Education and Growth with Endogenous Debt Constraints

David de la Croix[1] Philippe Michel

[1]dept. of economics & CORE
Univ. cath. Louvain

July 20, 2006

Borrowing Constraints and Education

For developing countries borrowing constraints limit private schooling expenditures

Effect on growth ?

Two possible frameworks:
- exogenous borrowing limit
- endogenous borrowing limit - incentive constraint

Borrowing Constraints

In the literature on education, either:
– exogenous (De Gregorio)
– endogenous in numerical models (Lochner and Monge)
– endogenous in small open economies (fixed prices) (Andolfatto and Gervais)

This paper: methodological contribution

Obtain analytical results with endogenous borrowing constraints

Endogenous Borrowing Constraints

Kehoe Levine (1993)

Households are allowed to borrow as long as it is in their interest to reimburse

If they default, they are penalized: banned from credit markets for the rest of their life

Azariadis and Lambertini (2003)

Intensity of borrowing limits depend on interest rate

Multiple Equilibria

Households

$$u\left(c_t,\ d_{t+1}\right) \tag{1}$$

$$h_t = A\ b_{t-1}^{\lambda}\ h_{t-1}^{1-\lambda} \tag{2}$$

In case of repayment:

$$c_t = h_t - s_t - R_{t-1} b_{t-1} \tag{3}$$
$$d_{t+1} = R_t s_t + \delta h_t \tag{4}$$

In case of default:

$$c_t = h_t$$
$$d_{t+1} = \delta h_t.$$

Individual Rationality Constraints

1. IRC old-age:

$$s_t \geq 0 \tag{5}$$

2. IRC middle-age:

$$\max_{s} u(h_t - s - R_{t-1}b_{t-1}, R_t s + \delta h_t) \geq u(h_t, \delta h_t) \tag{6}$$

Condition bearing on b_{t-1}:

$$\max_{s\geq 0} u(h_t - s - R_{t-1}b_{t-1}, R_t s + \delta h_t) \geq u(h_t, \delta h_t). \tag{IC}$$

Equilibrium

Production:

$$Q_t = L_t.$$

Labor market equilibrium:

$$L_t = h_t + \delta h_{t-1}$$

Assets market equilibrium:

$$b_{t-1} = s_{t-1}. \tag{7}$$

The Borrowing Limit - result

$$\text{Income share of debt repayment: } x_t = \frac{R_{t-1} b_{t-1}}{h_t}. \qquad (8)$$

Proposition

The constraint (IC) is equivalent to an upper bound $\bar{x}_t$ on x_t:

$$\bar{x}_t = 1 - g(R_t) \text{ with } g(R_t) = \left[\frac{u(1,\delta)}{u_2'(1,\mu(R_t))} - \delta \right] \frac{1}{R_t}.$$

$\bar{x}$ is equal to zero for $R_t \leq R_{min}$ – and it is positive for $R_t > R_{min}$. The function $g(.)$ and the threshold R_{min} only depend on preferences and δ.

The Borrowing Limit - intuition

For small interest rate, optimal consumption profile is flatter than income profile

In this case, optimal savings are non positive

It is never optimal to reimburse the loan

No access to borrowing $\bar{x} = 0$.

More Results on The Borrowing Limit

Proposition

The borrowing limit function $\bar{x}(R) = 1 - g(R)$ is increasing from 0 to 1 when R goes from R_{min} to $+\infty$. Its slope at R_{min} is equal to 0. For given $R > R_{min}$, $\bar{x}(R)$ decreases with respect to δ.

When future labor income prospects are high, δ is high, households can borrow less

Unconstrained Education Level

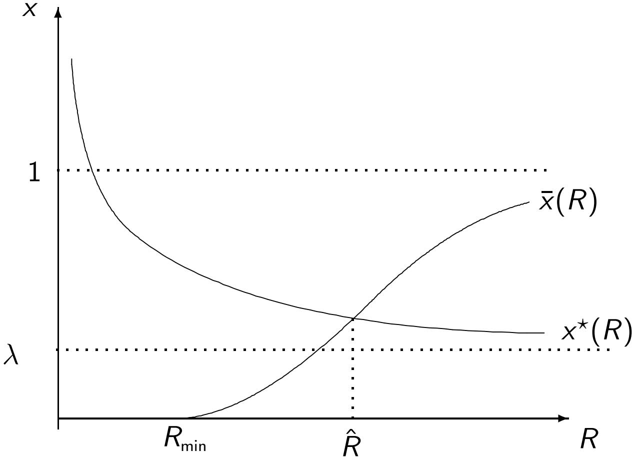
$$x_t^\star = \lambda \left(1 + \frac{\delta}{R_t}\right) \equiv x^\star(R_t). \qquad (9)$$

$x_t^\star$ is proportional to the factor $(1 + \delta/R_t)$ which transforms human capital into life-cycle income.

The constrained optimal level of x_t:

$$x_t = \min\left\{x^\star(R_t), \bar{x}(R_t)\right\} \equiv x(R_t). \qquad (10)$$

$\exists \hat{R} > R_{\min}$ s.t. $\bar{x}(\hat{R}) = x^{\star}(\hat{R}) \equiv \hat{x}$.



Growth rate

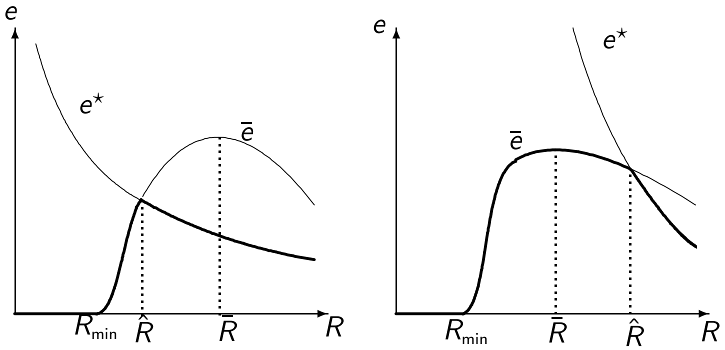
Income growth is proportional to investment in education $e_t = b_t/h_t$.

Link between e and x: (with a constant interest rate)

$$e = \left(\frac{A\, x(R)}{R}\right)^{\frac{1}{1-\lambda}}.$$

$\bar{x}(R)/R$ is hump-shaped, $x^\star(R)/R$ is decreasing.

Two possible configurations


e
e⋆
ē
R_{min}
$\hat{R}$
$\bar{R}$
R
e
e⋆
ē
R_{min}
$\bar{R}$
$\hat{R}$
R

Where is maximum growth attained ?

Proposition

If the elasticity of earnings to education, λ, is large enough, the maximum growth rate is attained in the interior of the constrained regime. Otherwise, it is attained at the frontier between the unconstrained and constrained regimes.

Steady State

Using the equilibrium condition on the asset market, dynamics are of the type:

$$x(R_{t+1}) = \frac{1}{A}\,\phi(R_t, x(R_t)). \tag{11}$$

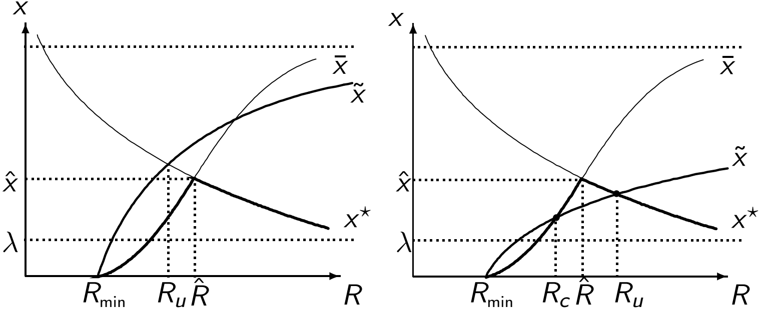
Steady states x satisfy: $x = \frac{1}{A}\,\phi(R, x(R))$.

This relationship implicitly defines a function $\tilde{x}(R)$, which describes the combinations R, x compatible with a steady state.

Lemma

The steady state function $\tilde{x}(R)$ is increasing from 0 to 1 when R goes from R_{min} to $+\infty$ and its derivatives satisfies: $d\tilde{x}(R_{min})/dR > 0$.

Possible Configurations



Existence of Steady States and Stability

When $A < \hat{A}$: weak productivity of education, no > 0 steady state.

The economy does not grow. Standard inescapable poverty trap.

When $A > \hat{A}$: two non-trivial steady states.
– R_c in the constrained regime is locally stable – infinite number of trajectories converging to it (local indeterminacy).
– R_u investment is unconstrained. R_u is locally unstable – unique trajectory leading to it: R_u from the initial period onward.

On the whole, global indeterminacy.

Comparing outcomes across the various equilibria

The relation between growth and the interest rate is hump-shaped.

What if financial markets are suddenly made perfect ?

unique equilibrium converging instantaneously to the steady state.

At this “perfect market” steady state, growth is higher than in the poverty trap, but not necessarily than in the “imperfect market” constrained steady state.

Moving towards perfect markets promotes growth for sure in the case of the poverty trap, (A low enough), while not necessarily otherwise.

Conclusion

Endogeneity of the borrowing limit is important

General equilibrium is important, give rise to complex dynamics (poverty trap, indeterminacy)

Conclusions from models with exogenous borrowing limits can be reversed

Example:
With exogenous limit, financial depth is always good for growth
Not always true here