Life expectancy and endogenous growth

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Introduction: Longevity and growth

Economic growth is responsible for the secular increase in longevity (Fogel, 1994).

We know little on the causal relationship going in the other direction, that is, from increased longevity to economic growth.

Barro and Sala-I-Martin (1995): a 13 year increase in life-expectancy is estimated to raise the annual growth rate by 1.4 %.
Explanation: life expectancy proxies for features that reflect desirable performance of a society.

There is however several channels through which life expectancy affects growth directly:

- when the probability of dying young is high, it is optimal for people to start working early in their life and not to stay at school too long.
- when life expectancy is short, the depreciation rate of human capital is high, making its accumulation more difficult.

Introduction: Vintage human capital

If the human capital accumulated at school is an important engine of growth, we should thus expect that the growth rate depends upon life expectancy.

We investigate this question in an overlapping generations model à la Blanchard (1985), in which agents decide the length of time devoted to schooling before starting to work.

Our assumption contrasts with Lucas (1988) which has the unrealistic implication that people invest a share of their time in education over all their life.

In our model, the aggregate human capital is built from a sequence of generations having different human capital levels – vintage human capital.

Demographics

Time is continuous and the equilibrium is evaluated from 0 onward.

At each point in time there is a continuum of generations indexed by their birth date, t.

the measure $V_{t,z}$ of the set of individuals born in t still living in z:

$$\mu(V_{t,z}) = e^{-\beta(z-t)}\,\pi \qquad \pi > 0, \beta > 0.$$

π: the measure of a new cohort.
β: the rate at which members of a given generation die.

The measure of each generation declines deterministically through time but each agent is uncertain about the time of his death.

For an individual born in t, $\mu(V_{t,z})/\pi$ is the expectancy at time t to live at least until time z.

The life expectancy does not depend on age (perpetual youth):

$$\int_t^\infty (z-t)\beta e^{-\beta(z-t)}\mathrm{d}z \quad = \quad \frac{1}{\beta}$$

The size of total population is π/β. (in Blanchard, $\pi = 1/\beta$).

Preferences

There is a unique material good, the price of which is normalized to 1, that can be used for consumption.

This good is produced from a technology using labour as the only input.

An individual born at time $t, \forall t \geqslant 0$, has the following expected utility:

$$\int_t^{\infty} c(z,t)e^{-(\beta+\theta)(z-t)}\mathrm{d}z \qquad \theta > 0, \tag{1}$$

the instantaneous utility function is assumed linear.

perfect insurance markets.

The intertemporal budget constraint:

$$\int_t^{\infty} c(z,t)R(z,t)\mathrm{d}z = \int_{t+T(t)}^{\infty} \omega(z,t)R(z,t)\mathrm{d}z \tag{2}$$

$R(z,t)$ is the discount factor.

The agent is assumed to go to school until time $t + T(t)$. After this education period, he earns a wage $\omega(z,t)$ per unit of time.

Technology

Wages depend on individual human capital, $h(t)$:

$$\omega(z,t) = h(t)w(z),$$

$w(z)$ is the wage per unit of human capital.

The individual's human capital is a function of the time spent at school $T(t)$ and of the *average* human capital $\overline{H}(t)$ at birth:

$$h(t) = A\,\overline{H}(t)T(t) \quad A > 0. \tag{3}$$

The parameter A is a productivity parameter.

The presence of $\overline{H}(t)$ introduces an externality as in Lucas (1988) and Azariadis and Drazen (1990): the cultural ambiance of the society at the time of the birth influences positively the future quality of the agent (through for instance the quality of the school).

Interpretation

It is as if each individual receives a private tutor at birth with the average human capital. Education depends on the private tutor's human capital.

Optimality conditions

The optimality condition for consumption is

$$r(z) = \theta$$

Reductio ad absurdum:
if $r(z) > \theta$ at some date z, consumption should be zero for all generations; as current workers are not allowed to go back to school and as consumption cannot be transformed into another good (like capital), a zero consumption level for all generations would violate the equilibrium condition on the goods market.
If $r(z) < \theta$ consumption would be infinite which is not compatible with goods market equilibrium.

The discount factor is

$$R(z,t) = e^{-(\theta+\beta)(z-t)}.$$

The first order condition for $T(t)$ is

$$\int_{t+T(t)}^{\infty} e^{-(\theta+\beta)(z-t)} w(z)\mathrm{d}z = T(t)e^{-T(t)(\theta+\beta)} w(t+T(t)). \qquad (4)$$

The left hand side is the marginal gain of increasing the time spent at school by one unit.
The right hand side is the marginal cost, i.e. the loss in wage income if the entry on the job market is delayed.

Labour market

The production function

$$Y(t) = H(t), \tag{5}$$

The equilibrium in the labour market

$$w(t) = 1.$$

Equation (4) becomes

$$T(t) = T \equiv \frac{1}{\theta + \beta}, \tag{6}$$

$\forall\, t \geqslant 0$.

The optimal time spent on education negatively affected by β.

The aggregate human capital stock is computed from the capital stock of all generations currently at work:

$$H(t) = \int_{-\infty}^{t-J(t)} \pi e^{-\beta(t-z)} h(z) \mathrm{d}z, \tag{7}$$

where $t - J(t)$ is the last generation that entered the job market at t. $J(t) = T(t - J(t))$.

Growth is exclusively linked to the appearance of new generations. Hence, the objective function of an individual is always finite.

Initial conditions (1)

To evaluate $H(t)$, for $t \geqslant 0$, we need to know an entire span of initial conditions for $h(t)$, from $-\infty$ to $J(0)$.

However $J(0)$ is endogenously determined at $t = 0$.

Let $J(t) = J_0(t), \ \forall t < 0$, with $\lim J_0(t)_{t \to 0^-} = J_0$, finite and strictly positive.

Additionnaly, we assume $h(t) = h_0(t), \ \forall t < -J_0$. Generations older than $-J_0$ have already entered the labour market before time $t = 0$.

Concerning existing generations still at school at $t = 0$, two possible cases could arise.

1. If $J_0 \geqslant T$, all generations $t \in [-J_0, -T[$ enter the labour market at $t = 0$ with an education $T(t) = -t$. Generations $t \in [-T, 0[$ choose a schooling time of T and will enter the labour market at period $t + T > 0$.
2. If $T \geqslant J_0$, all generations $t \in [-J_0, 0[$ decide $T(t) = T$ and nobody enters the labour market until time $t = T - J_0 > 0$.

Initial conditions (2)

In the next, we assume that $J_0 \geqslant \frac{1}{\theta+\beta}$.

We compute initial conditions for $H(t)$, $\forall t < 0$,

$$H(t) = H_0(t) \equiv \int_{-\infty}^{t-J(t)} \pi e^{-\beta(t-z)} h_0(z) \mathrm{d}z.$$

$\overline{H}(t) = H(t)\beta/\pi$

$$H(t) = H(0)e^{-\beta t} + \int_{-T}^{t-T} \beta e^{-\beta(t-z)}[A\ T\ H(z)]\mathrm{d}z, \qquad (8)$$

where

$$H(0) = \lim_{t \to 0^-} H_0(t) + \int_{-J_0}^{-T} e^{\beta z}[A(-z)H_0(z)]\mathrm{d}z. \qquad (9)$$

As generations $T \in [-J_0, -T[$ enter the labour market together, human capital jumps at $t = 0$, implying that $H(0) > \lim_{t \to 0^-} H_0(t)$.

Dynamics and steady state

Differentiating (8) with respect to time, we find the following DDE, $\forall t \geqslant 0$:

$$H'(t) = AT\beta e^{-\beta T} H(t-T) - \beta H(t), \tag{10}$$

with $H(0)$ given by (9).

- Aggregate human capital decreases at a rate β as time passes and people die.
- This is compensated by the entry of new generations in the job market. At time t, $\pi e^{-\beta T}$ individuals of generation $t-T$ enter the job market with human capital $A\ T\ H(t-T)\beta/\pi$.

The steady state growth rate of human capital γ is the solution to

$$\gamma + \beta = AT\beta e^{-(\beta+\gamma)T}.$$

Solving for γ leads to

$$\gamma = -\beta + \frac{W\left(T^2 A\beta\right)}{T} \tag{11}$$

where $W(.)$ is the Lambert W function: $W(z)e^{W(z)} = z$.

$W(T^2 A\beta)$ is unique and positive.

The growth rate does not depend on the size of the new cohorts π, as the externality has been specified in terms of average human capital.

Effect of longevity on steady state growth

the effect of the instantaneous probability of death β on the growth rate γ is given by

$$\frac{\mathrm{d}\gamma}{\mathrm{d}\beta} = -2 + \frac{\theta}{\beta} + W\left(T^2 A\beta\right) + \frac{\beta - \theta}{\beta(1 + W\left(T^2 A\beta\right))},$$

and the sign is indeterminate.

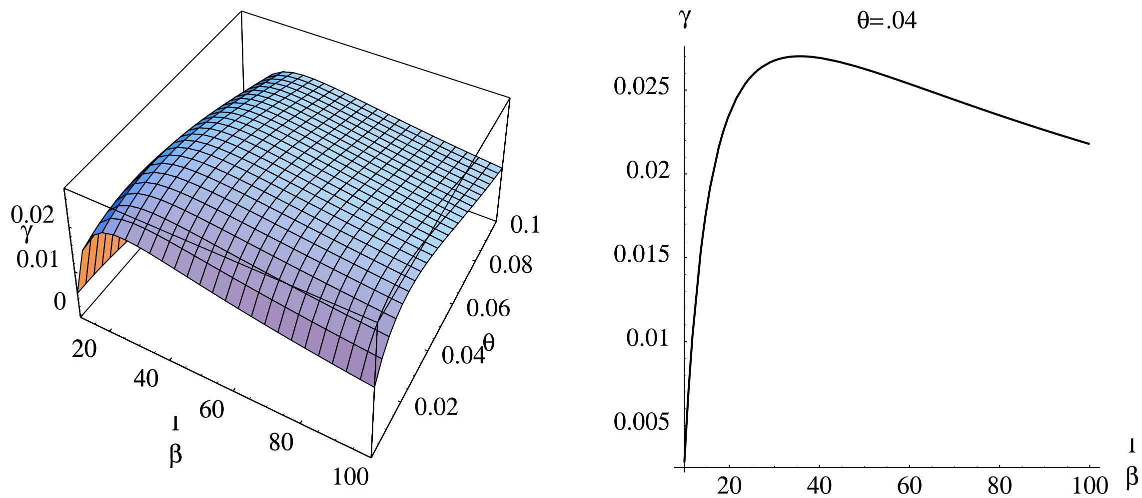
An increase in $1/\beta$ has three effects

- (+) agents die later on average, thus the depreciation rate of aggregate human capital decreases;
- (+) agents tend to study more because the expected flow of future wages has risen, and the human capital per capita increases;
- (-) agents enter the job market later in their life, thus the activity rate decreases.

Numerical computations show that we observe that the effect of $1/\beta$ on γ is hump shaped.

We should thus observe that the effect of life expectancy on growth is positive for countries with a relatively low life expectancy, but could be negative in more advanced countries.

Figure 1: Life expectancy and steady state growth rate ($A = .3$)



Stability

Detrended human capital:

$$z(t) = H(t)e^{-\gamma t}.$$

The DDE (10) becomes, $\forall t \geqslant 0$,

$$z'(t) = (\beta + \gamma)(z(t - T) - z(t)), \tag{12}$$

with $z(0) = H(0)$.

To solve it, we follow Bellman and Cooke (1963). We guess that $z(t) = e^{st}$ is a solution. Then,

$$s = (\beta + \gamma)(e^{-sT} - 1). \tag{13}$$

If s_k is a solution of equation (13), by linearity

$$z(t) = \sum p_k e^{s_k t}.$$

Equation (12) is identical to the one handled by Boucekkine, del Rio and Licandro (1997). Using the results of the later authors, we know that any root s_k of the equation (13) has non-positive real part. The only root with zero real part is $s_k = 0$.

The steady state is asymptotically stable.
The solution path is oscillatory.

Example

The initial conditions: $A = .3$, $\beta = .1$ and $\theta_0 = .02$.
$\Rightarrow \gamma_0 = 0.45\%$, $J_0 = 8.33$.

Permanent unexpected change in θ at $t = 0$.
$\theta = 0.05 \Rightarrow \gamma = 0.16\%, T = 6.66$.

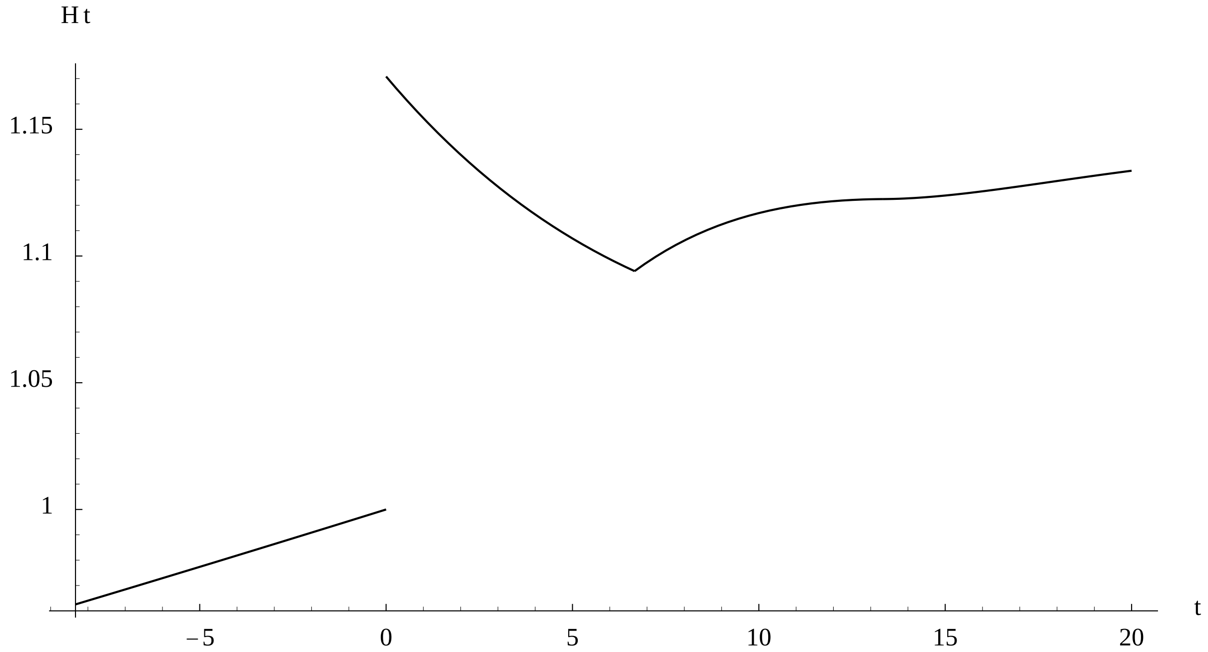
Generations from $-J_0$ to $-T$ enter the labour market at t=0, producing a discrete positive jump in $H(t)$.

New generations do not enter the labour market until $t = T$ and aggregate human capital decreases (initial h of the old is low).

At $t = T$, new generations start entering the labour market and this increases the aggregate stock.

After some time, human capital follows its new balanced growth path with a lower growth rate.

Figure 2: Dynamics



Conclusion

Life expectancy is a central factor that affects positively the optimal length of education, and hence, the growth rate of the economy.

However, the positive effect of a longer life on growth could be offset by a decrease in the participation rate.

Extentions:

- Optimal allocation
- Pensions
- Physical capital
- concave utility function

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