CART: the Constituent-oriented Age and Residence time Theory

A holistic tool to help understand complex reactive transport processes

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Nowadays marine models routinely produce large amounts of results. Making sense of all these real numbers is not a trivial task. This is why specific interpretation methods are needed. Estimating timescales is one of them. In this respect, a comprehensive theory (CART) is developed that allows for the estimation of timescales such as the age and the residence time from the solution of partial differential problems.

At any time and position, the age — a measure of the elapsed time — of every constituent, or group of constituents, of seawater can be estimated in such a way that

\[
\frac{\partial c_i}{\partial t} = \partial_i - \nabla \cdot (c_i \mathbf{v} - \mathbf{K} \cdot \nabla c_i) - \frac{\partial c_i}{\partial \tau}
\]

\[
C_i(t,x) = \int c_i(t,x,\tau) d\tau, \quad \alpha_i(t,x) = \int \tau c_i(t,x,\tau) d\tau
\]

\[
\frac{\partial c_i}{\partial t} = \partial_i - \nabla \cdot (c_i \mathbf{v} - \mathbf{K} \cdot \nabla c_i)
\]

\[
\frac{\partial \alpha_i}{\partial t} = C_i + \Pi_i - \nabla \cdot (\alpha_i \mathbf{v} - \mathbf{K} \cdot \nabla \alpha_i)
\]

\[
(\vartheta_i, \Pi_i) = \int (\partial_i, t \partial_i) d\tau
\]
Theoretical results are easier to obtain than in the Lagrangian formalism. For instance, the biases of the age derived from radioactive tracers (see figure opposite) or from the time lag method have been uncovered and investigated in depth. The age also turned out to be of use to diagnose reaction rates in ecosystem models, i.e. in models in which the reaction terms are non-linear. On the other hand, the age of tracers released by a point source in a number of shallow water domains has been simulated numerically and investigated in a theoretical manner, leading to the discovery of an intriguing symmetry property (Beckers et al. 2001).

The residence time is defined as the time needed for a particle to hit for the first time an open boundary of the domain. To account for the fact that particles may re-enter the domain of interest after leaving it, the concept of exposure time was introduced. The propensity of particles to re-enter the domain may be evaluated by means of the return coefficient. The residence time and the exposure time, \( \eta(t, \mathbf{x}) \), are the solution of an adjoint equation (Delhez et al. 2004), which is to be integrated backward in time,

\[
\frac{\partial \eta}{\partial t} = - \omega - \nabla \cdot (\eta \mathbf{v} + \mathbf{K} \cdot \nabla \eta), \quad \omega(\mathbf{x}) = \begin{cases} 1 & \text{if } \mathbf{x} \in \text{domain of interest} \\ 0 & \text{if } \mathbf{x} \notin \text{domain of interest} \end{cases}
\]

In the framework of idealised flow studies, it was seen that the residence/exposure time of sinking particles (e.g. diatoms) in the upper mixed layer is an increasing function of the eddy diffusivity. A generalisation of this approach allowed for the evaluation of the amount of light such particles are exposed to. Relevant inequalities and seemingly counterintuitive results were established and discussed.

The abovementioned time-scales proved to be particularly useful for investigating the water renewal of semi-enclosed domains. A general method was developed, which suggests that the age of the renewing water be estimated as well as the residence/exposure time of the water originally present in the domain of interest (de Brye et al. 2012). Several estuaries share the same property (see figure opposite): the variability of the residence and exposure time is much more pronounced at the period of the dominant tidal component than at the timescale of the spring-neap cycle though these diagnostic timescales are usually significantly longer than the duration of a spring-neap cycle.

The concept of age is being generalised, leading to the notion of partial age (Mouchet et al. 2016). The domain of interest is split into several subdomains and every constituent particle is henceforth “equipped” with several clocks (rather than only one), allowing for the time spent in each subdomain to be evaluated. This way, information about pathways is obtained without having recourse to Lagrangian calculations.
Publications related to the development and application of the
Constituent-oriented Age and Residence time Theory (CART)
http://www.climate.be/cart

Selected publications


Comprehensive list of publications


25. WHITE L. and E. DELEERSNIJDER, 2007, Diagnoses of vertical transport in a three-dimensional finite-element model of the tidal circulation around an island, *Estuarine, Coastal and Shelf Science*, 74, 655-669


31. PRIMEAU F. and E. DELEERSNIJDER, 2009, On the time to tracer equilibrium in the global ocean, 5, 13-28


35. MERCIER C. and E.J.M. DELHEZ, 2010, A modified TVD scheme for the advection of two or more variables with consideration for their sum, *Ocean Dynamics*, 60, 1157-1166


1 Quarterly Bulletin of CERF (Coastal & Estuarine Research Federation)