Identification of the Barycentric Parameters of Robot Manipulators from External Measurements

B. RAUCENT,† G. CAMPION,‡§ G. BASTIN,‡ J. C. SAMIN† and P. Y. WILLEMS‡

Key Words—Robot modelling; parameter estimation.

Abstract—An original procedure for the estimation of the barycentric parameters of a robot is presented. This procedure requires only the processing of measurements provided by an external experimental set-up. The procedure is based on the property that the relations between the robot motion and its reactions on the bedplate are completely independent of the internal joint forces. A convincing validation experiment on a PUMA 562 is reported.

1. Introduction

The control algorithms which are commonly implemented on industrial robots do not account for the inherent nonlinearities of the dynamical robot motion. For this reason these control laws break down for high speed operation when the nonlinear effects become important. High speed and high-precision control can however be achieved using advanced control algorithms, such as “the computed torque” control. Such control laws require an accurate and in-depth knowledge of the robot dynamical model. Under the assumption of a rigid links, the dynamical equations, derived for instance using Lagrange’s formalism, take the form of a set of nonlinear equations. The structure of these equations is well known but they involve characteristic parameters which have to be estimated with a good precision. These parameters can be classified in four sets.

(1) The geometrical parameters (lengths of the links, positions of the joints, ...), which are constant and can be assumed to be known with a good accuracy, because they result directly from the design and the construction of the manipulator.

(2) The inertial parameters of the links, characterizing the mass distribution for each link. The numerical values of these parameters are constant during the lifetime of the manipulator and can therefore be identified once and for all in the course of the acceptance tests.

(3) The terminal mass parameters which are constant for a given operation but can, of course, vary from operation to operation.

(4) The joint parameters (friction in the joints, ...) undergo a slow variation over the manipulator lifetime. Moreover the modeling of the friction effects (viscous and/or Coulomb friction) is rather complicated.

For most commercial robots the values of these parameters are unknown and parameter estimation is therefore necessary for advanced control algorithm implementation. This paper concentrates on the estimation of the inertial parameters independently of the friction effects and under the assumption that the geometric parameters are known.

In the classical identification approach (see e.g. Mayeda et al., 1984; Ferreira, 1984; Olsen and Bekey, 1985; Gautier, 1986; Armstrong et al., 1986) the values of the parameters are estimated from input (torques applied to the links) and output (positions, velocities and accelerations of the links) data provided by “internal” measurement devices located inside the arms. The dynamical model relating these inputs and outputs is described by a set of differential equations which are linear in a set of so-called barycentric parameters which are themselves nonlinear functions of the inertial and terminal mass parameters (see Raucent, 1990). This implies that the estimation of these barycentric parameters can be performed in principle by linear regression. The practical implementation however presents an important drawback: the torques applied to the links are not directly available but have to be evaluated as sums of the torques provided by the actuators and of the friction torques which may be important. Two problems then occur:

(a) For most commercial robots the torques provided by the actuators can be obtained from internal measurements, but with a poor accuracy. For instance, when the actuator is a DC motor, the torque is measured via the input current through a torque constant which is given from the manufacturer’s technical data, albeit with a low precision. Furthermore it can vary over the robot’s lifetime.

(b) The implementation of the parameter estimation requires an accurate model of the friction effects. The parameters involved in the friction model have to be estimated together with the barycentric parameters. This coupling can substantially degrade the accuracy of the estimation of the barycentric parameters.

In this paper, we present an alternative approach for the estimation of the barycentric parameters which avoids the two above-mentioned drawbacks (see also Raucent et al., 1988 and Raucent, 1990).

(i) The estimation is based on a reformulation of the dynamics of the system which relates the motion of the robot to the reaction forces and torques on the bedplate and is, therefore, totally independent from the internal torques (i.e. actuator torques and friction torques).
The estimation method makes use of external measurement only, obtained with a specific experimental set-up. The paper is organized as follows. The experimental set-up is described in Section 2. The auxiliary reaction model which relates the motion of the robot to the reaction forces and torques on the bedplate is presented in Section 3 while its parametrization is discussed in Section 4. Finally two validation experiments are reported in Section 5.

2. Sensors and instrumentation

The experimental set-up we have developed for this study is as follows (see Fig. 1).

(a) The robot is placed on a sensing platform (Kistler Instrumente AG) which is provided with sensors able to measure the three components of the forces and the three components of torques between the bedplate and the first link of the robot. The relative accuracy of this measurement is about 1%. The advantage of this experimental set-up is to provide data (which are processed by the estimation algorithm) with a much better accuracy than those obtained from the actuators.

(b) The measurement of the position of each link is performed by a high-precision visual position sensor (SELCOM AB). Several Light Emitting Diodes are attached to each link of the robot. The light emitted from each diode is captured by two special cameras and, after analogic-to-digital conversion, a computer programme converts the two images into a three-dimensional result. Positions of the joint are then computed. Finally, the velocity and acceleration are evaluated by numerical differentiation (with appropriate noise digital filtering). The resolution of the system is 1/4096 of the measuring range and the accuracy is about 1/500 of the full scale which depends on the location of the cameras.

By this experimental set-up, all the data which are necessary for the estimation are obtained externally and independently of the hardware of the robot control unit.

3. Robot models

In this section we derive the equations describing the dynamical model of the robot (relating the motion of the robot to the generalized forces applied to the links) and the reaction model (relating the motion of the robot to the forces and torques applied to the bedplate).

Consider a robot manipulator with \( n \) rigid links. The \( n \) joint coordinates are denoted \( q_d \). Consider in addition a virtual motion of the robot with respect to its bedplate, characterized by six extra coordinates, \( q_r \) (three for the virtual translation motion, and three for the virtual rotation motion). In this way we define a generalized system with \( (n+6) \)-degrees-of-freedom. Of course, any actual motion of the robot is such that \( q_r \) remains identically equal to zero: \( q_r(t) = 0 \) for all \( t \). Due to this constraint, the robot equations are derived using Lagrange multipliers.

Defining the kinetic energy by:

\[
T(q_d, q_r, \dot{q}_d, \dot{q}_r) = \frac{1}{2} (\dot{q}_d, M(q_d, q_r, \dot{q}_d, \dot{q}_r)),
\]

where

\[
M = \begin{pmatrix}
M_{dd}(q_d, q_r) & M_{dr}(q_d, q_r) \\
M_{rd}(q_d, q_r) & M_{rr}(q_d, q_r)
\end{pmatrix},
\]

is the \( (n+6) \times (n+6) \) symmetric definite positive inertia matrix of the generalized system; and denoting \( U(q_d, q_r) \) the potential energy associated with gravity, the robot motion satisfies the following Lagrange equations:

\[
\frac{d}{dt} \left( \frac{\partial T}{\partial \dot{q}_d} \right) - \frac{\partial T}{\partial q_d} + \frac{\partial U}{\partial q_d} = Q_d,
\]

\[
\frac{d}{dt} \left( \frac{\partial T}{\partial \dot{q}_r} \right) - \frac{\partial T}{\partial q_r} + \frac{\partial U}{\partial q_r} = Q_r,
\]

together with the constraints:

\[
q_r = 0.
\]
The dynamical model of any mechanical system made up of rigid bodies is characterized by a set of "inertial" parameters which describe the mass distribution in each link. For each rigid body in the robot there are 10 such basic parameters (one for the mass, three for the position of the center of mass and six for the inertia matrix of the body) but it is well known that there exists a reparametrization, called the barycentric parametrization (Fisher, 1906) which enters the model linearly. Actually, the D- and R-models involve only a few independent linear combinations of the basic set of barycentric parameters. Recursive methods (Raucent, 1990; Gautier, 1990) are used to calculate these linear combinations and lead to the definition of an identifiable parametrization of the link is as follows:

\[ \theta_d = S_d \theta_d, \quad \theta_r = S_r \theta_r, \]

where \( \theta \) is the full set of \( N \) barycentric parameters and \( S_d \) and \( S_r \) are two full rank constant matrices, respectively of dimension \((N_d \times N)\) and \((N_r \times N)\). It is easily shown in addition that:

1. \( N_d < N_r \)
2. there exists a full rank \((N_d \times N)\) matrix \( S \) such that:

\[ \theta_d = S \theta_r. \]  

(5)

This means that the values of the parameters of the D-model (i.e. \( \theta_d \)) can be deduced from the values of the parameters of the R-model (i.e. \( \theta_r \)). It must be kept in mind that we are interested mainly in the numerical values of \( \theta_d \), because control design is based on the D-model only.

The linearity of the D- and R-models with respect to the parametrizations \( \theta_d \) and \( \theta_r \) implies that \( M_d, M_a, f_d \) and \( f_r \) can be expressed linearly in the components of \( \theta_d \) and \( \theta_r \), as follows:

\[ M_d(q_d, \theta_d) = M_{a0}(q_d, 0) + \sum_{i=1}^{N_d} M_{a1}(q_d, 0) \theta_d, \]

\[ M_a(q_d, \theta_d) = M_{a0}(q_d, 0) + \sum_{i=1}^{N_d} m_d(q_d, 0) \theta_d, \]

\[ f_d(q_d, q_d, 0, 0) = f_{a0}(q_d, q_d, 0, 0) + \sum_{i=1}^{N_d} f_{a1}(q_d, q_d, 0, 0) \theta_d, \]

\[ f_r(q_d, q_d, 0, 0) = f_{r0}(q_d, q_d, 0, 0) + \sum_{i=1}^{N_r} f_{r1}(q_d, q_d, 0, 0) \theta_d, \]

where the \( M_{a0}, M_{a1}, M_{r0}, M_{r1}, f_{a0}, f_{a1}, f_{r0}, f_{r1} \) are known functions of \( q_d \) and \( \theta_d \). The R-model can therefore be rewritten as:

\[ Q_r - M_{a0}(q_d, 0) \dot{\theta}_d - f_{a0}(q_d, q_d, 0, 0) = \phi(q_d, q_d, 0, 0) \theta_d, \]

where \( \phi \) is a \( N_r \times 6 \) matrix whose \( i \)th column is given by:

\[ M_{a0}(q_d, 0) \ddot{q}_d + f_{a0}(q_d, q_d, 0, 0). \]

4. Barycentric parametrization

The dynamical model of any mechanical system made up of rigid bodies is characterized by a set of "inertial" parameters which describe the mass distribution in each link. For each rigid body in the robot there are 10 such basic parameters (one for the mass, three for the position of the center of mass and six for the inertia matrix of the body) but it is well known that there exists a reparametrization, called the barycentric parametrization (Fisher, 1906) which enters the model linearly. Actually, the D- and R-models involve only a few independent linear combinations of the basic set of barycentric parameters. Recursive methods (Raucent, 1990; Gautier, 1990) are used to calculate these linear combinations and lead to the definition of an identifiable parametrization of the link is as follows:

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1. \( N_d < N_r \)
2. there exists a full rank \((N_d \times N)\) matrix \( S \) such that:

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The linearity of the D- and R-models with respect to the parametrizations \( \theta_d \) and \( \theta_r \) implies that \( M_d, M_a, f_d \) and \( f_r \) can be expressed linearly in the components of \( \theta_d \) and \( \theta_r \), as follows:

\[ M_d(q_d, \theta_d) = M_{a0}(q_d, 0) + \sum_{i=1}^{N_d} M_{a1}(q_d, 0) \theta_d, \]

\[ M_a(q_d, \theta_d) = M_{a0}(q_d, 0) + \sum_{i=1}^{N_d} m_d(q_d, 0) \theta_d, \]

\[ f_d(q_d, q_d, 0, 0) = f_{a0}(q_d, q_d, 0, 0) + \sum_{i=1}^{N_d} f_{a1}(q_d, q_d, 0, 0) \theta_d, \]

\[ f_r(q_d, q_d, 0, 0) = f_{r0}(q_d, q_d, 0, 0) + \sum_{i=1}^{N_r} f_{r1}(q_d, q_d, 0, 0) \theta_d, \]

where the \( M_{a0}, M_{a1}, M_{r0}, M_{r1}, f_{a0}, f_{a1}, f_{r0}, f_{r1} \) are known functions of \( q_d \) and \( \theta_d \). The R-model can therefore be rewritten as:

\[ Q_r - M_{a0}(q_d, 0) \dot{\theta}_d - f_{a0}(q_d, q_d, 0, 0) = \phi(q_d, q_d, 0, 0) \theta_d, \]

where \( \phi \) is a \( N_r \times 6 \) matrix whose \( i \)th column is given by:

\[ M_{a0}(q_d, 0) \ddot{q}_d + f_{a0}(q_d, q_d, 0, 0). \]
The equations of the R-model are:
\[
\begin{pmatrix}
0 & g & 0 & 0 & \sin(q)q & \cos(q)q^2 \\
ge^{-q} & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & \cos(q)q - \sin(q)q^2 & 0 \\
0 & 0 & 0 & 0 & 0 & \cos(q)q - \sin(q)q^2 \\
g & 0 & -\sin(q)q & \cos(q)q^2 & 0 & 0
\end{pmatrix}
\begin{pmatrix}
\theta_1 \\
\theta_2 \\
\theta_3 \\
\theta_4 \\
\theta_5 \\
\theta_6
\end{pmatrix}
= Q_r - J_a \tau.
\]

with
\[
Q_r = (C_r, F_r, C_r, F_r, C_r, F_r)^T,
\]
and
\[
\theta_i = (m, m_{by}, m_{bz}, K_{yy}, K_{zz}, T_i)^T.
\]

It appears that five barycentric parameters are involved in the model and that the terms containing \(m\) and \(m_b\) are constant. The components of \(\theta_i\) are a subset of the components of \(\theta_0\) so that the value of \(\theta_i\) just by substituting \(S = \begin{pmatrix} 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix}\) in equation (5).

5. Experimental results

Two experimental applications on an industrial PUMA robot are reported in this section. The first one is presented to illustrate the identification method. The second one is a validation test where the parameter estimates are compared with their true values.

5.1. Description of the PUMA. The PUMA 562 (Unimation) is a serial six-degrees-of-freedom manipulator with six revolute joints (Fig. 3). We limit our attention to the first three joints. The other joints which rely on the wrist are assumed to be fixed. We introduce four reference frames:

(a) An inertial basis \((x^0, y^0, z^0)\) attached to the base of the robot, where \((x^0, y^0)\) define the horizontal plane of the bedplate and \(z^0\) is the vertical axis.

(b) A basis \((x^1, y^1, z^1)\) attached to the first link with \(z^1 = z^0\) the axis of rotation (angle \(q^1\)) and positioned by the vector: \(D^1 = d^1 z^0\).

(c) A basis \((x^2, y^2, z^2)\) attached to the second link with \(y^2 = y^1\) the axis of rotation (angle \(q^2\)).

(d) A basis \((x^3, y^3, z^3)\) attached to the third link with \(y^3 = y^2\) the axis of rotation (angle \(q^3\)) and positioned by the vector: \(D^3 = d^3 x^1 + d^3 y^1\).

The mass distribution is described as follows:

* the masses of the three links are denoted \(m^1, m^2\) and \(m^3\).
* the position of the centers of mass are expressed by:

\[
\begin{align*}
R^1 &= d^1 x^1 + \frac{m^1}{2} (\frac{d^1 x^1}{d^1 y^1})^2 \\
R^2 &= d^2 x^2 + \frac{m^2}{2} (\frac{d^2 x^2}{d^2 y^2})^2 \\
R^3 &= d^3 x^3 + \frac{m^3}{2} (\frac{d^3 x^3}{d^3 y^3})^2
\end{align*}
\]

* the central inertia matrices of the three links are expressed in the bases \((x^i, y^i, z^i)\) with \(i = 1, 2, 3\), by:

\[
J^i = \begin{pmatrix}
J_{xx}^i & J_{xy}^i & J_{xz}^i \\
J_{yx}^i & J_{yy}^i & J_{yz}^i \\
J_{zx}^i & J_{zy}^i & J_{zz}^i
\end{pmatrix}
\]

As in the example of Section 4, we note that some of these inertial parameters are known a priori to be zero. In particular, the inertia matrices of the links are diagonal because the chosen reference frames are supposed to be aligned with the principal axes of inertia. Moreover, it must be pointed out that the motor characteristics are given by the manufacturer and are therefore not to be estimated.

In order to satisfy the identifiability conditions presented in Section 5, the parametrizations \(\theta_d\) and \(\theta_r\) must be defined as follows (see Rauchent, 1990):

\[
\begin{align*}
\theta_d &= \begin{pmatrix}
J_{xx}^1 & J_{xy}^1 & J_{xz}^1 & J_{xx}^2 & J_{xy}^2 & J_{xz}^2 & J_{xx}^3 & J_{xy}^3 & J_{xz}^3 \\
J_{yx}^1 & J_{yy}^1 & J_{yz}^1 & J_{yx}^2 & J_{yy}^2 & J_{yz}^2 & J_{yx}^3 & J_{yy}^3 & J_{yz}^3 \\
J_{zx}^1 & J_{zr}^1 & J_{xz}^3 & J_{zx}^2 & J_{zr}^2 & J_{xz}^3 & J_{zx}^3 & J_{zr}^3 & J_{xz}^3
\end{pmatrix}
\end{align*}
\]

It appears that \(\theta_d\) is included in \(\theta_r\). The corresponding D and R-models equations have been automatically generated by the software ROBOTRAN developed by Maes (1990).

5.2. Identification of the link parameters. The identification of the PUMA 562 parameters has been performed from data obtained with the external experimental set-up described in Section 2. The results are compared in Table 1 with parameter values calculated on the basis of the data obtained with internal measurements given respectively by Tarn et al. (1985) and Armstrong et al. (1986).

Table 1 clearly shows that the values obtained with our approach are of the same order of magnitude as the values obtained with other methods. A more precise validation of the results is not possible in this case since the exact values of the parameters of the PUMA 562 are not available. In the next section we present a genuine validation experiment where the estimates can be compared to true values.

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<tbody>
<tr>
<td>(\theta_1) (kg m(^2))</td>
<td>1.665</td>
<td>1.920</td>
<td>1.357</td>
</tr>
<tr>
<td>(\theta_2) (kg m(^2))</td>
<td>2.888</td>
<td>2.384</td>
<td>2.829</td>
</tr>
<tr>
<td>(\theta_3) (kg m(^2))</td>
<td>2.234</td>
<td>2.786</td>
<td>2.174</td>
</tr>
<tr>
<td>(\theta_4) (kg m(^2))</td>
<td>-0.598</td>
<td>-0.558</td>
<td>-0.605</td>
</tr>
<tr>
<td>(\theta_5) (kg m(^2))</td>
<td>0.567</td>
<td>0.533</td>
<td>0.300</td>
</tr>
<tr>
<td>(\theta_6) (kg m(^2))</td>
<td>0.545</td>
<td>0.547</td>
<td>0.336</td>
</tr>
<tr>
<td>(\theta_7) (kg m(^2))</td>
<td>-0.103</td>
<td>-0.150</td>
<td>-0.142</td>
</tr>
<tr>
<td>(\theta_8) (kg m(^2))</td>
<td>3.212</td>
<td>3.702</td>
<td>3.790</td>
</tr>
<tr>
<td>(\theta_9) (kg m(^2))</td>
<td>0.802</td>
<td>1.061</td>
<td>0.864</td>
</tr>
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</table>
5.3. Validation test: identification of known load parameters. We reduce the PUMA to a one-degree-of-freedom robot by only moving the third link (Fig. 4). As shown in Section 4 the parameters of the D-model reduce, in this case, to:

\[ \theta_d = \left( \frac{m_{b_d}}{K_{rr}} \right) \]

We now modify the system by adding a known load mass \((M = 3.87 \text{ kg})\) attached to the third link. The position of the center of mass of the load is given by the vector \(\mathbf{R} = \mathbf{l} \mathbf{z}^2\) (with \(l = 0.514 \text{ m}\)). The parameters of the modified system differ from the previous ones as follows:

\[ \theta_d^* = \left( \frac{m_{b_d}}{K_{rr}} \right) + \left( \frac{Ml}{MI} \right). \]

We estimate \(\theta_d^*\) with the same test trajectory as before. Then, by the comparison of the estimates of \(\theta_d^*\) and \(\theta_d\), we can deduce estimates of \(MI\) and \(Ml^2\) which can in turn be compared to their exact values. This is done in Table 2. The results in this Table show an extremely good agreement between the exact and estimated values, and hence lends the credibility of this identification approach.

<table>
<thead>
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<th>Table 2. Identification of the Load</th>
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<tr>
<td>Estimated</td>
</tr>
<tr>
<td>(Ml) (kg m)</td>
</tr>
<tr>
<td>(Ml^2) (kg m^2)</td>
</tr>
</tbody>
</table>

6. Conclusions

We have shown that the estimation of the barycentric parameters involved in the dynamic model of a robot can be achieved by processing external measurements only (reactions at the bedplate; positions, velocities and accelerations provided by a vision device).

The originality of this method lies in the use of the auxiliary reaction model (R-model) which is completely independent of the internal forces and torques at the joints while the usual parameter estimation algorithms are based on internal measurements (torques applied by the actuators) which are noisy and inaccurate, due mainly to the friction effects. In contrast, the use of external measurements of positions, velocities and accelerations is clearly not an imperative requirement. The interest is to devise "acceptance tests" that can be carried out independently of the hardware of the robot control unit. But, obviously if the robot is provided with high accuracy position and velocity sensors, internal measurement can be used as well in our identification method.

Finally, we must mention that even if the barycentric parameters have been identified with a great accuracy, the dynamical model of the robot will still contain some uncertain time varying parameters, namely the friction coefficients, actuator parameters (torque constant) and the barycentric parameters that are affected by the transported load. As usual these remaining uncertain parameters can be compensated by using adaptive techniques for the design of advanced control systems (see Canudas et al., 1987 and Craig, 1988).

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References