Vintage human capital, demographic trends and endogenous growth

Raouf Boucekkine David de la Croix Omar Licandro

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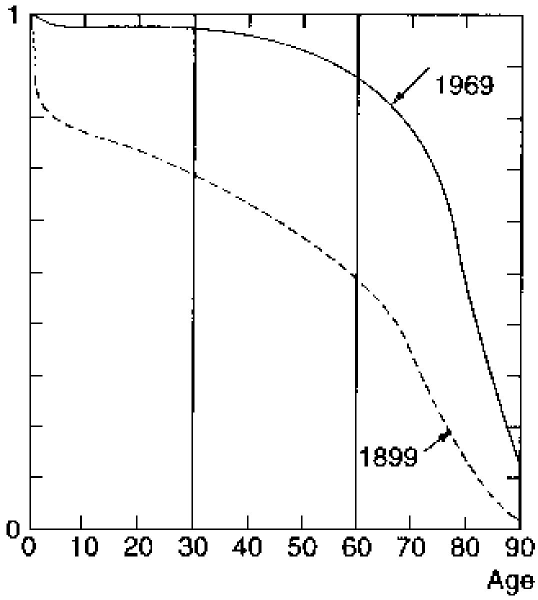
Figure 1: The decline in mortality – France – Survival laws



Source: Challier and Michel (1996).

Table 1: Long-run data

date	Growth rate of population		Years of schooling	
	Western Europe	U.S.	U.S.	France
1820-1870	0.8	2.9	2.8	n. a.
1870-1913	0.9	2.1	5.9	5.0
1913-1950	0.6	1.2	9.6	8.3
1950-1973	0.8	1.4	12.9	10.6
1973-1992	0.3	1.0	16.3	13.8

Source: Maddison (1995)

The set of individuals born in t:

$$\zeta \, e^{nt}$$

survival probability:

$$m(z-t) = \frac{e^{-\beta(z-t)} - \alpha}{1-\alpha}$$

$\alpha > 1, \beta < 0.$

Upper bound on longevity: $m(\mathrm{A}) = 0$:

$$\mathrm{A} = \frac{-\log(\alpha)}{\beta}.$$

life expectancy:

$$\Lambda = \frac{1}{\beta} + \frac{\alpha \log(\alpha)}{(1-\alpha)\beta}$$

Figure 2: Survival laws

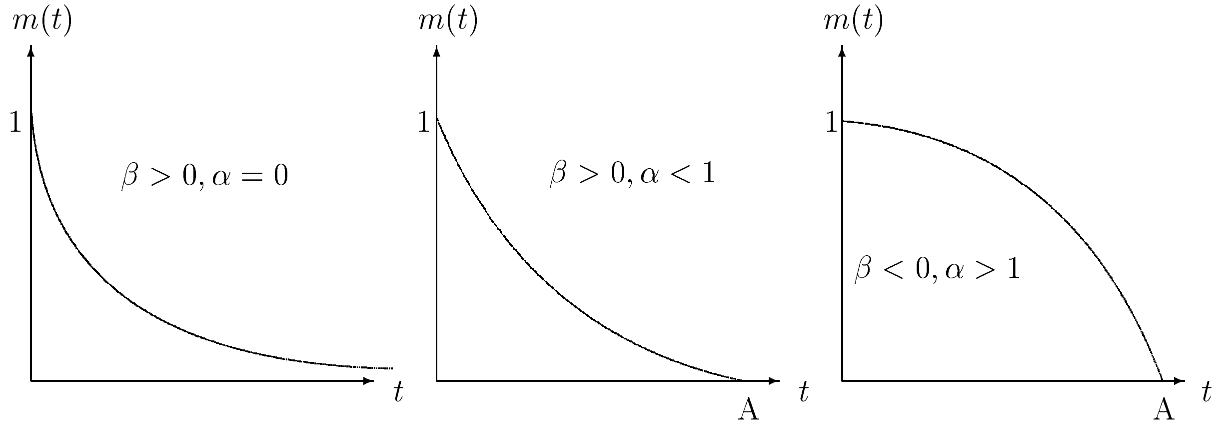
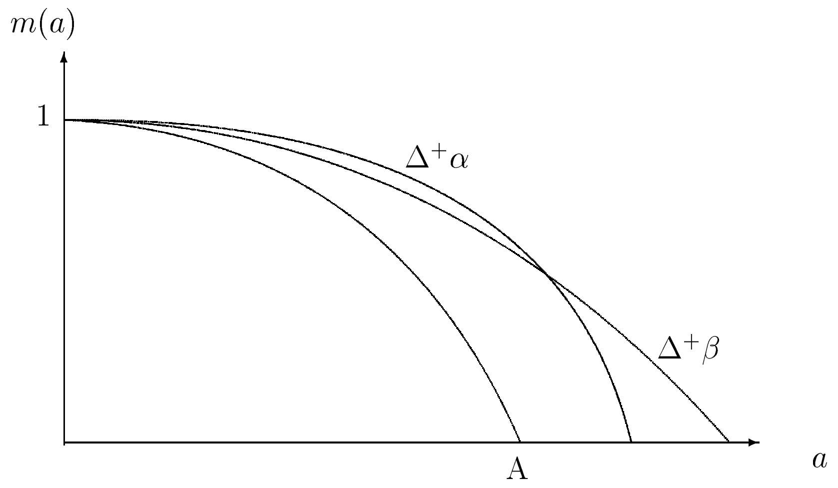


Figure 3: Changes in the survival laws



The size of the population at time t:

$$\int_{t-\mathrm{A}}^{t} \zeta\, e^{nz}\, m(t-z)\mathrm{d}z = \zeta\, e^{n\,t}\kappa$$

with

$$\kappa = \frac{n(1-\alpha) - \alpha\beta(1-\alpha^{n/\beta})}{n(1-\alpha)(n+\beta)}.$$

fertility rate: $1/\kappa$

An individual born at time t max

$$\int_t^{t+\mathrm{A}} c(t,z)\; m(z{-}t)\mathrm{d}z - \frac{\bar{H}(t)}{\phi} \int_t^{t+\mathrm{P}(t)} (z{-}t)\; m(z{-}t)\mathrm{d}z,$$

subject to

$$\int_t^{t+\mathrm{A}} c(t,z)R(t,z)\mathrm{d}z = \int_{t+\mathrm{T}(t)}^{t+\mathrm{P}(t)} \omega(t,z)R(t,z)\mathrm{d}z.$$

Spot wages :

$$\omega(t,z) = h(t)w(z),$$

Individual's human capital:

$$h(t) = \mu\bar{H}(t)\mathrm{T}(t).$$

At equilibrium we may rewrite contingent prices as

$$R(t,z) = m(z-t).$$

Schooling time

$$\mathrm{P}(t) = \min\left[\mathrm{T}(t)\ \mu\ \phi\ w(t+\mathrm{P}(t)), \mathrm{A}\right].$$

Production function:

$$Y(t) = H(t).$$

Labor market equilibrium:

$$w(t) = 1$$

Equilibrium schooling and retirement decisions

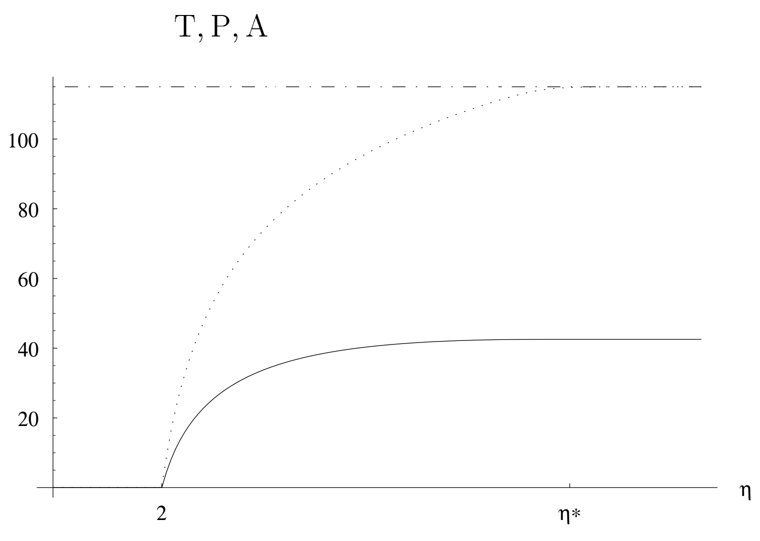
we define $\eta = \mu\phi$

Proposition 1 *(i) There exists a unique interior* $T^\star$*, and* $P^\star = \eta T^\star$ *if and only if* $2 < \eta < \eta^\star$.

(ii) If $\eta \geqslant \eta^\star$, $T^\star = T_{max}(\eta^\star)$ *and* $P^\star = A$.

(iii) If $1 < \eta \leqslant 2$, $T^\star = P^\star = 0$.

Figure 4: Optimal schooling and retirement as a function of η



Life expectancy and optimal schooling

Proposition 2 *A rise in life expectancy increases the optimal length of schooling.*

The balanced growth path

Productive aggregate human capital stock:

$$H(t) = \int_{t-\overline{\mathrm{P}}(t)}^{t-\overline{\mathrm{T}}(t)} \zeta\, e^{n\, z}\, m(t-z)h(z)\mathrm{d}z,$$

The average human capital:

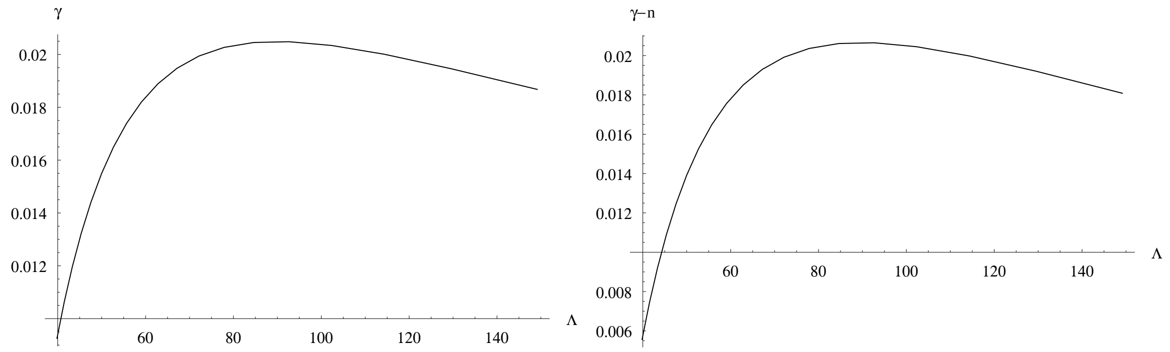
$$\bar{H}(t) = \frac{H(t)}{\kappa\, e^{n\, t}\zeta}.$$

The dynamics of human capital:

$$H(t) = \int_{t-\mathrm{P}}^{t-\mathrm{T}} m(t-z)\, \frac{\mu \mathrm{T} H(z)}{\kappa}\, \mathrm{d}z$$

Life expectancy and growth

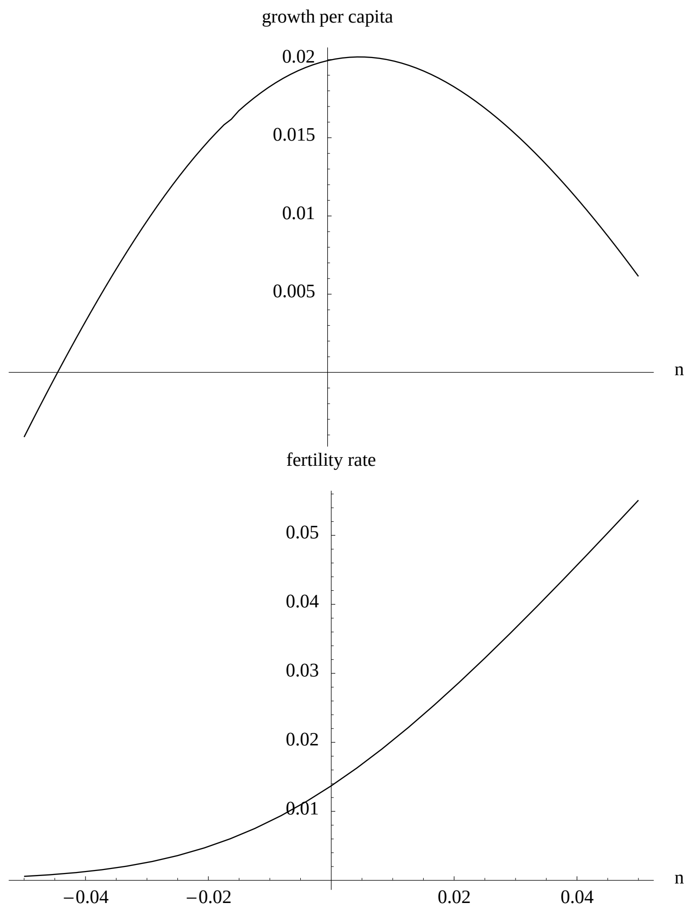
Figure 5: Growth and life expectancy at given n (left panel) and at given κ (right panel)



Proposition 3 *A rise in life expectancy through β at given population growth has a positive effect on economic growth for low levels of life expectancy and a negative effect on economic growth for high levels of life expectancy.*

Population growth and economic growth

Figure 6: Population and growth

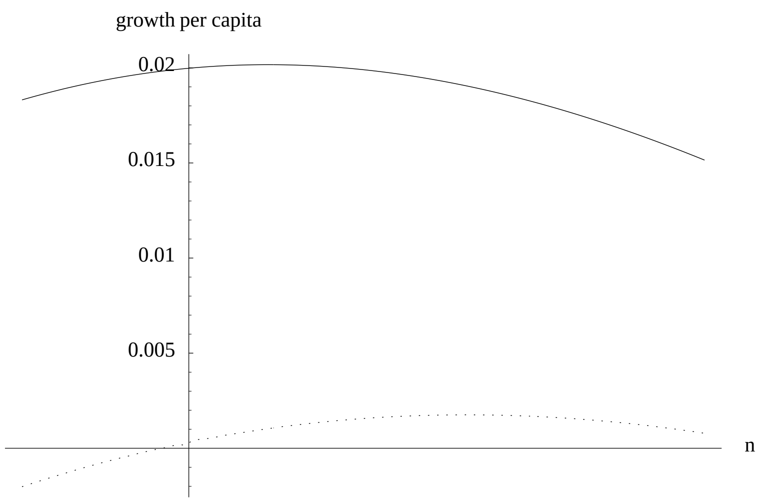


Proposition 4 *Assume that* $0 < \mathrm{T} < \mathrm{P} \leqslant A$. *There exists a population growth rate finite value* $n^\star$ *such that the long run per capita growth rate of the economy reaches its (interior) maximum at* $n^\star$.

From Malthus to Solow

regime	α	β	μ	ϕ	Λ	A	T	$\gamma_{n=0}$	T/Λ
Solow	5.44	-.0147	.2531	8.324	73	115	27	.0200	37%
Malthus	2.69	-.0147	.2531	8.324	39	67	13	.0004	33 %

Figure 7: From Malthus to Solow



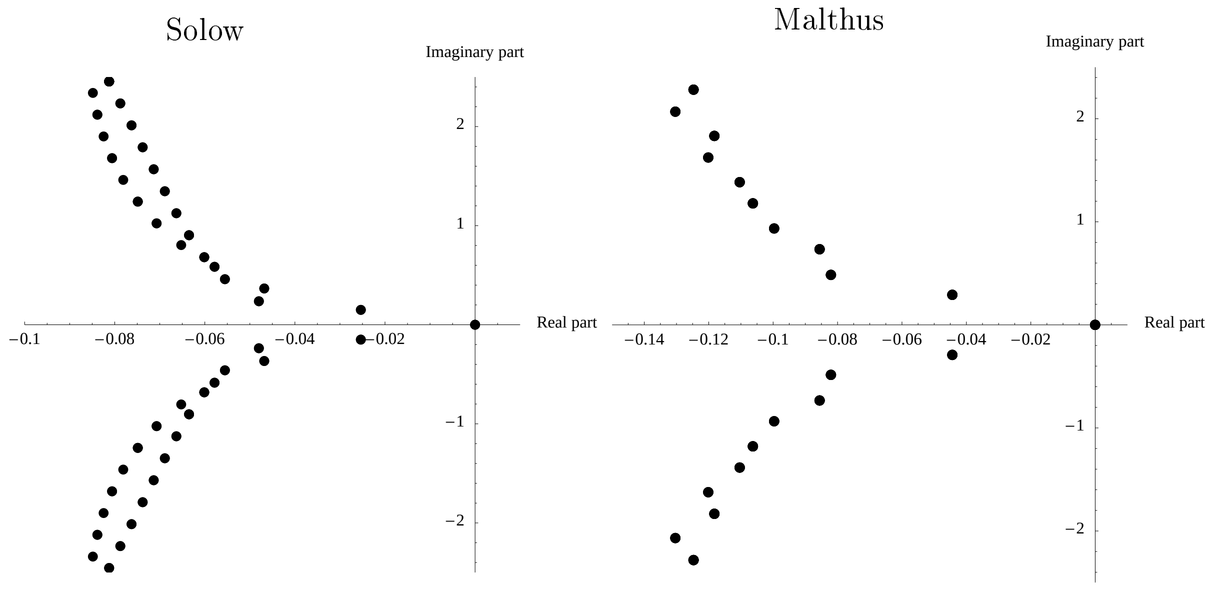
The dynamics of human capital

detrended human capital as

$$\hat{H}(t) = H(t)e^{-\gamma t},$$

$$\begin{aligned}
\hat{H}''(t) &= -\gamma(\beta+\gamma)\hat{H}(t) - (\beta+2\gamma)\hat{H}'(t) \\
&+ \frac{\mu \mathrm{T}}{(1-\alpha)\kappa}\Big[\Big(\gamma e^{-(\beta+\gamma)\mathrm{T}} - \alpha(\beta+\gamma)e^{-\gamma \mathrm{T}}\Big)\hat{H}(t-\mathrm{T}) \\
&\quad - \Big(\gamma e^{-(\beta+\gamma)\mathrm{P}} - \alpha(\beta+\gamma)e^{-\gamma \mathrm{P}}\Big)\hat{H}(t-\mathrm{P})\Big] \\
&+ \frac{\mu \mathrm{T}}{(1-\alpha)\kappa}\Big[\Big(e^{-(\beta+\gamma)\mathrm{T}} - \alpha e^{-\gamma \mathrm{T}}\Big)\hat{H}'(t-\mathrm{T}) - \Big(e^{-(\beta+\gamma)\mathrm{P}} - \alpha e^{-\gamma \mathrm{P}}\Big)\hat{H}'(t-\mathrm{P})\Big].
\end{aligned}$$

Figure 8: Eigenvalues of the calibrations “Solow” and “Malthus”



Dynamic simulation

Figure 9: Dynamic simulation of a drop in fertility in Solow