$ represents the depreciation rate. Wages are taken as given by the firm and, from (2), they are a known function of the shock. The firm also takes the interest rate as given.

At period $t-1$, the representative firm can forecast the labour market equilibrium as a function of the unknown shock μ_{it}. $\overline{l}_{it}$ represents the forecast labor supply: if the shock is bad, the firm knows that there will not be any rationing and that she will face an infinitely elastic labor supply; if the shock is good, firms will be proportionally rationed and each firm will face an infinitely inelastic labor supply (at $l_{it} = 1$).

As shown in the Appendix, the first order necessary conditions for x_{it} and k_{it} are respectively:

$$x_{it}\, f'(x_{it})\, \mathrm{E}_{t-1}(l_{it}) = \frac{k_{it}}{x_{it}} \int_0^{\overline{\mu}_{it}} f(x_{it})\, \mu_{it}\; \mathrm{dF}(\mu_{it}) \tag{4}$$

$$\delta + r_t = \frac{1}{x_{it}} \int_0^{\overline{\mu}_{it}} f(x_{it})\, \mu_{it}\; \mathrm{dF}(\mu_{it}) \tag{5}$$

To interpret these conditions let us call d_{it} at the ratio of expected production to expected capacities:

$$d_{it} = \frac{x_{it}\ \mathrm{E}_{t-1}(l_{it})}{k_{it}}. \tag{6}$$

Combining conditions (4) and (5) with the definition of d, one has

$$\delta + r_t = f'(x_{it})\; d_{it}. \tag{7}$$

It states that the user cost of capital must be, at the optimum, equal to the expected marginal benefit of capital. The marginal benefit is equal to the marginal productivity of capital times the ratio of expected production to expected capacities, i.e., marginal productivity is weighted by the probability that the new equipment be effectively utilized.

Since the economy is perfectly competitive and returns to scale are constant, from the optimality conditions we can easily show that expected profits are zero.

2 The aggregate equilibrium

Since all segments are ex-ante identical and the expected value of the shock is one, heterogeneity in the economy can be seen as deviations from the capital average productivity. But this heterogeneity exists only ex-post, after the realization of the idiosyncratic shocks. Since there is no heterogeneity ex-ante, all firms in all segments choose the same capital stock and capital-labor ratio. By symmetry, the optimality conditions for x and k are the same for all firms in all segments and they are given by (4) and (7), i.e.,

$$x_t f'(x_t)\, l_t = \frac{k_t}{x_t} \int_0^{\overline{\mu}_t} f(x_t)\mu\, \mathrm{dF}(\mu) \tag{8}$$

$$\delta + r_t = f'(x_t)\frac{x_t\, l_t}{k_t}, \tag{9}$$

where

$$\bar{\mu}_t = \frac{x_t}{k_t}. \tag{10}$$

x_t, k_t and l_t represent optimal capital-labor ratio, capital stock and expected employment respectively. Per-capita aggregate employment, which is equal to expected employment, results from the aggregation over μ of individual employment and it can be written as

$$l_t = \frac{k_t}{x_t}\int_0^{\bar{\mu}_t} \mu\, \mathrm{dF}(\mu) + \int_{\bar{\mu}_t}^{\infty} \mathrm{dF}(\mu). \tag{11}$$

From (2) and (3) aggregate labor income is given by

$$w_t l_t = f(x_t)\int_{\bar{\mu}_t}^{\infty} \mathrm{dF}(\mu).$$

Combined with the optimal condition for x (Eq. (8)), the aggregate labor income becomes:

$$w_t l_t = [f(x_t) - x_t f'(x_t)]\, l_t. \tag{12}$$

Note that the wage index is equal to the ex-ante marginal productivity of labor.

From the optimality conditions for capital and the capital-labor ratio, we know that aggregate pure profits are zero, even if some firms have negative profits and other firms have positive profits. A costless insurance contract is supposed to share the aggregate zero pure profits and to avoid that some firms be unable to repay their debts.

Finally, the equilibrium requires the clearing condition between savings and the capital stock, which from (1) and (12) is

$$\theta(r_{t+1})\,[f(x_t) - x_t f'(x_t)]\, l_t = k_{t+1}(1+n). \tag{13}$$

2.1 Underemployment and capacity utilization

Proposition 1. *Provided that the probability of being in the "bad (resp. good) state" is strictly positive, there is underemployment (resp. underutilization of capacities) at equilibrium.*

Proof.

- if $\mathrm{F}(\bar{\mu}) > 0$, then

$$l_t = \int_0^{\bar{\mu}} \frac{\mu}{\bar{\mu}}\, \mathrm{dF}(\mu) + \int_{\bar{\mu}}^{\infty} \mathrm{dF}(\mu) \quad < \quad \int_0^{\infty} \mathrm{dF}(\mu) = 1$$

since $\int_0^{\bar{\mu}} \frac{\mu}{\bar{\mu}}\, \mathrm{dF}(\mu) < \int_0^{\bar{\mu}} \mathrm{dF}(\mu)$.

- if $1 - F(\bar{\mu}) > 0$ then

$$d_t = \frac{x_t l_t}{k_t} = \int_0^{\bar{\mu}} \mu \, dF(\mu) + \int_{\bar{\mu}}^{\infty} \bar{\mu} \, dF(\mu) < \int_0^{\infty} \mu \, dF(\mu) = 1$$

since $\int_{\bar{\mu}}^{\infty} \bar{\mu} \, dF(\mu) < \int_{\bar{\mu}}^{\infty} \mu \, dF(\mu)$. □

At equilibrium, if a positive measure of firms are in an “underemployment equilibrium” (resp. “underutilization equilibrium”), there is underemployment of labor (resp. underutilization of capacities) in the aggregate. The coexistence of underemployment of labor and underutilization needs only that the probability of being in both the “bad” and the “good” states be strictly positive. For any non-degenerate continuous distribution function F defined in $]0, \infty[$, this property is verified if $0 < \bar{\mu} < \infty$.

Underemployment of production factors results from the fact that irreversible skill decisions of households and irreversible investment decisions of firms are taken without knowing with certainty firms productivity. This implies that some agents have invested their human capital in segments that are hit by a negative shock, generating underemployment of labor because there is a lack of productive capacities. On the other hand, some other agents have invested their physical capital in segments that are hitten by a positive shock, being unable to fully utilize their capacities because there is a lack of skilled workers in their segment. In this economy, uncertainty and irreversibilities generate a misallocation of capital and labor across segments, described as “structural mismatch” in the “quantity rationing literature” (see, e.g., Sneessens 1987 and de la Croix and Licandro 1995).

In a more general OLG model with many generations the time unit should be smaller implying that technological decisions should be taken more often than once in firm’s life, as in the vintage capital model, with firms taking these decisions at different moments in time. If idiosyncratic shocks have some persistence, the existence of resource underemployment could be partially offset by the birth of new generations investing in the human capital needed for segments in excess capacity. Moreover, if individual are not too old, they could reinvest in the human capital needed for these segments. However, the systematic replacement of old by new vintages of capital would reinforce the misallocation of resources. Unfortunately, we are far from being able to solve models with vintage and human capital simultaneously.[10]

2.2 The capital-labor ratio

We have in this framework three different definitions for the capital-labor ratio: x_t represents the optimal capital-labor ratio, which is incorporated in the machines;

[10] The difficulties for solving vintage physical capital models are pointed out by Boucekkine et al. (1997). A vintage human capital model is proposed by de la Croix and Licandro (1999).

k_t represents the capital stock per-capita; k_t/l_t represents the effective capital-labor ratio. From the definition of d_t in Eq. (6) and l_t in (11) we know that

$$x_t = d_t \left(\frac{k_t}{l_t}\right).$$

The effective capital-labor ratio is larger than the optimal one because some units of capital are not employed at equilibrium. Moreover, from the definitions of d_t and l_t, we know that

$$\left[\frac{k_t}{l_t}\right]^{-1} = [x_t]^{-1} \int_0^{x_t/k_t} \mu \, \mathrm{dF}(\mu) + [k_t]^{-1} \int_{x_t/k_t}^{\infty} l_t^s \, \mathrm{dF}(\mu).$$

This means that the effective capital-labor ratio (the one which is observed at the macroeconomic level) is a weighted average of (i) the ex-ante capital-labor ratio which is the effective ratio prevailing in the firms with a bad productivity shock and (ii) the capital stock per capital which is the capital-labor ratio prevailing in firms with a good productivity shock.

3 Equilibrium dynamics

The equilibrium path of this economy, $\forall t \geq 0$, is described by the first order difference equation system (8) to (13), with given initial condition k_0. As it is standard in OLG models, different types of equilibria are possible. Nevertheless, we argue that the non-linearities associated to irreversibilities and uncertainty contribute to enrich the dynamic properties of the Diamond model. We show it by the mean of an example in which equilibria and their qualitative properties depend on the variance of the idiosyncratic shock.

To analyze the effect of uncertainty on the equilibrium path, we impose the following particular assumptions. The utility function $U(c_1, c_2)$ is Cobb-Douglas, which implies that the propensity to save is constant $\theta(r) = \theta$. The function $f(x)$ is of the CES type,

$$f(x_t) = A(\alpha x_t^{-\gamma} + 1 - \alpha)^{-\frac{1}{\gamma}}$$

where $A > 0$, $\gamma \geq -1$ and $0 < \alpha < 1$. The idiosyncratic shock is log-normal distributed, i.e.,

$$\mu_{it} = \exp\left\{-\frac{\sigma^2}{2} + \epsilon_{it}\right\}$$

where $\epsilon_{it} \sim \mathrm{N}(0, \sigma^2)$.

Following Lambert (1988) Eq. (11) can be approximated by

$$l_t \approx \left(\left(\frac{x_t}{k_t}\right)^{\rho} + 1\right)^{-\frac{1}{\rho}} \quad \text{where} \quad \rho = -1 + \frac{2}{\sigma}\,\frac{\phi(-2/\sigma)}{\Phi(-2/\sigma)}, \tag{14}$$

ϕ represents the standard normal density and Φ the standard cumulative normal distribution. The parameter $\rho > 0$ and it is an inverse function of σ. Let us

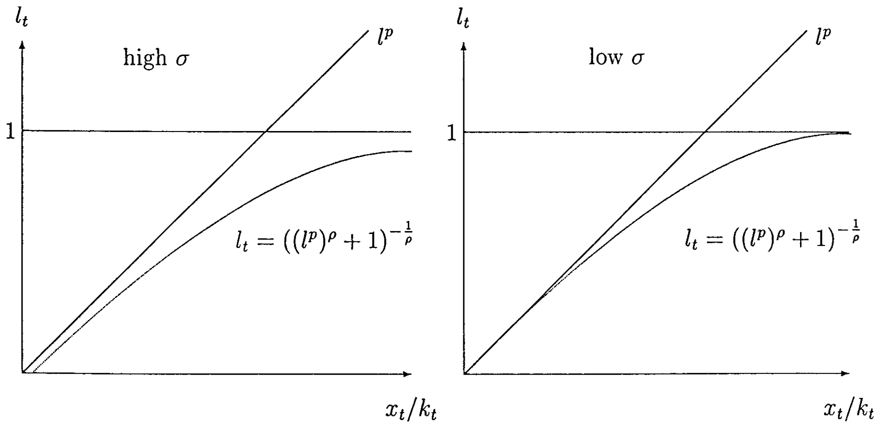

Fig. 2. Aggregate employment

call $1/\rho$ the "structural mismatch" parameter, which represents a measure of the misallocation of capital and labor across segments. Figure 2 displays aggregate employment l_t as a function of the two constraints, the aggregate full capacity employment $l^p = x_t/k_t$ and the labor supply $l^s = 1$. If the variance is higher (left panel), the structural mismatch parameter is higher too, and the distance between the two constraints and aggregate employment is large. Notice that if there is no uncertainty, $\sigma = 0$ and $\rho = +\infty$, the employment function boils down to $\min[l^p, 1]$.

Employment in our framework is a linear homogeneous function of labor demand and labor supply, as a result of explicit aggregation over labor market segments. This makes two important differences with respect to Pissarides (1990). The first is that Pissarides needs to suppose that such a function exists, by assuming that the transaction technology is part of the fundamentals of the economy. We derive it from a standard economy with market segmentation. Moreover, labor markets are competitive in our framework while Pissarides needs to impose a bargaining process on wages. Secondly, Eq. (14) is defined on the levels of employment, labor supply and labor demand and not on their variations (hiring, job searchers and vacancies) as in Pissarides. This allows us to have a much simpler dynamic model.

Additionally, with the same approximation it can be shown that

$$\int_0^{\bar{\mu}} \mu \, \mathrm{dF}(\mu) \approx \left(\frac{x_t l_t}{k_t}\right)^{\rho+1} .$$

Under these assumptions, from (8) and (11), there is a positive and simple relation between k_t and x_t,

$$x_t = \left(\frac{\alpha}{1-\alpha}\right)^{\frac{1}{\gamma+\rho}} k_t^{\frac{\rho}{\gamma+\rho}} . \tag{15}$$

From equations (8), (11) and (13) the equilibrium condition can be written as a first order difference equation for capital k_t: $\forall t \geq 0$,

$$k_{t+1} = \mathscr{G}(k_t;\rho) \equiv B\,(1-\alpha)^{-1/\gamma}\left[\left(\frac{\alpha}{1-\alpha}\right)^{\frac{\rho}{\gamma+\rho}} k_t^{\frac{-\gamma\rho}{\gamma+\rho}} + 1\right]^{-\frac{\gamma+\rho+\gamma\rho}{\gamma\rho}} \tag{16}$$

where

$$B \equiv \frac{A\theta}{1+n}$$

with given initial condition k_0. Under similar assumptions on preferences and technology, the transition function $\mathscr{G}(k_t;\infty)$ corresponds to the standard Diamond economy, that we call, in the sequel, the Diamond economy. As it is well-known, under these assumptions, the Diamond economy has two type of equilibrium paths: (a) when $0 \geq \gamma \geq -1$, there is only one strictly positive and stable steady state and (b) when $\gamma > 0$ there could be or a "poverty trap" equilibrium (two strictly positive steady state equilibria, one stable and the other unstable) or an "inescapable poverty trap" equilibrium (with none strictly positive steady state equilibrium).

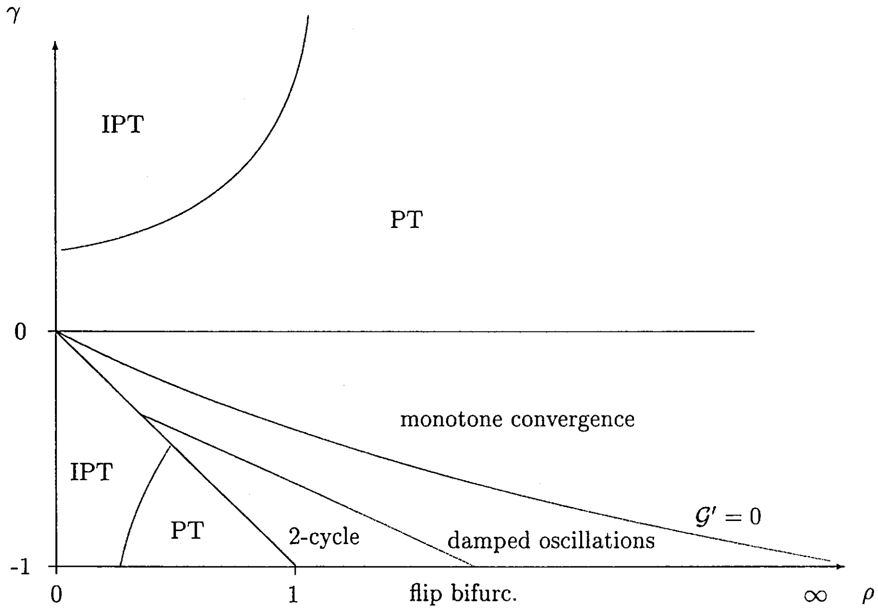

Fig. 3. Bifurcation diagram

Notice first that, in the Cobb-Douglas technology case ($\gamma = 0$), (15) is linear and (16) ensures that there exists a unique and stable positive steady state value for k, which is monotonically increasing in ρ. The rise in microeconomic uncertainty increases the misallocation of production factors across segments and the underemployment rates, and decreases the steady state capital stock.

In Fig. 3, we present the bifurcation diagram for parameters γ and ρ, given parameters B and α. As in the Diamond economy, the Cobb-Douglas technology case (when $\gamma = 0$) is the border between a monotone convergence region and a poverty trap (PT) region. When factors are ex-ante gross complement (i.e.,

$\gamma > 0$), an "inescapable poverty trap" (IPT) could arise when the mismatch parameter $1/\rho$ is sufficiently high (i.e., when uncertainty is large enough), even if the Diamond economy is characterized by a PT equilibrium for all γ. When factors are ex-ante gross substitutes (i.e., $0 > \gamma > -1$), an increase in uncertainty could also generate richer dynamics than in the Diamond economy: the unique strictly positive steady state, which is always monotonically convergent in the Diamond economy, could display damped oscillation before to degenerate in a stable two period cycle, a poverty trap equilibrium and/or in an inescapable poverty trap equilibrium. The borderline between the monotone convergence region and the damped oscillations region is given by the locus where the $\mathscr{G}$ function is horizontal, and corresponds to the pairs $\{\rho, \gamma\}$ for which $\mathscr{G}'(k;\rho) = 0$ for all k, implying $\gamma = -\rho/(1+\rho)$. Notice also that there is a locus involving discontinuity, when $\gamma = -\rho$. The transition function is decreasing ($\mathscr{G}'(k;\rho) < 0$) only between the $\mathscr{G}'(k;\rho) = 0$ locus and the discontinuity locus; a flip bifurcation could arise in this region. At the south-west of the discontinuity locus the transition function is increasing ($\mathscr{G}'(k;\rho) > 0$) and a saddle-node bifurcation could arise.[11]

Let us now consider in more detail two particular cases in which the presence of irreversibilities and uncertainty, reflected in the mismatch parameter $1/\rho$, changes the qualitative nature of equilibrium paths.

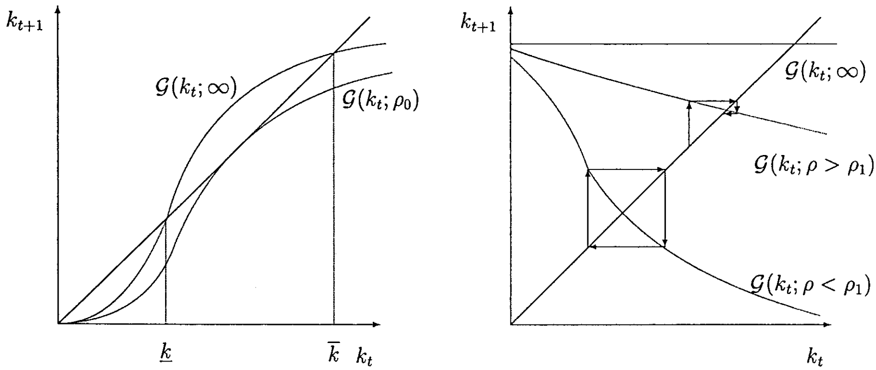

Fig. 4. The saddle-node and flip bifurcations

3.1 An inescapable poverty trap

When factors are ex-ante gross complement ($\gamma > 0$), a positive steady state is almost never unique in the Diamond economy.[12] The saving locus is shown in the left panel of Fig. 4. When there exist two strictly positive steady state equilibria $\underline{k}$ and $\overline{k}$, the steady state equilibria $\overline{k}$ and zero are asymptotically stable and $\underline{k}$

[11] The location of the flip and saddle-node bifurcations is based on numerical simulations. The presence of these bifurcations and the location of the borders depends on parameters B and α.

[12] It is a standard example in the OLG literature. See Azariadis (1996).

is unstable and can be interpreted as a "poverty trap": if the initial value k_0 is lower than $\underline{k}$ the economy converges to the zero steady state; If k_0 is larger than $\underline{k}$ the economy converges to the high equilibrium $\overline{k}$. When uncertainty increases, implying a decrease in ρ, $\mathscr{G}(k;\rho)$ moves down in the $\{k_t, k_{t+1}\}$ space and both positive equilibria move nearer. There could be a "saddle-node bifurcation" at $\rho = \rho_0$, where both positive equilibria become equal.[13] When $\rho < \rho_0$ the poverty trap becomes inescapable: there is no strictly positive steady state equilibrium and for any initial k_0 the economy converges to the zero steady state.

In this example, the rise in structural mismatch generates a reduction in revenues and savings, moving down the transition function. The highest steady state value of capital decreases and the poverty trap increases until the bifurcation point is reached, after what the poverty trap becomes inescapable.

3.2 Endogenous cycles

The richest dynamic behaviour occurs when there is ex-ante gross substitutability in production technology, which seems to be a plausible assumption for very long periods as in the two generations OLG model. Let us take the extreme case in which $\gamma = -1$, i.e., when the production factors are ex-ante perfect substitutes. In the Diamond economy, the transition function $\mathscr{G}(k;\infty)$ is always horizontal (see the right panel of Fig. 4) and convergence to the steady state is achieved in one step. When uncertainty appears, the capital stock at the unique strictly positive steady state decreases and the $\mathscr{G}(k;\rho)$ transition function moves down and becomes negatively sloped; the (local) dynamics of capital is characterized by damped oscillations. There is a "flip-bifurcation" at $\rho = \rho_1$, where the slope of $\mathscr{G}(k;\rho_1)$ is equal to -1 at the unique strictly positive steady state.[14] When ρ becomes smaller than ρ_1 the positive steady-state is still unique but becomes unstable. As the steady-state equilibrium looses stability, a stable two-cycle appears, in which the economy moves from (a) a period in which old individuals are poor, the technology is labor intensive, unemployment of young agents is low and savings are high, to (b) a period where old individuals are rich, the technology is capital intensive, unemployment of young agents is high and savings are low. Thus, if the economy has little capital at t, in spite of some segments facing large shocks labor intensive technologies will produce little unemployment and lots of saving by the young generation. At $t + 1$ there will be plenty of capital which will induce capital intensive technologies, which will produce a disastrous misallocation of resources, which will produce high unemployment and almost no savings, which will ensure labor intensive technologies at $t + 2$... We clearly realize in this example that high possibilities of ex ante capital-labor substitution are necessary for the existence of endogenous cycles.

[13] The saddle-node bifurcation point ρ_0 must verify $\mathscr{G}(k;\rho_0) = k$, $\mathscr{G}'(k;\rho_0) = 1$ and $\mathscr{G}''(k;\rho_0) \neq 0$; see Hale and Koçak (1991) for a complete characterization.

[14] The flip point ρ_1 must verify $\mathscr{G}(k;\rho_1) = k$, $\mathscr{G}'(k;\rho_1) = -1$ and $[\mathscr{G}^2(k;\rho_1)]''' \neq 0$; see Hale and Koçak (1991) for a complete characterization.

Considering that a unit of time in this model is typically of the order of 30 years, one can make a comparison with Europe in the end of the 20th century: in the sixties, unemployment was very low, producing a generation which accumulated much capital. Technology in the eighties became highly capital-intensive and we face now a generation of unemployed young people with rich parents. The bottom line being, as in all OLG models, that you do not choose how much capital you got, and you do not care how much there will be left when you die. Notice finally that the cycles that appear in our model economy should be related to long cycles rather than to business cycles. To investigate the implications of underutilization for short-term cycles, a Real Business Cycle approach is preferable (see Fagnart et al. 1999).

3.3 The mismatch parameter

A global appraisal of the role of the misallocation of resources can be made using the bifurcation diagram for the mismatch parameter ρ (Fig. 5). In this figure, we represent non-zero limit points of the sequence of capital stocks for the different values of ρ at given γ (the ex-ante elasticity of substitution between capital and labor). Starting with $\rho = 0$, we are in the inescapable poverty trap region and there is no limit points (except 0). Once ρ increases (mismatch decreases), we reach the region of the poverty trap in which the high steady state, which is locally stable, is a limit point. Above the discontinuity point $\rho = -\gamma = 1$, two non trivial limit points appear, which are the values of k along the two-period cycle. The amplitude of the cycle diminishes when ρ increases. For ρ large enough, the economy is characterized by a single limit point, which is a stable steady state.

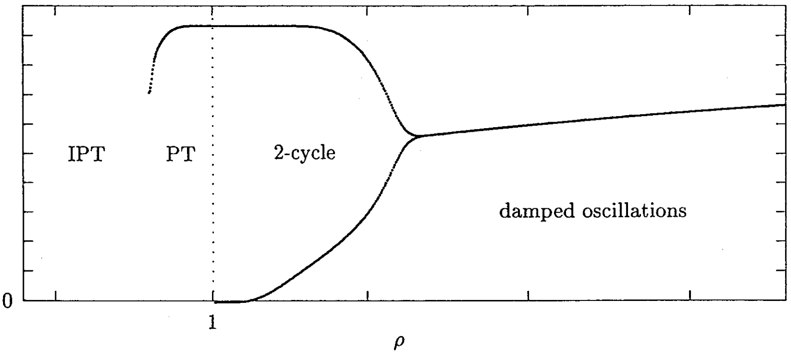

Fig. 5. Bifurcation diagram for ρ when $\gamma = -1$

In this example, the rise in structural mismatch moves down the transition function, as before, but affects also its slope around the steady-state, making the economy less and less quick to converge. At the flip-bifurcation point, the strictly positive steady-state is no longer stable and a stable two-period endogenous cycle appears. This shows that, even in cases where dynamics of the Diamond economy is very poor, the introduction of irreversibility and uncertainty may give rise to interesting dynamics.[15]

4 Conclusions

In an OLG economy, we show that technological irreversibilities and segmented labor markets, combined with idiosyncratic uncertainty, generate unemployment and underutilization of capacities. Because it takes one generation to reallocate resources among segments, idiosyncratic shocks produce misallocation, which takes the form of underemployment of production factors. This framework provides an alternative approach to the search model of Pissarides (1990) to analyze structural unemployment problems. The main differences with Pissarides are: our employment function is defined over stocks (and not flows); it is derived from the existence of segmented labor markets (and not simply assumed); our labor markets are Walrasian (no bargaining between firms and workers); wages differ across segments.

The interest of our approach is to link heterogeneity with the presence of uncertainty and irreversibility, and to make possible the analysis of the effect of idiosyncratic uncertainty on growth. Indeed, the variance of the idiosyncratic shock, which also measures heterogeneity and mismatch in the economy, plays a crucial role. When the production factors are ex-ante gross complements, the variance of the shock can present a “fold bifurcation,” where a rise in structural mismatch pushes the economy in an “inescapable poverty trap.” When production factors are sufficiently gross substitute, the variance of the shock can present a “flip bifurcation,” where an increase in uncertainty may generate endogenous cycles.

Appendix

The firm h of segment i chooses x_{it}^h and k_{it}^h in order to maximize expected profits, i.e.

$$\max_{k_{it}^h, x_{it}^h} \mathrm{E}_{t-1}\left[(f(x_{it}^h) - w_{it})\, l_{it}^h - (\delta + r_t)k_{it}^h\right]$$

where

$$l_{it}^h = \min\left[\frac{k_{it}^h}{x_{it}^h}\mu_{it}, \bar{l}_{it}\right],$$

and

$$\bar{l}_{it} = \begin{cases} \infty & \text{if} \quad \mu_{it} \leq \bar{\mu}_{it} \\ 1 & \text{otherwise} \end{cases}$$

[15] Unemployment cycles are also generated by Pissarides (1990) and Diamond and Fudenberg (1989) in alternative equilibrium unemployment set-ups.

Wages and the interest rate are taken as given by the firm. Since we are assuming that the rationing scheme is uniform, firms can not manipulate it: at the symmetric equilibrium, if the labor demand is greater than 1 (i.e., $\mu_{it} > \bar{\mu}_{it}$) each firm will be constrained to hire no more than one unit of labor. Otherwise, the firm faces an infinitely elastic labor supply, as in the standard competitive framework.

Two different situations can arrive, depending on the relation between firm capacities and market capacities, i.e., k_{it}^h/x_{it}^h and k_{it}/x_{it} respectively.

- If $k_{it}^h/x_{it}^h < k_{it}/x_{it}$, the h firm needs less workers than other firms to produce at full-capacity. She could not be rationed even when other firms are. In this case, expected profits must be written as:

$$\frac{k_{it}^h}{x_{it}^h}\int_0^{\bar{\mu}_{it}^h}(f(x_{it}^h)-w_{it})\,\mu_{it}\ \mathrm{dF}(\mu_{it})+\int_{\bar{\mu}_{it}^h}^{\infty}(f(x_{it}^h)-w_{it})\ \mathrm{dF}(\mu_{it})-(\delta+r_t)k_{it}^h$$

where

$$\bar{\mu}_{it}^h=\frac{x_{it}^h}{k_{it}^h}.$$

- In the alternative case ($k_{it}^h/x_{it}^h \geq k_{it}/x_{it}$), the h firm can eventually employ more than one worker when other firms are not rationed in this market. In this case, expected profits must be written as:

$$\frac{k_{it}^h}{x_{it}^h}\int_0^{\bar{\mu}_{it}}(f(x_{it}^h)-w_{it})\,\mu_{it}\ \mathrm{dF}(\mu_{it})+\int_{\bar{\mu}_{it}}^{\infty}(f(x_{it}^h)-w_{it})\ \mathrm{dF}(\mu_{it})-(\delta+r_t)k_{it}^h$$

with $\bar{\mu}_{it}$ given by the market equilibrium conditions, i.e., $\bar{\mu}_{it}=\frac{x_{it}}{k_{it}}$.

The only difference between both problems is in the limits of integration, i.e., $\bar{\mu}_{it}^h$ or $\bar{\mu}_{it}$. By applications of the Leibnitz' rule for the derivation of integral functions, it is easy to show that in both cases the solution is formally equivalent. First order necessary conditions for x_{it}^h and k_{it}^h are respectively:

$$x_{it}^h f'(x_{it}^h)\,\mathrm{E}_{t-1}(l_{it}^h)=\frac{k_{it}^h}{x_{it}^h}\int_0^{\tilde{\mu}_{it}^h}(f(x_{it}^h)-w_{it})\,\mu_{it}\ \mathrm{dF}(\mu_{it})$$

$$\delta+r_t=\frac{1}{x_{it}^h}\int_0^{\tilde{\mu}_{it}^h}(f(x_{it}^h)-w_{it})\,\mu_{it}\ \mathrm{dF}(\mu_{it}).$$

where

$$\tilde{\mu}_{it}^h=\max\{\bar{\mu}_{it},\bar{\mu}_{it}^h\}.$$

After computing $\mathrm{E}_{t-1}(l_{it}^h)$, we can see that all parameters in this equation system do not depend on h, implying that $k_{it}^h=k_{it}$ and $x_{it}^h=x_{it}$, for all h.

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