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de la Croix, D., and Michel, P.: A Theory of Economic Growth – Dynamics and Policy in Overlapping Generations. XVII, 378 pp. Cambridge University Press, Cambridge. 2002. Softcover £ 20.95.

This book analyzes a variety of macroeconomic issues in an overlapping generations (OLG) model with production, known in the literature as the Diamond OLG model. The book presents the model in detail, which is used extensively in macroeconomic theory, and its properties. Throughout the book the authors present many variations of this model and derive all of the results rigorously. Particularly, definitions, assumptions, propositions and their proofs are brought in detail, resulting in a very technical text. This is a major advantage for graduate students, for example, who wish to study this important framework and its applications in depth. The authors develop many relevant tools to be used in the analysis of the various issues in this model, such as optimal equilibrium allocations, dynamic evolution and convergence, intergenerational transfers and their economic implications, optimal policy in financing public spending and its economic consequences. The material is organized in an efficient manner. It begins with the basic model, existence of equilibria and their uniqueness under various conditions, the validity of the two welfare theorems in this infinite horizon economy, and then proceeds with applications of this machinery to analyze various economic issues. This framework is very well suited to conduct such an analysis as one has an overview of the macro literature of the last three decades.

In chap. 1, the homogeneous OLG model with production, the Diamond (1965) framework, is presented in various forms. The presentation is very clear, accompanied by many examples which are essential to exposing the readers to this type of economies. The notions of temporal and intertemporal equilibria with existence and uniqueness results (spelling out the assumptions required) are included. The analysis of convergence to a steady state and the stability results are basically similar to the well-known neoclassical one-sector growth model. This includes the discussion related to the data (from a few countries) showing the relationship between saving rates and growth

rates. Some of the clumsy conditions guaranteeing the existence and uniqueness results, in some variations of this model, become clear and intuitive when the examples are presented. In particular, the choice of Cobb-Douglas production function and constant intertemporal elasticity of substitution (CIES) utility functions are rich enough, when parameters are varied, for example, they give rise to bifurcation cases as well. The authors describe briefly a generalization of the model to heterogenous agents but ignore the resulting difficulties that arise with respect to its dynamical and convergence properties. Thus, it becomes clear that the choice of a "representative agent" model is justified only under restrictive conditions. Examining this framework under a borrowing constraints assumption indicates how inequality issues can be handled in these models.

It is well known that in OLG models competitive equilibria may not be Pareto optimal due to the "double infinity" property. Chapter 2 deals with the issue of dynamic inefficiency due to overaccumulation of capital along an intertemporally efficient path and which is analyzed extensively. The chapter includes illuminating examples showing how in a long run equilibrium the interest rate may be *lower* than the rate of population growth (which results in dynamic inefficiency). Most of this chapter is devoted to allocations attained by a central planner's optimization. This type of intergenerational planner optimal allocations is closely related to the optimal growth literature, due to the infinite horizon optimization. Hence, the technique used in this case to characterize them is similar. The authors apply dynamic programming methods, that is, value functions, optimal policy functions and Euler equations, to derive characterization and properties of the optimal planner's allocations in this framework (as in the widely-discussed one-sector optimal growth model). However, the economic interpretations of the various conditions, which *differ* in this case (in the absence of altruism) due to the intergenerational structure, are clear and insightful. The presentation, motivation and interpretations of the results here are extremely efficient and the examples play a major role in achieving that.

In recent years economists have considered growth models where the equilibrium capital stocks may be unbounded. In chap. 2 the authors examine such cases for CIES utility functions. The analysis of such cases, basically via examples, exposes the readers to the analytic difficulties and the way to obtain results in such unbounded equilibria without going through the clumsy treatment of the general model. It is demonstrated how different constellations of the parameters, related to utility and production functions, result in a clear-cut long run behavior of the capital stocks.

Redistribution of resources between generations is an important issue studied extensively in recent years by many economists. Chapter 3 deals with such a problem in various setups. The framework developed here is very suitable for analyzing macroeconomic problems such as intergenerational transfers, debt and optimal governmental policies. The authors always define clearly the equilibrium notion that will be used (in this case, competitive equilibrium either with transfers or with exogenously given public spending) and the conditions that guarantee its existence. Comparison of the optimal capital accumulation, given the two well known social security systems, fully-funded vs. pay-as-you-go, has been obtained with homogeneous population. However, redistribution issues (which require heterogeneity) are absent from this discussion. The conditions assumed to obtain equilibrium with pensions, e.g., assumptions related to highest and lowest sustainable transfers, are messy and could have been simplified. Similarly, this framework is utilized to address the issue of optimal fiscal policy: choosing optimal taxation on labor and income assuming that exogenously given governmental expenditures are present. Comparative static results are derived first when lump-sum tax levels are used along the equilibrium path. The analysis conducted here in solving these second best problems is detailed and well done. In addition, following the initial treatment of the model, various important issues are raised and discussed. In some cases technical difficulties arise (such as non-convexities) and the results are attained only for examples.

Chapter 4 studies major issues that have captured much attention in the literature: how to finance public spending and the sustainability of government debt. The framework developed here is suitable for analyzing such problems, as was done throughout the theoretical papers studying these issues. The authors carefully examine the analytical part (existence and characterization of intertemporal equilibria under various conditions), as well as the conceptual issues related to policy; for example, constant per-capita debt vs. keeping the deficit constant. The questions related to decentralization in the presence of debt and public spending are discussed thoroughly, considering several methods of taxations. Some of the results derived here are quite general, for example, in decentralizing any optimal path there are no restrictions on the sequence of public debts as long as the government can tax the "young" and the "old" freely. However, it seems to me that the authors are considering too many cases here which differ only slightly from one another, hence with only minor added value. Thus, the readers' attention is abstracted from the major issues, e.g., arbitrarily variable debt vs. constant deficit policies. Nonetheless, this chapter provides an insightful examination of important issues and essential analytical tools to handle such questions.

This includes valuable examples and graphical explanations that facilitate understanding of the long run consequences, sustainability, overaccumulation of capital, the role of asset bubbles and the emergence of cycles.

Important extensions of the main framework used throughout this book are brought in chap. 5. So far we abstracted from linkage between generations and externalities due to human capital accumulation. In motivating transfers from parents to children, such as bequests and inter-vivos, the authors choose to model altruism between parents and children as in the dynastic model: the parents' utility function includes the utility of their offspring. Although the equilibrium considered is *perfect foresight*, this results in maximizing an infinite-horizon problem for each agent in choosing consumption, saving and bequest. Thus, even though competitive equilibria are considered, the Bellman equation is applicable in characterizing the optimum in this case. However, the choice of the dynastic model is essential to attaining the neutrality of government financing policy, i.e., the Ricardian equivalence result. This is derived after establishing the existence of intertemporal equilibrium with positive bequests. I find this approach to be appropriate, namely, to start the section with the neutrality result before moving to other versions of this model where neutrality does not hold, e.g., other form of altruism, uncertainty etc.

In recent years many economic researchers discussed various forms of endogenous growth models where human capital serves as an engine to growth. Since education plays a crucial role in these models the assumptions regarding preferences, the intergenerational transmission of human capital, financing of education and market constraint, are important in deriving results. A change in the preferences of the parents to the "*joy of giving*" case will affect the analysis tremendously. The authors briefly consider only a few cases, rather than considering several human capital production functions and concentrating on growth and human capital accumulation issues. They deviate from the homogeneity assumption, which is maintained almost throughout the book, in order to deal with the issue of *distribution*. I find this approach inefficient since the issues dealing with provision of human capital, the way it is financed and its effect on income inequality deserves a separate section by itself!

Choosing a lognormal distribution of human capital it is shown that under public funding of education income inequality declines. The "mixed" case, namely, when education is provided publicly in part and privately as well was not addressed. When education is financed solely by loans ("market funding") altruism plays no role, however, the human capital of parents affects the human capital of their offspring. Admittedly, the discussion of the various

models in this chapter provides some understanding of the vast literature on these topics and gives the reader an idea about the additional difficulties generated by such externalities to the analysis within this economy. The way transmission of preferences is treated is limited since altruism between parent and child is absent; however, the interested reader derives information about the basic cases treated here. Addressing the problems, where a current decision depends on past actions or the parent-child relationship, is an important step forward, although it involves much more complex analysis, since it brings macroeconomic models closer to reality.

I find the material covered in this book to be an excellent introduction to an essential tool in economics, which can be applied to analyzing many macroeconomic problems. Such issues include debt policy, intergenerational transfers and the effects of human capital formation on growth and distribution. The strategy of the authors is to conduct the analysis rigorously, which implies defining each notion and stating explicitly all of the assumptions needed to prove their results. This book is highly recommended for advanced macroeconomics graduate courses, since it covers a wide range of important issues which can properly be analyzed by different variations of the basic model. The rigor and clarity of the analysis make it valuable to understanding macroeconomic policy.

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