Chapter 11
The tradeoff between growth and redistribution: ELIE in an overlapping generations model

David de la Croix and Michel Lubrano

Abstract The ELIE scheme of Kolm taxes labour capacities instead of labour income in order to circumvent the distortive effect of taxation on labour supply. Still, Kolm does not study the impact of ELIE on human capital formation and investment. In this paper, we build an overlapping generations (OLG) model with heterogenous agents and endogenous growth driven by investment in human capital. We study the effect of ELIE on education investment and other aggregate economic variables. Calibrating the model to French data, we highlight a tradeoff between growth and redistribution. With a perfect credit market, ELIE is successful in reducing inequalities and poverty, but it is at the expense of lower investment in education and slower growth. In an economy with an imperfect credit market where individuals cannot borrow to educate, the tradeoff between growth and redistribution is not overturned but is less severe. However, it is possible to overturn completely that trade-off simply by changing the base of taxation for the young generation which is equivalent to subsidising education.

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11.1 Introduction

Equal **L**abour **I**ncome **E**qualisation or ELIE is a form of taxation and transfer imagined by Kolm (2005) in his theory of macrojustice. In this scheme, labour is the only source of income. ELIE proposes to tax labour capacities, and not labour supply in order to circumvent the distortive effect of taxation on labour supply.[1]

Kolm does not consider capital income by arguing that physical capital is itself produced by labour, so that for macrojustice (not for microjustice) capital can be neglected. Lubrano (2009) builds a simple neo-classical growth model à la Solow (1956) and analyses how the ELIE scheme reduces disposable savings and slows down physical capital accumulation.

In the most simple model considered in Kolm (2005), capacities are considered as given. So that the individual has no action on them and labour is reduced to labour time. In more complex versions of ELIE, see e.g. Kolm (2005, chap. 8), labour is considered to be multidimensional, which means that individuals can devote a part of their labour time to improve their capacities, initial education being an outstanding example. In this case, education is simply added to actual working time, Kolm (2005, p. 142). However, no formal model of education decisions is given, because "in most cultures educational choices are little affected by taxes on earnings to be paid decades later in unknown situations" (Kolm 2005, p. 180). In this paper, we consider on the contrary that when individuals can modify their capacities and their gross wages by investing in human capital, taxing capacities is likely to have an effect on their incentives to educate.

The aim of this paper is thus to evaluate what are the effects of taxation and redistribution on human capital accumulation. We introduce an ELIE-like scheme in an overlapping generations (OLG) model where heterogenous agents choose how much to invest in education when young. The initial model comes from Azariadis and de la Croix (2006), itself based on an extension of Azariadis and Drazen (1990) to a world with heterogeneous agents. This model has two important characteristics. First, both growth and the income distribution are endogenous. We can therefore study how these two variables co-move facing changes in the environment. Second, individuals differ by their abilities, but not by their inherited wealth. Taxing labour income will affect incentives to educate and will redistribute resources from the rich to the poor, and not the opposite as it is the case when agents differ in their initial endowments of physical capital, as in García Peñalosa and Turnovsky (2007).

[1] See Cardia et al. (2003) for an estimation of this distortion based on a general equilibrium model.

We will focus on long-run income growth and inequality without paying attention to possible losers and gainers along the transition path. For that purpose, we will characterise the balanced growth path of our overlapping generations model and analyse the effect of introducing an ELIE-like scheme on growth and inequality.

We will first consider the effect of an ELIE-like scheme on inequality, education and growth in a world where the credit market is perfect, i.e. where all individuals can freely borrow for their educational investment. We shall then consider a situation where human capital cannot be collateralised and where individuals cannot borrow. In this case, we expect a taxation-redistribution scheme to be less harmful for growth by redistributing resources towards those who are constrained in their education decision. Indeed, as stressed by Bénabou (2005), the tradeoff between growth and redistribution generated by a taxation scheme can depend on the availability of a credit market. A scenario of "growth-enhancing redistribution" may seem relevant when the capital market is less well-functioning or even unavailable. This scenario might also be relevant if the young and old generations are taxed differently in order to subsidise education.

The paper is organised as follows. The model is presented in Section 11.2. The extension to the case of an imperfect credit market is proposed in Section 11.3. Section 11.4 is devoted to the calibration and simulation of the long-run equilibrium. The tradeoff between growth and redistribution is analysed in Section 11.5. The tradeoff between growth and redistribution is overturned by a different implementation in Section 11.6. The last section concludes.

11.2 An overlapping generations model

The model is set up in discrete time, with time going from 0 to ∞. A unit of time measures the length of a generation. At each point in time, two generations of workers are alive. Junior workers (aged 18-39 to fix ideas), and senior workers (aged 40-62). Assuming that individuals are born at age 18 and die at age 62, we abstract from childhood and old-age.[2] The number of individuals born at time t is N_t. At one date t, total population includes N_t young workers and N_{t-1} old workers. Young workers chose either to work

[2] The complete model should include four generations (childhood, junior worker, senior workers and old age). But as our main concern is the trade-off between growth and intra-generational redistribution, we neglect for simplicity the childhood and old-age generations. Moreover, a four generation model would not be feasible because generations are constrained to be of the same length.

directly or to devote one part of their time to specialise with an advanced program of education. Each young individual i born at t is endowed with one unit of time. δ_{it} is the proportion of this time devoted to further education while $1-\delta_{it}$ the proportion of time devoted to work and earn money. The tradeoff they face is therefore between studying to improve their human capital for getting more money when old and working for getting more money right now. Workers benefit from their further education during their second period of life, when reaching seniority (i.e. age 40). This modeling choice reflects the idea that the skill premium becomes much more important after 40. For the old generation, there is a retirement age which is determined by a parameter ν that can be supposed to be the same for everybody and to be constant over time. This exogenous and policy parameter is a device to modulate the length of the activity of the old generation. For instance if the length of a generation is 22 years, the period of activity of the old generation will be $\nu\,22$. If people retire at 59, the value of ν can be determined as follows: $1-\nu = (62-60)/22 \rightarrow \nu = 10/11$. The use of this parameter is simply to allow for early retirement, knowing that in France the official age of retirement is 60, while the mean retirement age is most of the time around 59 and that workers retire at 65 in some professions.

Heterogeneity is introduced by supposing that each agent i born at time t has a different ability. His ability vector denoted $\varepsilon_{it} = (\varepsilon_{it}^{Y}, \varepsilon_{it}^{O})$ is drawn from a distribution defined over $\mathbb{R}_{+}^{2}$ (a bivariate lognormal for instance) with mean $(1,1)'$ and a variance-covariance matrix Σ. ε_{it}^{Y} is related to physical strength and is attached to the working ability when young. For the same individual, ε_{it}^{O} incorporates elements related to his intellectual capacities (IQ) and thus to his ability to learn and to make education profitable when he will be old in $t+1$. We have two generations living at the same time. The old generation, born at $t-1$ is characterised by a vector ε_{t-1} drawn at $t-1$ while the younger generation is characterised by a second bivariate vector ε_{t}, drawn at t.

11.2.1 Human capital and growth

At each date t the old generation has an average human capital stock $\overline{h}_t$. Along a balanced growth path, it is growing over time at rate g. Average human capital determines a cultural environment from which everyone draws benefits. The stock of human capital of a member of the young generation (say at age 18) results from the combination of his environment $\overline{h}_t$ and of his personal characteristics ε_{it}^{Y}

$$h_{it}^{Y} = \varepsilon_{it}^{Y}\,\overline{h}_t. \tag{11.1}$$

There are stronger and weaker individuals and that makes differences in the wage they can earn. The wage rate per efficient unit of human capital is denoted w and it is time independent along the balanced growth path. The difference in earnings across young individuals results from differences in abilities ε_{it}^{Y} and differences in the number of hours worked $1-\delta_{it}$. However, because the mean of ε_{it}^{Y} was supposed to be unity, the average wage rate in the young generation is equal to w. A young individual has to decide which proportion $1-\delta_{it}$ of his time he will work in order to earn

$$(1-\delta_{it})w\varepsilon_{it}^{Y}\overline{h}_t \tag{11.2}$$

and which proportion δ_{it} of his time he will devote for advanced studies in order to increase his future human capital stock at time $t+1$ when he will be old:

$$h_{it+1}^{O} = \varepsilon_{it}^{O}\psi(\delta_{it})\overline{h}_t. \tag{11.3}$$

Coupled with ε_{it}^{O}, $\psi(\delta_{it})$ tells how much education can be transformed into true future capacities. It will determine the expected earning in $t+1$:

$$\nu w\varepsilon_{it}^{O}\psi(\delta_{it})\overline{h}_t. \tag{11.4}$$

When old, the individual will earn money by working the first fraction ν of his second period of life. He will rely on his savings for the last $1-\nu$ part of his life.[3] The function ψ is assumed to be increasing, concave and satisfies boundary conditions

$$\lim_{\delta\to 0}\psi'(\delta) = +\infty, \quad \lim_{\delta\to 1}\psi'(\delta) = 0, \tag{11.5}$$

implying that it is always optimal to spend a strictly positive time span for building human capital.

$\overline{h}_t$ is the average human capital of the old generation at time t (hence of individuals born in $t-1$), while the average capital stock for the next generation is

$$\overline{h}_{t+1} = \frac{1}{N_t}\sum_{i=1}^{N_t} h_{it+1}^{O}. \tag{11.6}$$

Along a balanced growth path, the growth factor, denoted by G, is constant. Using (11.3), we can now characterise G as

$$G = \frac{\overline{h}_{t+1}}{h_t} = \frac{1}{N_t}\sum_{i=1}^{N_t}\varepsilon_{it}^{O}\psi(\delta_{it}). \tag{11.7}$$

[3] The parameter ν is here only to analyse the sensitivity of the model to the retirement age. The actual period of retirement beyond 62 is not included in the model.

In this model human capital is the sole engine of growth. Growth depends on two terms. The first term involves the heterogenous abilities of old workers ε_{it}^{O}. Thus growth depends in a way on the results of a genetic lottery. The second term is the decision to invest in human capital when young δ_{it}. This second factor results from the profitability of education. It can be influenced by economic policies, taxation and redistribution.

11.2.2 Income and education decisions

Individuals have to take decisions concerning investment in education (δ_{it}), consumption c_{it}^{Y} and saving s_{it}. Young individuals at present time t consume c_{it}^{Y} and save s_{it}. At future time $t+1$, they will consume c_{it+1}^{O} and will not save any longer; they are not altruistic and it is therefore optimal for them to consume all their wealth when old. Their preferences are represented by a utility function which depends on present and future consumption only. It has the form of:

$$\ln c_{it}^{Y} + \beta \ln c_{it+1}^{O}. \tag{11.8}$$

The old generation does not take into account the fact that the young generation benefits from its human capital. The intergenerational transmission channel operates with (11.1) and is totaly involuntary. The utility function is simple and short-sighted.

Earnings when young are devoted to consumption and saving

$$w(1-\delta_{it})h_{it}^{Y} = c_{it}^{Y} + s_{it}. \tag{11.9}$$

When getting old, the young generation will consume c_{it+1}^{O} that will be financed partially by future labour income (in a proportion equal to $\nu < 1$ because of early retirement age), and partially by past savings,

$$c_{it+1}^{O} = \nu w h_{it+1}^{O} + R s_{it} \tag{11.10}$$

where w is the wage per unit of human capital and R is the return on capital between the two periods; both are constant along a balanced growth path. We now determine the life cycle total income Ω_{it} for the young generation:

$$\Omega_{it} = [(1-\delta_{it})w\varepsilon_{it}^{Y} + \nu \frac{w}{R} \varepsilon_{it}^{O} \psi(\delta_{it})]\bar{h}_{t}. \tag{11.11}$$

Since preferences do not depend on leisure, and as long as the capital market is perfect, the individual decision problem is separable. We first maximise life-cycle income to determine optimal education. Second, we maximise utility given income to determine optimal saving and optimal consumption.

The optimal δ_{it} maximising the life cycle income is given by:

$$\psi'(\delta_{it}) = \frac{\varepsilon_{it}^{Y}}{\nu\varepsilon_{it}^{O}} R. \quad (11.12)$$

This implicit equation gives the optimal value of δ_{it} and thus represents the tradeoff between studying and working. The opportunity cost of an additional year of education is $\varepsilon_{it}^{Y} w$ while its discounted benefit is $\nu\varepsilon_{it}^{O}\psi'(\delta_{it})w/R$. Up to now, in the absence of any redistribution mechanism, the decision of educating depends solely on the ratio between physical and intellectual abilities and on the retirement age.

11.2.3 Firms

Firms produce the final good with the following CES production function:[4]

$$Y_t = A(\alpha K_t^{-\theta} + (1-\alpha)L_t^{-\theta})^{-1/\theta}, \quad (11.13)$$

where K_t is the stock of physical capital, L_t is the labour input in efficiency units, A is a parameter measuring total factor productivity, $\alpha \in (0,1)$ is related to the capital share[5] and $1/(1+\theta)$ is the elasticity of substitution between the two factors. It is convenient for the rest of the paper to note the production function in its intensive form, which means explaining $y = Y/L$ as a function of $\kappa = K/L$:

$$y = f(\kappa) = A(\alpha\kappa^{-\theta} + 1 - \alpha)^{-1/\theta}. \quad (11.14)$$

If w is the wage rate, the labour share is defined as

$$\frac{w}{y} = 1 - \frac{\alpha}{\kappa^{\theta}(1-\alpha) + \alpha}. \quad (11.15)$$

For $1/(1+\theta) = 1$, i.e. $\theta = 0$, we have the Cobb-Douglas case with its constant labour share. For $1/(1+\theta) < 1$, i.e. $\theta > 0$, the wage share depends positively on the evolution of the wage rate compared to the total factor productivity. For $1/(1+\theta) > 1$ and $\theta < 0$, this is just the reverse. A non-constant labour share might be justified for developed countries as made apparent in Duffy and Papageorgiou (2000) with an elasticity of substitution slightly greater than 1.

[4] For more details on the algebra of the CES production function, see e.g. de la Croix and Michel (2002).

[5] Arrow et al. (1961) call α the distribution parameter.

We assume that capital depreciate fully after one period. The competitive behaviour of firms leads to the equalisation of marginal productivity to prices so that the rate of return of capital and the wage rate are given by:

$$R = A^{-\theta}\alpha(y/\kappa)^{(\theta+1)}$$
$$w = A^{-\theta}(1-\alpha)(y)^{(\theta+1)}.$$

The capital stock is formed by the aggregation of all savings

$$K_{t+1} = \sum_{i=1}^{N_t} s_{it}. \tag{11.16}$$

Labour market clears so that labour input in efficiency units is given by:

$$L_t = \sum_{i=1}^{N_t}(1-\delta_{it})\varepsilon_{it}^{Y}\overline{h_t} + \sum_{i=1}^{N_{t-1}} \nu\varepsilon_{it-1}^{O}\psi(\delta_{it-1})\overline{h_t}. \tag{11.17}$$

In this model, labour supply in efficiency units is for one part a function of personal abilities, and is thus partly exogenous, but for the other part it results from an endogenous decision depending on the profitability of education. Any taxation redistribution scheme is going to modify labour supply because it will alter the profitability of education for the old generation. Let us investigate now how an ELIE-like scheme can be introduced in an overlapping generations model and under which conditions it can keep its neutrality on labour supply in a dynamic setting.

11.2.4 Implementing ELIE in an OLG model

In order to have a self contained presentation, let us briefly summarise the main characteristics of a simplified ELIE-like scheme. In a given society, the ELIE scheme is a self financed distributive system where taxes and subsidies, both denoted t_i in Kolm (2005) and in this volume, balance with $\sum t_i = 0$ where i is an index covering all the individuals at a given period of time. An equal amount of labour is taken from each individual i measured in terms of his productivity while an equal monetary amount, $k\tilde{w}$, is redistributed to all, so that the net transfer is $t_i = -k(w_i - \tilde{w})$. The variable $\tilde{w}$ is determined so as to balance the system. k is a parameter which measures the taxation-redistribution rate. One of the main characteristics of ELIE, is that income is not taxed, but capacities are taxed in order to have the most inelastic taxation base as possible. In our OLG model, the modeler directly observes the working capacities of the individuals as the vectors ε_t and ε_{t-1} are exogenous and given a priori. So they are a perfect inelastic base for taxation.

Finally, as ELIE concerns only active age individuals, we are not going to apply this scheme to the fraction of time $1-\nu$ during which people are pre-retired. Note that ν is a fixed policy parameter on which the individual has no action.

The introduction of an ELIE scheme is going however to modify indirectly and in the long term the incentives faced by individuals in this economy. *First*, taxation of individual i is based on his labour capacities which are partially exogenous with the observed abilities ε_{it}^{Y} and ε_{it}^{O} and partially endogenous because there is a decision to be taken for δ_{it}, the degree of higher education. Taxation will reduce the future return on education. It should be noted however, that when considering the generations separately, ELIE has no distortive effect because education decisions are taken in the first period while the resulting enhanced human capital is taxed in the second period. So in the short term, the taxation base is perfectly inelastic as can be seen from (11.18). The *second* aspect of ELIE is that there is an equal redistribution to everybody. In the initial model, young people might have to borrow in order to get supplementary education. Here, the ELIE redistribution increases the incentives to educate by providing grants to young individuals who have a weak endowment ε_{it}^{Y} of physical strength.

For *the young generation*, we can either decide to tax their physical strength capacity whatever their decision of getting educated during a fraction δ_{it} of the period. Or, we can decide to tax only the fraction of their time devoted to work and leave aside from the taxation base the fraction of their time during which they decide to educate. This last option is a subsidy to education. In this section, we shall present the benchmark model where the whole capacities of the young generation are taxed. This is in a way a *pure* implementation of ELIE which taxes capacities independently of labour supply decisions,[6] here represented by δ. Refinements and by the way more realistic cases are considered in Section 11.6, where in particular we examine the consequences of using as a taxation base only the fraction of the strength capacities that is devoted to actual work and not to education. While being a priori contrary to the philosophy of ELIE, we shall see in section 11.6 that this option is particularly attractive.

For the simple benchmark case, when the whole capacities are taxed taxed, the young age budget constraint is

$$\overline{h}_t[(1-\delta_{it})\varepsilon_{it}^{Y}w - k(\varepsilon_{it}^{Y}w-\tilde{y})] = c_{it}^{Y} + s_{it}. \tag{11.18}$$

The net transfer to a young individual i is given by:

[6] In his chapter 8, Kolm (2005, p. 142) considers multidimensional labour where education is part of labour supply and where education time is added to labour, at least in a first approximation: “Another natural way of introducing training, formation, or education in the considered structure consists in adding the corresponding time to the duration of labour *stricto sensu*.”

$$t_{it}^{Y} = -k(\varepsilon_{it}^{Y} w - \tilde{y})\overline{h}_t. \tag{11.19}$$

The old generation born at time $t-1$ also receive an identical transfer $k\tilde{y}\overline{h}_t$, which can be written as $k\tilde{y}G\overline{h}_{t-1}$. This transfer must be multiplied by ν because only active workers are concerned by ELIE transfers. Symmetrically, only the fraction ν of their capacities is taxed. Once they have left the labour market, old workers only consume their previous savings. There is no taxation nor redistribution. ELIE is kept independent of any type of early retirement system here. The budget constraint for old people born at time $t-1$ writes:

$$\nu\overline{h}_{t-1}[\varepsilon_{it-1}^{O}\psi(\delta_{it-1})w - k(\varepsilon_{it-1}^{O}\psi(\delta_{it-1})w - G\tilde{y})] = c_{it}^{O} - Rs_{it-1}. \tag{11.20}$$

The net transfer to an old individual i is:

$$t_{it}^{O} = -k(\varepsilon_{it-1}^{O}\psi(\delta_{it-1})w - G\tilde{y})\nu\overline{h}_{t-1}. \tag{11.21}$$

The taxes he has to pay are a direct function of his human capital and of the decision to educate he took in the previous period. But what he receives depends on the the level of both $\tilde{y}$ and the growth rate g which are a function of past collective decisions to educate.

We have to determine the constant $\tilde{y}$ which will balance the budget of the system jointly for the young born at time t and for the old born at time $t-1$, because both live at time t. This means

$$k\overline{h}_t\left[\sum_{i=1}^{N_t}(\varepsilon_{it}^{Y} w - \tilde{y}) + \nu\sum_{i=1}^{N_{t-1}}\left(\frac{\varepsilon_{it-1}^{O}\psi(\delta_{it-1})}{G}w - \tilde{y}\right)\right] = 0 \tag{11.22}$$

because $\overline{h}_{t-1}/\overline{h}_t = 1/G$. As the mean of ε_{it}^{Y} is one, we get a simplified expression for $\tilde{y}$:

$$\tilde{y} = \frac{w}{N_t + \nu N_{t-1}}\left[N_t + \frac{\nu}{G}\sum_{i=1}^{N_{t-1}}\varepsilon_{it-1}^{O}\psi(\delta_{it-1})\right]. \tag{11.23}$$

We retrieve the usual result of Kolm that $\tilde{y}$ is equal to the mean wage $\overline{w}$ in the case of a stationary economy ($G=1$) and a degenerate ψ function with $\psi(.) = 1$. We are here is a formal dynamic model which of course has properties which are different from the original ELIE. This model is nevertheless compatible with ELIE when it is reduced to the static case ($G = 1$ and $\psi(.) = 1$).

11.2.5 Optimal education and savings with ELIE

To determine optimal education in the presence of the ELIE redistribution scheme, we maximise income over the life cycle as a function of δ:

$$\max_{\delta_{it}} \overline{h}_t \left[(1-\delta_{it})\varepsilon_{it}^{Y} w - k(\varepsilon_{it}^{Y} w - \tilde{y}) + \nu \frac{(1-k)\varepsilon_{it}^{O}\psi(\delta_{it})w + kG\tilde{y}}{R}\right].$$

Proposition 11.1 (Optimal education with perfect credit market).
Life cycle income of individual i is maximised for δ_{it} satisfying

$$\psi'(\delta_{it}) = \frac{\varepsilon_{it}^{Y}}{\nu\varepsilon_{it}^{O}} \frac{R}{1-k}. \tag{11.24}$$

Proof. The first order condition (11.24) corresponds to a maximum because the function $\psi(.)$ is concave. □

The individual choice for δ depends on his capacities, on the taxation rate k and on the endogenous rate of return on capital. Since ψ is a concave function, education is increasing in the ratio of IQ to strength and in the age of retirement ν, and decreasing in the tax rate k and in the rate of return on capital r. There is thus a clear distortive effect of ELIE on the decision of educating. The previous case (11.12) can be recovered of course with $k=0$. But it can also be recovered if ν, the retirement age, is made a function of k with for instance $\nu = \tilde{\nu}/(1-k)$. Other solutions are also possible and will be studied in Section 11.6.

Saving is determined by young people, taking into account the old people that they will become. It is convenient to rewrite the income of the young for the first period as

$$\omega_{it}^{Y} = [(1-\delta_{it})\varepsilon_{it}^{Y} w - k(\varepsilon_{it}^{Y} w - \tilde{y})]\overline{h}_t \tag{11.25}$$

and the income of the young when they will become old in the second period

$$\omega_{it+1}^{O} = \nu[\varepsilon_{it}^{O}\psi(\delta_{it})w - k(\varepsilon_{it}^{O}\psi(\delta_{it})w - G\tilde{y})]\overline{h}_t. \tag{11.26}$$

Optimal saving s_{it} is determined by a utility maximisation under two constraints:

$$\max \log c_{it}^{Y} + \beta \log c_{it+1}^{O}$$

$$\text{subject to} \qquad \begin{aligned} \omega_{it}^{Y} &= c_{it}^{Y} + s_{it} \\ \omega_{it+1}^{O} &= c_{it+1}^{O} - R s_{it}. \end{aligned}$$

The solution is given by

$$s_{it} = \frac{\beta}{(1+\beta)}\omega_{it}^{Y} - \frac{1}{R(1+\beta)}\omega_{it+1}^{O}. \tag{11.27}$$

As usual in OLG models where individuals work during two periods, savings depends positively on income when young ω_{it}^{Y} and negatively on the discounted income when old ω_{it+1}^{O} (see de la Croix and Michel 2002). In this first version of the model, we do not impose a positivity constraint on s_{it}. This means that individuals can freely borrow in the first period. The sole constraint is that their savings are zero at the end of the second period. This implies that the credit market is perfect. In particular, individuals can decide to borrow for educating.

11.2.6 Equilibrium

We shall now collect the different parts of the solution in order to provide a formal definition of the equilibrium. For that purpose, it is convenient to define the following intensive variables, following the notations adopted for the CES production function:

$$\hat{\omega}_i^{Y} = \omega_{it}^{Y}/\overline{h_t} \qquad \hat{\omega}_i^{O} = \omega_{it}^{O}/\overline{h_t} \qquad \hat{s}_i = s_{it}/\overline{h_t}$$

$$\hat{K} = K_t/\overline{h_t} \qquad \hat{L} = L_t/\overline{h_t}.$$

For defining a stationary equilibrium, we suppose that the size of the young generation N_t is kept proportional to N^Y while the size of the old generation N_{t-1} is kept proportional to N^O so that their relative size is constant.

Definition 11.1. Given the policy parameter k, an equilibrium with a perfect credit market is

- a vector of individual variables $\{\delta_i, \hat{\omega}_i^{Y}, \hat{\omega}_i^{O}, \hat{s}_i\}$ satisfying for $i = 1 \dots N^Y$:

$$\hat{\omega}_i^{Y} = (1-\delta_i)\varepsilon_i^{Y} w - k(\varepsilon_i^{Y} w - \tilde{y}), \tag{11.28}$$

$$\hat{\omega}_i^{O} = \nu\varepsilon_i^{O}\psi(\delta_i)w - \nu k(\varepsilon_i^{O}\psi(\delta_i)w - G\tilde{y}), \tag{11.29}$$

$$\psi'(\delta_i) = \frac{\varepsilon_i^{Y}}{\nu\varepsilon_i^{O}}\,\frac{R}{1-k}, \tag{11.30}$$

$$\hat{s}_i = \frac{\beta}{1+\beta}\hat{\omega}_i^{Y} - \frac{1}{(1+\beta)R}\hat{\omega}_i^{O}. \tag{11.31}$$

- a vector of aggregate variables $\{G, \tilde{y}, \hat{K}, \hat{L}, \kappa\}$ satisfying

$$G = \frac{1}{N^Y} \sum_{i=1}^{N^Y} \varepsilon_i^O \psi(\delta_i), \tag{11.32}$$

$$\tilde{y} = \frac{w}{N^Y + \nu N^O} \left[N^Y + \frac{\nu}{G} \sum_{j=1}^{N^O} \varepsilon_j^O \psi(\delta_j) \right], \tag{11.33}$$

$$\hat{K} = \sum_{i=1}^{N^Y} \frac{\hat{s}_i}{G}, \tag{11.34}$$

$$\hat{L} = \sum_{i=1}^{N^Y} (1-\delta_i)\varepsilon_i^Y + \sum_{j=1}^{N^O} \nu \varepsilon_j^O \psi(\delta_j), \tag{11.35}$$

$$\kappa = \hat{K}/\hat{L}. \tag{11.36}$$

- and a vector of prices $\{R, w\}$ satisfying

$$R = A^{-\theta} \alpha (f(\kappa)/\kappa)^{\theta+1} \tag{11.37}$$

$$w = A^{-\theta} (1-\alpha)(f(\kappa))^{\theta+1}. \tag{11.38}$$

11.3 Imperfect credit market

We define an imperfect credit market as an environment in which young households cannot credibly commit their future labour income as a collateral against current loans. As in Kehoe and Levine (1993), we assume that individuals are allowed to borrow up to the point where they are indifferent between repaying loans and suffering market exclusion. Since everyone dies at the end of the second period, default involves no penalty and is individually optimal. As in this context it is optimal for them never to pay back their credits, banks will always refuse to lend them money. The borrowing constraint then takes the very simple form: $s_{it} \geq 0$.

Let us first identify the individuals who are going to be affected by this constraint.

Proposition 11.2 (Earnings profile and borrowing constraint).
There exist a function $\Gamma(\varepsilon^Y, \varepsilon^O)$, such that individual i is credit constrained if and only if $\Gamma(\varepsilon_{it}^Y, \varepsilon_{it}^O) < 0$. The function $\Gamma(.)$ is implicitly defined by:

$$\Gamma(\varepsilon_{it}^Y, \varepsilon_{it}^O) = \beta(1-k-\delta_{it})\varepsilon_{it}^Y - \frac{\nu}{R}(1-k)\varepsilon_{it}^O \psi(\delta_{it}) - \left(\frac{\nu G}{R} - \beta \right) \frac{k\tilde{y}}{w}, \tag{11.39}$$

with δ_{it} given by (11.24). The function $\Gamma(.)$ is increasing in ε_{it}^{Y} and decreasing in ε_{it}^{O}.

Proof. The function $\Gamma(\varepsilon_{it}^{Y},\varepsilon_{it}^{O})$ is derived from the condition $s_{it} \geq 0$ using the saving function (11.27) and the definitions of incomes (11.25) and (11.26). Since δ_{it} is an increasing function of ε_{it}^{Y} and a decreasing function of ε_{it}^{Y}, the sign of the partial derivatives of Γ are not ambiguous. □

As in De Gregorio and Kim (2000) and in de la Croix and Michel (2007), households with a steep potential earning profile would like to borrow in order to study longer, but credit rationing prevents them from doing so. All others have positive saving and study as long as they wish. *Hence constrained individuals are those with a relative low strength and high IQ.*

Note that the threshold function Γ depends on prices through (11.39). For example, when yields r are high, there will be fewer constrained households, other things being equal. Hence, although our borrowing constraint is very simple, the proportion of rationed people depends on equilibrium prices.

For the constrained households, Equation (11.24) no longer determines their education choice. Instead, these households maximise an autarkic utility, i.e. the utility they could reach without being able to use the credit market to smooth consumption. More explicitly, they choose education in order to maximise the utility function (11.8) where the consumption arguments were replaced by actual wages, with no possibility of saving: $c_{it}^{Y} = \omega_{it}^{Y}$ and $c_{it+1}^{O} = \omega_{it+1}^{O}$ so that

$$\max_{\delta_{it}} \ln\left((1-\delta_{it})\varepsilon_{it}^{Y} w - k(\varepsilon_{it}^{Y} w - \tilde{y})\right) \\ + \beta \ln\left((1-k)\varepsilon_{it}^{O} \psi(\delta_{it}) w + kG\tilde{y}\right) + (1+\beta)\ln \overline{h}_t + \beta \ln \nu.$$

Proposition 11.3 (Optimal education with imperfect credit market). *The autarkic utility of an individual i with $\Gamma(\varepsilon_{it}^{Y},\varepsilon_{it}^{O}) < 0$ is maximised for the unique value of δ_{it} satisfying*

$$\left(1-\delta_{it} - k\left(1 - \frac{\tilde{y}}{\varepsilon_{it}^{Y} w}\right)\right)\beta\psi'(\delta_{it}) = \psi(\delta_{it}) + \frac{k}{1-k}\frac{G\tilde{y}}{\varepsilon_{it}^{O} w}. \qquad (11.40)$$

Proof. The left hand side of (11.40) is decreasing in δ_{it}, going from $+\infty$ to 0 as δ_{it} goes from 0 to 1. The right hand side of (11.40) is increasing in δ_{it}. Hence there exists a unique δ_{it} equalising these two terms. □

We can now define the equilibrium with an imperfect credit market.

Definition 11.2. Given the policy parameter k, an equilibrium with an imperfect credit market is

- a vector of individual variables $\{\hat{\omega}_i^Y, \hat{\omega}_i^O, \delta_i, \hat{s}_i\}$ satisfying for $i = 1 \dots N^Y$ (11.28)-(11.29) and

$$\psi'(\delta_i) = \frac{\left[\psi(\delta_i) + \frac{k}{1-k}\frac{G\tilde{y}}{\varepsilon_i^O w}\right]}{\left[\beta\left(1 - \delta_i - k\left(1 - \frac{\tilde{y}}{\varepsilon_i^Y w}\right)\right)\right]} \qquad \text{if } \Gamma(\varepsilon_i^Y, \varepsilon_i^O) < 0$$
$$= \frac{\varepsilon_i^Y}{\nu \varepsilon_i^O}\frac{R}{1-k} \qquad \text{if } \Gamma(\varepsilon_i^Y, \varepsilon_i^O) \geq 0,$$
$$\hat{s}_i = 0 \qquad \text{if } \Gamma(\varepsilon_i^Y, \varepsilon_i^O) < 0$$
$$= \frac{\beta}{1+\beta}\hat{\omega}_i^Y - \frac{1}{1+\beta}\frac{\hat{\omega}_i^O}{R} \qquad \text{if } \Gamma(\varepsilon_i^Y, \varepsilon_i^O) \geq 0.$$

- a vector of aggregate variables $\{G, \tilde{y}, \hat{K}, \hat{L}, \kappa\}$ satisfying (11.32)-(11.36).
- and a vector of prices $\{R, w\}$ satisfying (11.37)-(11.38)

When the credit market is perfect, ELIE acts as a obstacle to the decision of educating. When the credit market is imperfect, the ELIE scheme can help the constrained individuals in their decision to educating. The ability of ELIE to promote education will then depend on the proportion of constrained individuals.

11.4 Numerical simulation of the equilibrium

The objective of this section is to calibrate and simulate the benchmark version of the model. Doing so will allow us to assess the size of the tradeoff between growth and redistribution in the perfect market case and to determine whether it is modified by the presence of borrowing constraints (imperfect credit market).

Assumed that one period of the model is 22 years. It is then useful to define the annual growth rate of income and the annual interest rate as:

$$g = G^{1/22} - 1, \qquad r = R^{1/22} - 1$$

11.4.1 A priori information

We first choose a functional form for the production function of human capital and the distribution of abilities. The production of human capital has to satisfy the two limit conditions (11.5) to guarantee an interior solution for all agents. We use:

$$\psi(\delta) = b\left(\frac{1}{\gamma}\delta^{\gamma} - \delta\right),$$

where $\gamma \in (0,1)$ and b is a scale parameter used as a degree of freedom for calibrating the model. In Figure 11.1, we have graphed this function for a

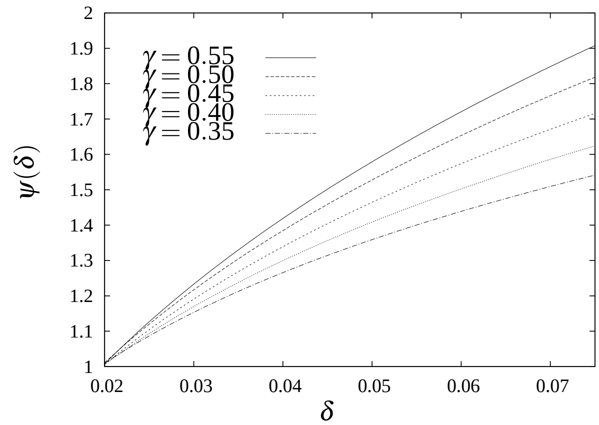

Fig. 11.1 Production function of human capital

range of values for γ that are within the domain compatible with our calibration exercise. The scale parameter b was adjusted accordingly to obtain a nice graph.

The psychological discount factor of individuals is set to 3% per year. As we have assumed that one period of the model is 22 years, we have: $\beta = 0.97^{22} = 0.512$. The growth rate of population $n = N^Y/N^O - 1$ can be directly computed from official data which yields $1 + n = 1.177$.[7] Finally, we have taken $N^Y = 10000$, which implies that $N^O = N_Y/1.177 = 8496$.

The abilities bivariate index $(\varepsilon^Y, \varepsilon^O)$ is assumed to be distributed over a generation according to a bivariate lognormal distribution. The usual way of obtaining a lognormal distributed random variable is to take the exponential of a normal random variable. Let us thus consider a bivariate normal distribution with mean $\mu = [\mu_1, \mu_2]'$, and variance-covariance matrix

$$\begin{pmatrix} \sigma_{\text{O}}^2 & \rho\sqrt{\sigma_{\text{Y}}^2\sigma_{\text{O}}^2} \\ \rho\sqrt{\sigma_{\text{Y}}^2\sigma_{\text{O}}^2} & \sigma_{\text{O}}^2 \end{pmatrix} = \sigma_{\text{Y}}^2 \cdot \begin{pmatrix} 1 & \rho\sqrt{\sigma_{\text{O}}^2/\sigma_{\text{Y}}^2} \\ \rho\sqrt{\sigma_{\text{O}}^2/\sigma_{\text{Y}}^2} & \sigma_{\text{O}}^2/\sigma_{\text{Y}}^2 \end{pmatrix}. \qquad (11.41)$$

[7] The total population in France is available from the Web site of INSEE http://www.insee.fr/fr/ffc/pop_age2.htm, *Population totale par sexe et âge au 1er janvier 2007, France métropolitaine*. From these annual data, we computed the ratio between the population born between 1960 and 1981 and the population born between 1938 and 1959. The value of this ratio is 1.177.

This matrix has three parameters: the correlation ρ and the two variances σ_{Y}^2 and σ_{O}^2. The resulting lognormal distribution has marginal means equal to $\exp(\mu_i+\sigma_i^2)$, marginal variances equal to $\varsigma_i^2=(\exp(\sigma_i^2)-1)\exp(2\mu_i+\sigma_i^2)$. The correlation coefficient ρ is independent of the means and equal to

$$\rho=\frac{\exp(\rho\sigma_{\text{Y}}\sigma_{\text{O}})-1}{\sqrt{(\exp(\sigma_{\text{Y}}^2)-1)(\exp(\sigma_{\text{O}}^2)-1)}}. \tag{11.42}$$

When $\rho=0$, $\rho=0$, but when $\rho\neq0$, then $|\rho|<|\rho|$. Even if ρ is kept fixed, ρ varies with σ_i. It is convenient, for elicitation purposes, to reparametrise this matrix in σ_{Y}^2, ρ and the relative variances $\sigma_{\text{O}}^2/\sigma_{\text{Y}}^2$. We do not have much information to calibrate this variance-covariance matrix. The parameter σ_{Y}^2 can be adjusted to match a measure of inequality for the observed income distribution in France. The Gini coefficient obtained on French gross income data and equal to 0.327 in 1998.[8] This will be matched with the Gini coefficient implied by the model (computed in Appendix A). But we have no precise procedure to calibrate the two other parameters. It seems reasonable to assume that the ability to work when young is equally dispersed as the ability to work when old. However, ability in youth only reflects different endowments in efficient labour, while ability in old age also embodies the ability to accumulate human capital. We select $\sigma_{\text{O}}^2/\sigma_{\text{Y}}^2=1$ in a first step and will carry some sensitivity analysis for $\sigma_{\text{O}}^2/\sigma_{\text{Y}}^2=1.5$. The parameter ρ directly influences to proportion of types in society. With $\rho=0$, the four possible types detailed below are in equal proportion. We will take $\rho=0$ as a benchmark and we will carry sensitivity analysis for $\rho=-0.9$ which maximises the proportion of the type for which education makes an important difference.

We assume that people retire at the age of 59 as reported by the OECD in 2002. This imply that $\nu\simeq10/11$.

The productivity parameter b governs the long-term growth rate of output per capita. We shall adjust it on the observed growth rate of GDP per capita that we collected from Maddison (2007) data over the period 1981-2003. We have $G=1.44$, which gives an annual growth rate g of 1.67%. The parameter γ determines the time spent on education in the first period of life. We shall adjust it so as to match the observed share of time devoted to education. We assume that the first period of the model covers ages 18-39. Doing so supposes that higher education is an alternative to working, but elementary and secondary education is not. The percentage of time devoted to schooling is computed using Education at a Glance from OECD (2006) (Indicator A3, page 53). We use Tertiary type A and B graduation rates and obtain $\delta=0.075$.

[8] This figure comes from the Human Development Report of the United Nations http://hdrstats.undp.org/indicators/147.html. It can also be found elsewhere, such as in the World Fact Book of the CIA.

As far as technology is concerned, we borrow from Duffy and Papageorgiou (2000) the conclusion that, in developed countries, the elasticity of substitution between capital and labour adjusted human capital is of the order of 1.1 and we set $\theta = -0.1$. From Askenasy (2003) we take that the share of capital in value added, κ/y, is 0.35 (it fluctuates between 0.32 and 0.38 in the last 30 years). The physical capital share parameter α will we set to match this value. We also learn from this study that the rate of return on capital in the manufacturing sector fluctuates between 9% and 14%. We do not use this information directly but it will serve as a benchmark to check whether our equilibrium r is in line with the data. Finally, the scale parameter A is normalised to 1. Varying A leaves everything else unchanged provided that we adjust α to keep the same capital share.

We now summarise the available a prior information in Table 11.1.

Table 11.1 A priori information used for calibration

σ_O^2/σ_Y^2	ρ	N^Y/N^O	ν	κ/y	β	θ	g	δ	Gini
1.0	0.0	1.177	0.91	0.35	0.512	-0.1	1.67%	0.075	0.327
1.5	0.0	1.177	0.91	0.35	0.512	-0.1	1.67%	0.075	0.327
1.0	-0.9	1.177	0.91	0.35	0.512	-0.1	1.67%	0.075	0.327

The first line is used for calibrating the benchmark model. The last two lines are used for sensitivity analysis.

11.4.2 Calibration of the model

In order to impose the a priori information on growth, education and inequality, we have four parameters of adjustment, the two parameters of the production function of human capital $\psi(\delta)$, a scale parameter for the variance covariance matrix of the lognormal distribution σ_Y^2, and the capital share parameter α. Given starting values for the rate of return on capital r and the wage rate w, the model is solved iteratively using the fixed point algorithm described in Appendix B, conditionally on initial given values for γ, b, σ_Y^2 and α. As a by-product, the model produces a vector δ, a growth rate g, an income distribution for which a Gini coefficient is computed, and a capital share in value added κ/y. The four adjustment parameters are then updated using the following scheme

$$\gamma = \gamma + (0.075 - \bar{\delta})$$
$$b = b + (.0167 - g)$$
$$\sigma_{Y}^{2} = \sigma_{Y}^{2}(0.327 - \text{Gini})$$
$$\alpha = \alpha + (0.35 - \kappa/y)$$

and the process is iterated until convergence is reached. We found the following solution displayed in Table 11.2. The obtained rate of return on capital is 10.14% on an annual basis which is within the range provided by Askenasy (2003).

Table 11.2 Calibration and solutions of the initial model

	α	γ	b	σ_{Y}^{2}	r	g	δ	Gini	κ/y
Perfect credit	0.475	0.456	2.387	0.204	10.14%	1.67%	0.075	0.327	0.350
Imperf. credit (0)	0.475	0.456	2.387	0.204	9.94%	1.59%	0.070	0.316	0.352
Imperf. credit (1)	0.475	0.456	2.421	0.223	10.02%	1.67%	0.070	0.327	0.351
Imperf. credit (2)	0.475	0.456	2.431	0.204	10.04%	1.67%	0.070	0.316	0.351
Imperf. credit (3)	0.473	0.503	3.027	0.211	10.02%	1.67%	0.075	0.327	0.350

In (0), no parameter is adjusted. In (1), b and σ_{Y}^{2} are adjusted, in (2), only b is adjusted, in (3) the four parameters are adjusted.

Let us now calibrate the model with an imperfect credit market. When we keep the same parameters, we see from the second line of Table 11.2 that credit rationing entails a drop in education and growth, and incidently in inequality too. The capital share increases. In order to make comparisons between the two cases, we have to recalibrate some of the parameters. We recalibrate b in order to match the same growth rate as before, which requires an increase in this coefficient to compensate for the loss of growth due to the imperfection of the credit market. We also recalibrate σ_{Y}^{2} to match the required level of inequality. Matching the same growth-inequality pair in the two versions of the model allows to compare the trade-off between growth and redistribution across them. We do not alter the parameters γ and α to make the two models similar in this respect.

The results are reported in the line labeled (1) of Table 11.2. We also report a calibration where we only adjust parameter b and leads to a similar result. In the last line of Table 11.2, the one labeled (3), we report a calibration of the imperfect credit market model where we compute the four parameters γ, b, σ_{Y}^{2} and α in order to match the four targets g, δ, Gini and the capital share. We see that doing so requires an important rise in the elasticity of human capital to education γ.

11.4.3 Heterogenous behaviour without redistribution

To better grasp the logic of the model, we distinguish four groups of individuals, depending on their abilities ε^Y when young and ε^O when old. Given the median of each marginal of the joint distribution of $(\varepsilon^Y, \varepsilon^O)$, we classify each individual in a two by two entry table. Type 00 has a physical strength ε^Y lower than the median and an intellect ε^O lower than the median. For convenience, we call this type *white collars*. Type 10 has a physical strength ε^Y greater than the median and an intellect ε^O lower than the median. We call this type *blue collars*. Type 01 a physical strength lower than the median and an intellect greater than the median. We call this type *academics*. Finally, type 11 has a higher physical strength and a higher intellect. We call this type *managers*. Table 11.3 presents some characteristics of these different groups. As $\rho = 0$, each type represents 25% of our sample.

Table 11.3 Education and saving decisions

	Education		Net savings		Borrow. prop.		Income young		Life cycle income	
ε^Y	ε^O 0	1	0	1	0	1	0	1	0	1
	Perfect credit market									
	(0.075)		(0.017)		(0.116)		(0.073)		(0.086)	
0	0.062	0.154	0.011	0.001	0.026	0.410	0.048	0.043	0.056	0.067
1	0.021	0.063	0.032	0.023	0.000	0.027	0.102	0.098	0.107	0.115
	Imperfect credit market									
	(0.070)		(0.018)		(0.000)		(0.074)		(0.087)	
0	0.064	0.129	0.011	0.003	0.000	0.000	0.047	0.044	0.055	0.065
1	0.021	0.066	0.032	0.023	0.000	0.000	0.105	0.100	0.110	0.117

Physical strength when young is indicated in column and intelligence when old is indicated in line. The mean value for each small two-two table is indicated between brackets. The total life cycle income for a young individual is given by $\tilde{\omega}_{it} = \omega^Y_{it} + [(1-k)\varepsilon^O_{it}\psi(\delta_{it})w + kG\tilde{y}]\nu/R$.

All types decide to educate, but according to different degrees. Types 00 and 11 choose to educate around the mean, type 01 (academics) chooses to educate twice the mean, while type 10 (blue collars) has the lowest decision of education. These decisions have marked consequences. Type 10 (blue collars) are major savers because they will earn well above the mean when young, but below the mean when old. Type 01 (academics) net saving is roughly zero. 41% of this group borrow to finance longer education and have the prospect of earning a very high wage when old. Notice that the group of academics will earn the minimum when young (roughly the same as white collars), but will receive the maximum when old so that over the life cycle their earn more than white collars. Managers and blue collars receive a similar average income when young, but a quite different one when old.

Considering now the equilibrium with an imperfect credit market, we observe that the academics 01 are strongly hit by the impossibility to borrow. Their education is reduced, and their life-cycle income as well.

We have analysed the sensitivity of these results to the choice of ρ. Increasing correlation up to $\rho = 0.50$ and recalibrating the model to fit observed growth, education and inequality, we observe that this higher correlation between abilities greatly diminishes the proportion of borrowers. The type proportion is changed to 0.333 (white collars and managers) and 0.167 (blue collars and academics). In this world, 33.1% of the academics borrow money for educating which relates to an average of 7.8% in the whole economy (instead of 11.6% in the benchmark calibration).

We do a similar exercise for $\sigma_{\text{O}}^2/\sigma_{\text{Y}}^2 = 1.5$ while keeping $\rho = 0$ to investigate the robustness of the results to different relative variances. In this case the type proportions are only very slightly modified compared to the benchmark. 36.6% of the academics borrow money for educating which relates to an average of 10.4% in the whole economy. Results are close to those of the benchmark.

11.5 The tradeoff between growth and redistribution

We now introduce the ELIE transfer system. We do so by letting k vary between 0 and 0.40. Let us recall that $k = 0.40$ means that for a working week of five days, the product of two days is taken for redistribution. Remember that in our model both inequality and growth are endogenous. We have seen in Proposition 11.1 that individual investment in education is negatively affected by taxation k, but this was only a partial equilibrium effect, for a given rate of return on capital. We will now investigate whether this partial equilibrium effect carries over to the general equilibrium framework; the numerical simulation will also allow us to quantify this effect. In section 11.6, we investigate how this trade-off can be overturned by a different implementation of ELIE.

As the system balances, money is taken from some individuals and distributed to others. If the focus of the analysis was on the life cycle of one generation in the previous section, it has now to be on the two generations together. This means that at time t, we have to study the interaction between young and old and detail the possible intergenerational transfers. The ELIE transfer system has the particularity of reducing inequality in the income distribution. We will investigate by how much does the ELIE scheme affect the Gini coefficient of young and old incomes.

11.5.1 Simulations results

Let us first analyse the impact of ELIE on macroeconomic variables, before analysing its impact in term of inequality and poverty. First consider the case of a perfect credit market. The impact of ELIE on the growth rate is negative as shown in Table 11.4. The young generation decides to educate less and growth in our model is affected solely by the growth of human capital. However, ELIE also decreases the proportion of young individuals that are obliged to borrow to finance their supplementary education. The lower investment in education allows an increasing capital labour ratio which in turn implies an increase in the wage rate per efficient unit of human capital and a lower rate of return on capital. The decrease in the rate of return on capital dampens the negative effect of k on the decision of education but does not overturn it.

Considering now the imperfect credit market case, the question here is whether the negative effect of redistribution on growth via the distortive highlighted above can be overturned by a positive effect of redistribution on growth through the easing of borrowing limits bearing on poor people. The answer is no. Table 11.4 shows that growth is still decreasing in k, indicating that the predominant effect is still the distortion one. But the drop in growth is slightly less severe than in the previous case, indicating that the effect of ELIE on borrowing limits help to limit the cost of taxation in terms of growth. If the proportion of constrained individuals on the credit market were larger, the compensating effect of ELIE would have been larger.

The right panel of Table 11.4 provides inequality measures for different k in the two economies. Not surprisingly, ELIE manages to reduce inequality in the population (this is also true for within group inequality, but inequality remains greater in the older generation.) We define a poverty level as 60% of the mean income.[9] We compute a head count measure of poverty as the proportion of individuals below the poverty level. Increasing k from 0.00 to 0.40 allows to decrease poverty from 28.1% of the population to 8.2%. Comparing the model with perfect credit market to the one with imperfect credit market, we observe that ELIE diminishes the Gini coefficient

[9] There are various ways of defining a relative poverty line. EUROSTAT defines the poverty level as 60% of the median income. France and INSEE use 50% of the median income. The European Commission once used 50% of the mean in its reports. See Atkinson (1998) for a discussion.

Table 11.4 Macroeconomic impact of ELIE

k	g (% annual)	r	δ	Saving rate	Percent. borrowers	% credit constr.	Gini	Headcount poverty
				Perfect credit market				
0.0	1.67	10.14	0.075	22.88	11.63	0.00	0.327	0.281
0.1	1.52	9.95	0.069	22.86	9.28	0.00	0.295	0.242
0.2	1.35	9.73	0.063	22.85	7.04	0.00	0.264	0.195
0.3	1.15	9.48	0.056	22.83	4.99	0.00	0.232	0.143
0.4	0.91	9.19	0.050	22.82	3.39	0.00	0.200	0.082
				Imperfect credit market				
0.0	1.67	10.02	0.070	23.61	0.00	13.77	0.327	0.287
0.1	1.54	9.89	0.066	23.42	0.00	11.05	0.298	0.251
0.2	1.39	9.72	0.061	23.25	0.00	8.27	0.268	0.204
0.3	1.21	9.51	0.055	23.11	0.00	5.85	0.237	0.153
0.4	0.99	9.25	0.049	23.00	0.00	3.87	0.205	0.090

in the same way in both cases, but is slightly less efficient at reducing poverty when credit market is imperfect. Notice that, if one wishes to totally remove poverty, one needs to push k as high as 0.60.

The elimination of poverty by the ELIE scheme is further illustrated in Fig. 11.2. We observe that the income distribution is fairly regular and corresponds to the shape of a log-normal distribution when there is no redistribution. The ELIE scheme shifts the whole distribution to the right (poverty reduction), except for the extreme right tail which is dampened (inequality reduction). We only report the graph for the perfect market case. The imperfect market case produces a very similar graph.

11.5.2 Assessing the size of the tradeoff

We have seen in the previous section that increasing the value of k reduces growth but promotes redistribution and reduces poverty. We measure inequality using a Gini index computed on the total income distribution and poverty using the P_0 index of Foster et al. (1984). The importance of the trade-off between growth and redistribution is illustrated in Fig. 11.3 where the graph of 1−Gini and $1-P_0$ against the annual growth rate is displayed. We give this graph both when credit market is perfect and when it is imperfect. The difference between these two cases is not negligible, but not large either. We can measure the difference in the tradeoff by comparing the slope

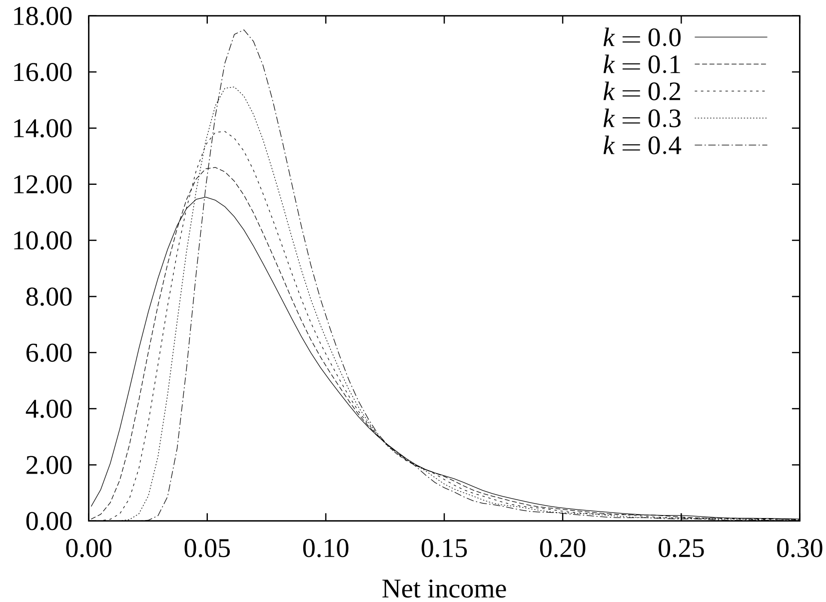

Fig. 11.2 Income distribution with perfect credit market

of the two curves. The slope with perfect credit market is equal to -5.90, which implies that reducing the Gini by 1 point costs 0.059 in term of annual growth rate. In the imperfect market case, the slope is lower in absolute value and equal to -5.58. The ratio of two slopes is 0.95. Hence the tradeoff between growth and redistribution is slightly less severe with an imperfect credit market, but is far from being overturned.

As far as poverty is concerned, reducing poverty by 1 point costs 0.0376 in terms of annual growth rate. This cost drops to 0.0347 when the credit market is imperfect. Here the ratio of the two slopes is 0.92. These numbers indicate that ELIE is quite good at reducing poverty at a relatively low cost in terms of growth, and that this is even more true if individuals face borrowing constraints. Still again, we are far from a case of “growth-enhancing redistribution”, where redistribution lifts so much the credit constraints that the disincentive effect on growth is overturned, as suggested by Bénabou (2005) for instance.

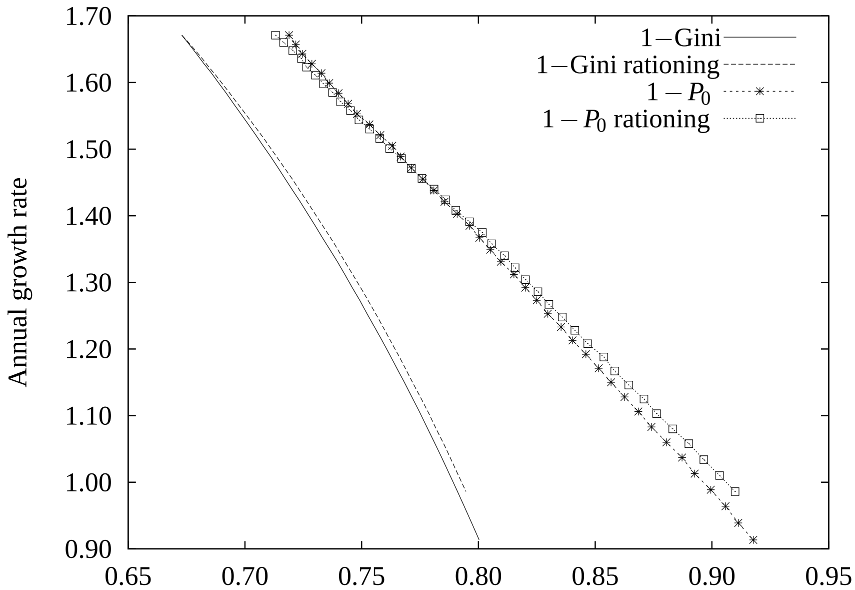

Fig. 11.3 Trade-off between redistribution and growth for $k = 0$ to $k = 0.4$

11.5.3 The trade-off with a larger proportion of 01 type

When the credit market is imperfect, the ELIE scheme helps some poor individuals to invest in education. These are the 01 types (the academics) who have a strong potential in terms of future income growth but little resources when young. This is why the tradeoff between growth and redistribution is less severe when the credit market is imperfect. But ELIE also redistributes to the 00 types (white-collars), which is fine as far as equality is concerned, but is of no help as far as growth is concerned. This explains why the trade-off is not modified much when the credit market is imperfect. Unless ELIE is targeted towards the 01 households, its effect on borrowing constraints is not strong enough to suppress its negative effect on growth.

To illustrate this point, we consider a calibration of the model with a strong negative correlation between the two ability shocks: $\rho = -0.9$. In that case the economy is mostly composed of academics and blue collars (43% of population each). Among young individuals, ELIE will redistribute in favour of academics, without "wasting" too much resources on white collars, who form a small 7% fraction of the population. Hence, ELIE is much more targeted towards persons with a strong growth potential. Assuming such a strong negative correlation is of course unrealistic, but this simulation is meant to illustrate the properties of the ELIE scheme as a function of the type of ability distribution in the population.

We recalibrate the model with a perfect market using $\rho = -0.9$. This gives $\gamma = 0.413$, $b = 1.890$, $\sigma_{Y}^2 = 0.159$ and $\alpha = 0.475$. The empirical correlation between the two shocks is $\rho = -0.74$. We also recalibrate the model with an imperfect credit market to obtain the same growth and inequality without ELIE ($b = 1.931$ and $\sigma_{Y}^2 = 0.188$). Then we simulate various levels of redistribution by letting k vary between 0.00 and 0.40.

The distance between the curves with perfect and imperfect credit market is now more important. This is because ELIE is now more targeted towards 01 people and plays therefore a greater role in alleviating the credit constraints for the individuals with a strong growth profile. The ratio between the two slopes is now 0.87 (against 0.95) for the 1−Gini slopes and 0.81 (against 0.92) for the $1 - P_0$ slopes.

Notice finally that the case with $\rho = -0.9$ is the most favourable situation to generate a positive influence of ELIE on growth. Although we know little on the distribution of abilities in the population, and hence the parameters of this distribution are subject to a large uncertainty, it seems pretty clear now that no parameter configuration would be able to reverse the tradeoff between growth and redistribution. We have to find something else.

11.6 How to overturn the trade-off

The original ELIE has no distortive impact because the tax base is chosen independently of labour supply decisions. Once we introduce a decision for educating in a two generation model, the distortive effect reappears. We have studied up to now the least favourable case. We have given indications on how to reduce the distortive effect of ELIE by an alternative implementation. We now explore two possibilities which are equivalent to either subsidising education in the first period or making it more profitable in the second period.

11.6.1 Education subsidies

The crucial decision of educating has to be taken in the first period. ELIE had a disincentive effect on that decision, because labour capacities when senior are taxed at a proportional rate while the opportunity cost when young is not tax deductible (the whole physical capacity ε_{it}^{Y} was taken as a basis for taxation). In doing so, we had a dogmatic vision of ELIE where the taxation basic must be independent of labour supply decisions and thus of δ. What happens if we now decide to apply ELIE only to the sole fraction of ε_{it}^{Y} that is devoted to actual work and to exclude the fraction which is devoted to education? The taxation base is no longer ε_{it}^{Y}, but $(1-\delta_{it})\varepsilon_{it}^{Y}$. The young age budget constraint becomes

$$\overline{h}_t[(1-\delta_{it})\varepsilon_{it}^{Y}w - k((1-\delta_{it})\varepsilon_{it}^{Y}w - \tilde{y})] = c_{it}^{Y} + s_{it}, \tag{11.43}$$

instead of Equation (11.18). The net transfer to a young individual i is now:

$$t_{it}^{Y} = -k((1-\delta_{it})\varepsilon_{it}^{Y}w - \tilde{y})\overline{h}_t, \tag{11.44}$$

replacing Equation (11.19) of the benchmark. As in the benchmark, they receive $k\tilde{y}\overline{h}_t$ (while equilibrium $\tilde{y}$ will be different). But in the benchmark case, they had to pay $k\varepsilon_{it}^{Y}w\overline{h}_t$, while here, they have only to contribute to the system for $k(1-\delta_{it})\varepsilon_{it}^{Y}w\overline{h}_t$. The more they educate, the less they contribute to the system in the first period. There is thus a subsidy to educating and implicitly a transfer from the old generation to the young generation.

Let us now determine the constant $\tilde{y}$ which will balance the budget of the system. Balanced budget implies

$$k\overline{h}_t\left[\sum_{i=1}^{N_t}((1-\delta_{it})\varepsilon_{it}^{Y}w - \tilde{y}) + \nu\sum_{i=1}^{N_{t-1}}\left(\frac{\varepsilon_{it-1}^{O}\psi(\delta_{it-1})}{G}w - \tilde{y}\right)\right] = 0 \tag{11.45}$$

because $\overline{h}_{t-1}/\overline{h}_t = 1/G$. This implies

$$\tilde{y} = \frac{w}{N_t + \nu N_{t-1}}\left[\sum_{i=1}^{N_t}(1-\delta_{it})\varepsilon_{it}^{Y} + \frac{\nu}{G}\sum_{i=1}^{N_{t-1}}\varepsilon_{it-1}^{O}\psi(\delta_{it-1})\right]. \tag{11.46}$$

We can no longer simplify the expression using the assumption that the mean of the ε_{it}^{Y} is one. To determine optimal education in this new scheme, we maximise income over the life cycle as a function of δ:

$$\max_{\delta_{it}} \overline{h}_t\left[(1-\delta_{it})\varepsilon_{it}^{Y}w - k((1-\delta_{it})\varepsilon_{it}^{Y}w - \tilde{y}) + \nu\frac{(1-k)\varepsilon_{it}^{O}\psi(\delta_{it})w + kG\tilde{y}}{R}\right].$$

The first-order condition for a maximum is given by

$$\psi'(\delta_{it}) = \frac{\varepsilon_{it}^{Y}}{\nu \varepsilon_{it}^{O}} R, \tag{11.47}$$

which is the same expression as in the case when there is no ELIE scheme (Equation (11.12)). Hence, when education time is deductible from taxes, the distortive effect of ELIE should disappear. The implicit subsidy implied by the deductibility exactly offset the effect of the tax bearing on future income. Compared to the benchmark, we will no longer have the distortive on education choices; but we will have a lower transfer $\tilde{y}$ since the tax basis has been shrunk.

11.6.2 Linking early retirement to redistribution

As an alternative to subsidising education in the first period, we can make it more profitable in the second period, simply by increasing the early retirement age ν. Equation (11.24) suggests to link this age to the intensity of redistribution by implementing

$$\nu = \frac{\tilde{\nu}}{1-k},$$

where $\tilde{\nu}$ is the retirement age in the case without ELIE scheme. Then, the optimal rule for education (11.24) becomes Equation (11.47) again. Increasing the length of active live raises the return on education investment. Here, by letting the retirement age increase with redistribution, we compensate the negative effect of the ELIE tax on education by increasing length of active life and, hence, the return on education. Again here, the distortive effect of ELIE disappears. We are left with a rising labour supply as redistribution increases.

11.6.3 Numerical assessment

The two alternative efficient implementations of ELIE have clearly different macroeconomic properties, despite the fact that they both imply the same decision function for educating. They are not equally feasible. If subsidising education is possible whatever the value of k, postponing retirement as a function of k can be implemented only for a small range of values of k. Here as ν is already close to 1, this solution can work only for $k \leq 0.1$.

Table 11.5 illustrate the macroeconomic properties of these two implementations using the same calibration as before with a perfect credit market.

We give between brackets results for the option consisting in postponing retirement. When education is subsidised, the rate of growth of the economy is no longer decreasing with k, but is even slightly increasing with it (more increasing). The rate of return of capital remains more or less constant (increases) as well as the capital share (decreases). The wage rate increases (decreases). Education slightly increases (strongly increases). The percentage of borrowers decreases and is lower than in the benchmark model (decreases slowly and less than in the benchmark model). The share of the young generation in total income slightly increases (decreases). If we now look at inequality and poverty, they are both slightly more reduced compared to the benchmark model. But postponing retirement decreases inequality less in the young generation but more in the old generation, compared to the solution of subsidising education.

Table 11.5 Macroeconomic impact of ELIE with subsidies to education

k	g (% annual)	r	δ	Saving rate	Percent. borrowers	Gini total	Headcount poverty
	Subsidising education						
0.0	1.67	10.14	0.075	22.88	11.63	0.327	0.281
0.1	1.68	10.13	0.075	22.91	9.20	0.294	0.241
0.2	1.68	10.12	0.075	22.94	6.64	0.262	0.193
0.3	1.68	10.11	0.076	22.98	4.41	0.229	0.140
0.4	1.69	10.11	0.076	23.01	2.76	0.196	0.078
	Decreasing pre-retirement						
0.0	1.67	10.14	0.075	22.88	11.63	0.327	0.281
0.1	1.73	10.53	0.078	22.39	10.18	0.294	0.242

It is thus fairly possible to find an implementation of ELIE that has no distortive effect. On the contrary, that new implementation can even be *growth enhancing*, even if the credit market is perfect. The solution of subsidising education is much easier to implement and certainly more politically feasible than postponing retirement. In the benchmark model, the equilibrium wage rate and the pivot for redistribution $\tilde{y}$ are roughly equal. When education is subsidised, w is 3.5% higher than $\tilde{y}$. So there is slightly less to redistribute, but in both cases w and $\tilde{y}$ increase with k at exactly the same pace. Moreover, inequality in the young generation is unaffected by subsidising education. Inequality in the old generation is significantly reduced when education is subsidised in the first period so that overall inequality and poverty are more reduced in that case than in the benchmark model.

11.7 Conclusion

The ELIE scheme of Kolm (2005) proposes to tax labour capacities instead of labour income in order to promote social freedom. Once the individual has paid kw_i to society, he is free to dispose of his supplementary earnings as he pleases. As a secondary effect, the taxation redistributive scheme ELIE has no distortive effect on labour supply. The question of human capital formation and investment is addressed by Kolm (2005) only marginally as a dimension of labour (its quality and efficiency, chapter 8), or as a piece of information (chapter 10) for practically implementing ELIE. Otherwise, ELIE is confined to a static world with no consideration for dynamics, growth and inter-temporal optimisation.

In this paper, we have built an overlapping generations model with heterogenous agents and endogenous growth driven by investment in human capital. We have studied the effect of the ELIE scheme on education investment decisions and other aggregate economic variables. The fundamental question is to decide how to implement the ELIE scheme in this growth model and which part of the capacities to use as an inelastic basis for taxation. Clearly, the whole capacities can be taxed for the old generation. For the young generation, theory shows that, ceteris paribus, the implemented ELIE has a negative effect on investment decisions in education if the whole capacities are taxed. This effect arises because the implemented ELIE taxes future labour income, which reduces the return to investment in human capital in an inter-temporal optimisation. The distortive effect of this implementation of ELIE is completely overturned if the part of the capacities that are used for financing education are taken out of the tax base. This is a form of subsidy to education. This result can be seen paradoxical, because the implemented ELIE looses its distortive effect in a dynamic setting just when its basis is made elastic for the young generation.

Calibrating the model on French data, we illustrate the traditional trade-off between growth and inequality when whole capacities are taxed. In its crude implementation, ELIE is successful at reducing inequality and poverty, but at the expense of a lower investment in education and a slower growth rate. In a world with an imperfect market where individuals cannot borrow to educate in the first period, the tradeoff between growth and redistribution is modified. Indeed, in such a world, ELIE helps poor students to finance their education which counteracts partly its negative effect on the future return to education. But since ELIE redistributes to all poor people, and not only to those with a strong growth potential, the beneficial effect of ELIE obtained by releasing borrowing constraints is quantitatively small.

Using an alternative implementation of ELIE, growth can remain constant while inequality is reduced. This variant of the model, calibrated on French data, shows that education has to be subsidised if we want to escape from the traditional trade-off between growth and redistribution. Moreover, the usual argument according to which students should pay high fees at the university because those fees are partly compensated by their discounted future earnings is wrong. Our model shows that when there is redistribution, high fees have a disincentive effect on education decisions. And it also shows that subsidising education when there is redistribution enhance growth and reduces inequality in a better way.

Appendix A. Model's income distribution

We give here the formula to derive the net income distribution of the population living at time t. It is formed by the concatenation of the vector of income of the young generation and of the vector of income of the old generation living at the same time. For the young generation the budget constraint gives:

$$\omega_i^Y = (1-\delta_i)\varepsilon_i^Y w - k(\varepsilon_i^Y w - \tilde{y}) \quad \text{for } i = 1...N^Y.$$

For the old generation, the net income is, still up to the multiplicative factor h_t, established for the working part of that generation given by the budget constraint (11.20)

$$\omega_j^O = [\varepsilon_j^O \psi(\delta_j) w - k(\varepsilon_j^O \psi(\delta_j) w - G\tilde{y})]/G \quad \text{for } j = 1...N^O.$$

Taking into account that $h_{t-1} = h_t/G$. The net income distribution is thus given by:

$$\omega' = \left[\left([\omega_i^Y]_{i=1}^{N^Y} \right)', \left([\omega_j^O]_{j=1}^{N^O} \right)' \right].$$

The income distribution is computed for the age group 18-62. Accordingly, the relevant income of the old generation is here ω_{it}^O, and v does not enter this formula, contrary to (11.20). We compute the Gini coefficient for ω.

Appendix B. Numerical methods

The model is solved using a traditional fixed point algorithm. We give below the procedure to compute the equilibrium with credit constraints for given parameters. The case without constraints is just a simplification of this more complicated case. We first have to fix starting values for the aggregate variables r, w and $\tilde{y}$. Then we apply the following algorithm.

- **Step 1** identify constrained agents running (11.39) for the first generation using ε_t, store the results in id_1. Do the same for the second generation using ε_{t-1} and store the result in id_2.
- **Step 2**
  - compute the optimal δ using (11.24)
  - compute the constrained optimal δ_c solving (11.40) using a fixed point algorithm
  - $\delta_j = id_j \delta + (1 - id_j)\delta_c$ for $j = 1, 2$.
- **Step 3** compute the growth rate g, the different income and transfers vectors, and the vector of savings. Deduce K, L, r, w and $\tilde{y}$.
- **Step 4** Check the change in δ_1 and $\tilde{y}$. If the sum of the absolute changes is greater than 10^{-6}, go to step 1. Otherwise deliver the needed vectors and equilibrium values.

In the unconstrained case, step 1 does not exist and step 2 does not involve computing δ_c.

References

Arrow, K. J., Chenery, H. B., Minhas, B. S., and Solow, R. M. (1961). Capital-labor substitution and economic efficiency. *Review of Economics and Statistics*, 43:225–250.

Askenasy, P. (2003). Partage de la valeur ajoutée et rentabilité du capital en France et aux Etats-Unis: une reévaluation. *Economie et Statistique*, 363-364-365:167–189.

Atkinson, A. B. (1998). *Poverty in Europe*. Yrjö Jahnsson Lectures. Blackwell, Oxford.

Azariadis, C. and de la Croix, D. (2006). Financial institutional reform, growth, and equality. In Eicher, T. S. and García-Peñalosa, C., editors, *Institutions, Development, and Economic Growth*, CESifo Seminar Series, pages 33–64. The MIT Press, Cambridge (USA).

Azariadis, C. and Drazen, A. (1990). Threshold externalities in economic development. *Quarterly Journal of Economics*, 105(2):501–526.

Bénabou, R. (2005). Inequality, technology and the social contract. In Aghion, P. and Durlauf, S., editors, *Handbook of Economic Growth*, volume 1, chapter 25, pages 1595–1638. Elsevier.

Cardia, E., Kozhaya, N., and Ruge-Murcia, F. J. (2003). Distortionary taxation and labor supply. *Journal of Money, Credit and Banking*, 35(3):350–73.

De Gregorio, J. and Kim, S.-J. (2000). Credit markets with differences in abilities: education, distribution and growth. *International Economic Review*, 41(3):579–607.

de la Croix, D. and Michel, P. (2002). *A Theory of Economic Growth: Dynamics and Policy in Overlapping Generations*. Cambridge University Press, Cambridge.

de la Croix, D. and Michel, P. (2007). Education and growth with endogenous debt constraints. *Economic Theory*, 33:509–530.

Duffy, J. and Papageorgiou, C. (2000). A cross-country empirical investigation of the aggregate production function specification. *Journal of Economic Growth*, 5:87–120.

Foster, J., Greer, J., and Thorbecke, E. (1984). A class of decomposable poverty measures. *Econometrica*, 52:761–765.

García Peñalosa, C. and Turnovsky, S. (2007). Growth, income inequality, and fiscal policy: What are the relevant trade-offs? *Journal of Money, Credit and Banking*, 39(2-3):369–394.

Kehoe, T. and Levine, D. (1993). Debt-constrained asset markets. *Review of Economic Studies*, 60(4):865–888.

Kolm, S.-C. (2005). *Macrojustice. The Political Economy of Fairness*. Cambridge University Press, Cambridge (UK).

Lubrano, M. (2009). The redistributive aspects of ELIE: a simulation appoach. In Gamel, C. and Lubrano, M., editors, *Macrojustice : A pluridisciplinary evaluation of Kolm's theory*. Springer, Heidelberg. (this volume).

Maddison, A. (2007). *Contours of the World Economy, 1-2030AD; Essays in Macroeconomic History*. Oxford University Press, Oxford. http://www.ggdc.net/maddison.

OECD (2006). *Education at a Glance*. OECD Publications, Paris.

Solow, R. M. (1956). A contribution to the theory of economic growth. *The Quarterly Journal of Economics*, 70(1):65–94.