How to Strongly Link Data and its Medium: 
– the Paper Case –

Philippe Bulens∗, François-Xavier Standaert†, and Jean-Jacques Quisquater

Université catholique de Louvain, B-1348 Louvain-la-Neuve.

Abstract

Establishing a strong link between the paper medium and the data represented on it is an interesting alternative to defeat unauthorized copy and content modification attempts. Many applications would benefit from it, such as show tickets, contracts, banknotes or medical prescripts. In this paper, we present a low cost solution that establishes such a link by combining digital signatures, physically unclonable functions [12, 13] and fuzzy extractors [7]. The proposed protocol provides two levels of security that can be used according to the time available for verifying the signature and the trust in the paper holder. In practice, our solution uses ultra-violet fibers that are poured into the paper mixture. Fuzzy extractors are then used to build identifiers for each sheet of paper and a digital signature is applied to the combination of these identifiers and the data to be protected from copy and modification. We additionally provide a careful statistical analysis of the robustness and amount of randomness reached by our extractors. We conclude that identifiers of 72 bits can be derived, which is assumed to be sufficient for the proposed application. However, more randomness, robustness and unclonability could be obtained at the cost of a more expensive process, keeping exactly the same methodology.

1 Introduction

Securing documents is an important topic in our everyday life. Bank notes are probably the most obvious example and it is straightforward to detect, e.g. micro-printing, ultra-violet inks, ... that are aimed to make their falsification difficult. But in fact, many other documents are concerned, e.g. show tickets, legal papers or medical prescripts. Even the passports that now embed an RFID chip are still enhanced with such physical protections of which the goal is to prevent counterfeiting. In other words, they aim to render the effort for producing good-looking fakes prohibitively high. In general, any field where the authenticity of a document is important would benefit from a way to prevent duplication and/or modification. But in practice, the cost of the proposed solutions also have to be traded with the security level that has to be reached.

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†Associate researcher of the Belgian Fund for Scientific Research (FNRS - F.R.S.)
As a matter of fact, the topic of anti-counterfeiting techniques is very broad. Even when restricting the concern to paper documents, various contributions can be found in the scientific literature and in products of companies. For example, Laser Surface Authentication (LSA) extracts a unique fingerprint from almost any document and packaging, based on intrinsically occurring randomness measured at microscopic level using a laser diode, lenses and photodetectors [3]. The scheme is shown to be highly resistant to manipulation. Moreover, the authors of [8] suggest that the fingerprint could be stored on the document itself through an encrypted or digitally signed 2D barcode or a smart chip. Another approach to fingerprint paper is presented in [4]. Using a standard scanner and by capturing a sheet of paper under 4 different orientations, authors are able to estimate the shape of the papers’ surface. A unique fingerprint is derived from those captured physical features which is shown to be secure and robust to harsh handling. Similarly, printed papers can be authenticated using the inherent non-repeatable randomness from a printing process [21]. Here, the misplacement of toner powder gives rise to a print signature that is experimentally shown to be unique and random. It is then explained how to exploit such a print signature in order to design a scheme ensuring the authentication of a document.

A similar idea is developed in this paper, i.e. we aim to bind the fingerprint of the medium and the data lying on it. Like in the print signature, this idea is achieved by performing a digital signature on these information as a whole. But contrary to [21] where the fingerprint is printed on the paper and analyzed using shape matching methods (a well-studied problem in computer vision), we make the fingerprint intrinsic to the paper. For this purpose, we incorporate ultra-violet fibers during the fabrication process of the paper. The proposed solution relies on the combination of a Physically Unclonable Function (PUF) with robust cryptography and randomness extraction schemes. That is, we use fuzzy extractors to build unique identifiers from the fiber-enhanced papers. Importantly, we mention that using fibers as a PUF was motivated by low-cost applications (e.g. medical prescriptions, typically). Hence, the actual unclonability of the proposal is only conjectured for low-cost adversaries. But increasing the unclonability by considering more complex physical sources of randomness would be feasible at the cost of a more expensive process (techniques such as presented in [4] could typically be used in this way).

Summarizing, the following results mainly aim to evaluate a coherent application of existing techniques. Additionally to the description of our protocol for copy or modification detection, we pay a particular attention to the careful statistical analysis of the robustness and amount of randomness that are extracted from the papers. We believe that such an analysis is interesting since most published works on PUFs (e.g. based on microelectronic devices [14]) are limited in the number of samples they use for their estimations.

Note that from a theoretical point of view, such a Sign(content, container) scheme could be applied to any object. To make it practical only requires a way to robustly measure the intrinsic features of a medium and to embed a digital signature. But such an adaptation should also be considered with care since each ingredient of the protocol could be tuned in function of the target application. In other words, the solution we propose is general, but finding the best tradeoff between cost and security for a given application is out of our scope.
The rest of the paper is organized as follows. Section 2 gives a global overview of the proposed method and points out its requirements. The ingredients of our protocol are described in Section 3. The main contribution of the paper is then presented in Section 4 in which the paper case study is investigated and evaluated. Eventually, conclusions are given in Section 5.

2 Overview

In this section, a general overview of the process that we propose for paper authentication is sketched. The components of the scheme will be discussed afterwards. First, the signature of a document works as follows.

1. Some additional agent is poured into the paper paste to make secure sheets (Fig. 1). For example, we use ultra-violet fibers in our application.

2. The physical features of the paper are then extracted, encoded into a tag $T_1$, and printed on the paper to authenticate (Fig. 2).

3. Some actual content is printed on the paper (Fig. 3).

4. Eventually, this content is concatenated to the physical information and signed. The digital signature is encoded in a second tag, $T_2$ (Fig. 4).

![Figure 1: Making paper.](image1)

![Figure 2: Extracting physical features.](image2)

![Figure 3: Adding content.](image3)

![Figure 4: Signing container + content.](image4)
Second, in order to check if the document is a genuine one, two levels of security can be considered. These two levels are neither mandatory nor exclusive. Whether one or both should be used is driven by the application. For example, in the context of medical prescription, the pharmacist may decide to apply the first level of security for his usual customers and the second level for a person he has never seen before. The verification works as follows.

**Level 1.** The verifier trusts the second step of the previous procedure and only performs the verification of the digital signature using $T_1$, $T_2$ and the content of the document (Fig. 5). Interestingly, this process does not necessarily require the use of optical character recognition since the paper content might have been summarized in $T_2$. Of course, this implies that the paper content fits into the tag size constraints.

**Level 2.** A full verification is performed (Fig 6).

1. The physical features of the medium are extracted and encoded in a new tag $T^*$. This tag $T^*$ is then compared with $T_1$ and the document is rejected if the two tags do not match.
2. If both tags do match, then the verification proceeds like before. Using $T^*$ or $T_1$ makes no difference at this stage.

3 Ingredients

To be implemented in practice, the previous authentication process requires different ingredients, namely a digital signature algorithm, a tag encoder/decoder, a physical unclonable function (PUF) and an extraction scheme. Proposals for each of these ingredients are discussed in the present section. We mention that these choices are not expected to be optimal but to provide a solution that reasonably fits to the low cost requirements of our target applications. Hence, they could be improved or tuned for other applications.
3.1 Signature

Digital signature is a well studied field in cryptography and a variety of solutions are available for different applications. When devising a real application, using a standard is a natural way to go. For this reason, the Elliptic Curve Digital Signature Algorithm (ECDSA [1]) was chosen. Because it provides relatively short signatures, this lets space for possible addition of content to the tag.

3.2 Tag En-/De- coding

As for digital signature, a large number of visual codes exist, from the old bar code to more complex 2-dimensional codes. A list of commonly used 2D bar codes is available on the web [16]. The Datamatrix [9] was chosen for its high density within a small area (up 1500 bytes in a single symbol whose size can reduce to a bit more than a 1-inch square with a 600 dpi printer, see Fig. 7).

Figure 7: A datamatrix encodes large amount of data in small areas.

3.3 PUF: Physical Unclonable Function

PUF or equivalently physical one-way functions were introduced by Pappu [12, 13] in 2001. A PUF is an object whose function can easily be computed but is hard to invert. For a more in-depth view of PUF and their use in cryptography, we refer to Pim Tuyls et al.’s book [15]. As it will be used in this paper, every challenge sent to a PUF (i.e. each time a given sheet of paper is scanned) should be answered by almost the same response (i.e. picture). It should then be ensured that the size of the response set is large enough to prevent collisions (i.e. different sheets of paper should not output the same response). Also, the protected papers should be hard enough to clone. With this respect, PUF generally rely on some theoretical arguments borrowed from physics.

In our case and as mentioned in the introduction of this paper, we consider a weaker type of PUF that is just expected to be hard to clone by a low-cost adversary. According to the papermaker [2], systematically generating twins of fiber-enhanced papers is an expensive process. But there is still the option for an attacker to scan the sheet of paper under ultra-violet illumination and attempt to carefully reproduce the fibers on a clear sheet of paper. This is exactly what we assumed to be hard enough to be considered as a real threat in our context. We mention again that the focus of this paper is not in finding the best PUF but in devising a complete solution for preventing paper modification and copy and evaluating its reliability in terms of randomness and robustness.
3.4 Physical Extraction

The major problem when extracting fingerprints from physical objects is that they are usually not usable for cryptographic purposes. This difficulty arises from the fact that (1) the extracted fingerprint may not be perfectly reproduced, due to small variations in the measurement setup or in the physical source and (2) the extracted fingerprints from a set of similar objects may not produce the uniform distributions required by cryptography. Hopefully, turning noisy physical information into cryptographic keys can be achieved using fuzzy extractors.

3.4.1 Theory

The idea of fuzzy extractors arose from the need to deal with noisy data. Therefore, the building parts were somehow spread throughout the literature until Dodis et al. [7] gathered them all into a general and unified theory. Instead of devising with fuzzy extractors immediately, the notion of secure sketches is introduced. A secure sketch is a pair of functionalities: sketch and recover. First, upon input $t$, sketch outputs a string $s$. Then, when recover receives input $t'$ and the sketch-computed $s$, it outputs $t$ provided $t'$ is close enough to $t$. The required property that $t$ remains largely unknown even though $s$ is available ensures the security of the sketch. The main result of [7] is to prove that (and show how) fuzzy extractors can be built from secure sketches using strong randomness extractors. The fuzzy extractor builds upon secure sketches as depicted in Fig. 8. In an enrollment phase, the input to the sketch procedure is also sent to a strong extractor, together with randomness $u$, which generates output $R$. The pair $(u, s)$ is stored as data helper, $W$. Then, in a reconstruction phase, the data helper is used to regenerate the output $R$ from a new input $t'$ through recover and extract. In practice, the construction of secure sketches requires a metric to quantify closeness, e.g. hamming distance, set difference or edit distance. An example using the hamming distance metric is discussed next.

![Figure 8: Turning a secure sketch into a fuzzy extractor.](image)

3.4.2 Practice

In this section, an overview of the practical approach developed by Tuyls et al. [14] is given. The same approach will be used for the paper case. Their article deals with proof-read hardware, i.e. a piece of hardware within which the key is not stored but is regenerated when needed. In order to achieve this, two additional layers are placed on the top of an integrated circuit. The first one is a grid of capacitive sensors and the second one is an opaque coating containing two kinds of dielectric particles. The capacitances of this coating are characterized both across the set of built chips and within a set of measures for the same
chip in order to approximate their inter- and intra-class distributions (that are generally assumed to be Gaussian). Comparing the standard deviations of these distributions (as in Fig. 9) already gives an intuitive insight on the feasibility to discriminate different chips. The fuzzy extractor then works as theoretically described in the previous section: an enrollment is performed once to compute the extracted fingerprint and its data helper; then reconstruction is performed any time the key is needed. The global scheme is depicted in Fig. 11.

Figure 9: Comparing the capacitance inter- and intra-class distributions.

Figure 10: Using the data helper $W$ to recover a key from a measurement.

During the enrollment phase (left part of Fig. 11), all sensors on a chip measure the local capacitance of the coating. A first part of the data helper, denoted as $w^*$, is built as the shift to center the measures in the interval they lie in. Those intervals are also used to convert each measured value to a short sequences of bits. The concatenation of those short sequences, the fingerprint $X$ is used together with the codeword $C_K$ (hiding the key $K$) to generate the second part of the data helper, denoted as $W = X \oplus C_K$.

Figure 11: Global view of the scheme: Enrollment and Reconstruction phases.

When reconstructing the key (right part of Fig. 11), each output value of the sensors is corrected with the first part of the helper data and then mapped to a short sequence of bits whose concatenation is denoted $Y$. Provided that $Y$ is not too far away from $X$ (in the sense of hamming distance), $C_K$ can be recovered by decoding $Y \oplus W$. As the map between the key $K$ and the codeword $C_K$ is uniquely defined, $K$ is immediately identified by $C_K$ (see the right part of Fig. 10). Hence, $K$ can be regenerated at will without worrying about the
measurement variations. But if the measures are too far from the ones used during the enrollment, K won’t be recovered. And this is exactly the expected behavior: if the measures are too different, the chips assumed to be attacked and hence should prevent access to the key. We refer to [14] for more details.

4 The Paper Case

To avoid confusion, it is worth clarifying that in this work, fingerprint denotes the X or Y bit string built from the physics (during enrollment or reconstruction), whereas identifier stand for the K bit string generated from a random source. In Tuyls et al., K is a key that could be used for encryption. In our case, K is an identifier that can be recovered. This difference will be reminded later.

From the description of previous section, there are three main steps in the extraction of a fingerprint from random physical measurements, namely the measurement of the physical feature, the characterization of its probability distribution and the generation of the bit sequences (X or Y). For the paper case, the obvious measurement tool is the one mimicking the human eye. The approach that we chose was to slightly modify a scanner by replacing its white tube by a fluorescent lamp. The measurement that is performed is thus a uv-scan of the paper sheet outputting a 24-bit color picture. Using image processing techniques, a list of fibers is then established, each of which is described as tuple containing position, orientation, surface and color (YUV-components). Given this as input, the characterization of the probability distributions depends on mainly three parameters that we detail in the rest of this section.

Number of sensors. As described in Fig. 12, a sheet of paper can be divided in different equal-area sub-sheets. By analogy with the previous coating PUF example, we denote each of those sub-sheets as a sensor. Quite naturally, one may decide to extract (a lot of) information from a single sensor or (less) information from several sensors considered separately.

Number of features per sensor. Given one sensor, we can try to measure different features from its fibers. For example, we could measure the amount of fibers N, their orientation O, their luminance L or their overall surface S. In the following, we will consider those four exemplary features.

Number of bins per sensor. Eventually, one has to determine how much information we try to extract from each sensor and feature, i.e. the number of bins used to partition the inter-class distributions. It generally results in a tradeoff between information and robustness. For example, Fig. 13 depicts an (imaginary) inter-class distribution. This distribution is sliced in 4 in its bottom part and in 8 in its upper part. These 4-bin and 8-bin strategies will result to a maximum entropy of 2 or 3 bits of information per sensor and feature. Note that the short bit sequences attached to each of the bins are the binary strings of a Gray code (the same way as in [14]), which allow improving the robustness of the extraction: if one of the sensor is slightly deviating from its enrollment value, moving the measure from one bin to its neighbor will result in a string Y that only differs in 1 bit position from the enrollment string X. Hence, such an error will be easily corrected when decoding.
In order to characterize the paper, the whole set of available sheets has been scanned (1000 different sheets) as well as 16 randomly chosen sheets that were scanned 100 times. The Lilliefors test was applied to all the samples to check whether the measurements match a normal distribution, which was actually the case. As an illustration, the intra- and inter-class distributions of two features (amount of fibers $N$ and orientation $O$) are provided in Appendix. Note that we performed measurements with and without rulers in order to additionally evaluate the impact of two slightly different setups. A small but noticeable improvement when using rulers can be seen between the two columns on the left and the two on the right of the appendix figures (with the exception of the bottom right picture that represents the inter-class distribution).

### 4.1 Evaluation Criteria

To evaluate the protocol, two different aspects need to be evaluated: the robustness of the process and the entropy provided by the sheets of secure paper.

**Robustness.** We first need to ensure that a correctly scanned paper will be recognized immediately (without requiring additional scan of the same sheet). It is estimated through the success rate (SR). This later one is simply computed as the ratio between the amount of correct measures and the amount of measures.

**Entropy.** We then need to measure the amount of information that can be extracted from the secure paper. Entropy estimations will be used to evaluate the number of different fingerprints that we can expect from the process.

### 4.2 Analysis

#### 4.2.1 Robustness

We first evaluated the success rate, i.e. the robustness, at the output of the scheme, see Fig. 11. This was done using the 16 sheets that were scanned 100 times. For each of those 16 sheets, the enrollment phase is performed with one of the scan to build both parts of the data helper. Then, the reconstruction phase is carried out for the remaining 99 scans. Each time the identifier generated
during the enrollment is correctly recovered, it increases the success rate. The result for the N feature is shown in Fig. 14 without error-correcting codes (ECC) and in Fig. 15 when a BCH(7, 4, 1) code is applied to improve the robustness.

![Figure 14: Success rate without error-correcting codes (N feature).](image1)

![Figure 15: Success rate with error-correcting codes (N feature).](image2)

When multiple features and multiple sensors are taken into account, the fingerprint is built as the concatenation over the sensors of the concatenation over the features of the Gray codes, namely:

\[
X = s \left( \big|| \ F \ GC(F, S) \big| \right) = \underbrace{NOLS_{S_0}}_{S_0} \bigg| \underbrace{NOLS_{S_1}}_{S_1} \bigg| \cdots \bigg| \underbrace{NOLS_{S_S}}_{S_S}
\]

The resulting success rate is pictured Fig. 16 and 17, without and with BCH code.

![Figure 16: Success rate without error-correcting codes (4 features).](image3)

![Figure 17: Success rate with error-correcting codes (4 features).](image4)
Increasing either the number of sensors or the number of bins decreases the success rate. However, it is also clear that increasing the number of sensors is a better approach than increasing the number of bins with respect to robustness.

Other parameters for the error-correcting code can be plugged in. For example, Fig. 18 and Fig. 19 use BCH codes able to correct up to 2 and 3 errors, respectively. In the case of 64 sensors, 2 bins and BCH(31, 16, 3), this improves the success rate up to 95%, a reasonable target value for real applications.

![Figure 18: Success rate with ECC: BCH(15, 7, 2) (4 features).](image)

![Figure 19: Success rate with ECC: BCH(31, 16, 3) (4 features).](image)

Finally, we also evaluated the impact of embedding a ruler in the scanner to ensure proper positioning of the sheet before scanning. Out of the 16 sheets scanned 100 times, 6 were scanned after the ruler was setup.

![Figure 20: Success rate without ruler (4 features, 10 sheets, BCH(7, 4, 1)).](image)

![Figure 21: Success rate with a ruler (4 features, 6 sheets, BCH(7, 4, 1)).](image)
The difference already mentioned in the appendix is now easily observed in Fig. 20 and 21 that have to be compared with Fig. 17 where all the sheets have been evaluated without distinguishing the use (or not) of the ruler.

### 4.2.2 Entropy

In order to estimate the entropy of the physical source, we used 1000 fingerprints generated as pictured in Fig. 11. We started with a simple sensor-based approach in which we evaluated the entropy using histograms. We note that computing the entropy per sensor is meaningful as long as these sensors can be considered independent. This seems a reasonable physical assumption in our context. By contrast, it is obviously not a good solution to evaluate the entropy when exploiting different features that are most likely correlated. Anyway, the histogram-based approach was just applied for intuition purposes and combined with the more advanced evaluations described in the end of this section.

![Figure 22: One set to estimate the distribution and build the bins.](image1)

In practice, let us assume a single sensor per sheet and 4 bins per sensor. We first used 500 scans to determine the positions of the bins as in Fig. 22. Then, we used the second set of 500 scans to evaluate the probabilities of the previously determined bins as in Fig. 23. Eventually, we estimated the entropy per sensor using the classical formula:

$$H = -\sum_i p_i \log p_i$$

where the $p_i$ are the bin probabilities. Such a naive method gives the results shown in Fig. 24. Note that for some choices of parameters, the entropy was stuck to zero. This is simply because the number of samples was not sufficient to fill all the bins in those cases. Indeed, given the size of the sample set, one can determine the amount of bins that should not be crossed to keep meaningful results, e.g. using Sturges rule\(^1\): $\lceil 1 + \log_2 M \rceil$, with $M$ the size of the set. In our example, it states that there should be no more than 10 bins. The entropies stuck to ground in Fig. 24 can be seen as the limits for the given sample size. Quite naturally, we see that the informativeness of an extractor increases with the number of sensors and bins, contrary to its robustness in the previous section.

\(^1\)Scott’s formula gives a similar result: $\frac{3.5 \sigma}{M^{1/3}} = \frac{3.5 \times 23}{500^{1/3}} = 10.146\ldots$
In order to confirm these first estimations, we then ran more complex test suites, in particular: ent [17], Maurer’s test including Coron’s modification [11, 6, 5] and the Context Tree Weighting (CTW) method [18, 19, 20]. The main idea behind these entropy estimation tools is to compare the length of an input sequence and its corresponding compressed output. For the details about how they actually process the data, we refer to the previous links.

These final results achieved are given in table 1, where $X(F, S, B)$ denotes the fingerprint built upon features $F$ when cutting the paper in $S$ sensors sliced in $B$ bins. As previously explained, when multiple features and multiple sensors are involved, the fingerprint is built as the concatenation over the sensors of the concatenation over the features of the Gray codes, $X = \|_F \big( \|_S GC(F, S) \big)$. The first column where fingerprints are only built from the amount of fibers $(N)$ shows that almost 32 bits of entropy can be extracted from the 32-bit strings which essentially confirms that different sensors are indeed independent. By contrast, when using 4 different features as in the right part of the table, we clearly see that the entropy extracted per bit of fingerprint is reduced, i.e. the features (amount of fibers, orientation, luminance and surface) are actually correlated. Most importantly, we see that overall the proposed solution allows to generate fingerprints with slightly more than 96 bits of entropy while ensuring a good robustness. In other words, this solutions largely fulfills the goal of a low-cost authentication process that we target in this paper.

<table>
<thead>
<tr>
<th></th>
<th>$X(N, 32, 2)$</th>
<th>$X(NOLS, 32, 2)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ent</td>
<td>$1 \cdot 32 \cdot 0.99 = 31.68$</td>
<td>$4 \cdot 32 \cdot 0.99 = 126.72$</td>
</tr>
<tr>
<td>Maurer*</td>
<td>$1 \cdot 32 \cdot 0.99 = 31.68$</td>
<td>$4 \cdot 32 \cdot 0.63 = 80.64$</td>
</tr>
<tr>
<td>CTW</td>
<td>$1 \cdot 32 \cdot 0.99 = 31.68$</td>
<td>$4 \cdot 32 \cdot 0.75 = 96.53$</td>
</tr>
</tbody>
</table>

Table 1: Entropy estimations in entropy bits per fingerprint $X$. Note finally that our use of fuzzy extractors significantly differs from the one of Tuyls et al.. We use physical features to build unique (but public) identifiers while [14] aims to generate cryptographic keys. Therefore, we do not have to
evaluate the secrecy of our identifiers but only their randomness. This is because in our protocol, the overall security comes from the digital signature that is applied both to the identifiers and to the content printed on a paper. An attack against our scheme would require to find a sheet of paper that gives the same identifier to perform a copy/paste attack. This is supposed to be hard enough in view of the 96 bits of entropy that the physical features assumably provide.

5 Conclusion

In this paper, a proposal to secure documents is presented that combines previously introduced robust cryptographic mechanisms and information extractors with a source of physical randomness. It has the interesting feature to provide two levels of verification, trading rapidity for trust. The scheme is quite generic and could be tuned for different application needs. Our case study was developed for low-cost standard desktop equipment. But the robustness, randomness and (mainly) unclonability of our proposal could be improved at the cost of a more expensive infrastructure. We also provide a detailed and motivated statistical analysis of the information extraction scheme. In the studied case, embedded ultra-violet fibers allows extracting 128-bit strings that correspond to an entropy of approximately 96 bits while providing 72-bit identifiers when applying an error correcting code. The resulting identifiers can be extracted with high robustness. This is considered to provide a sufficient security since an adversary would have to scan a prohibitive amount of secure paper to find a collision.

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References


Figure 25: Intra- and inter-class distributions for the amount of fibers $N$. 

$\mu = 385.51, \sigma = 8.62, \frac{\sigma_e}{\sigma_a} = 2.67$

$\mu = 388.13, \sigma = 7.22, \frac{\sigma_e}{\sigma_a} = 3.19$

$\mu = 388.42, \sigma = 4.73, \frac{\sigma_e}{\sigma_a} = 4.87$

$\mu = 369.22, \sigma = 8.42, \frac{\sigma_e}{\sigma_a} = 2.73$

$\mu = 349.87, \sigma = 6.60, \frac{\sigma_e}{\sigma_a} = 3.49$

$\mu = 374.40, \sigma = 5.33, \frac{\sigma_e}{\sigma_a} = 4.32$

$\mu = 402.58, \sigma = 8.24, \frac{\sigma_e}{\sigma_a} = 2.79$

$\mu = 363.44, \sigma = 7.18, \frac{\sigma_e}{\sigma_a} = 3.20$

$\mu = 407.99, \sigma = 4.61, \frac{\sigma_e}{\sigma_a} = 4.99$
Figure 26: Intra- and inter-class distributions for the orientation of the fibers $O$. 