Early Literacy Achievements, Population Density and the Transition to Modern Growth

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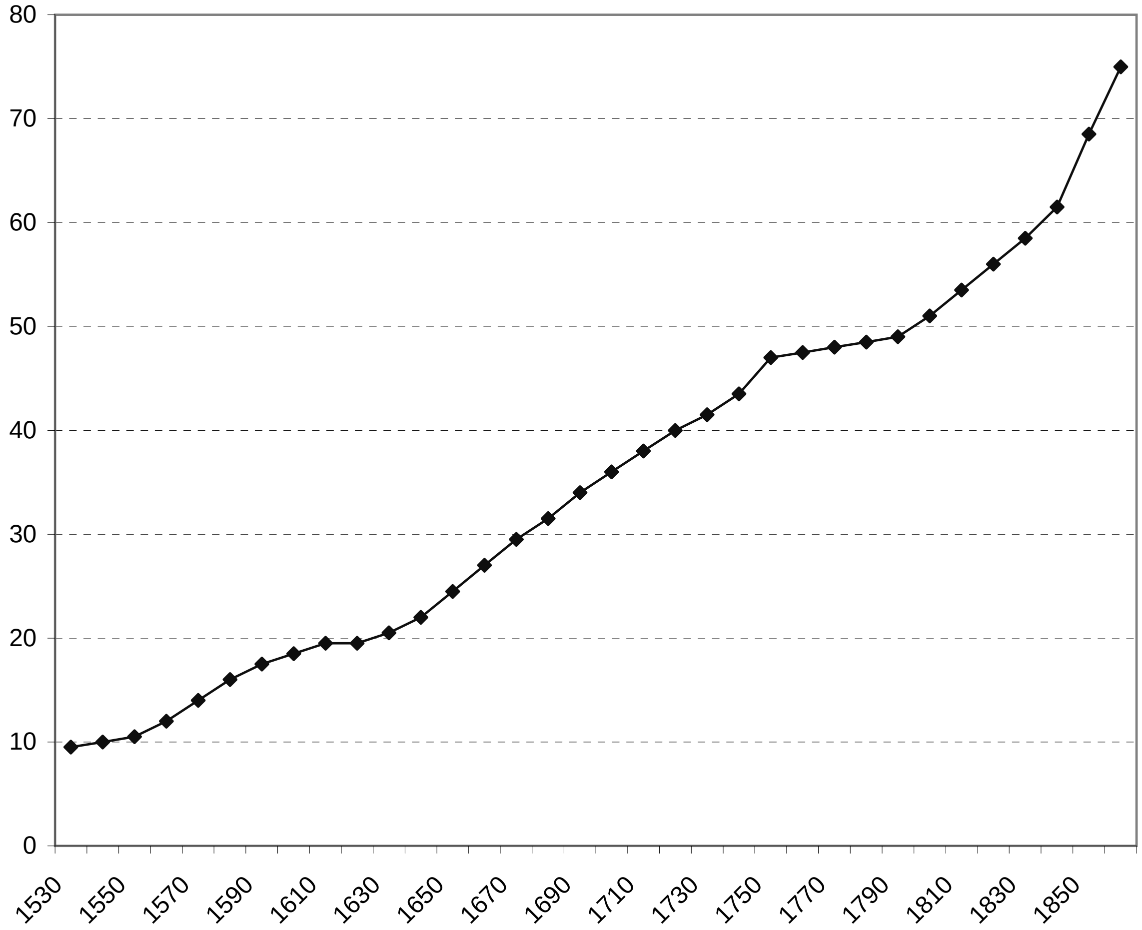
Improvements in literacy paved the way for the Industrial Revolution (human capital theory)
Education makes people more adaptable to new circumstances and receptive to new ideas
Why did people start to invest in education?

In England, improvements in literacy started as early as in the sixteenth century.

Literacy achievements (% population) [Estimation: Cressy (1980).]



Additional evidence: (A) Literacy surveys.

In England, half of the schoolage children received education (18th cent.). Between a fifth and a third on average in Europe. (Houston, 2002)

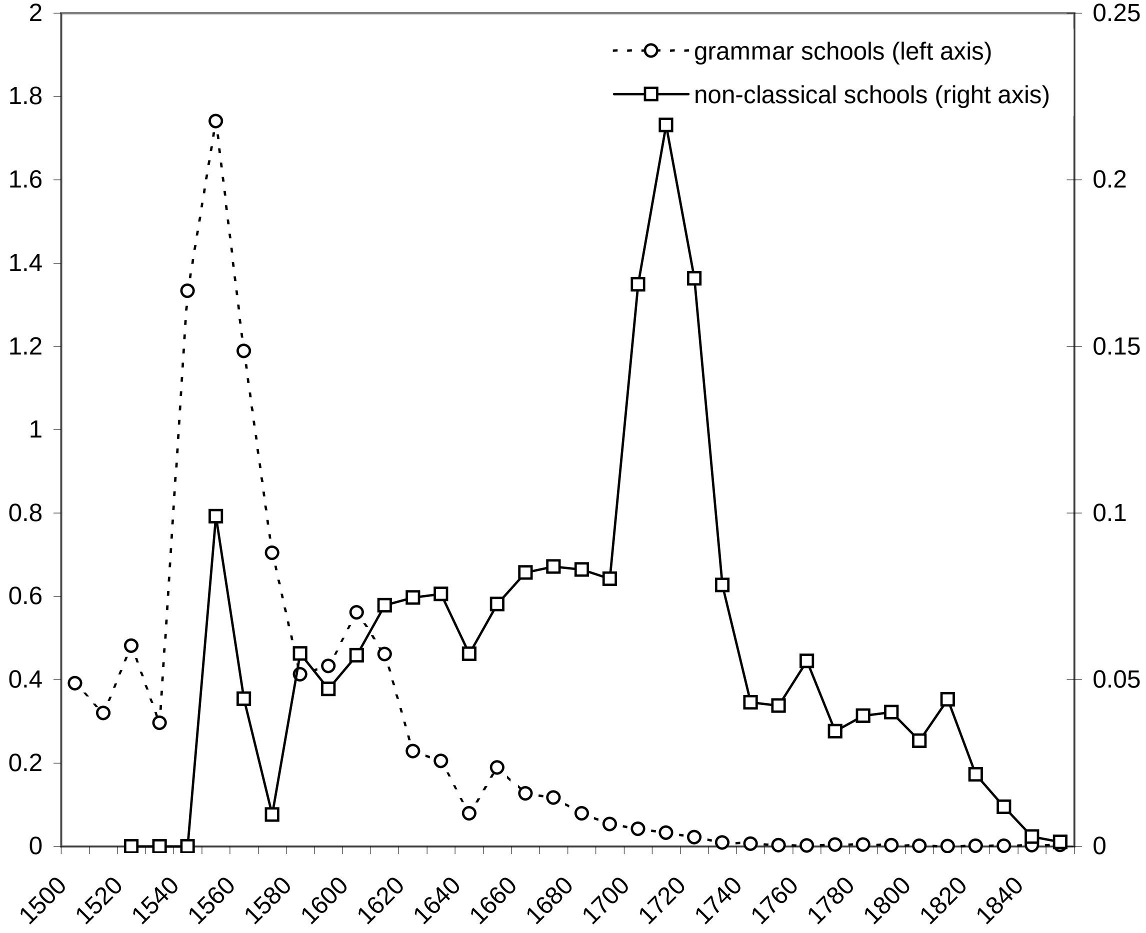
A key determinant of the English success: accessibility of schools (O Day, 1982) Small share of rural population was geographically distant to schools

(B) School foundations data we built from the School Inquiry Commission 1868.

High creation rate of Grammar schools over 1540-1620.

Creation of non-classical schools after 1700.

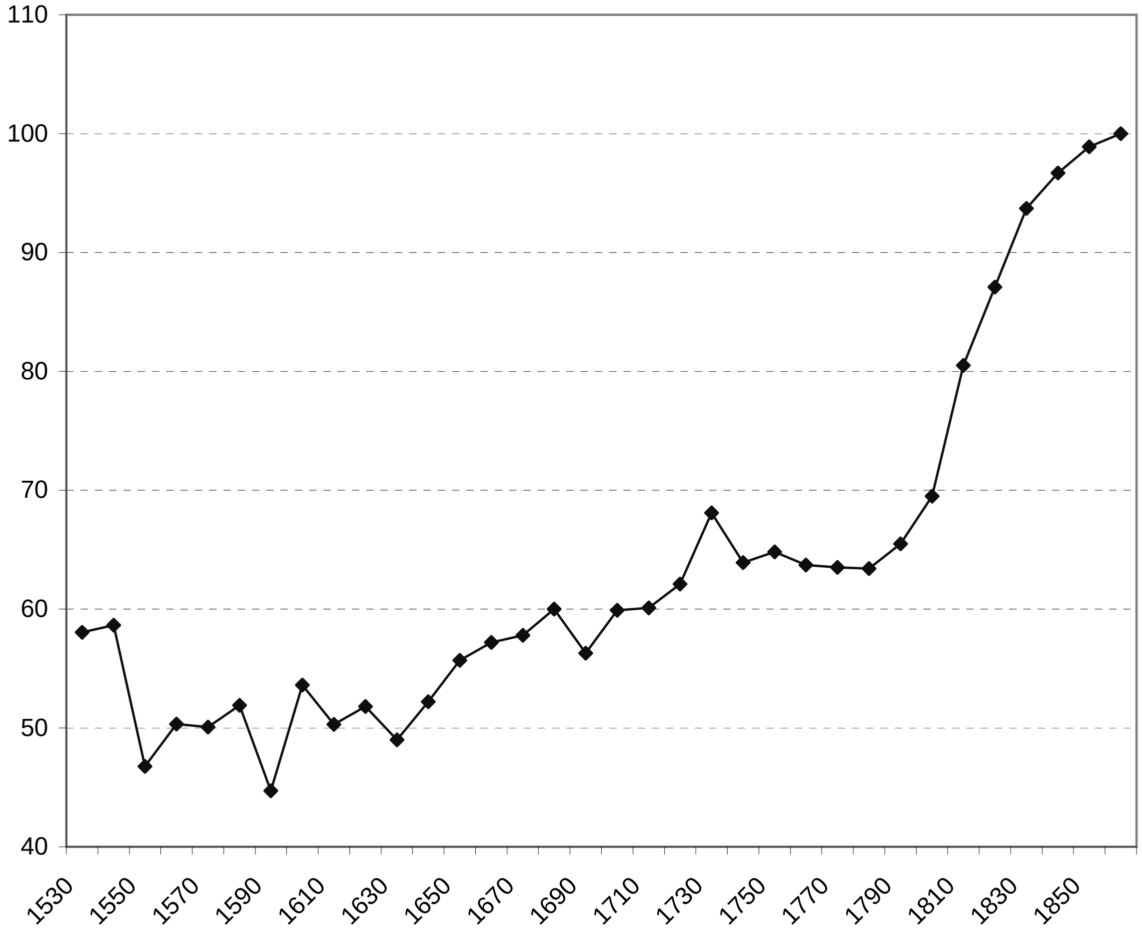
Creation rates of schools (own estimation)



Explanatory factors ?

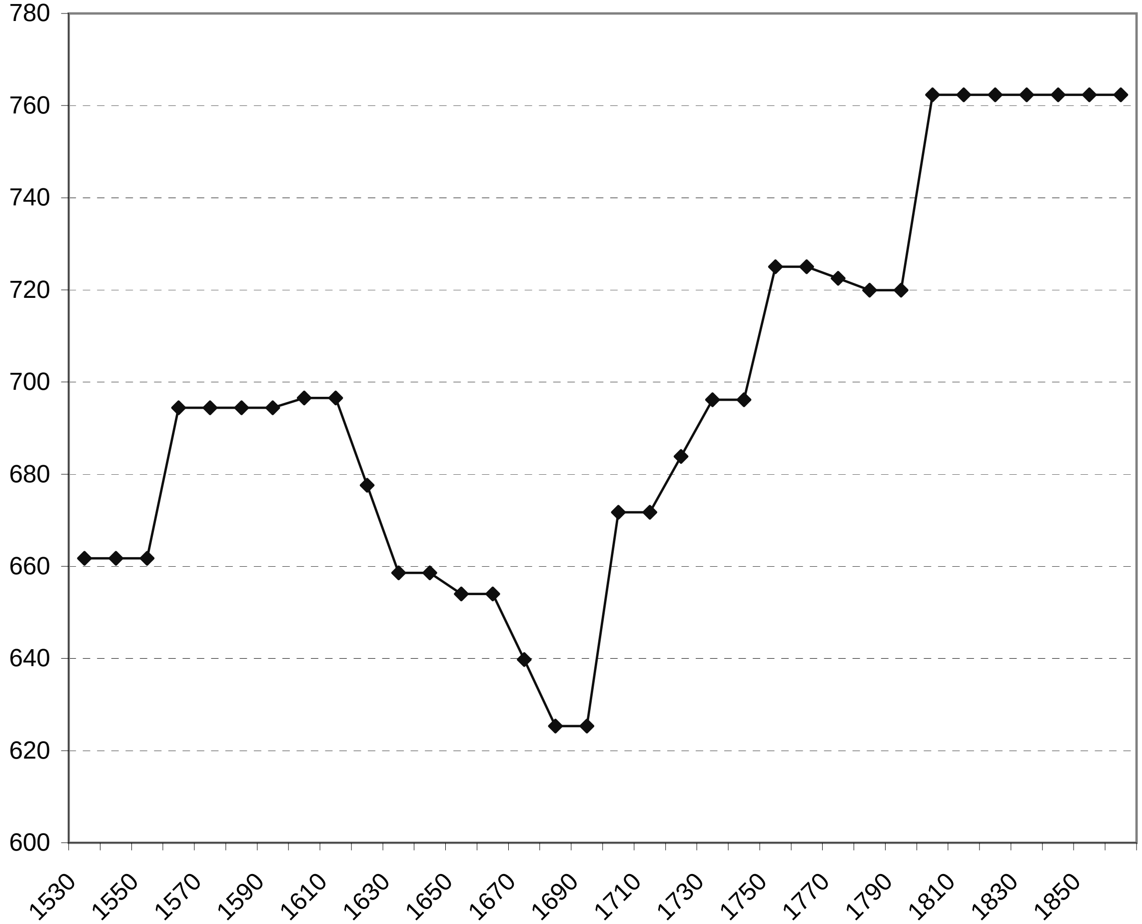
Technical progress in the modern sector (Hansen/Prescott 2002)
$\rightarrow$ return to investment in education increased (Doepke 2004)
but timing is wrong: Little productivity gains before 1700

Total factor productivity [Estimation: Clark 2001]



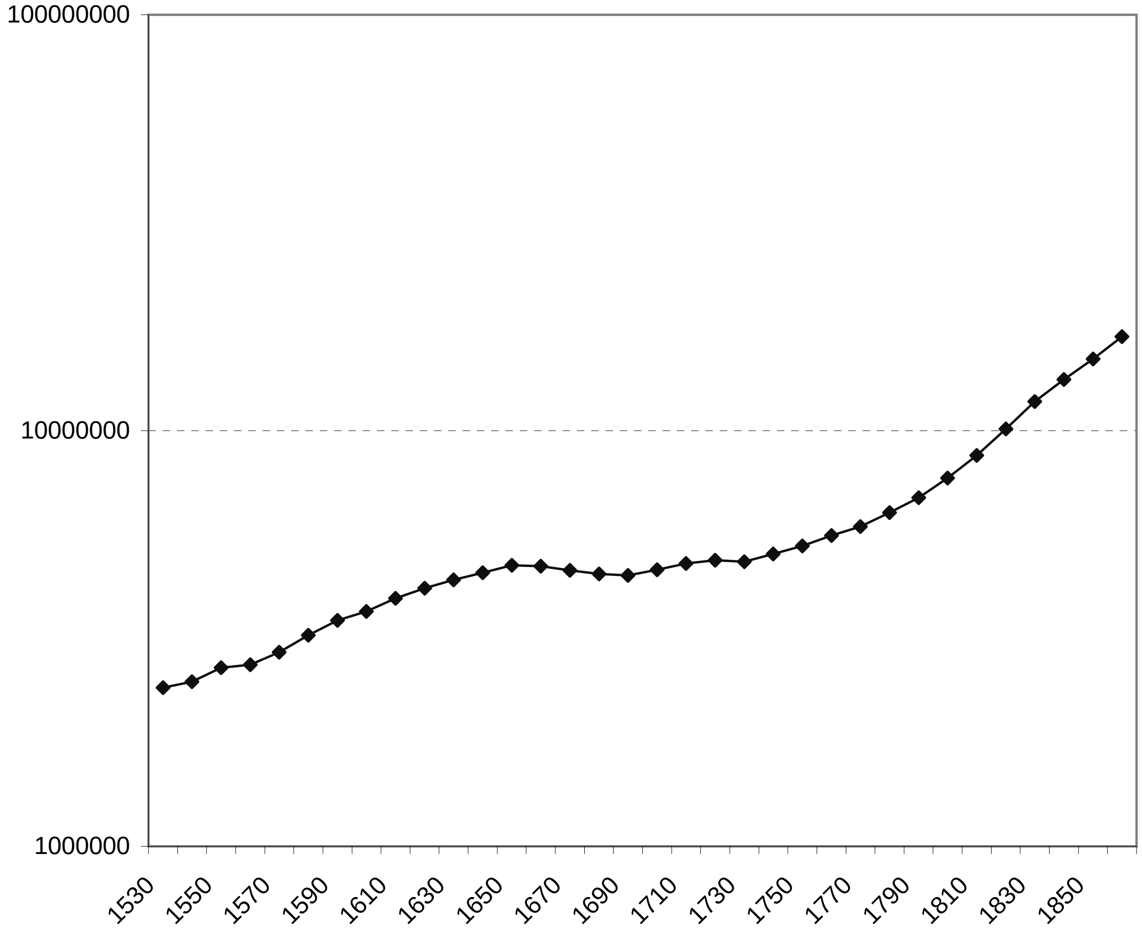
Longevity improvements increased the return to education.
Problem with England: longevity was actually stagnant over the period 1500 to 1700 or even declining after 1600
($\neq$ Europe because faster urbanization in England)

Mortality: number of survivors at age 40 from 1000 individuals at age 5 [Source: Wrigley et al. (1997)]

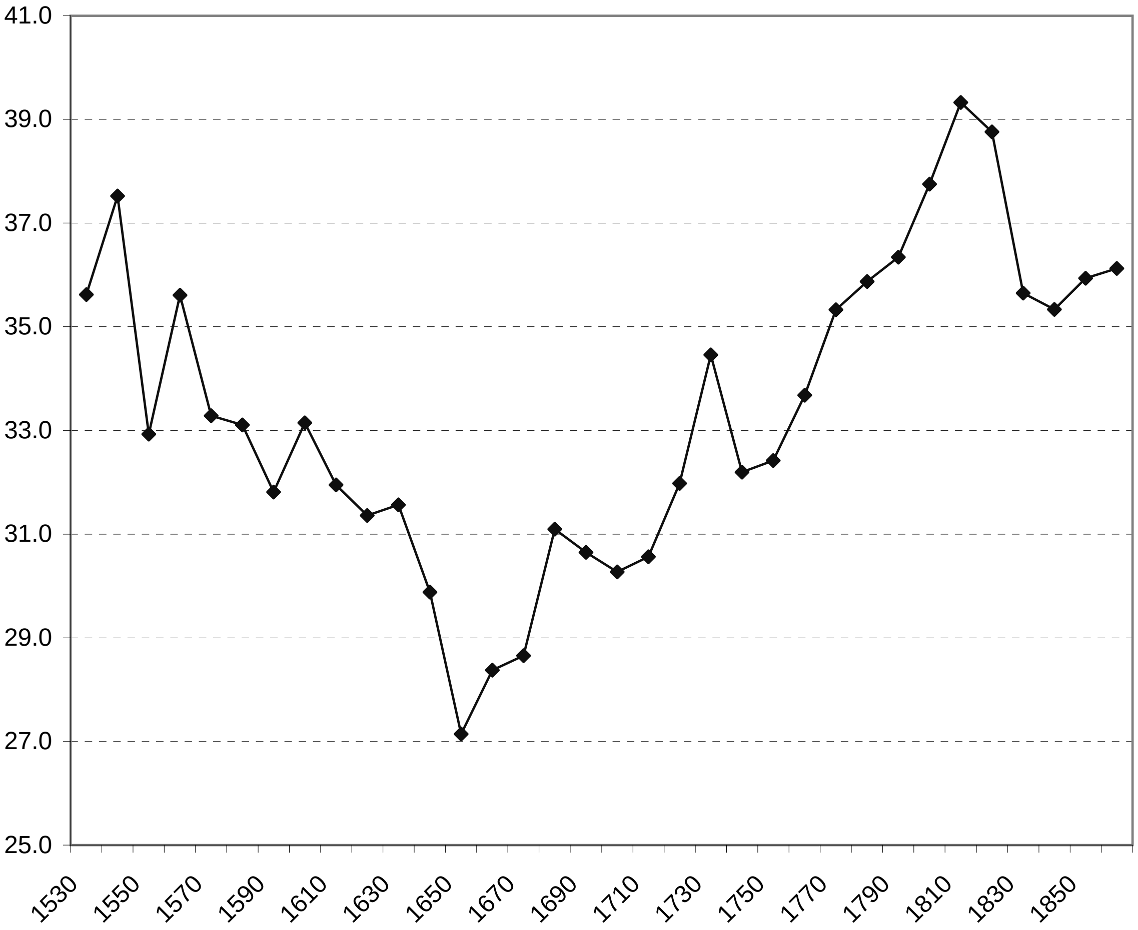


Higher *density of population* stimulated the return to education. Becker et al. (1999): larger populations encourage greater specialization and increased investments in knowledge. Galor and Weil (2000): "population-induced" technical progress which raised the return to human capital.

Population of England, age 5+ [Estimation: Wrigley et al. (1997)]



Crude birth rate [Estimation: Wrigley et al. (1997)]



Model to evaluate the role of the three factors:

growth theory (human capital, inter-temporal optimization)

$+$

geography (space dimension, choice of location of facilities)

In Galor and Weil's paper the effect of population on productivity is assumed instead of being derived from primary assumptions.

We want to derive the effect of population on productivity from some maximizing behavior...

... through the optimal choice of the number and location of education facilities.

Higher population density $\rightarrow$ more schools, $\rightarrow$ higher educational levels.

Time: continuous. At each point in time a new generation of size ζ_t is born.

Individuals have different innate abilities, μ, and location, i.

Space: circle of unit length. Each new generation is uniformly spread over the circle. Same technology set available everywhere.

$x(i)$: distance between the individual at i and the closest school.

Demographics: Concave survival function

$$m_t(a) = \frac{e^{\beta_t\, a} - \alpha_t}{1 - \alpha_t}, \qquad \alpha_t > 1, \beta_t > 0 \tag{1}$$

Maximum age $\mathsf{L}_t = \log(\alpha_t)/\beta_t$.

Technology

Material good, produced through two different technologies.

In the "modern sector", the technology employs human capital H_t with constant returns:

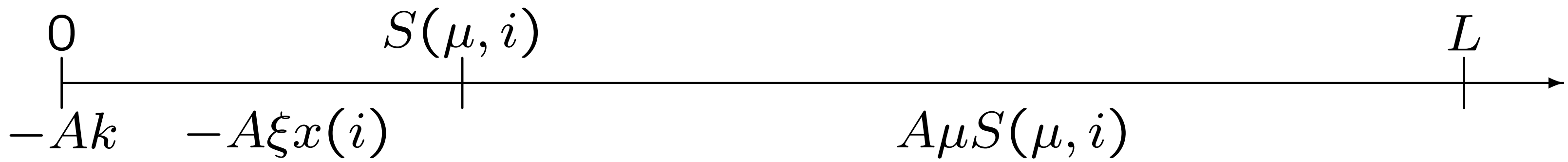
$$Y_t = A_t\, H_t \quad \text{where} \quad A_t = e^{\gamma_t t}. \tag{2}$$

In the "traditional sector", individuals have a productivity w^h per unit of time, independent of their level of human capital.

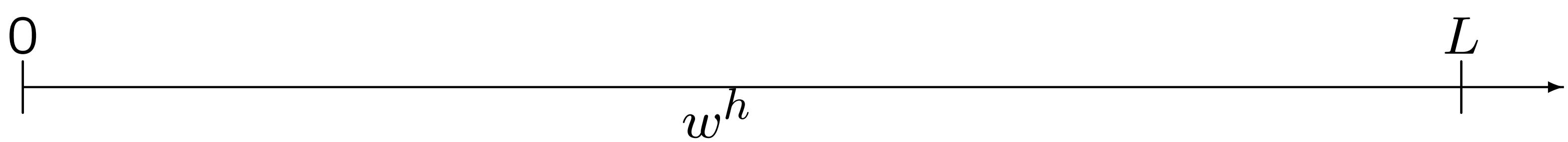
If $\gamma_t > 0$ the modern sector becomes more attractive (Hansen Prescott 2000)

Individuals:

Educated households:



Home production:



Individuals (μ, i) born at time t

Maximization of lifetime resources W:

$$W[\mathsf{S}] = \int_{t+\mathsf{S}_t(\mu,i)}^{t+\mathsf{L}_t} \omega_t(\mu, i, z)\, m_t(z-t) e^{-\theta(z-t)} \mathrm{d}z$$
$$- \int_{t}^{t+\mathsf{S}_t(\mu,i)} \xi\, x(i)\, e^{\gamma_z z}\, m_t(z-t) e^{-\theta(z-t)}\, \mathrm{d}z - k\, e^{\gamma_t t}\, \delta[\mathsf{S}_t(\mu, i)], \quad (3)$$

k is a fixed cost to be paid only if the individual decides to go to school:

$$\delta[\mathsf{S}_t(\mu, i)] = 1 \text{ if } \mathsf{S}_t(\mu, i) > 0, \text{ and } = 0 \text{ otherwise.}$$

Spot wage:

$$\omega_t(\mu, i, z) = h_t(\mu, i) A_z,$$

Education technology:

$$h_t(\mu, i) = \mu \, \mathsf{S}_t(\mu, i). \tag{4}$$

For education to be an optimal outcome:

$$W[\mathsf{S}] > \int_t^{t+\mathsf{L}_t} w^h m_t(z-t) e^{-\theta(z-t)} \mathsf{d}z \equiv W^h. \tag{5}$$

School location

At each date a number of classrooms is created to serve the newborn generation.

From the School Enquiry Commission (1865), three facts:
–all schools were independent but subject to rules from above
–in endowed schools the founders were obedient to a superior authority
– profit was a motivation for many schools (private)

We consider four different models of school creation

Baseline (M1): A central authority determines the optimal number of classrooms to maximize profits. Tuition fee k is exogenous. Attendance rate for each school: $R(E_t, k)$

$$\max_{E_t} A_t \left(k\zeta_t R(E_t, k) - f\right) E_t \tag{6}$$

(M2) tuition fee is endogenous: $\max_{E_t, k_t} A_t \left(k_t\zeta_t R(E_t, k_t) - f\right) E_t$

(M3) free entry process instead of central authority

$$E_t \text{ such that } k\zeta_t R(E_t, k) - f = 0$$

(M4) free entry + each school determines its tuition fee

$$E_t \text{ solves } k_t\zeta_t R(E_t, k_t) - f = 0 \text{ for a given } k_t$$

$$k_t \text{ is the solution of } \max_{k_t} k_t\zeta_t R(E_t, k_t) - f \text{ for a given } E_t.$$

Equilibrium

Given exogenous demographic and technological trends α_t, β_t, γ_t and ζ_t, an equilibrium consists of

– A path of optimal education decision $\{\mathsf{S}_t(\mu, i)\}_{t \geqslant 0}$ maximizing life-time resources;

– A path of optimal number of schools $\{E_t\}_{t \geqslant 0}$ and tuition fee $\{k_t\}_{t \geqslant 0}$ following M1, M2, M3, or M4.

Resolution:

– the individual problem

– the school creation problem

Solution to the individual's choice (for $\theta = 0$)

Existence and uniqueness of the interior solution

The first-order necessary condition is:

$$\mu \int_{t+\mathsf{S}_t(\mu,i)}^{t+\mathsf{L}_t} A_z\, m_t(z-t)\mathrm{d}z = m_t(\mathsf{S}_t(\mu,i))\; A_{t+\mathsf{S}_t(\mu,i)}\; (\mu\, \mathsf{S}_t(\mu,i) + \xi\, x(i))\,. \tag{7}$$

Under a steady technological progress:

$$\mu \int_{\mathsf{S}(\mu,i)}^{\mathsf{L}} e^{\gamma a}\, m(a)\mathrm{d}a = (\mu\, \mathsf{S}(\mu,i) + \xi x(i))\;\, e^{\gamma \mathsf{S}(\mu,i)}\, m(\mathsf{S}(\mu,i)). \tag{8}$$

Proposition 1 *For γ small enough, there exists a solution to (8) such that $0 < \mathsf{S}(\mu, i) < \mathsf{L}$ if and only if $\mu > \underline{\mu}(i)$. The solution is unique. This solution tends to zero as μ gets closer to $\underline{\mu}_i$.*

Corollary 1 *The threshold $\underline{\mu}_i$ is an increasing function of ξ, $x(i)$ and β. It is decreasing in α and γ.*

The interior solution may not exist under huge transport costs and distances to schools, or under a poorly efficient education sector.

For fixed ξ, μ and $x(i)$, this solution neither exists if the demographic parameters induce markedly low life expectancy and maximal age figures.

Comparative statics for schooling:

Proposition 2 *Under the conditions of Proposition 1, the interior solution* S *is a strictly increasing function of* γ *and* α*, and a strictly decreasing function of* β *and* $x(i)$*.*
For $x(i) > 0$*, it is strictly decreasing in* ξ*, and strictly increasing in* μ*.*
It is independent from ξ *and* μ *when* $x(i) = 0$*.*

Is the interior optimal schooling decision dominated by a corner solution ?

Possible corner solutions: $\mathsf{S}_t(\mu, i) = 0$ and $\mathsf{S}_t(\mu, i) = \mathsf{L}_t$,

$\mathsf{S}_t(\mu, i) = \mathsf{L}_t$ is always dominated because of costs of schooling.

$\mathsf{S}_t(\mu, i) = 0$: compare the two level of utilities:

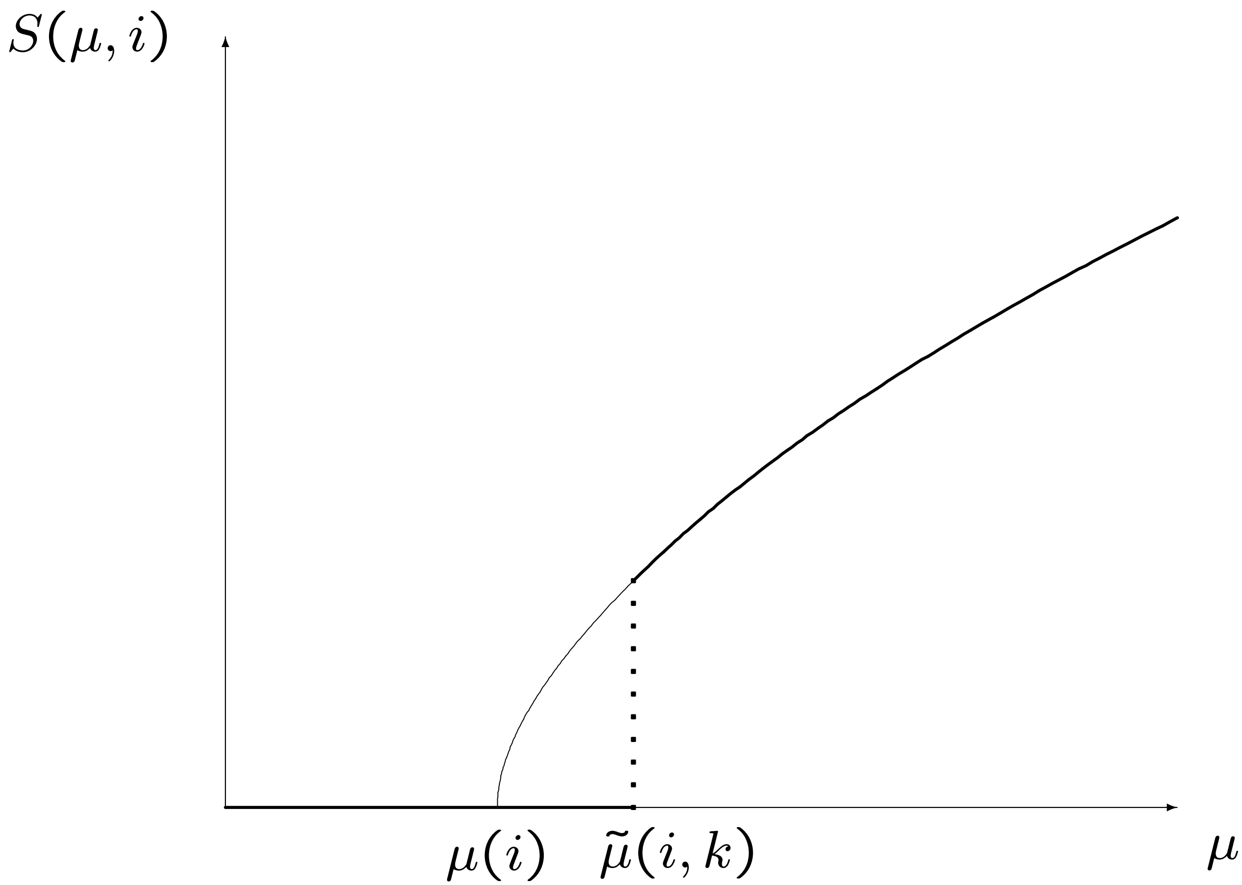
$$W[\mathsf{S}] > W^h.$$

For any fixed cost $k > 0$, there exist a threshold $\tilde{\mu}(i,k) > \underline{\mu}(i)$ checking:

(i) If $\mu > \tilde{\mu}(i,k)$, then the interior schooling solution is optimal: $\mathsf{S}^{\star} = \hat{\mathsf{S}}$.

(ii)If $\mu < \tilde{\mu}(i,k)$, then the interior schooling solution is dominated: $\mathsf{S}^{\star} = 0$.

For any t, the threshold $\tilde{\mu}$ is a strictly increasing function of k, w^h, ξ, $x(i)$ and β, and a strictly decreasing function of α and γ.


$S(\mu, i)$
$\underline{\mu}(i)$
$\tilde{\mu}(i, k)$
μ

Solution to the school location problem

Assume that, provided schools are created, there will be at least one at 0. Hence, the schools are located at $(j-1)/E$, with $j = 1, \ldots, E$.

The potential catchment area of the school at 0 is the circular segment $[-1/2E, 1/2E]$.

The distance function $x(i)$ is the arc length between location i and the closest school, hence in the catchment area of 0, $x(i) = i$.

Two cases may occur: separated or contiguous catchment areas

The members of the new-born cohort who attend school have

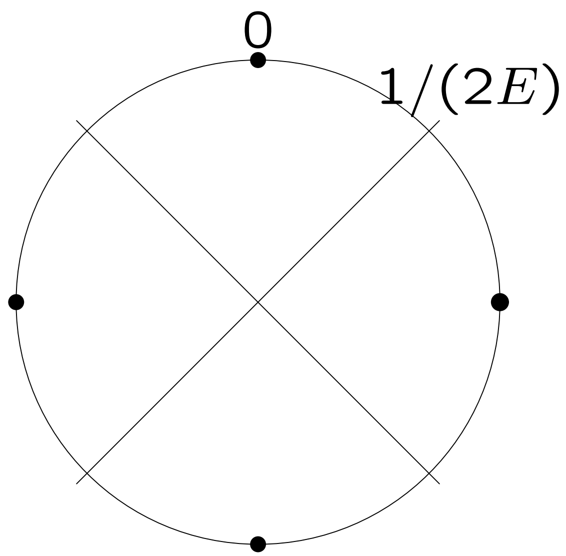
$$\mu > \tilde{\mu}(i,k)$$

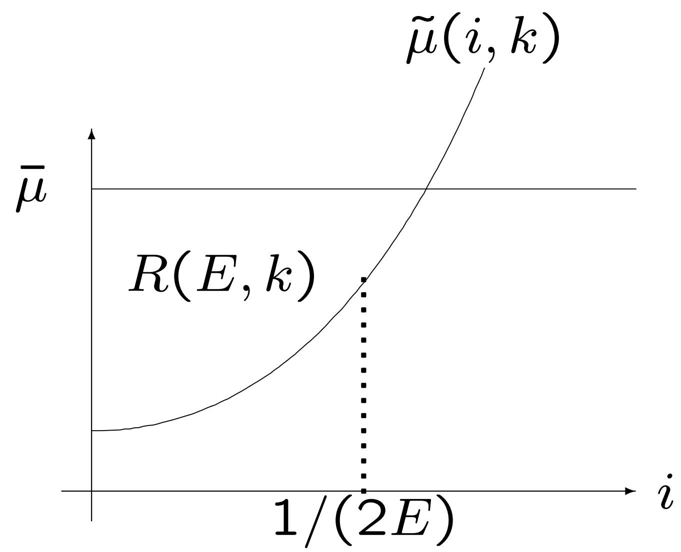
$\tilde{\mu}(\cdot)$ is increasing in i: for a population very close to the school, many students are likely to attend courses, while for very distant populations, only the most skilled ones will attend.

The attendance rate of population located at i is given by

$$\begin{aligned} r(i,k) &= \int_{\tilde{\mu}(i,k)}^{\overline{\mu}} g(\mu)d\mu \qquad \text{if } \tilde{\mu}(i,k) \leqslant \overline{\mu} \\ &= 0 \qquad \text{otherwise} \end{aligned}$$

Contiguous catchment areas





The attendance rate in the catchment area of the school at 0 is

$$R(E,k) = 2\int_0^{\frac{1}{2E}} r(i,k)di \tag{9}$$

The profit function:

$$B(E,k) = [k\zeta R(E,k) - f]\, E \tag{10}$$

Concave in E. Maximum reached at $E^\star \geqslant \tilde{E}$.

Solution

1. If $\zeta < f/(\overline{R}k)$, then no schools will be created.

2. If $\zeta = f/(\overline{R}k)$, then $\tilde{E} = 1/(2\ell)$ is the optimal number of schools, with ℓ such that $\tilde{\mu}(\ell) = \overline{\mu}$.

3. If $\zeta > f/(\overline{R}k)$, then E^* determined by $\partial B(E,k)/\partial E = 0$ is the optimal number of schools.

Non-linearity: low density, no school. Threshold at $f/(\overline{R}k)$ with jump in E.

Calibration & simulation

objective: what are the main factors behind the rise in literacy : fertility, mortality or technical progress ?

Simulation period: 1530-1860

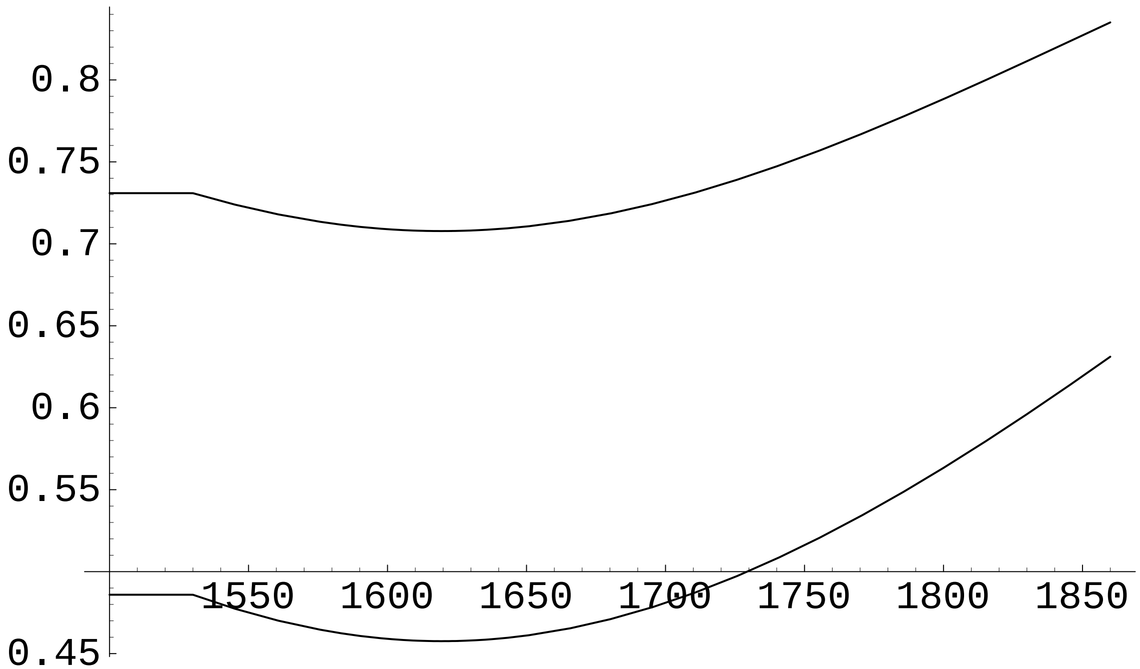
Birth date in the model: age 5 in the data

Calibrate exogenous processes α_t, β_t, ζ_t and γ_t to data.

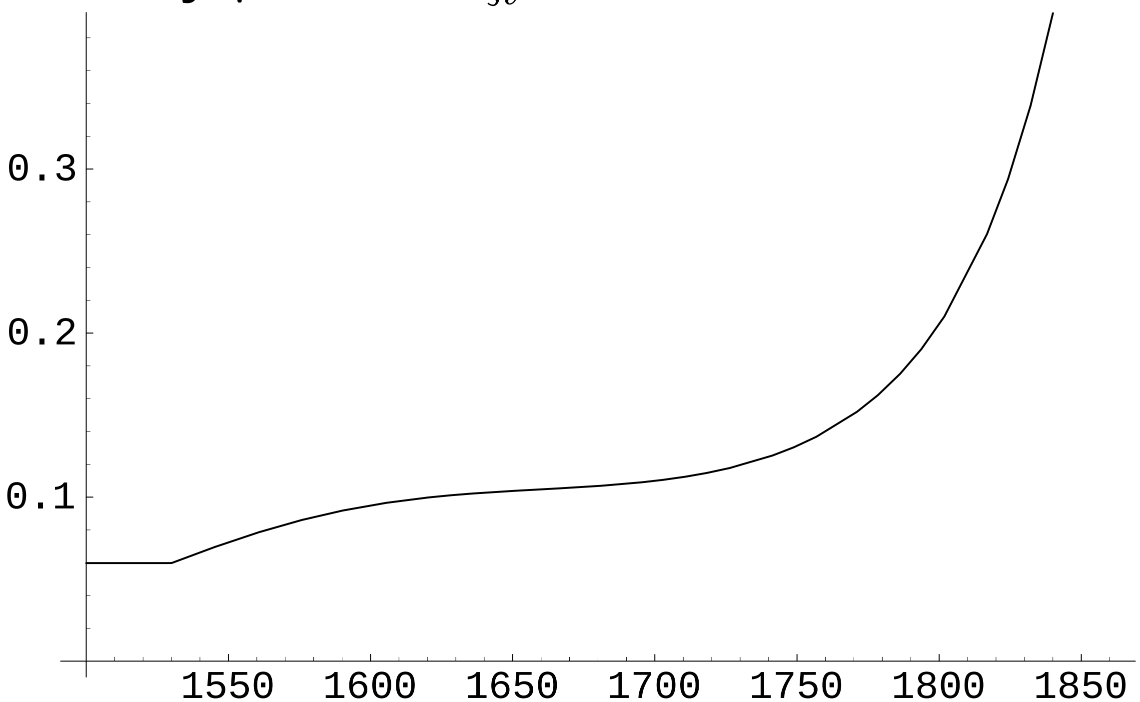
Polynomials of order 3 in time.
Parameters of the polynomial chosen by minimizing the distance with data.

Choice of the other parameters: θ, k, ξ, f, w^h, $g(\mu)$: largely arbitrary $\rightarrow$ sensitivity analysis.

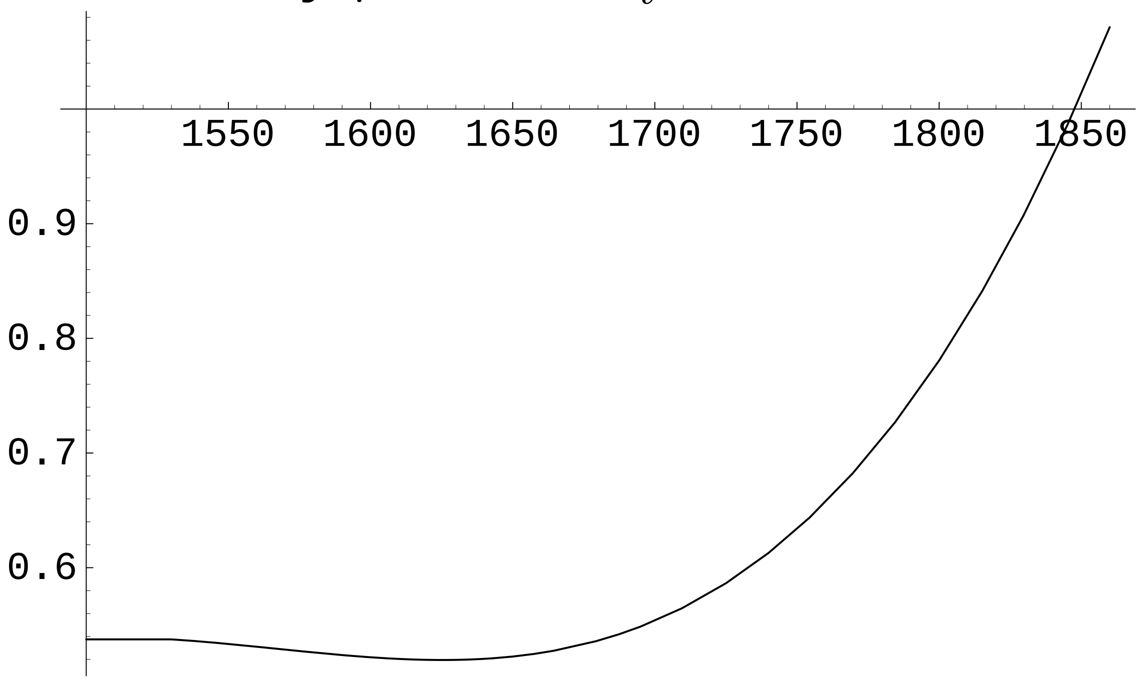
Mortality process: $m_t(30)$ and $m_t(50)$



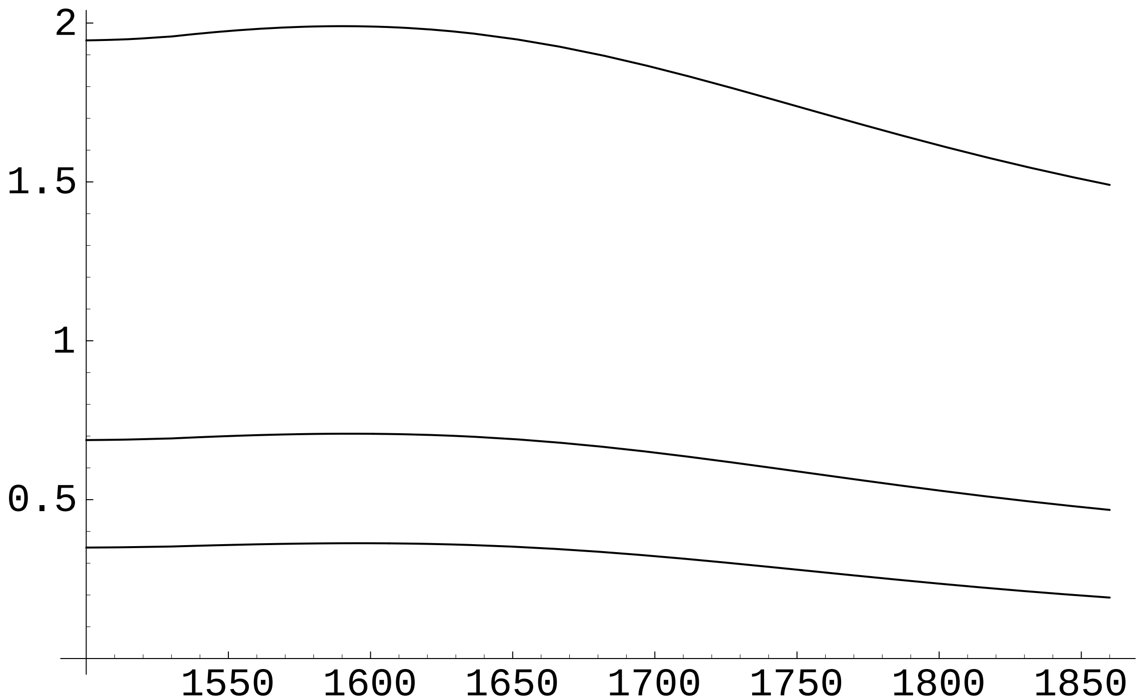
Fertility process: ζ_t



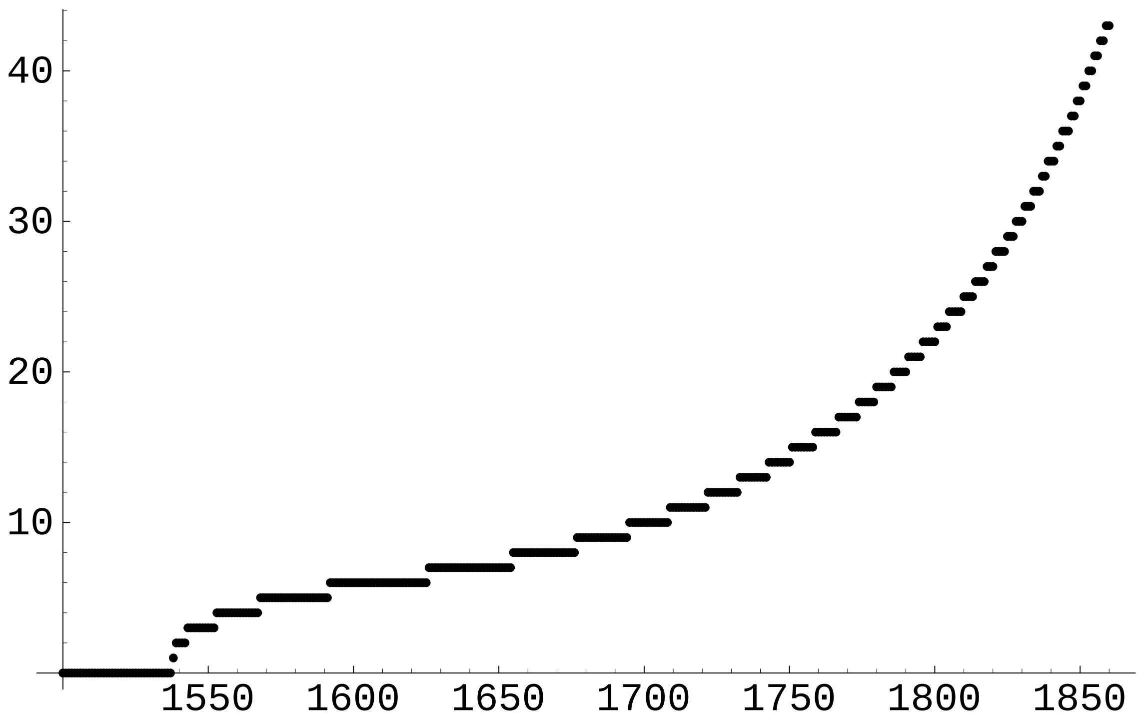
Productivity process: A_t



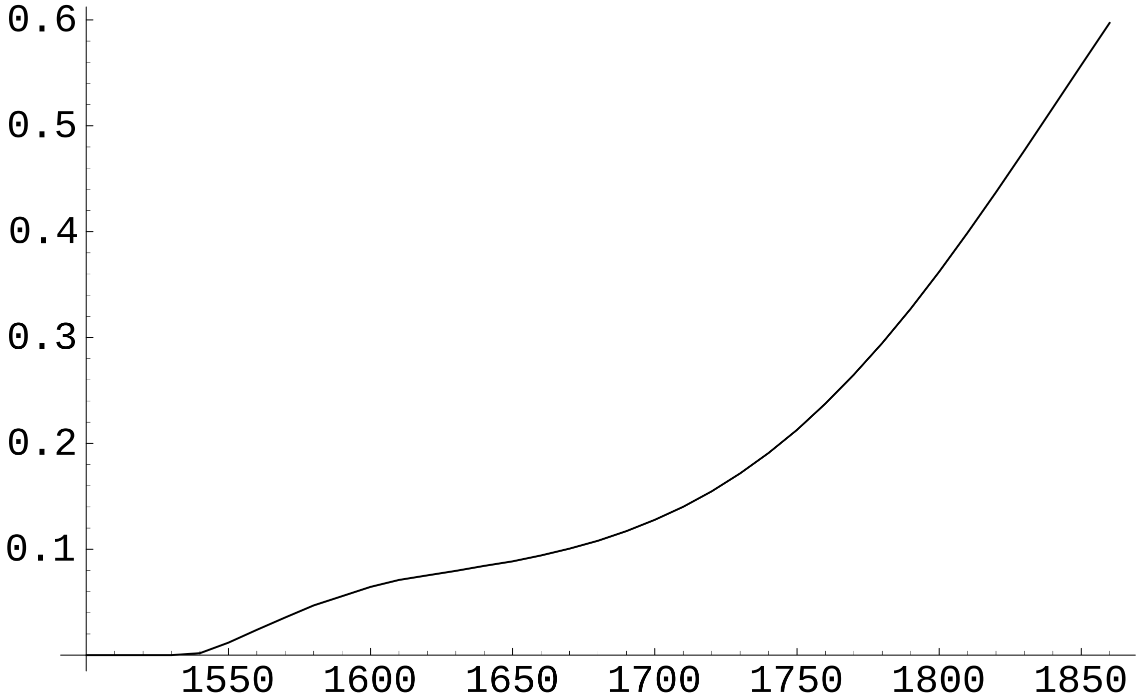
Baseline simulation - Functions $\tilde{\mu}_t(0.01, 10)$ (bottom), $\tilde{\mu}_t(0.1, 10)$, and $\tilde{\mu}_t(0.5, 10)$ (top)



Baseline simulation - school density



Baseline simulation - literacy



$$\Lambda_t = \frac{1}{P_t} \int_{t-\overline{L}}^{t} \zeta_z \, m_z(t-z) \int_0^1 \int_{\tilde{\mu}_z(i,k)}^{\infty} g(\mu) \, d\mu \, di \, dz.$$

Baseline simulation - GDP per capita

Human capital:

$$H_t = \int_{t-\overline{\mathsf{L}}}^{t} \zeta_z \, m_z(t-z) \int_0^1 \int_{\tilde{\mu}_z(i,k)}^{\overline{\mu}} \delta[t-z-\mathsf{S}_z(\mu,i)] \; h_z(\mu,i) g(\mu) \; d\mu \; di \; dz.$$

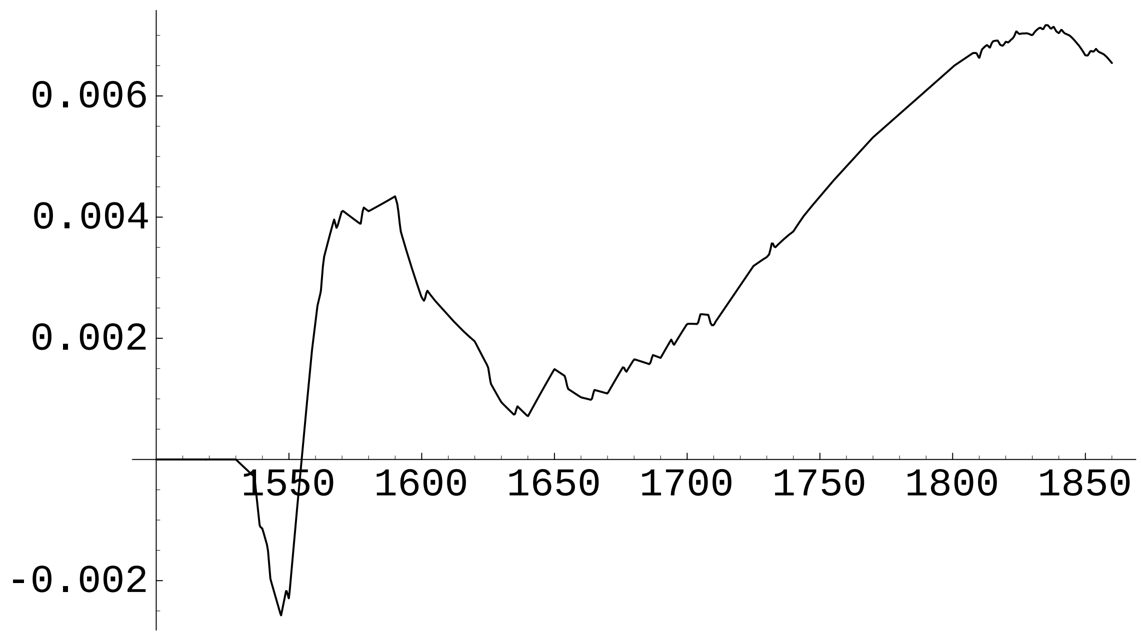
Total transportation costs:

$$\Xi_t = -\xi \int_{t-\overline{\mathsf{L}}}^{t} \zeta_z \, m_z(t-z) \int_0^1 \int_{\tilde{\mu}_z(i,k)}^{\overline{\mu}} \delta[\mathsf{S}_z(\mu,i)-(t-z)] \; x(i) g(\mu) \; d\mu \; di \; dz.$$

Total GDP

$$I_t = w^h(1-\Lambda_t)P_t + A_t(H_t - \Xi_t - kE_t). \tag{11}$$

Baseline simulation - growth rate



GDP per capita multiplied by 3.15 from 1500 to 1860. (3.94 with Maddison data)
difference reflects the accumulation of physical capital.

Counterfactual experiments	First school created	School creation	Change in Literacy	Average growth
Mortality	1730	13.2%	15.4%	12.2%
Birth density	1540	65.8%	31.5%	20.7%
Technical progress	1650	28.9%	59.7%	77.6%
Interaction terms		-7.9%	-6.6%	-10.6%

Rise in literacy: 1/6 due to mortality, 1/3 to density, 1/2 to technical progress.

neither productivity increases nor mortality improvements can explain the high rate of school foundations in the sixteenth century. Only the rise in population density can.

Remarks:

Effect of mortality: weak. Because in England, the fast urbanization process prevented longevity to increase much (compared to other places in Continental Europe)

Estimation of the density effect: it is a lower bound.
It we had assumed an externality:

$$A_t = e^{(1-\rho)\gamma_t t} H_t^{\rho}, \tag{12}$$

stronger effect of density.

Robustness analysis

to the parameters

to the model of school creation (M1 to M4)

Robustness analysis - results for 1500-1850

	School creation	Change in Literacy	I_t/P_t in 1850
Baseline	38	55.73%	1.77
No indexation of transportation costs	28	57.97%	1.84
Risk free interest rate 3%	17	79.46%	1.96
Lower variance $g(\mu)$	80	66.19%	1.71
Higher home productivity w^h	46	48.62%	1.77
w^h indexed on A_t	30	29.57%	1.77

Other assumptions on school formation - model M2

The central authority also determines the tuition fee:

$$\max_{E,k}(k\zeta R(E,k) - f)E$$

We can prove the the existence of an optimal solution

Other assumptions on school formation - model M3

Free entry process

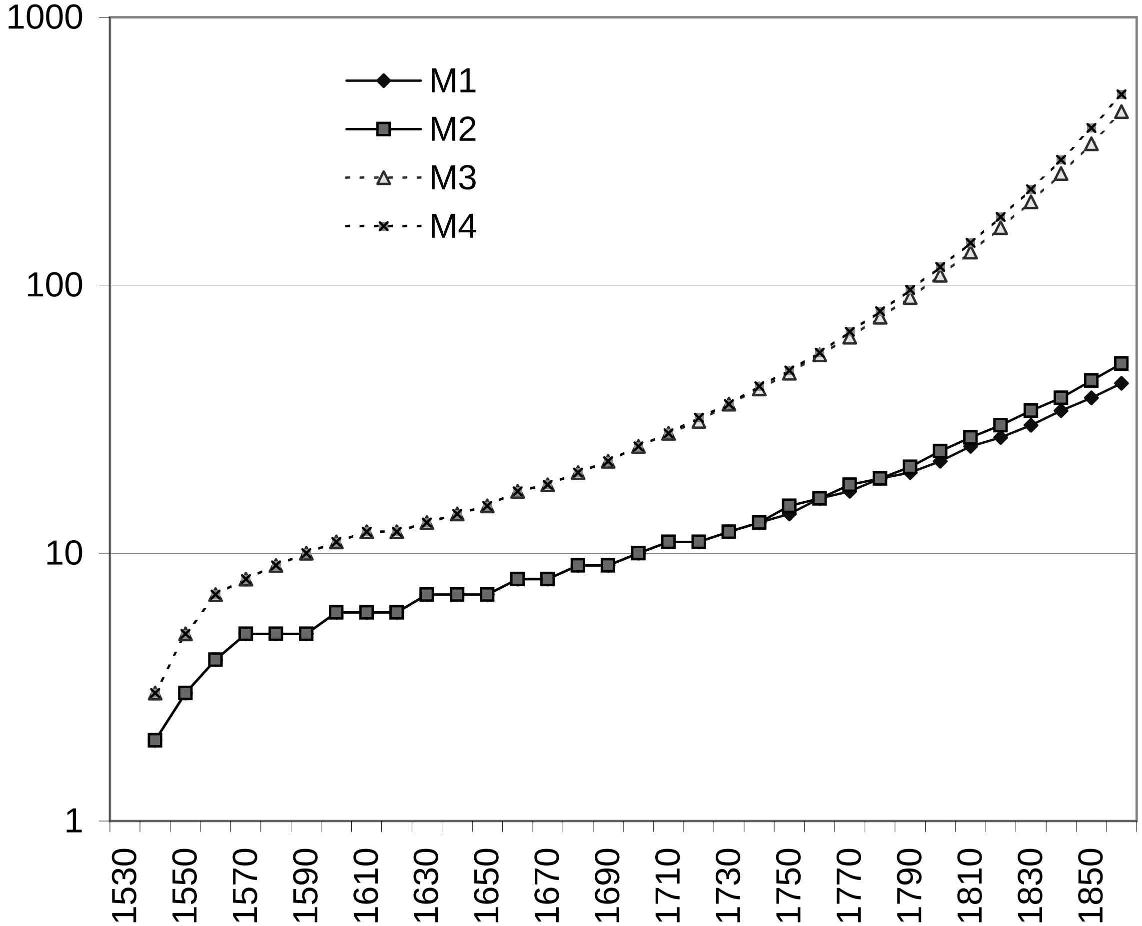
$$k\zeta R(E^{\diamond}) - f = 0 \tag{13}$$

since $B'(E^{\diamond}, k) < 0$, the density of schools with free entry is larger than the one chosen by a central authority.

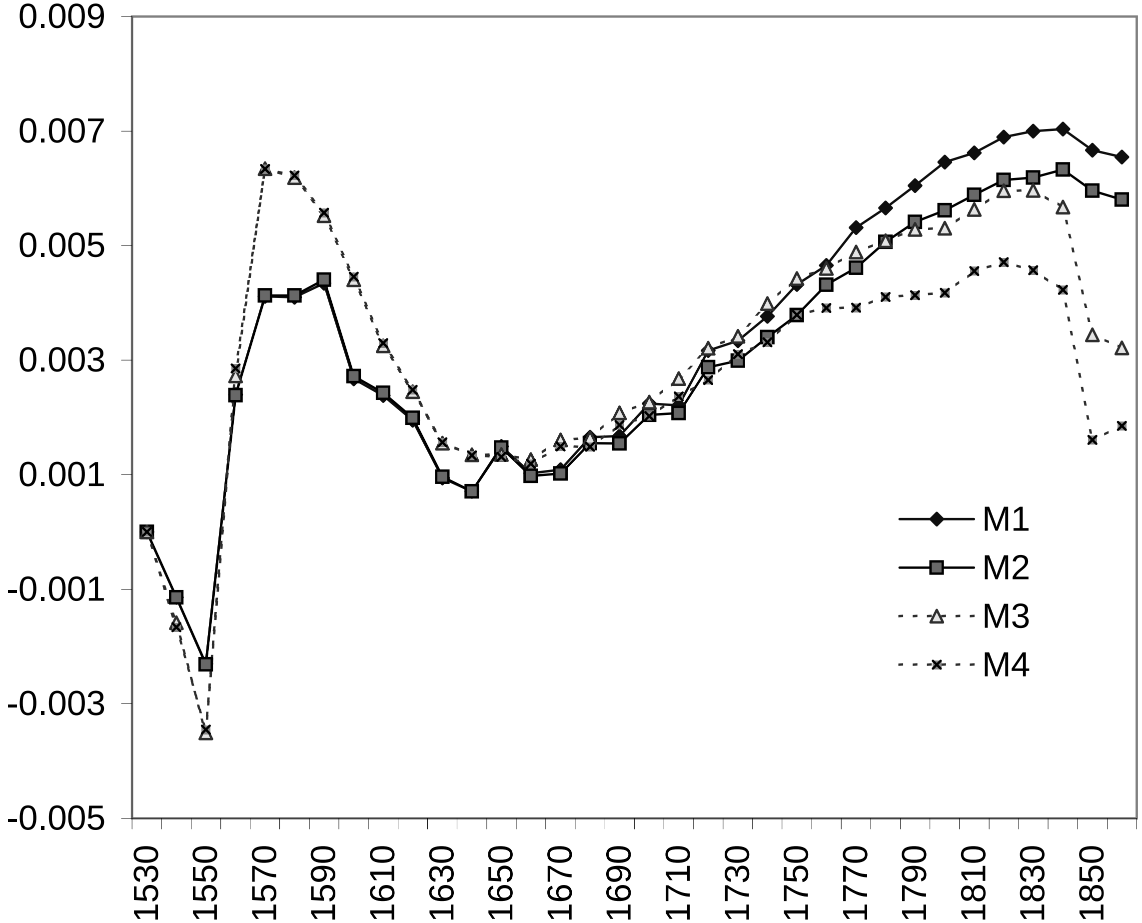
Other assumptions on school formation - model M4

Free entry process + endogenous tuition fee

School density (log) in the four models



growth in the four models



Lessons from the four models

Endogenizing the tuition fee k does not change much the results

Assuming free entry makes a big difference:
– log school density follows an exponential pattern
– growth is slowed down by this boom in the number of schools

Conclusion - Contributions of the paper

Micro-foundations for a model of the take-off with "population-induced" productivity gains

Explanation of the rise in literacy: 1/6 due to mortality, 1/3 to density, 1/2 to technical progress.

Comparing with school foundations data:
Only the rise in population density can explain the early school creations.

Sensitivity analysis to parameters and models of school foundations.
– with free entry of schools, hard to reproduce the acceleration in growth.

Final word

Common belief: rise in education was related to cultural and religious factors (Protestantism).

Here, can also be understood as an optimal response to population density passing a given threshold.