Income Growth in the 21st Century: Forecasts with an Overlapping Generations Model

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Main question

Rise in life expectancy
Drop in fertility rates
$\Rightarrow$ share of the elderly in population will increase

Aging is forecasted by demographers:
– inescapable
– little can be done to modify its magnitude for the next 50 years
– Strength varies across countries

Effects of these demographic changes upon the economy ?
–in theory, many effects whose weights are unknown a priori
–need for a quantitative approach

Theoretical effects of aging

Reduces the growth rate of the labor force
(become negative in FR and CA)

Stimulates savings $\rightarrow$ more capital per worker,
lower interest rates

Changes the characteristics of the labor force:
experience vs education

Public finance effects: sustainability of pension systems ?

What we do

Take demographic forecast for France, Canada, USA.

Build an overlapping generations model capturing some key features to study aging $\Rightarrow$ growth.

Calibrate the parameters over the period 1960-2000.

Simulate the model to forecast GDP per capita over 2000-2050.

Three scenarios:
constant policies, rise in retirement age, drop in pensions

Same methodology for the three countries $\rightarrow$ comparability.

Our model compared to existing studies

Strong demographic block with life uncertainty and migrations (exogenous)

Link the evolution of taxes and expenditures to demographic changes (use of generational accounting studies)

Labor market: interactions between education and experience

The model – population (1)

Model time is discrete and goes from 0 to $+\infty$.

At each date, some individuals die and a new generation appears.

Households reaching age 15 at year t belong to generation t.

Each household lives a maximum of 8 periods ($a = 0, ..., 7$)

The model – population (2)

The size of the young generation:

$$N_{0,t+1} = N_{0,t} m_t$$

m_t: exogenous demographic growth rate (fertility + migration)

The size of each generation:

$$N_{a,t+a} = N_{0,t} \beta_{a,t+a} + M_{a,t+a}$$

cumulative survival probability decreasing with age: $\beta_{a,t+a}$.

Total population at time t

$$N_t = \sum_{a=0}^{7} N_{a,t}$$

The model – households – objective

The expected utility:

$$E(U_t) = \sum_{a=0}^{7} \beta_{a,t+a} \ln(c_{a,t+a}) \tag{1}$$

The budget constraint:

$$\sum_{a=0}^{7} p_{a,t+a} \left[c_{a,t+a}(1+\tau^c_{t+a}) - T_{a,t+a}\right]$$

$$= \sum_{a=0}^{7} \left(\omega^L_{a,t+a} + \omega^E_{a,t+a} e_{a,t+a} + \omega^H_{a,t+a} h_{a,t+a}\right) \ell_{a,t+a} \tag{2}$$

Households choose their consumption spending and their investment in education.

The model – households – labor supply

Labor supply:

$$\overline{\ell}_t = (q_t(1-u_t),\ q_{t+1},\ q_{t+2},\ q_{t+3},\ q_{t+4}(1-\alpha_{t+4}),\ 0,\ 0,\ 0) \tag{3}$$

Experience:

$$\begin{aligned}\overline{e}_t \;=\; & (0,\ (1-u_t)q_t,\ (1-u_t)q_t + q_{t+1},\ (1-u_t)q_t + q_{t+1} + q_{t+2}, \\ & (1-u_t)q_t + q_{t+1} + q_{t+2} + q_{t+3},\ 0,\ 0,\ 0)\,, \end{aligned} \tag{4}$$

Education capital:

$$\overline{h}_t = \left(0,\ \epsilon u_t^{\psi},\ \epsilon u_t^{\psi},\ \epsilon u_t^{\psi},\ \epsilon u_t^{\psi},\ 0,\ 0,\ 0\right), \tag{5}$$

The model − households − transfers

Public transfers:

$$\begin{aligned}\overline{T}_t &= \Big(v_t q_t u_t \omega_{0,t}^L + \gamma_0 g_t,\ \gamma_1 g_{t+1},\ \gamma_2 g_{t+2},\ \gamma_3 g_{t+3},\\ &\quad \alpha_{t+4} b_{t+4} + \gamma_4 g_{t+4},\ b_{t+5} + \gamma_5 g_{t+5},\\ &\quad b_{t+6} + \gamma_6 g_{t+6},\ b_{t+7} + \gamma_7 g_{t+7}\big), \qquad (6)\end{aligned}$$

v_t: rate of subsidy on the cost of education

b_t: pension benefit

γ_a: share of total transfer g_t in favor of age a.

The model – households – first-order-conditions

The optimal education investment:

$$u_t^* = \left(\frac{\epsilon\psi \sum_{a=1}^{4} \left[\omega_{a,t+a}^{H} q_{t+a} \ell_{a,t+a} \right]}{(1 - v_t) q_t \left[\omega_{0,t}^{L} + \omega_{0,t}^{H} \right] + \sum_{a=1}^{4} \left[\omega_{a,t+a}^{E} q_{t+a} \ell_{a,t+a} \right]} \right)^{\frac{1}{1-\psi}} \tag{7}$$

Optimal consumption:

$$c_{a+1,t+a+1} = \frac{(1 + r_{t+1})(1 + \tau_t^c)}{(1 + \tau_{t+1}^c)} \; c_{a,t+a} \qquad \forall a = 0, ..., 6 \tag{8}$$

The model – firms – technology

Production function:

$$Y_t = A_t K_t^{1-\varphi} Q_t^{\varphi} \tag{9}$$

total factor productivity:

$$\frac{A_t}{A_{t-1}} \equiv G_t = (1-\lambda)\overline{G} + \lambda G_{t-1} + \varepsilon_t \tag{10}$$

efficiency units of labor:

$$Q_t = L_t^{1-\delta}\left[\mu E_t + (1-\mu) H_t\right]^{\delta} \tag{11}$$

The model – firms – first-order-conditions

firms maximize profits:

$$Y_t - (r_t + d)K_t - w_t^L L_t - w_t^H H_t - w_t^E E_t \tag{12}$$

FOC:

$$\begin{aligned}
r_t &= (1-\varphi)A_t Y_t / K_t - d & (13)\\
w_t^L &= \varphi(1-\delta)A_t Y_t / L_t & (14)\\
w_t^E &= \varphi\delta\mu A_t K_t^{1-\varphi} Q_t^{\varphi-1} L_t^{1-\delta} \left[\mu E_t + (1-\mu)H_t\right]^{\delta-1} & (15)\\
w_t^H &= \varphi\delta(1-\mu) A_t K_t^{1-\varphi} Q_t^{\varphi-1} L_t^{1-\delta} \left[\mu E_t + (1-\mu)H_t\right]^{\delta-1} & (16)
\end{aligned}$$

The model – public sector

Government budget constraint:

$$
\begin{aligned}
& \tau_t^w(w_t^L L_t + w_t^E E_t + w_t^H H_t) + \tau_t^c C_t + \tau_t^k r_t K_t + D_{t+1} - (1+r_t)D_t \\
= \; & N_{0,t} v_t q_t u_t w_t^L (1-\tau_t^w) + \sum_{a=0}^{7} N_{a,t}\gamma_a g_t \\
& + \vartheta_t Y_t + (N_{4,t}\alpha_t + \sum_{a=5}^{7} N_{a,t}) b_t \qquad (17)
\end{aligned}
$$

D_t: public debt at the beginning of period t

The model – equilibrium conditions (1)

Definition of the net discount factor:

$$R_{a,t+a} \equiv \prod_{s=t+1}^{t+a} (1 + r_s(1 - \tau_s^k))^{-1}$$

Equilibrium prices:

$$p_{a,t+a} = R_{a,t+a}\beta_{a,t+a}p_{t+a} = R_{a,t+a}\beta_{a,t+a} \tag{18}$$

$$\begin{aligned} \omega_{a,t+a}^{L} &= R_{a,t+a}\beta_{a,t+a}w_{t+a}^{L}(1 - \tau_{t+a}^{w}) \\ \omega_{a,t+a}^{E} &= R_{a,t+a}\beta_{a,t+a}w_{t+a}^{E}(1 - \tau_{t+a}^{w}) \\ \omega_{a,t+a}^{H} &= R_{a,t+a}\beta_{a,t+a}w_{t+a}^{H}(1 - \tau_{t+a}^{w}) \end{aligned} \tag{19}$$

The model – equilibrium conditions (2)

Goods market:

$$Y_t + K_t^\star = \underbrace{\sum_{a=0}^{7} N_{a,t} c_{a,t}}_{C_t} + \underbrace{K_{t+1} - (1-d)K_t}_{I_t} + \underbrace{\vartheta_t Y_t}_{G_t} \qquad (20)$$

Labor market:

$$L_t = \sum_{a=0}^{7} N_{a,t}\ell_{a,t}, \qquad E_t = \sum_{a=0}^{7} N_{a,t}\ell_{a,t}e_{a,t}, \qquad H_t = \sum_{a=0}^{7} N_{a,t}\ell_{a,t}h_{a,t} \qquad (21)$$

The quantitative experiment - method

Get data for observed exogenous variables,

Fix some parameters (same for the 3 countries),

Choose paths for other unobserved exogenous variables and parameters in order to match a series of characteristics (calibration).

Calibration not done at steady state but dynamically, over the period 1960-2000.

The equilibrium is computed as a transition from one steady state in 1900 to one another in 2250.

By starting in 1900, the stocks of education and experience around 1960 reflect the correct history of the population.

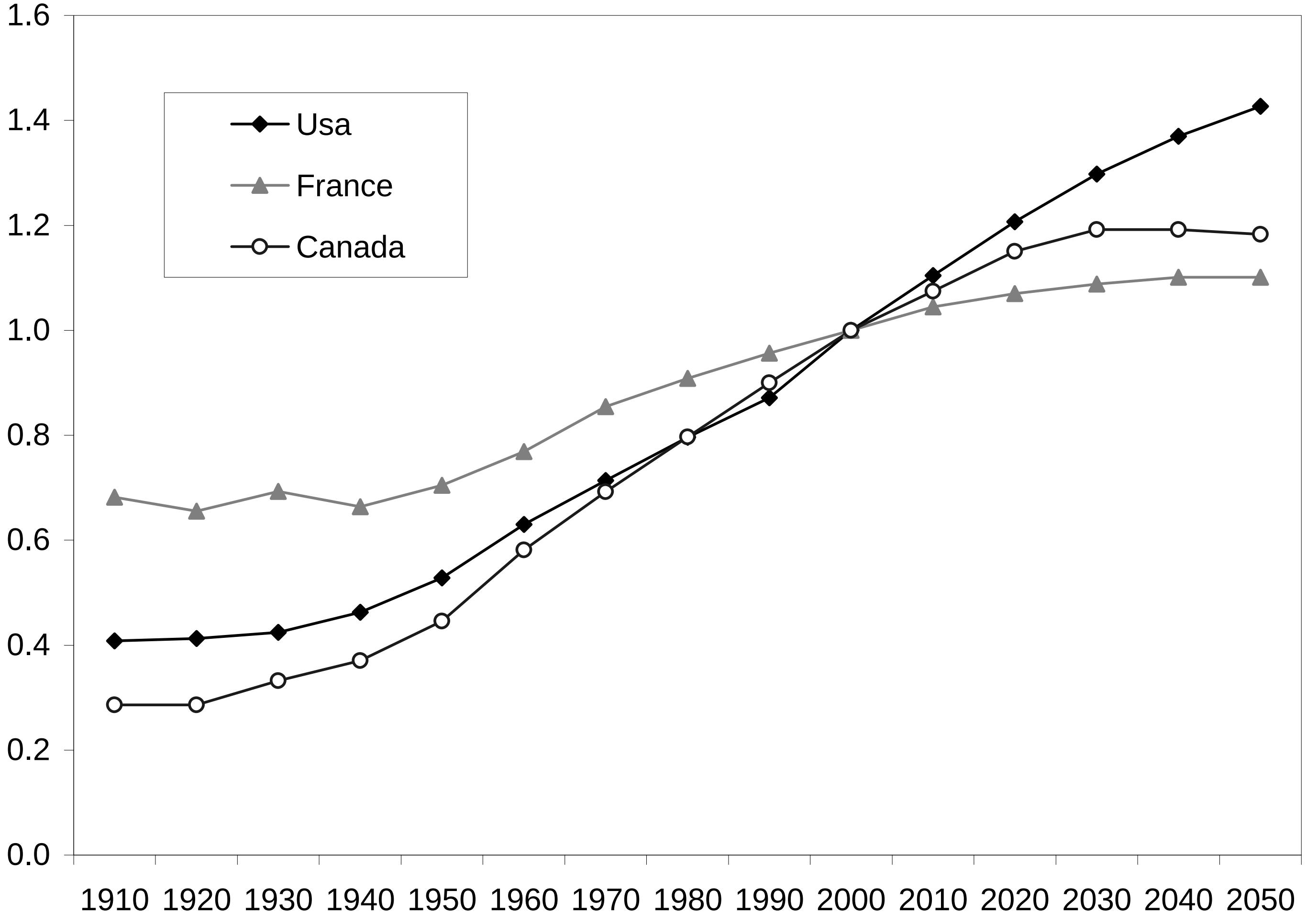
Observed exogenous variables

Demography

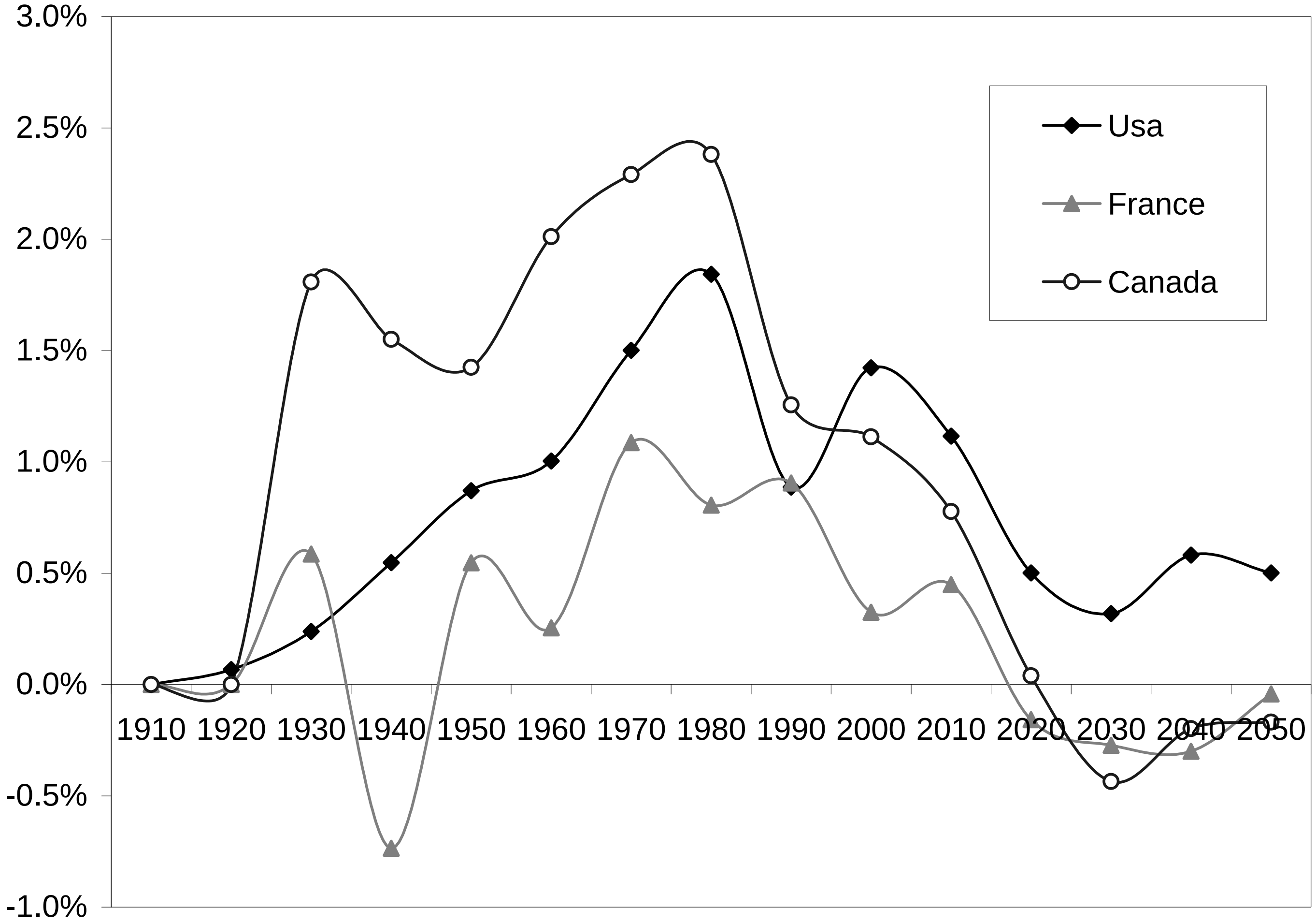
Education and participation rates

Public finance

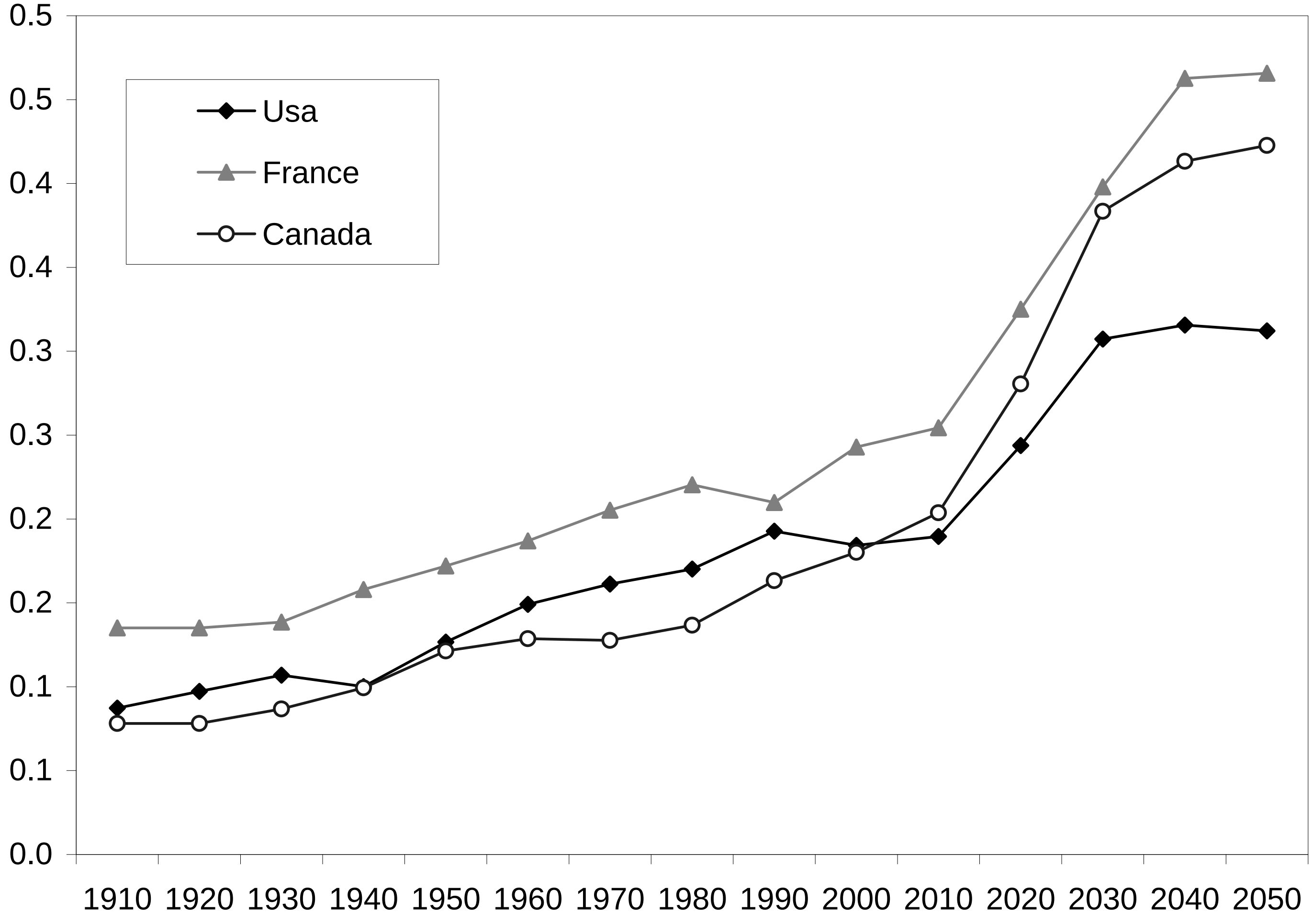
Population size (index 2000=100)



Growth rate of the population aged 15-64



Old Age Dependency (65+ / 15-64)



Parameters

The labor share in output, φ, is set to 0.7

The depreciation rate of capital d equals 0.4. (5% annual)

Share of raw labor in labor income $(1 - \delta)$ is set to 0.4

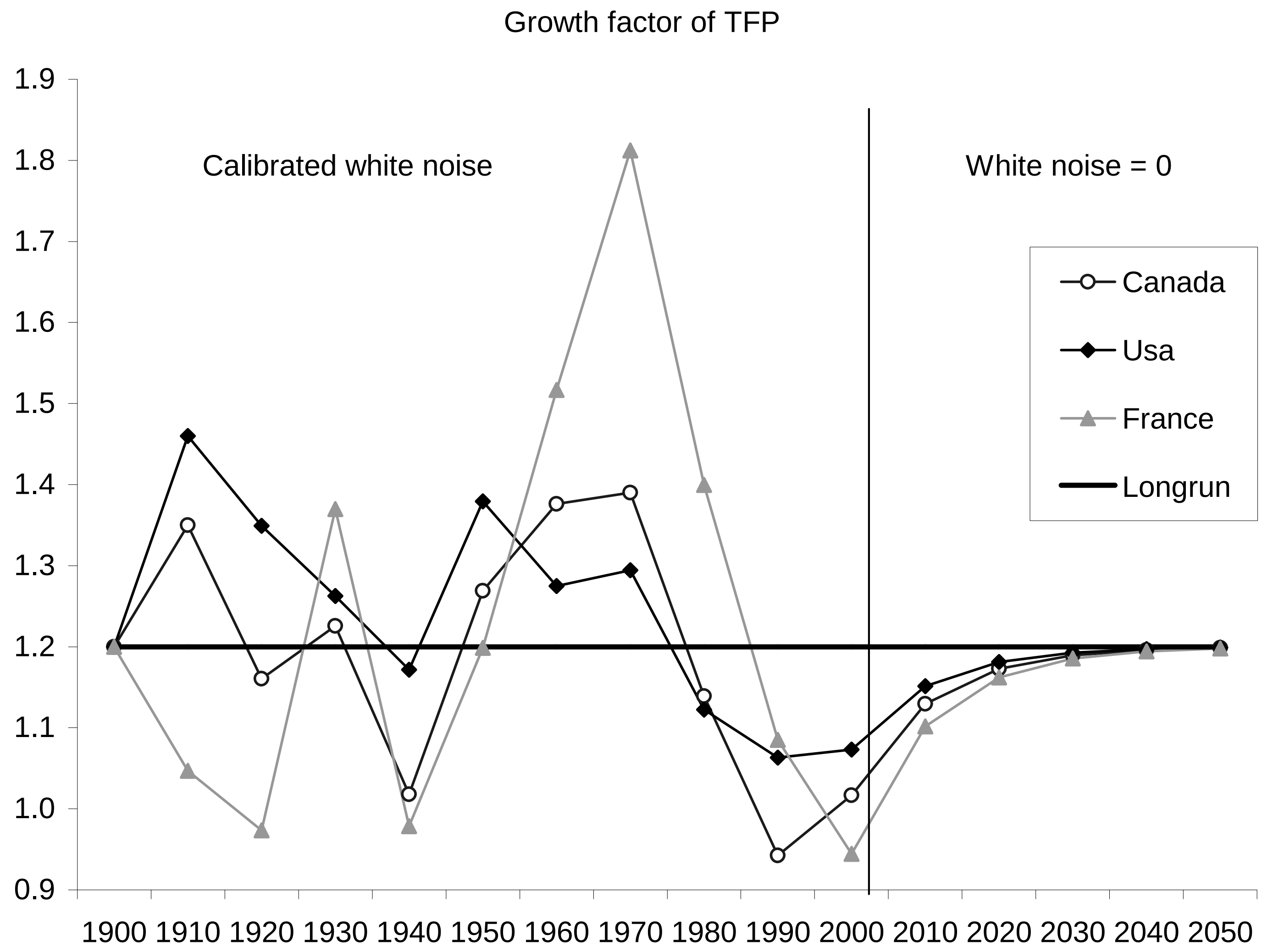
ϵ is set to 2.1 to deliver an adequate wage profile

$\psi = 0.6$ is in accordance with a return to an additional year of schooling of 11.5%

Identification of unobserved exogenous variables

Four unobserved exogenous variables:
– total factor productivity, A_t
– the rate of subsidy on education expenditures, v_t
– the level of pension benefit, b_t
– the scale of the age-specific transfer profile, g_t.

Chosen to match
– the GDP growth rate, –the share of social security in GDP
–the share of other transfers in GDP
–the education investment of young cohorts.


Growth factor of TFP
1.9
1.8
1.7
1.6
1.5
1.4
1.3
1.2
1.1
1.0
0.9
Calibrated white noise
White noise = 0
Canada
Usa
France
Longrun
1900 1910 1920 1930 1940 1950 1960 1970 1980 1990 2000 2010 2020 2030 2040 2050

Wage and assets profile per age

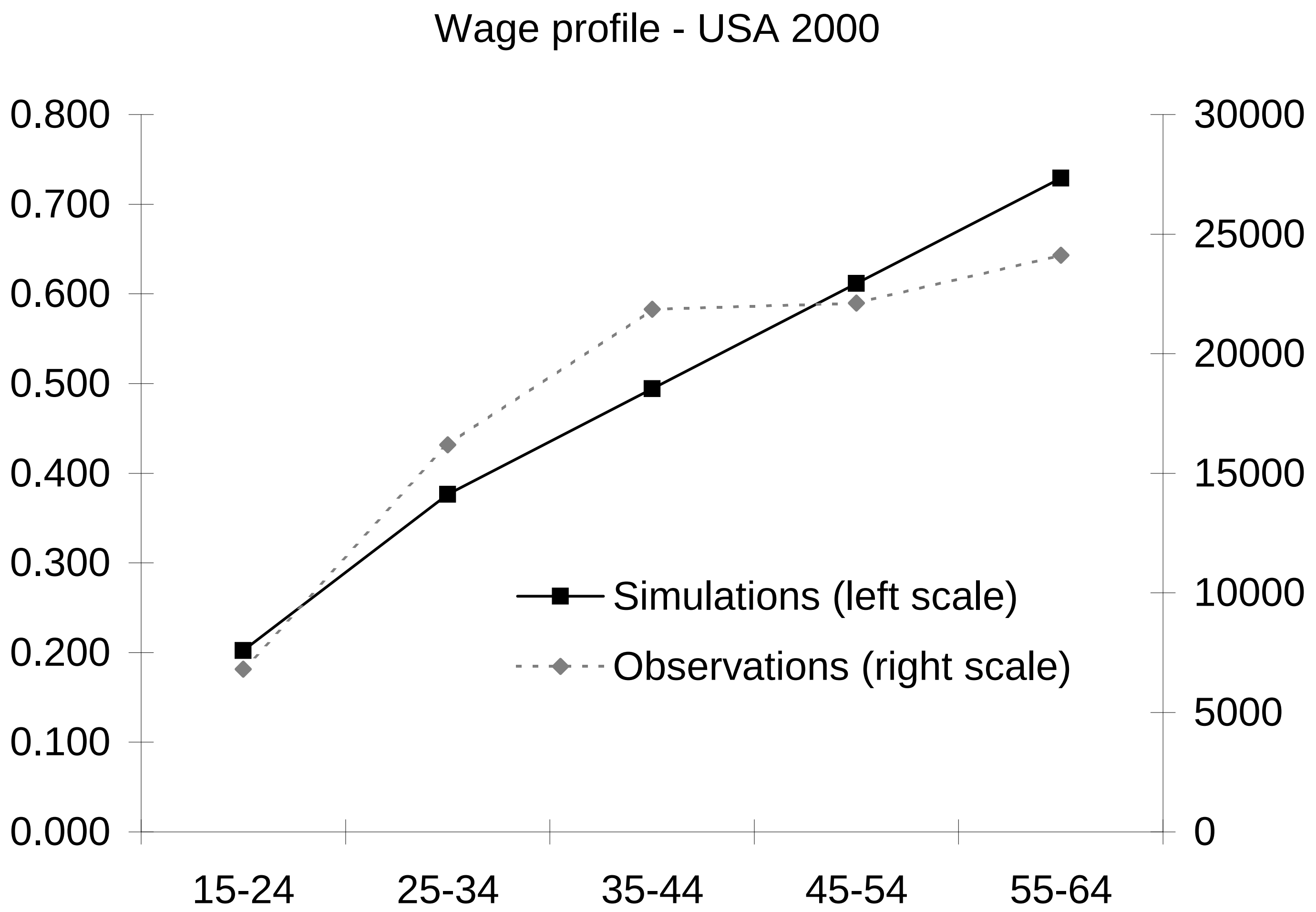
Important to have relatively correct wage and asset age-profiles.

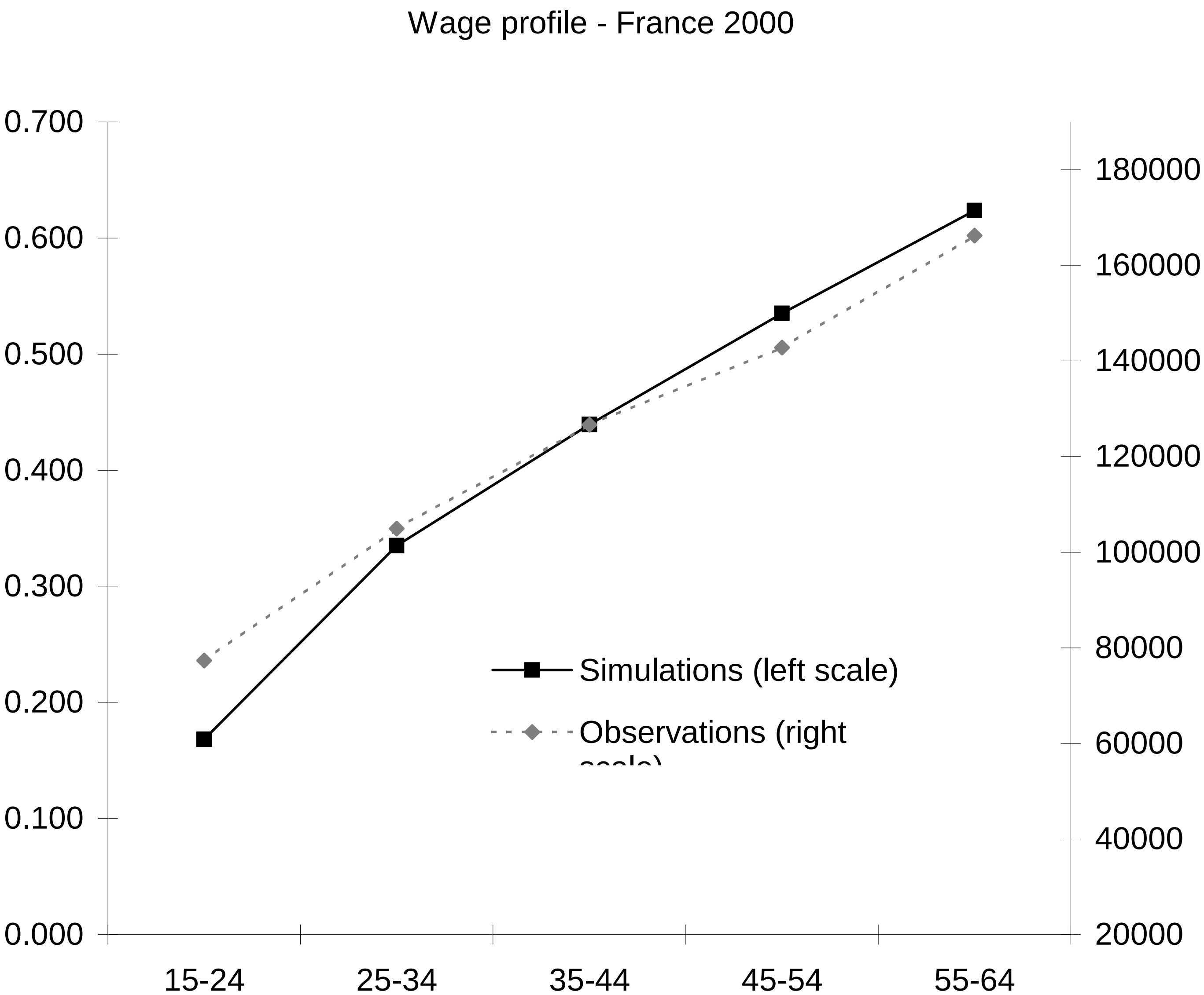
The shape of the wage profile per age is fully determined by the accumulation of experience; no exogenous trend.

Good match for France, OK for the US.

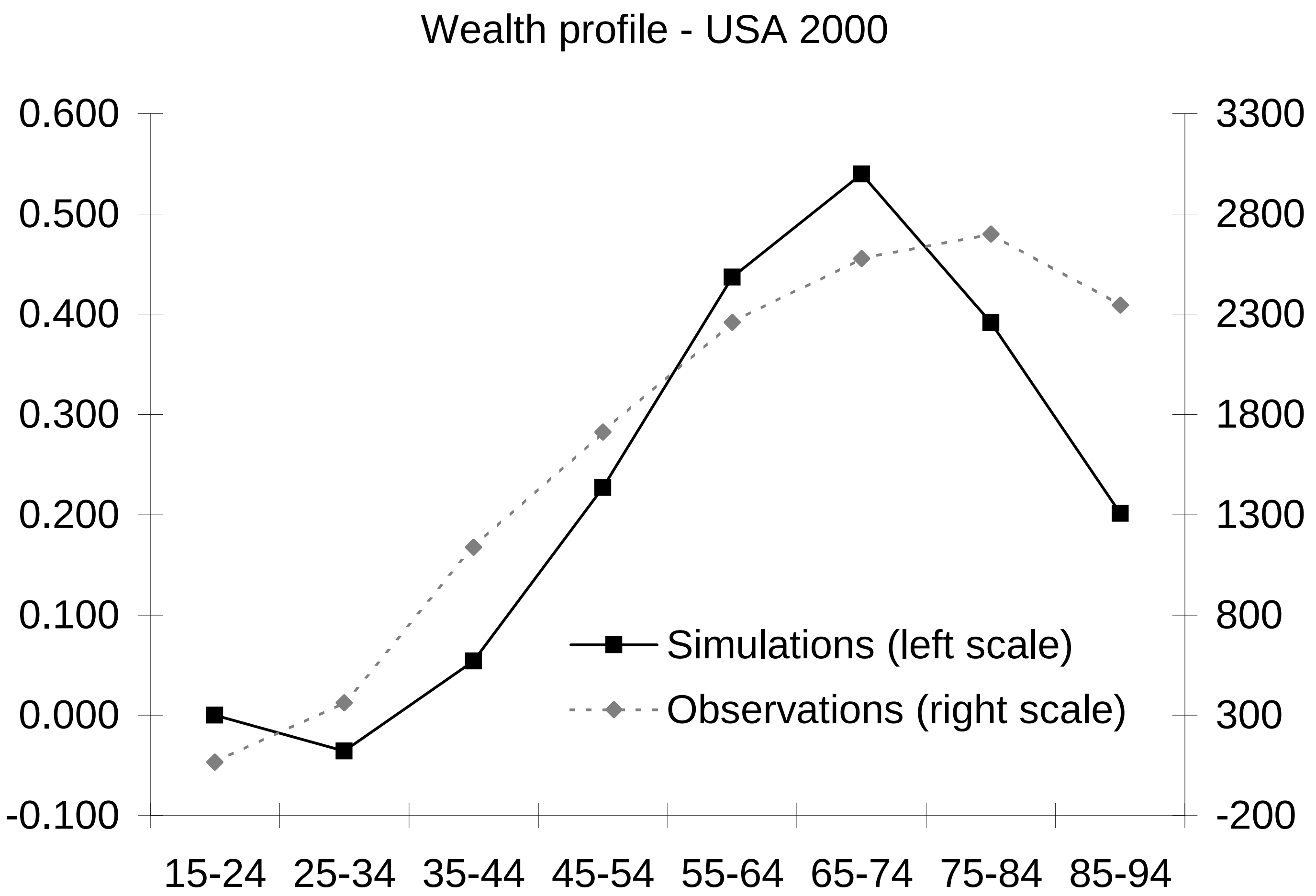
Good job also for the asset profile.

No need to suppose a pure time preference parameter on top of the mortality rate. The annuity market is also helpful to avoid poverty in the old age.

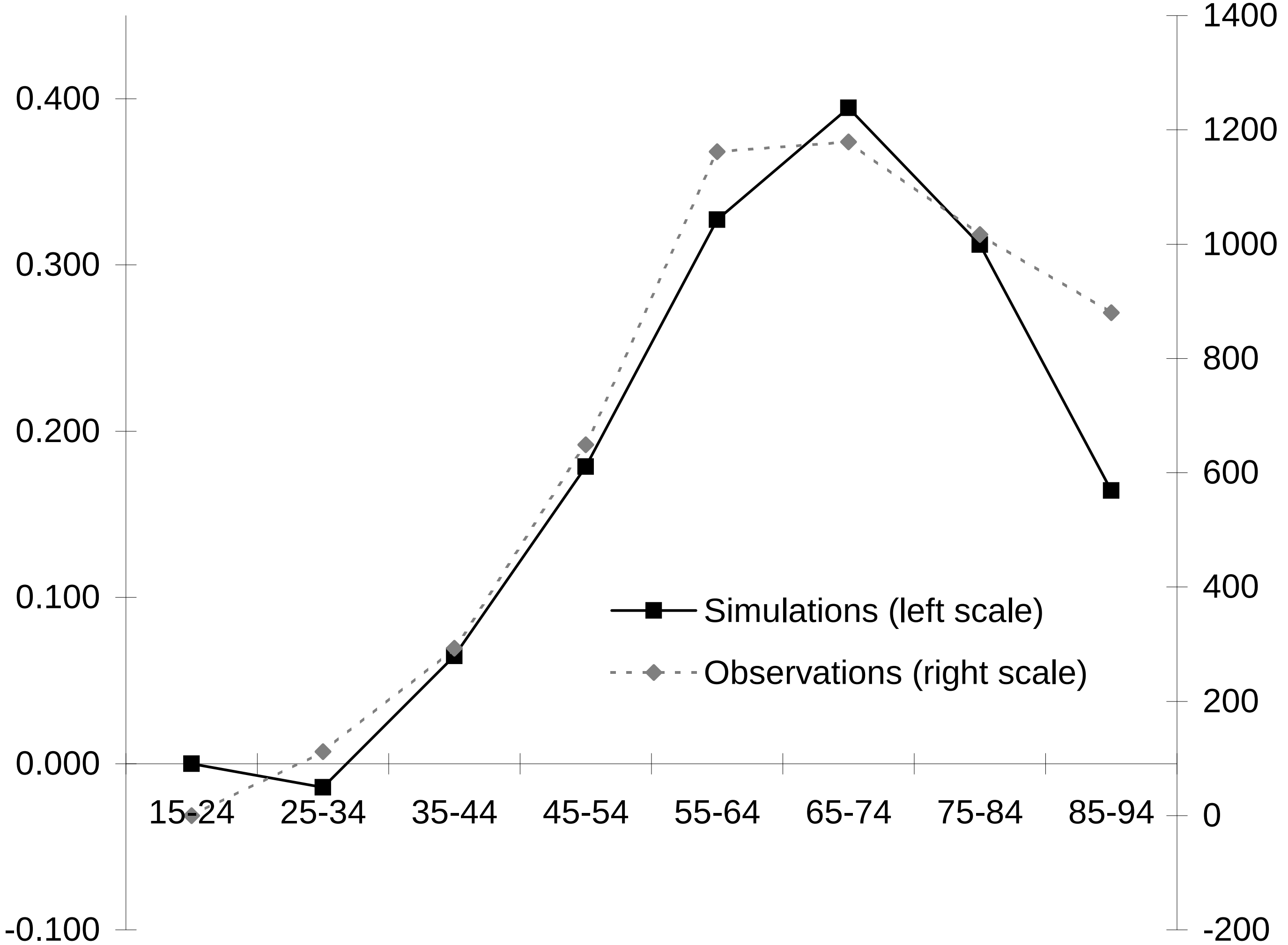

Wage profile - USA 2000
0.800
0.700
0.600
0.500
0.400
0.300
0.200
0.100
0.000
30000
25000
20000
15000
10000
5000
0
15-24
25-34
35-44
45-54
55-64
Simulations (left scale)
Observations (right scale)



Wage profile - France 2000
0.700
0.600
0.500
0.400
0.300
0.200
0.100
0.000
180000
160000
140000
120000
100000
80000
60000
40000
20000
15-24
25-34
35-44
45-54
55-64
Simulations (left scale)
Observations (right


Wealth profile - USA 2000
0.600
0.500
0.400
0.300
0.200
0.100
0.000
-0.100
3300
2800
2300
1800
1300
800
300
-200
15-24
25-34
35-44
45-54
55-64
65-74
75-84
85-94
Simulations (left scale)
Observations (right scale)

Asset profile - France 2000



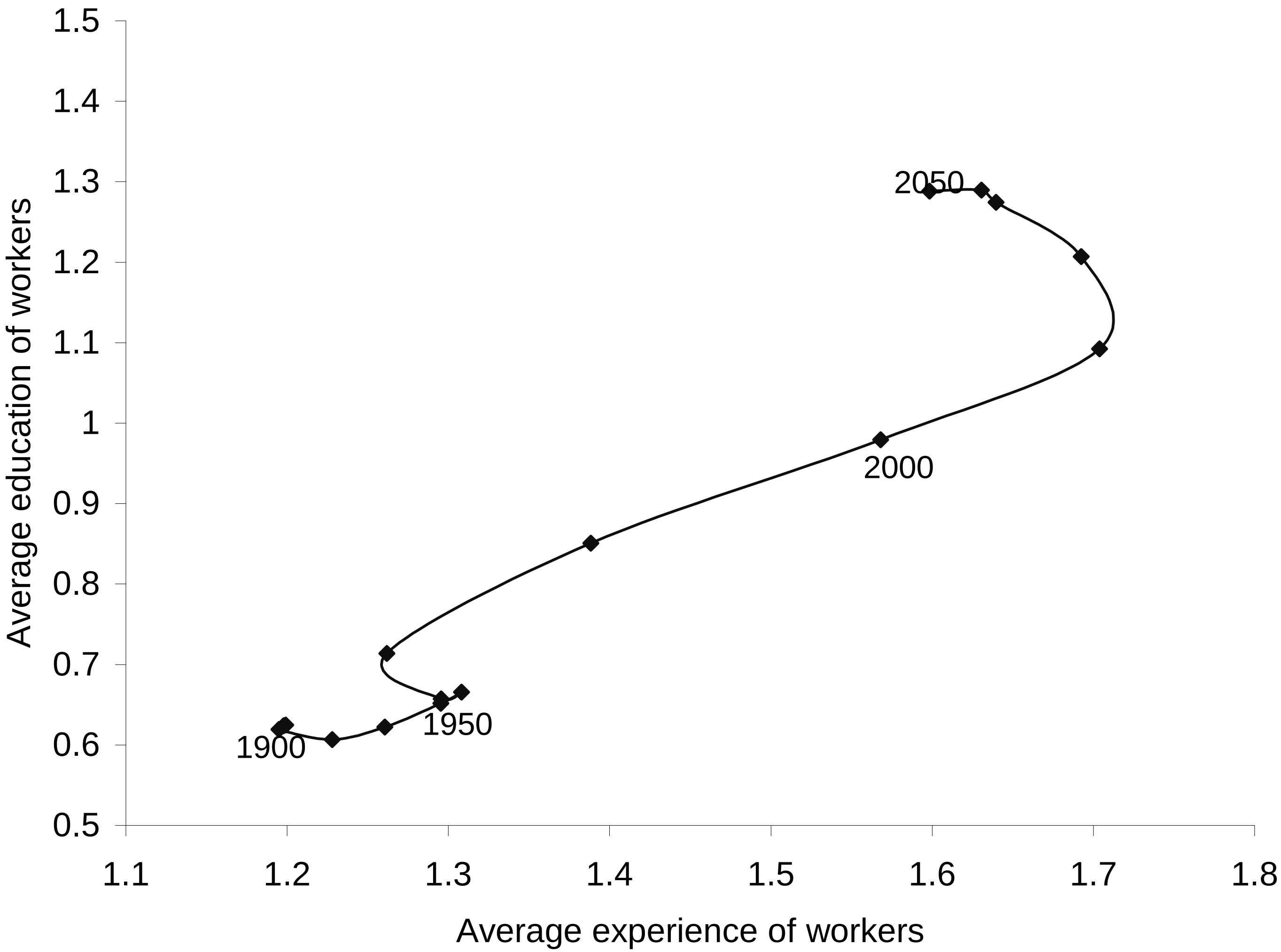
Baseline forecast

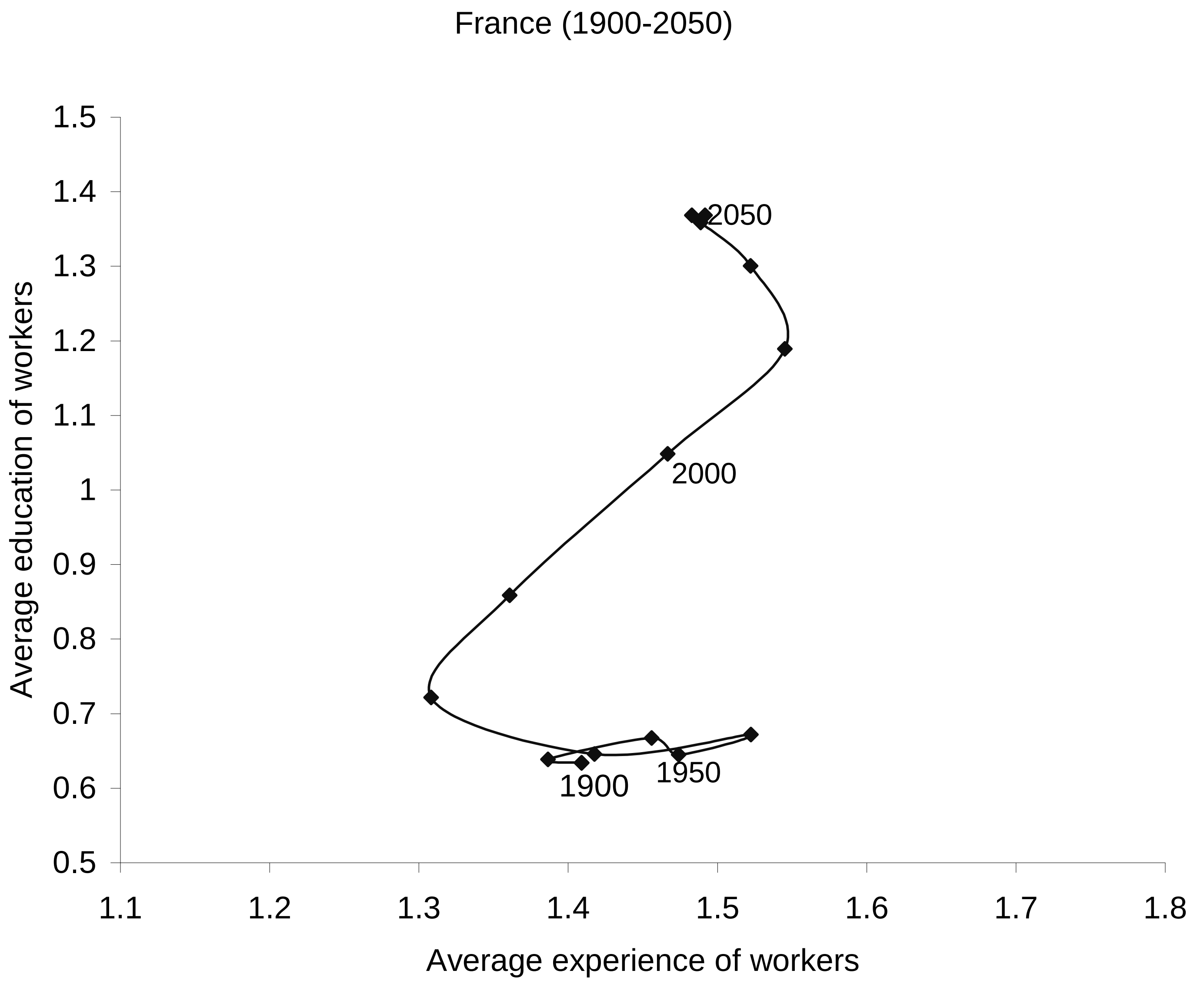
Future values of policy variables are kept at their 2000 level

Growth is driven by total factor productivity, the accumulation of physical capital and the dynamics of employment in efficiency units.

Employment in efficiency units: labor force, education, experience.

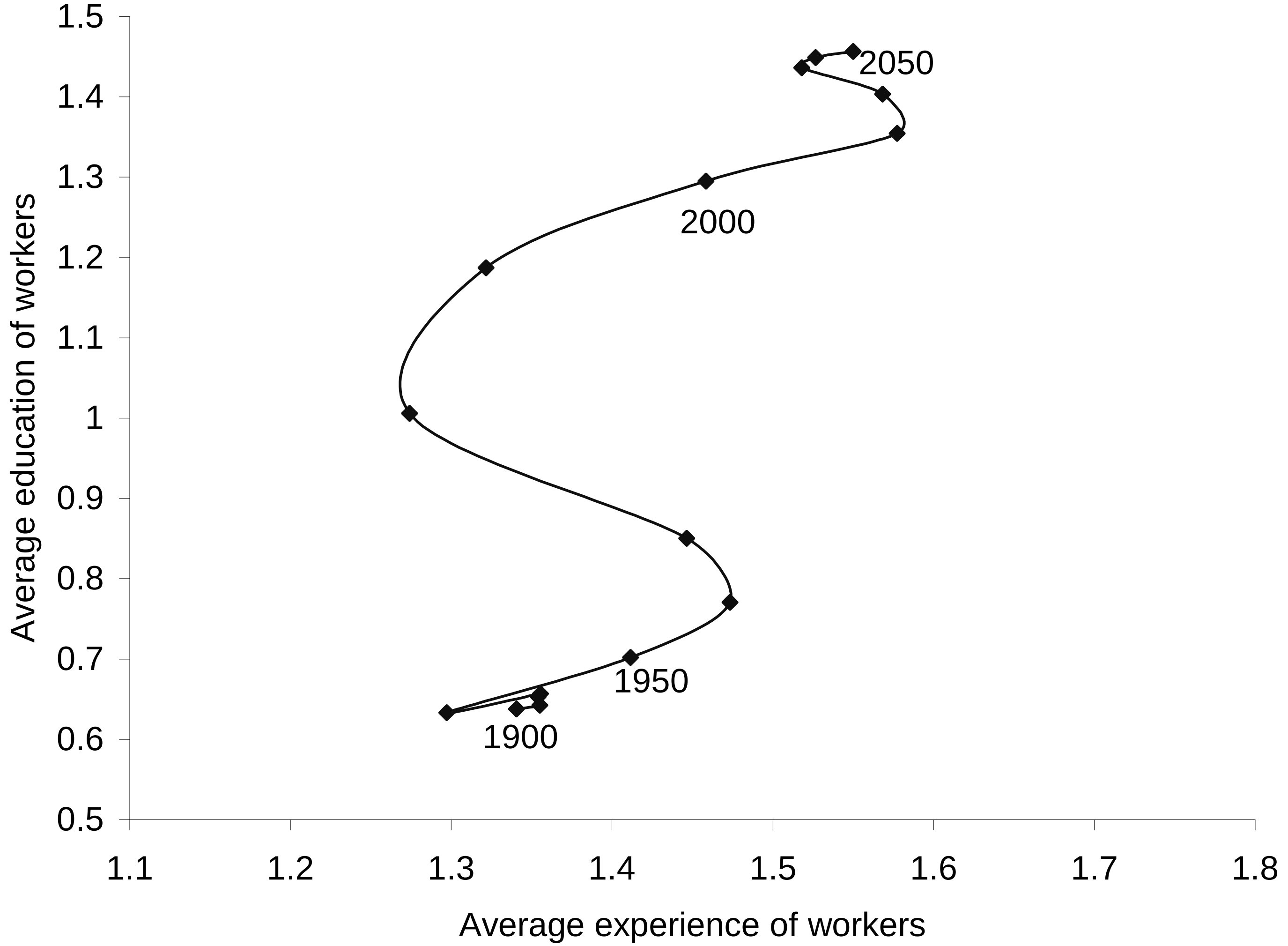
Canada (1900-2050)

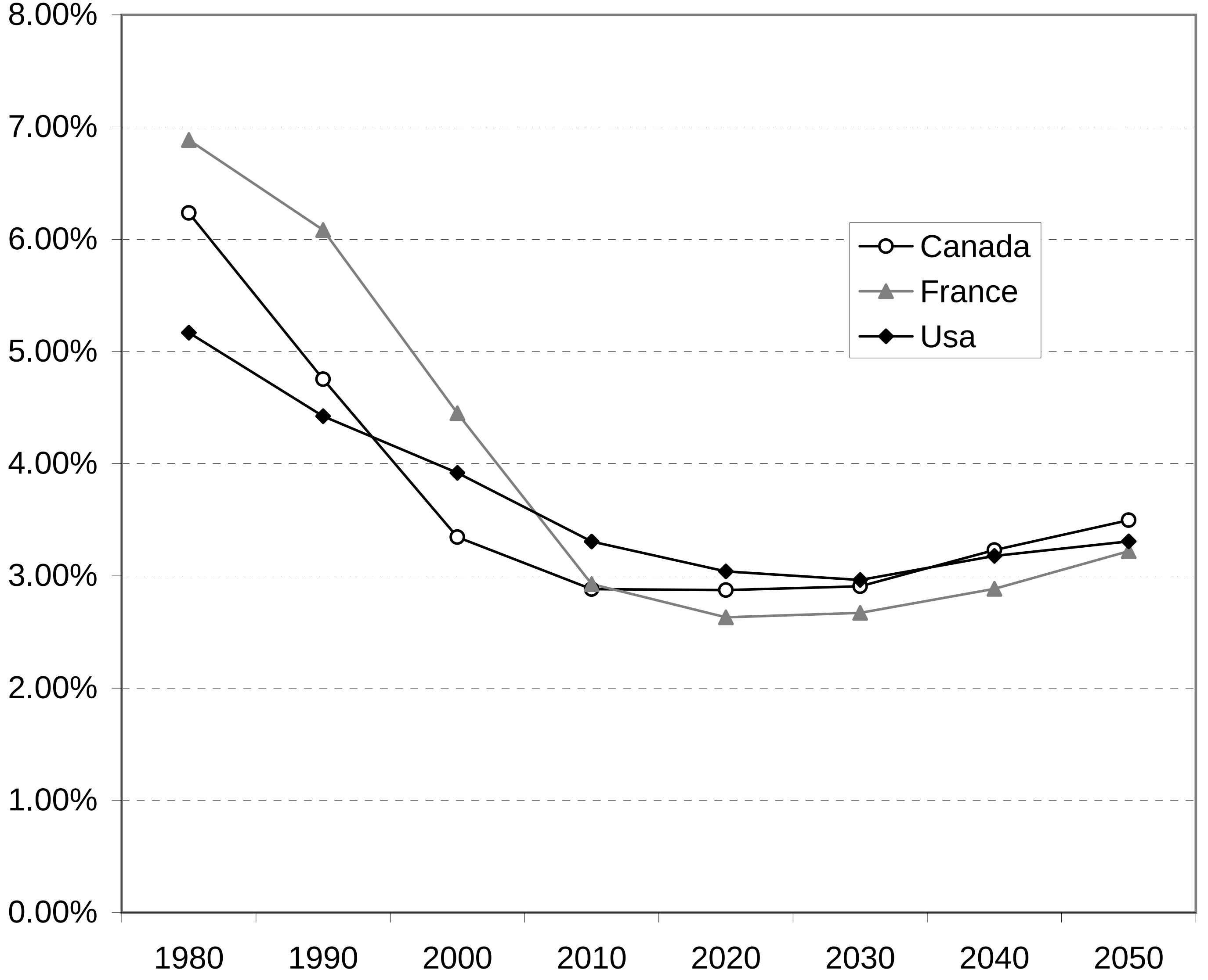


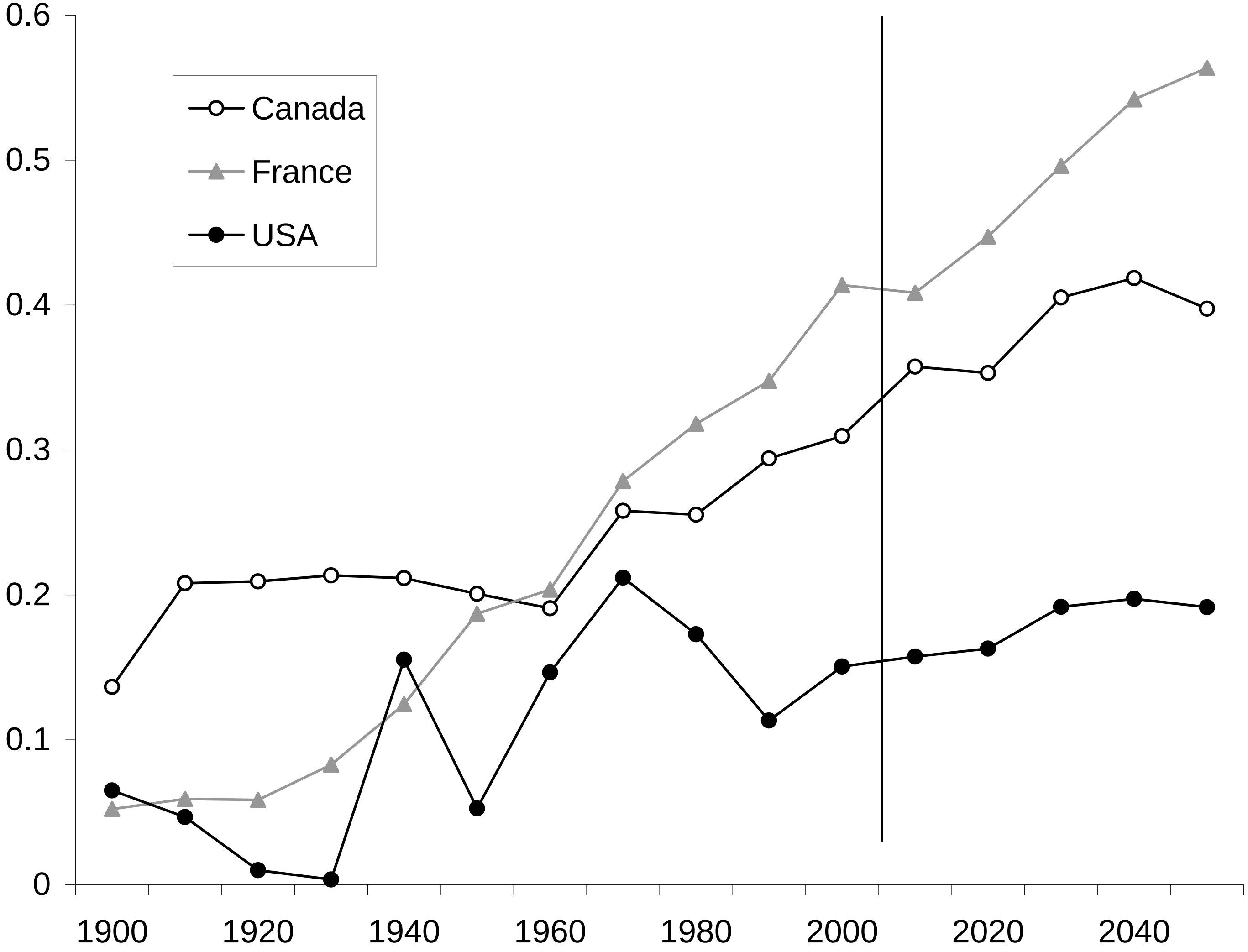


France (1900-2050)
1.5
1.4
1.3
1.2
1.1
1
0.9
0.8
0.7
0.6
0.5
Average education of workers
2050
2000
1950
1900
1.1
1.2
1.3
1.4
1.5
1.6
1.7
1.8
Average experience of workers

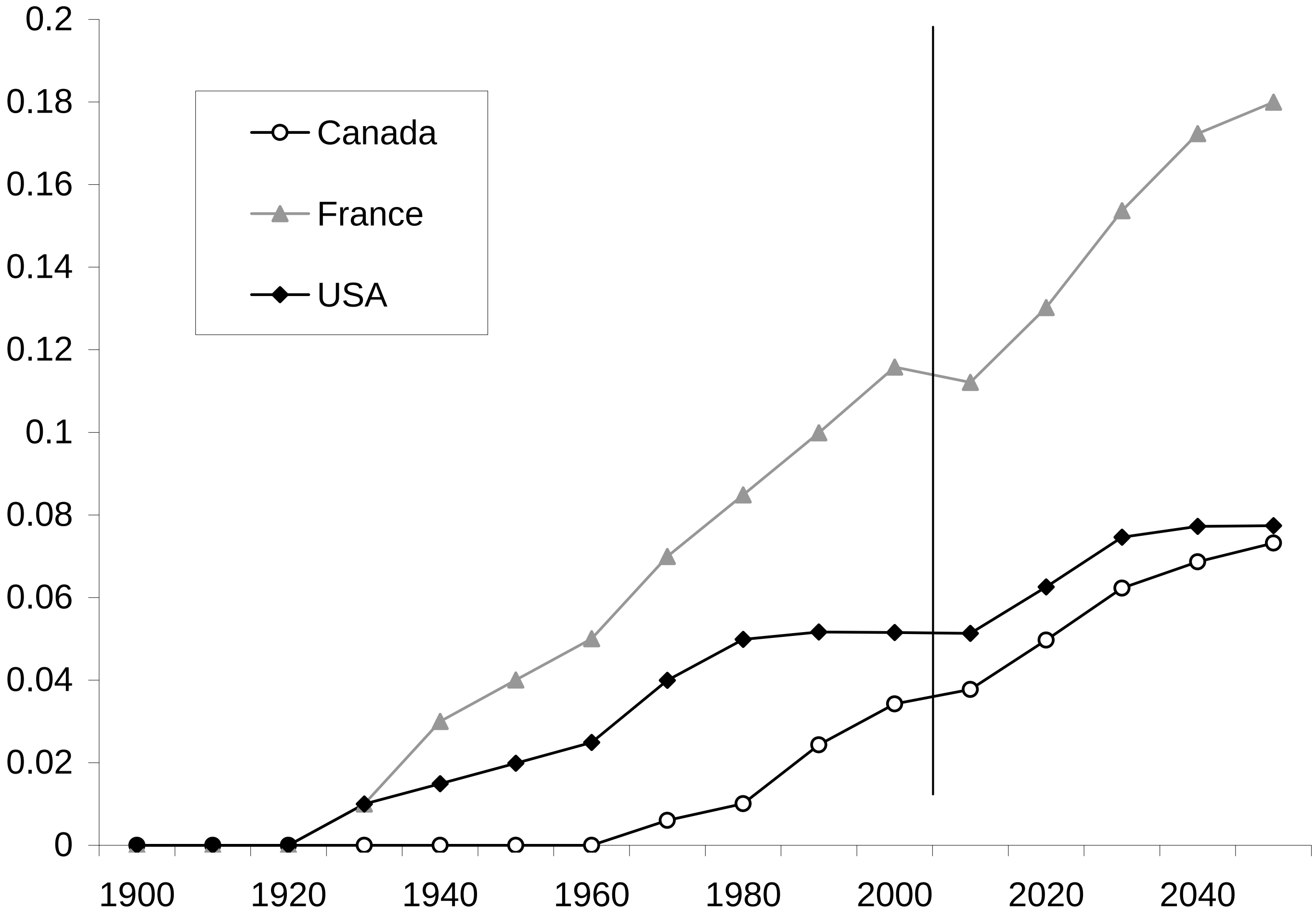
USA (1900-2050)




8.00%
7.00%
6.00%
5.00%
4.00%
3.00%
2.00%
1.00%
0.00%
1980
1990
2000
2010
2020
2030
2040
2050
Canada
France
Usa


0.6
0.5
0.4
0.3
0.2
0.1
0
Canada
France
USA
1900
1920
1940
1960
1980
2000
2020
2040

Public pensions in % of gdp
(for Canada, CPP only)



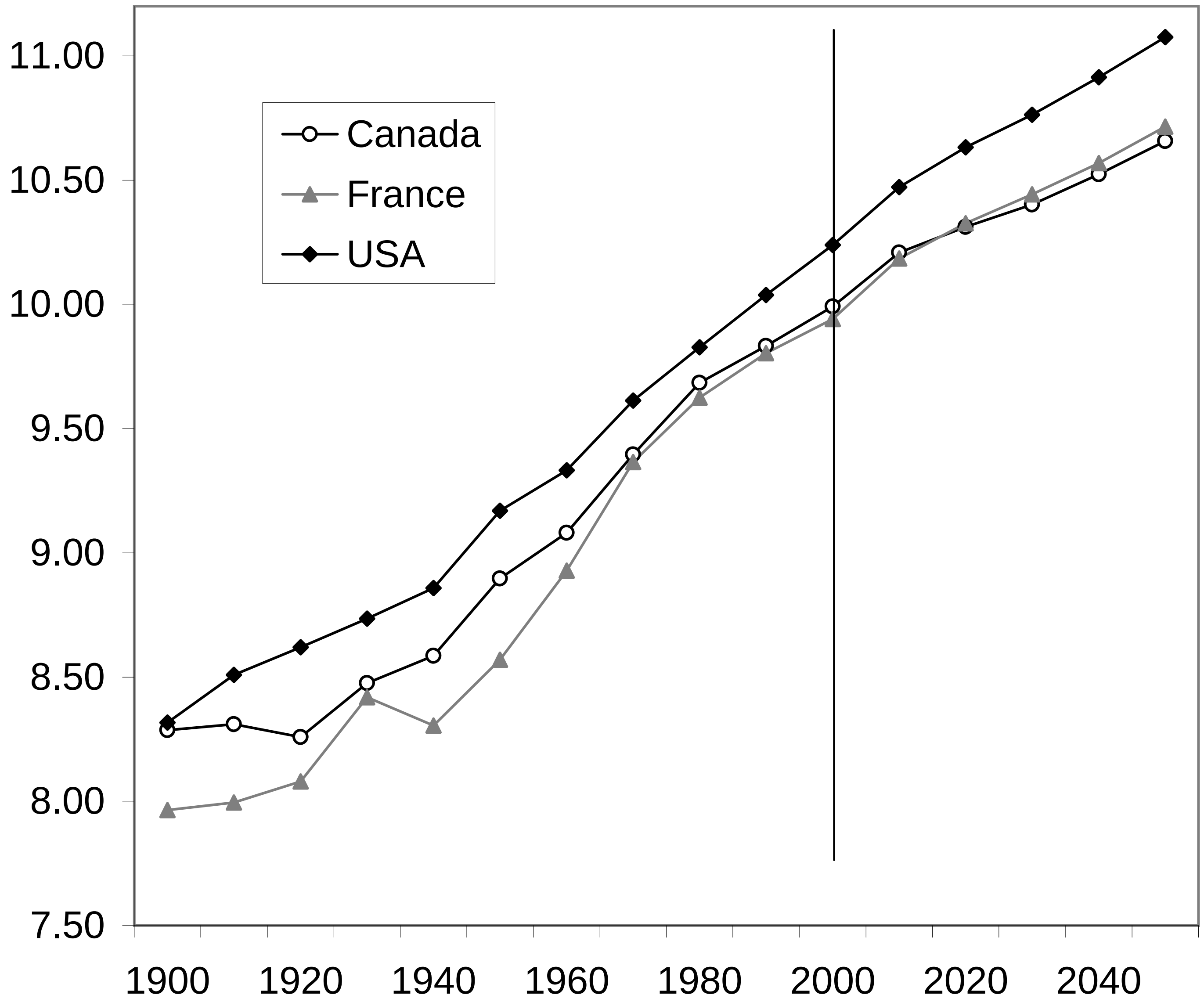
Forecasts of GDP per capita

forecasts of GDP per capita combines all the preceding elements

	Canada	France	USA
2000	21,843	20,769	27,954
2010	27,126	26,490	35,292
2020	30,091	30,539	41,466
2030	32,908	34,289	47,277
2040	37,175	38,899	54,944
2050	42,522	45,035	64,554

Population changes reinforce the leadership of the US.

France will catch-up and overtake Canada in 2020.


Canada
France
USA
11.00
10.50
10.00
9.50
9.00
8.50
8.00
7.50
1900
1920
1940
1960
1980
2000
2020
2040

Forecasts of growth

Demographic movements have a stimulating effect on growth between 2000 and 2010.

Thereafter, annual growth rates of GDP per capita are positive but become lower than the TFP growth after 2010.

The minimal growth rates are experienced between 2010 and 2030.

	Canada	France	USA
2000	1.61%	1.39%	2.02%
2010	2.19%	2.46%	2.36%
2020	1.04%	1.43%	1.63%
2030	0.90%	1.16%	1.32%
2040	1.23%	1.27%	1.51%
2050	1.35%	1.48%	1.63%

Alternative policy (1)

Gain in growth rates of rising the effective retirement age to 63 years

	Canada	France	USA
2000	-0.01%	0.00%	-0.01%
2010	0.20%	0.24%	0.10%
2020	0.35%	0.32%	0.04%
2030	0.06%	0.39%	0.00%
2040	0.04%	0.33%	0.00%
2050	0.05%	0.17%	0.01%

Alternative policy (2)

Gain in growth rates of adjusting pension benefits to keep income tax constant

	Canada	France	USA
2000	0.05%	0.04%	0.02%
2010	0.06%	0.05%	0.03%
2020	0.05%	0.07%	0.03%
2030	0.07%	0.09%	0.03%
2040	0.08%	0.11%	0.03%
2050	0.08%	0.13%	0.03%

Conclusion

Growth of GDP per capita will be positive but lower than total factor productivity growth over the period 2010-2040.

The gap between the leading country (the USA) and the two other countries (France and Canada) increases significantly.

France will catch-up and overtake Canada in 2020.

Convergence in retirement age would be very profitable for France.

A decrease in social security benefits would slightly stimulate growth but little impact on the gap between the countries.