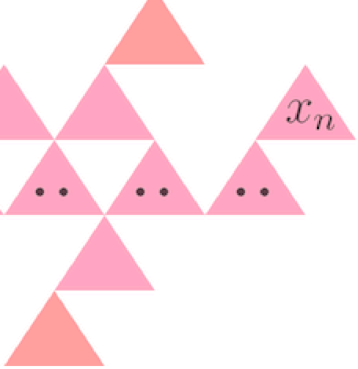

Non-Markovian Models of Networked Systems

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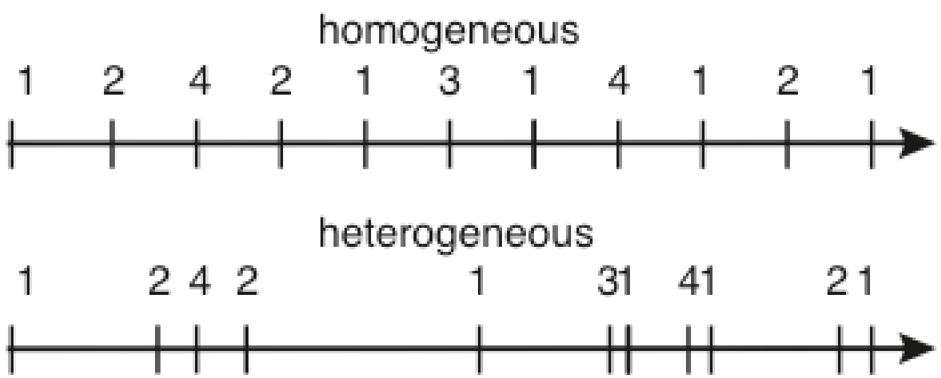
Memory in network flows and its effects on spreading dynamics and community detection, Martin Rosvall, Alcides V. Esquivel, Andrea Lancichinetti, Jevin D. West, Renaud Lambiotte, Nature Communications (2014)

Non-Markovian Models of Networked Systems: Time



$$\boldsymbol{x}(t) = \boldsymbol{x}_0 \int_t^{\infty} \rho(t')dt' + \int_0^t P\boldsymbol{x}(t-t')\rho(t')dt'$$

Diffusion on networked systems, a question of time or structure, J-C Delvenne, Luis Rocha and R. Iambiotte

Non-Markovian Models of Networked Systems: Pathways

New York → Chicago → New York	49,632
New York → Chicago → San Francisco	1,031
San Francisco → New York → Chicago → San Francisco	120
Atlanta → Chicago → Atlanta	17,207
Jacksonville → Atlanta → Chicago → Atlanta → Jacksonville	418

Memory in network flows and its effects on spreading dynamics and community detection, Martin Rosvall, Alcides V. Esquivel, Andrea Lancichinetti, Jevin D. West, Renaud Lambiotte, Nature Communications (2014)

I. Introduction

Network science in a nutshell

Construction of a network from empirical data, e.g. airline transportation network

Impact on dynamics
Definition of a model for epidemic spreading, e.g. meta-population model

Algorithms
Modularity, clustering, ranking, etc.

Pathways of diffusion, typically generated by a random walk process

Network science in a nutshell

Construction of a network from empirical data, e.g. airline transportation network

Impact on dynamics
Definition of a model for epidemic spreading, e.g. meta-population model

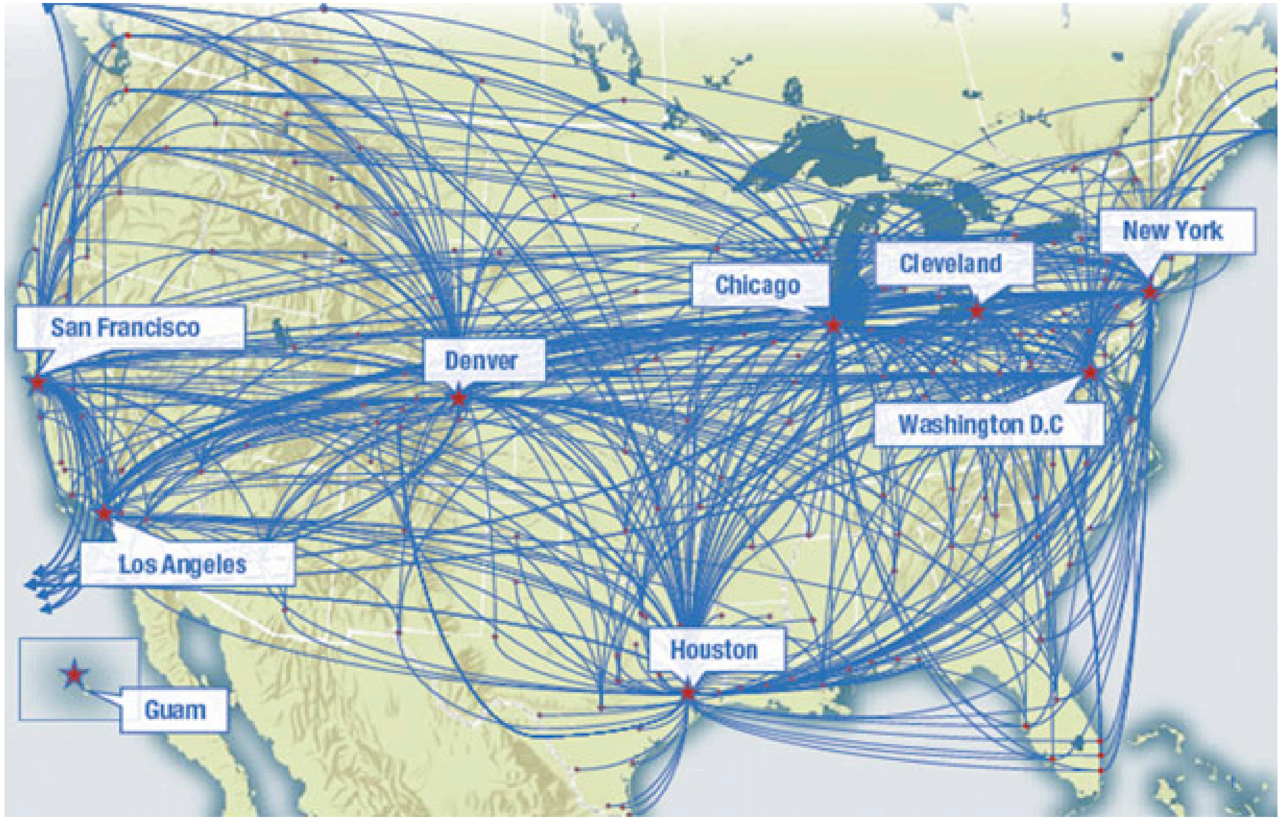
Algorithms
Modularity, clustering, ranking, etc.

Pathways of diffusion, typically generated by a **memoryless** random walk process

What if real-world pathways are available?

Networks with memory

Where you go to depends on where you come from
Mathematics of pathways instead of edges



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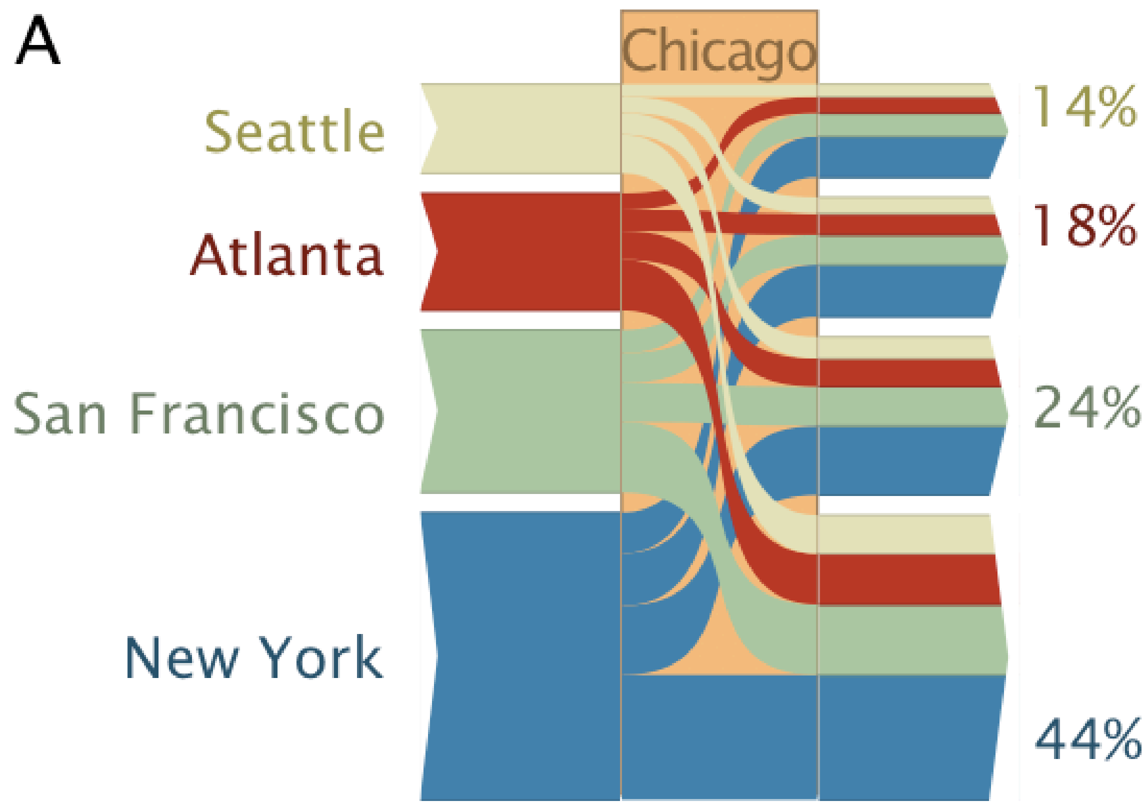

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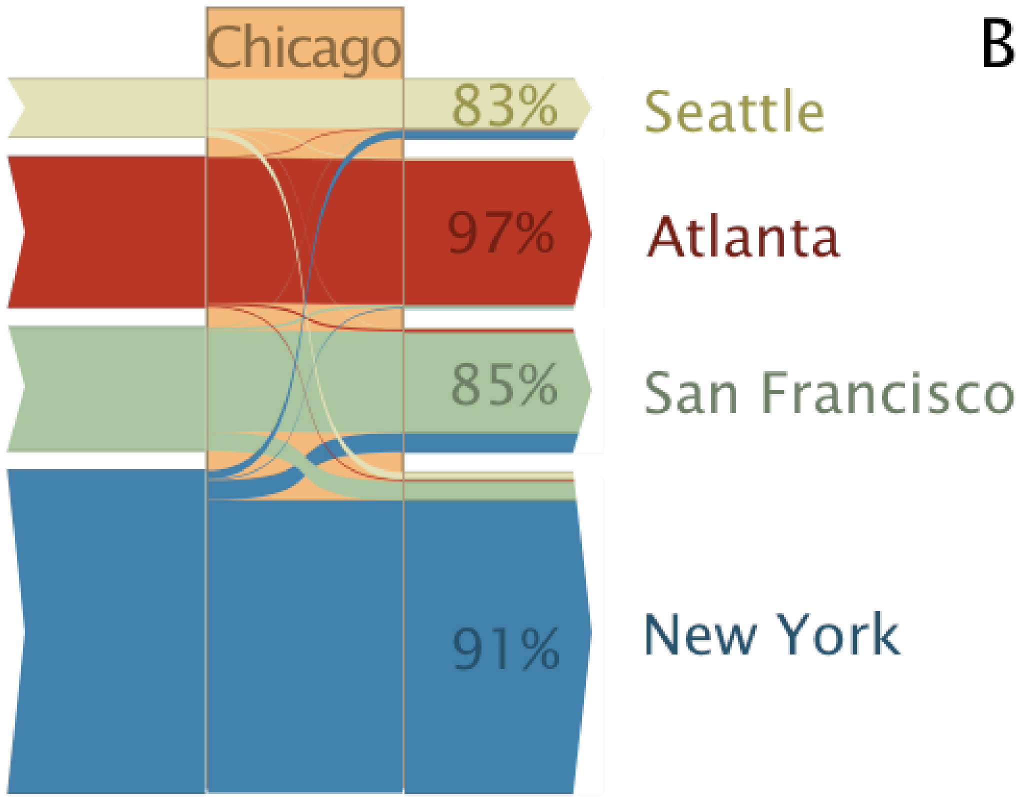

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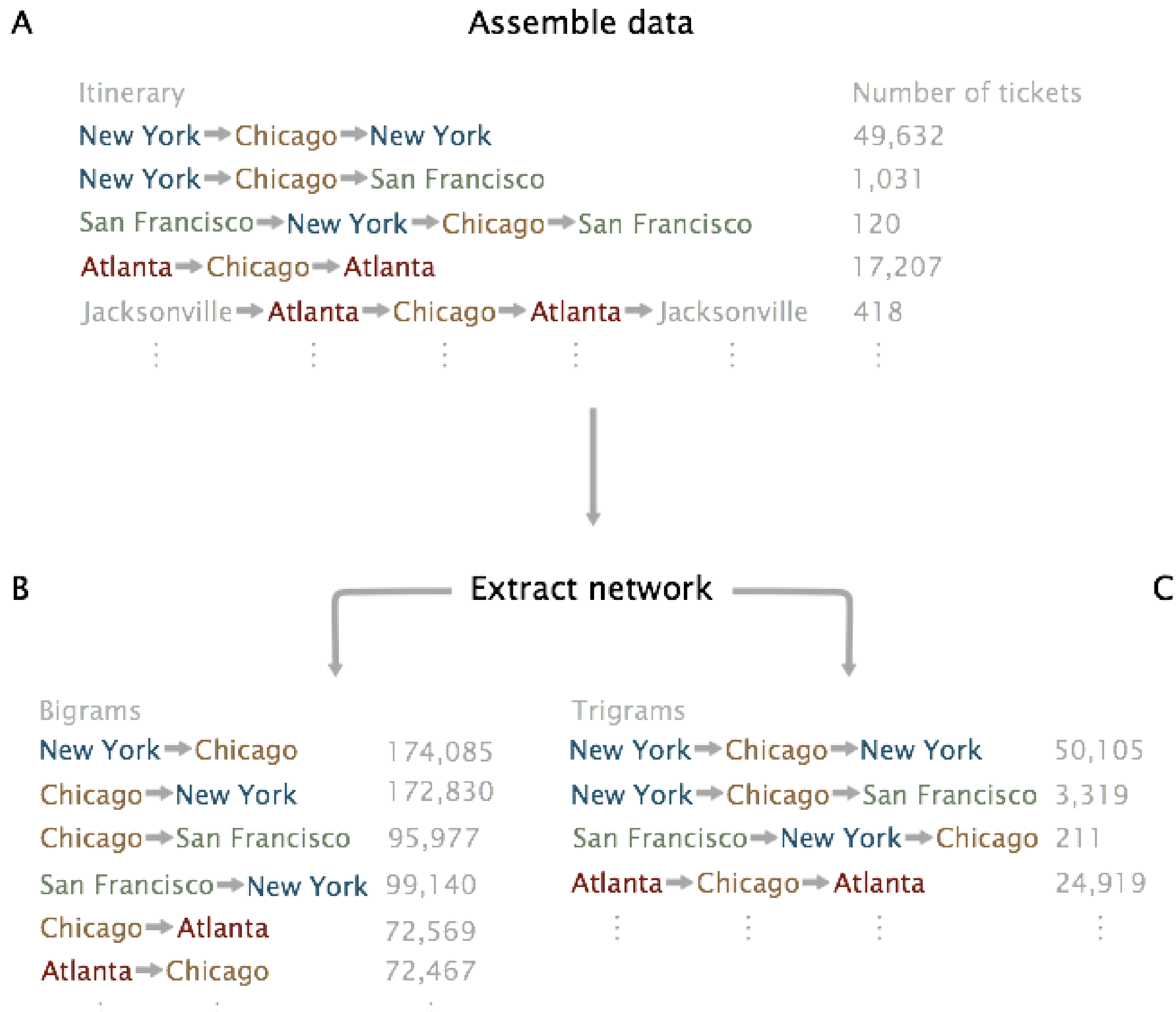
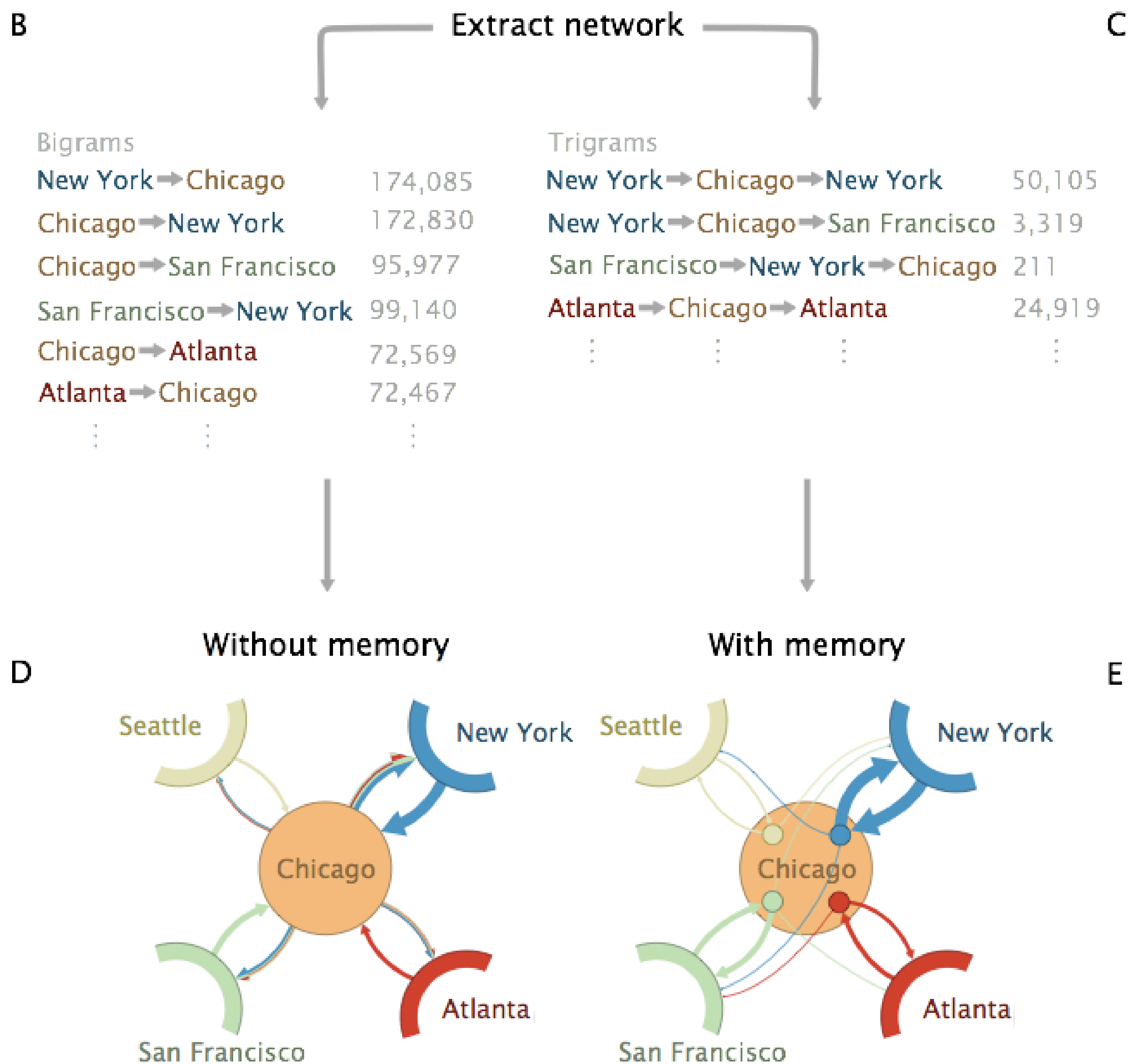
II. Data Extraction


A
Assemble data
Itinerary
Number of tickets
New York → Chicago → New York 49,632
New York → Chicago → San Francisco 1,031
San Francisco → New York → Chicago → San Francisco 120
Atlanta → Chicago → Atlanta 17,207
Jacksonville → Atlanta → Chicago → Atlanta → Jacksonville 418
B
Extract network
C
Bigrams
New York → Chicago 174,085
Chicago → New York 172,830
Chicago → San Francisco 95,977
San Francisco → New York 99,140
Chicago → Atlanta 72,569
Atlanta → Chicago 72,467
Trigrams
New York → Chicago → New York 50,105
New York → Chicago → San Francisco 3,319
San Francisco → New York → Chicago 211
Atlanta → Chicago → Atlanta 24,919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New York → Chicago → San Francisco 3,319
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Without memory
With memory
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San Francisco
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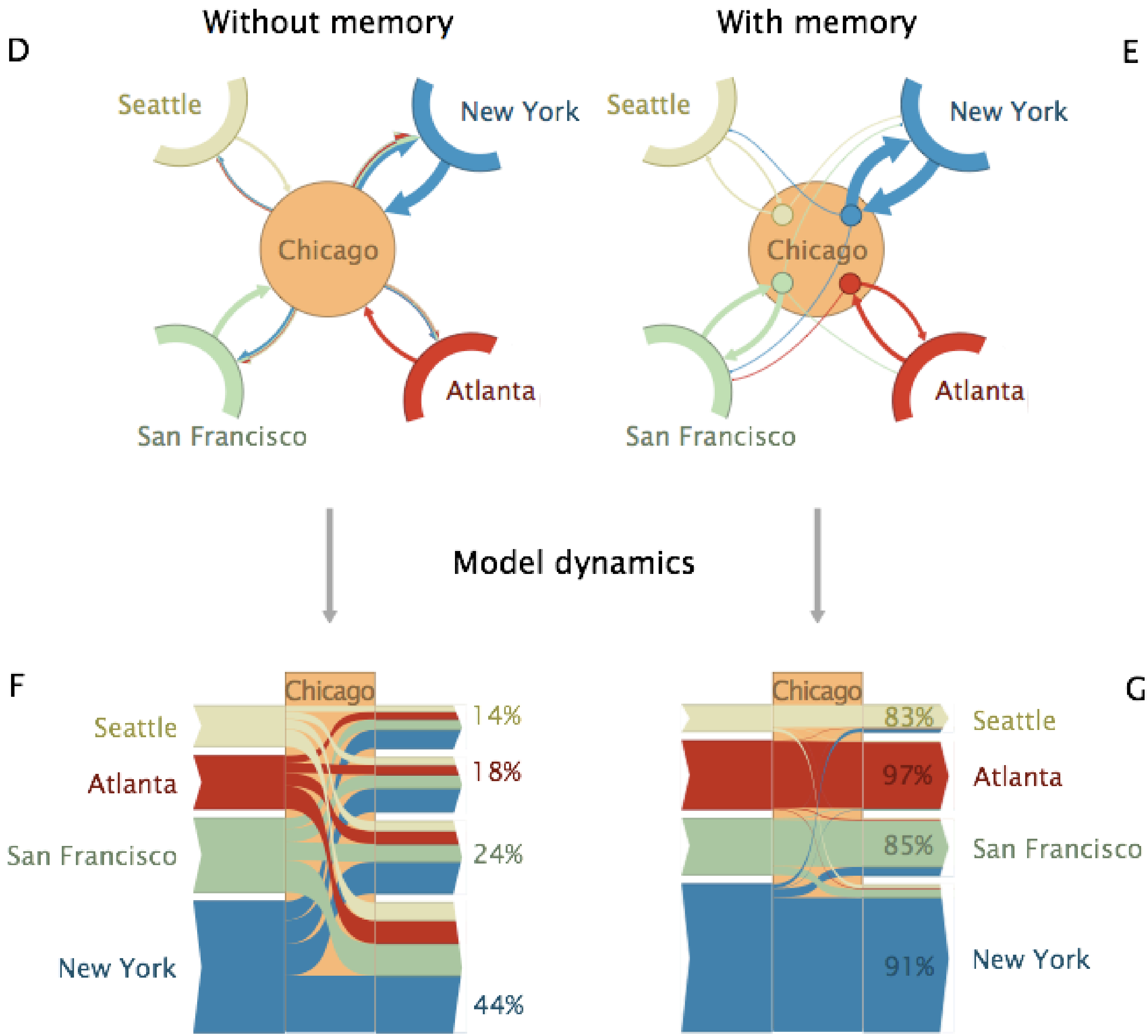
Seattle
New York
Chicago
San Francisco
Atlanta


Without memory
With memory
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San Francisco
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Model dynamics
F
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San Francisco
New York
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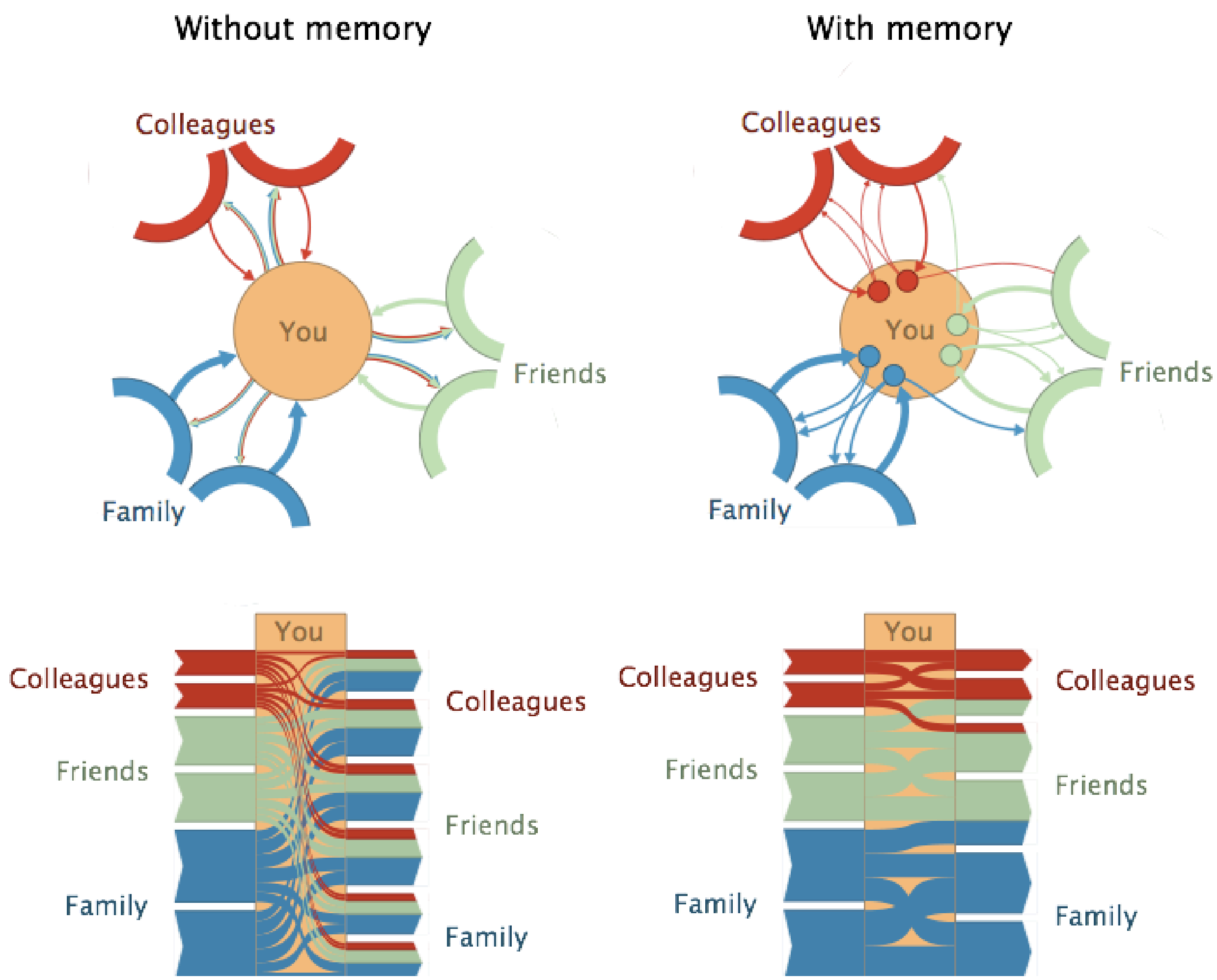
18%
24%
44%
Chicago
G
83%
97%
85%
91%
Seattle
Atlanta
San Francisco
New York

Networks with memory

Where information goes to depends on where it comes from
Mathematics of pathways instead of edges



Networks with memory

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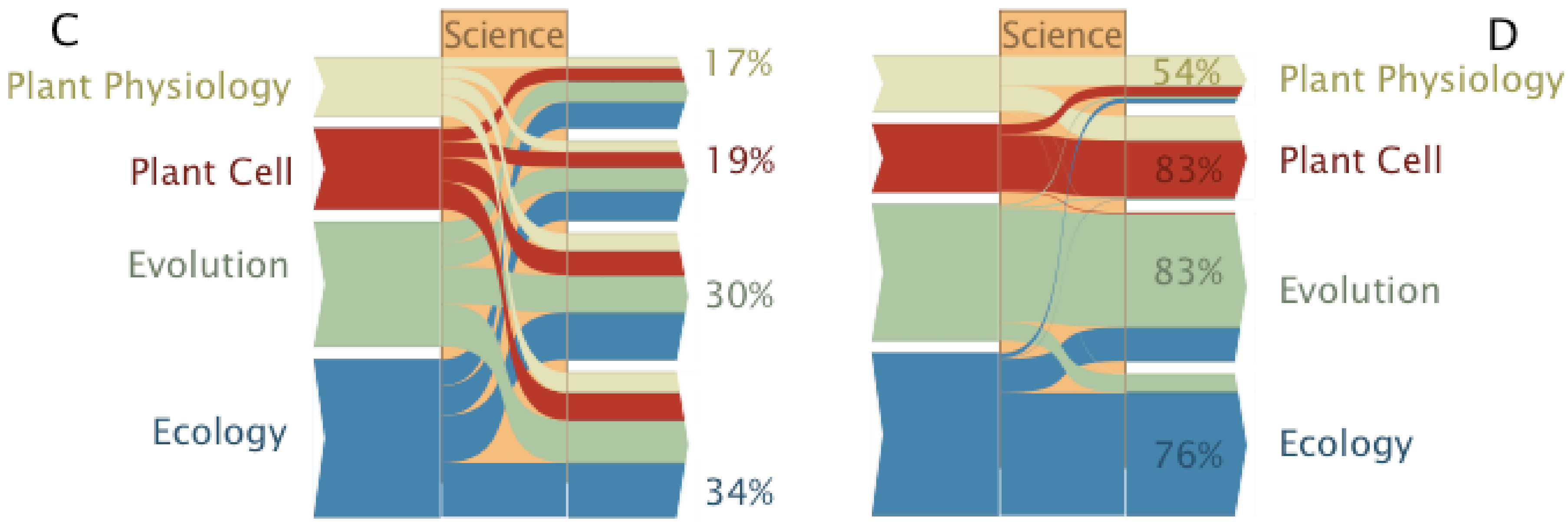

III. Significance

Memory affects pathways

In a broad range of social and information systems, pathways differ from those produced on (memoryless) networks

Table 1: Table 1. Summary of second-order Markov effects in real-world networks. M1 and M2 for results obtained with a first- and second-order Markov model, respectively. In the Supplementary Information, we provide 10th and 90th percentiles from a bootstrap analysis

Network	Number of nodes		Two-step return (%)		Three-step return (%)		Entropy rate (bits)		Module size (%)		Module assignmt.		Compression gain (%)	Ranking diff. (%)
	M1	M2	M1	M2	M1	M2	M1	M2	M1	M2	M1	M2	M1→M2	M1→M2
Airports	464	17,983	5.7	47	2.1	0.63	5.2	3.4	93	5.1	1.2	6.2	13	8.2
Cities	413	15,368	6.5	48	2.8	0.62	4.7	3.5	32	5.3	1.8	3.7	5.2	3.7
Journals	1,983	201,349	11	21	4.7	5.4	4.5	3.5	14	15	1.8	3.4	4.7	9.7
Patients	402	4,987	16	54	1.9	3.4	3.0	1.0	7.3	1.9	5.0	4.7	30	22
Taxis	416	2,763	20	10	6.8	10	2.2	1.1	3.1	2.2	1.5	1.7	7.1	6.5
Emails	144	1,432	14	58	5.2	2.7	3.0	1.3	12	5.8	1.3	3.0	26	18

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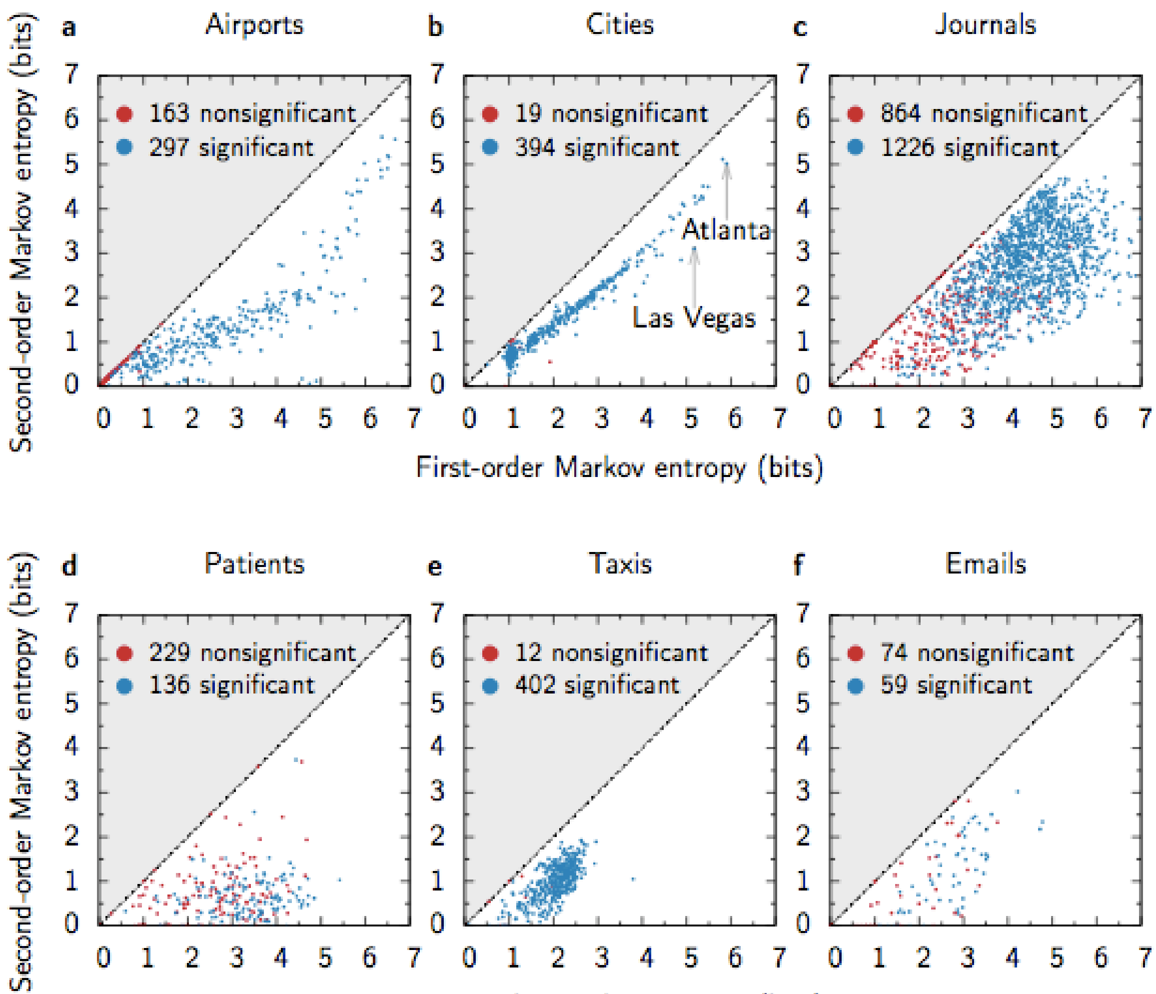

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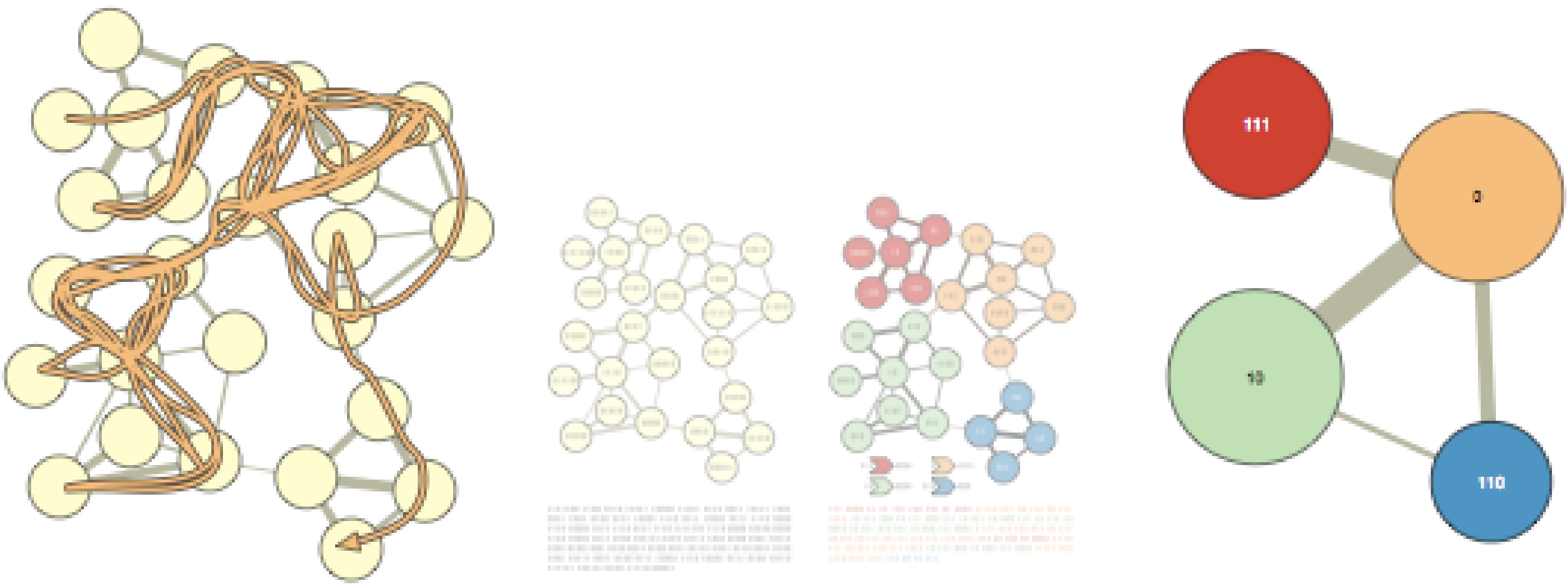
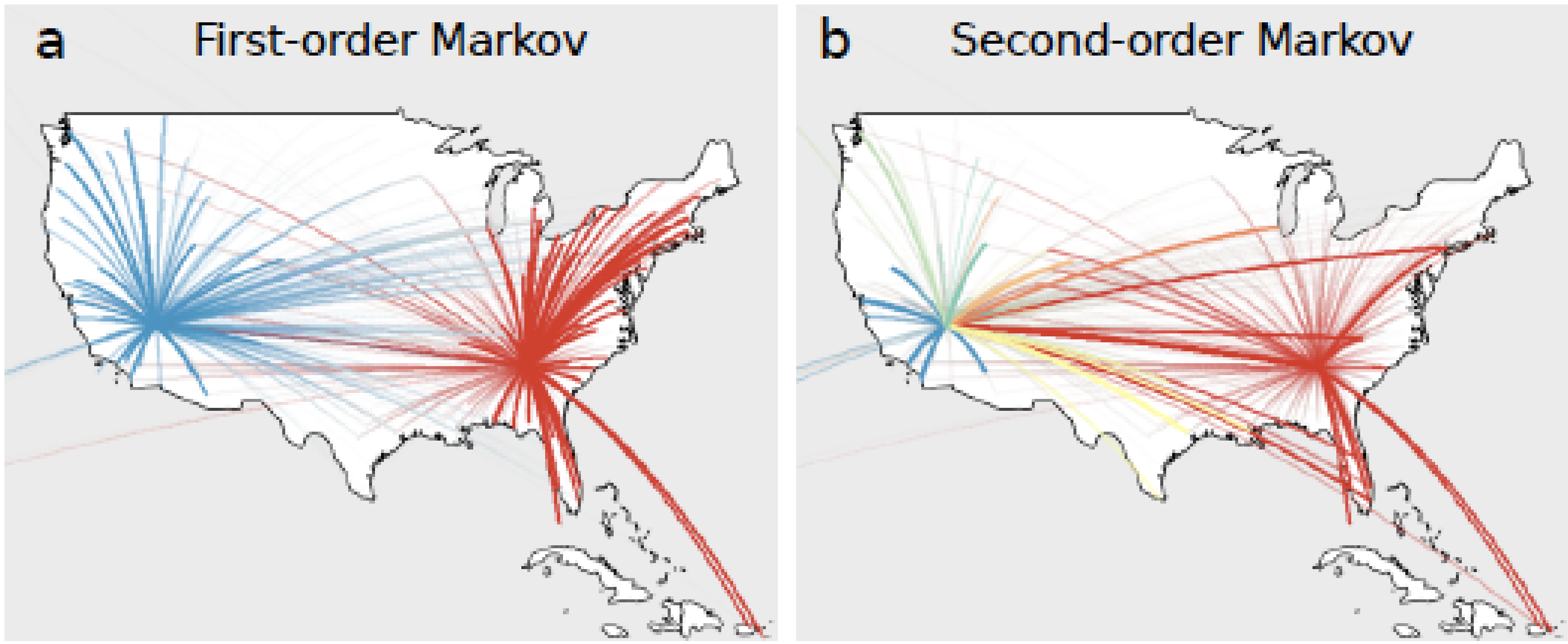
III. Algorithms

Algorithms for memory networks: community detection



Partition the system in order to optimise the compression of pathways of a random walker

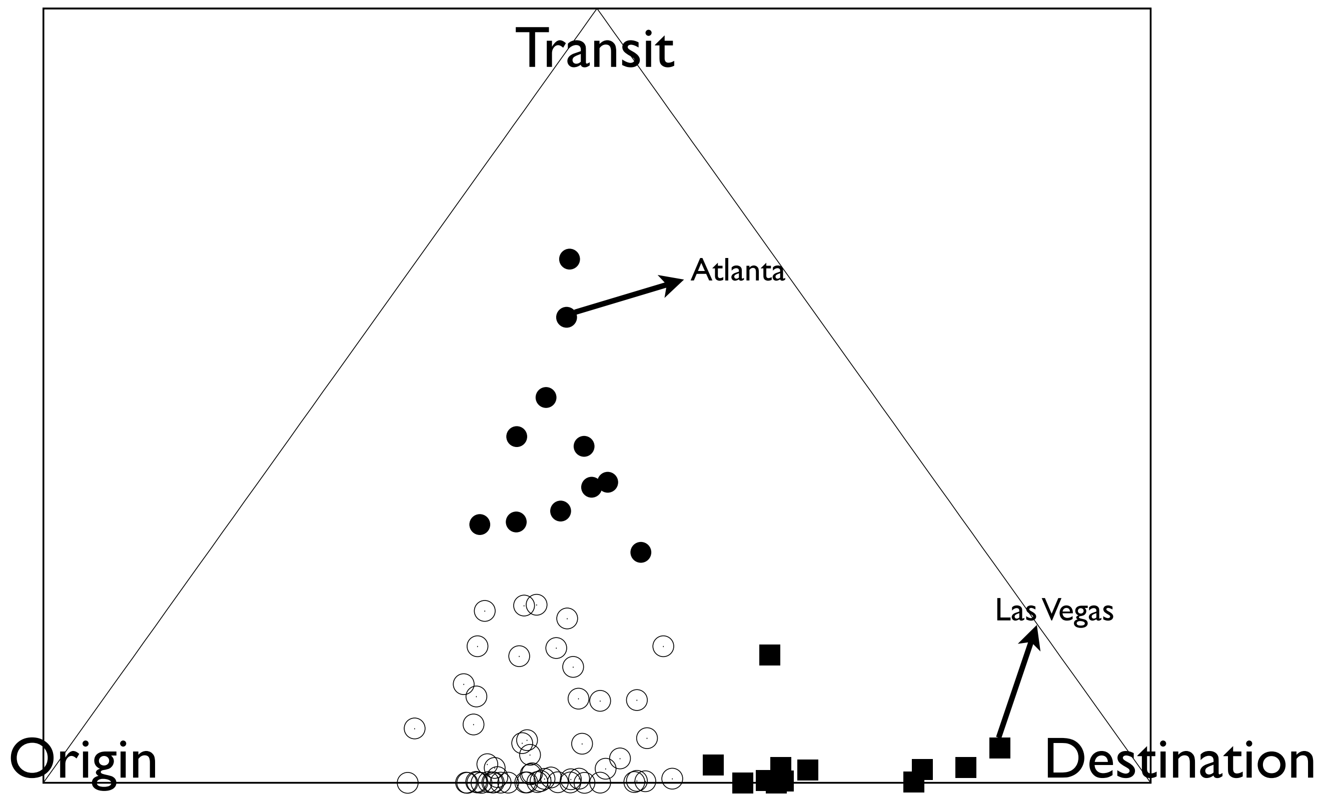
Algorithms for memory networks: community detection



More realistic pathways -> more realistic modules.
Second-order Markov dynamics allow for better compression, because random dynamics on networks obscure essential structural information.
The method reveals smaller, more overlapping networks.
Connections with link partitioning and clique-percolation.

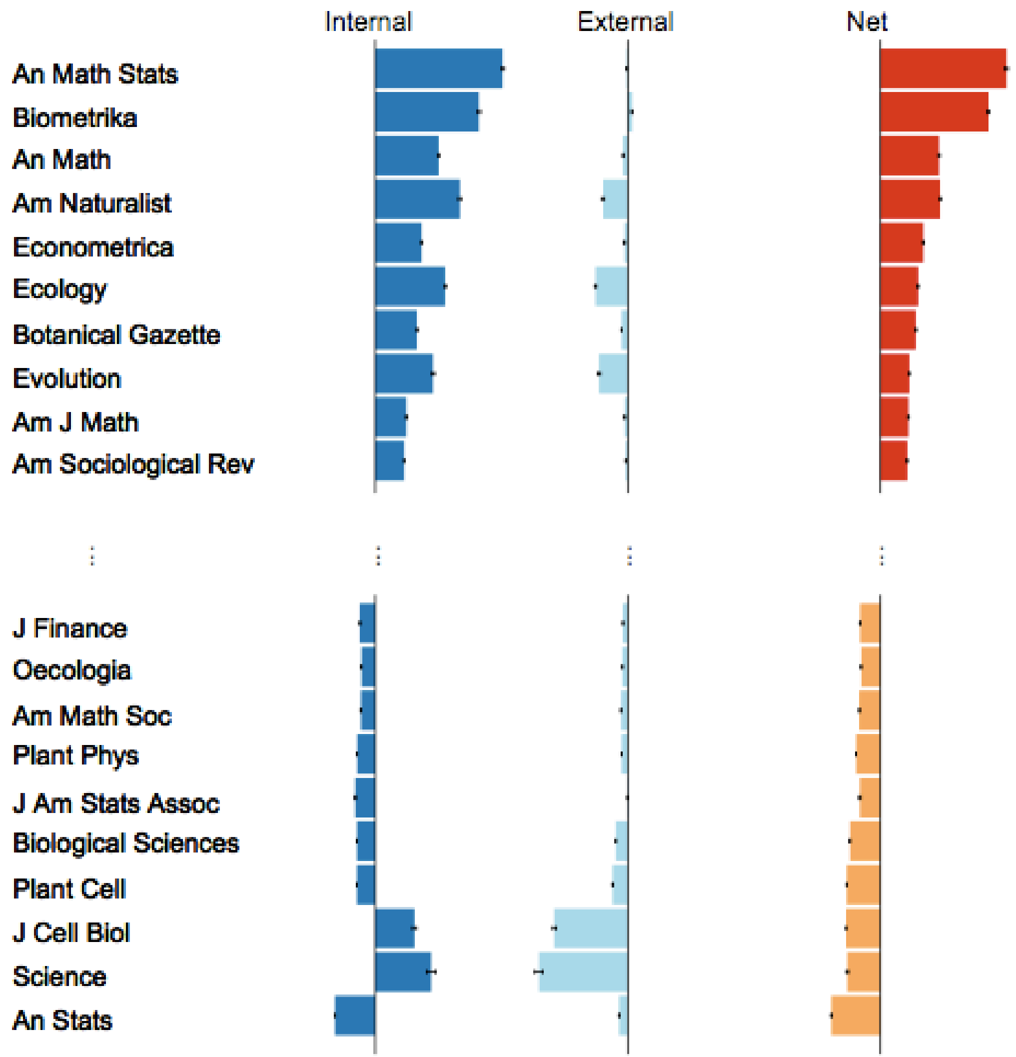
Evans, T. & Lambiotte, R. Line graphs, link partitions, and overlapping communities. Phys. Rev. E 80, 016105 (2009).
Ahn, Y., Bagrow, J. & Lehmann, S. Link communities reveal multiscale complexity in networks. Nature 466, 761–764 (2010).

Algorithms for memory networks: role detection



Detection of roles based on the location of nodes in the pathways

Algorithms for memory networks: ranking



Pagerank based on empirical flows instead of random ones

IV. Models

Model for memory networks

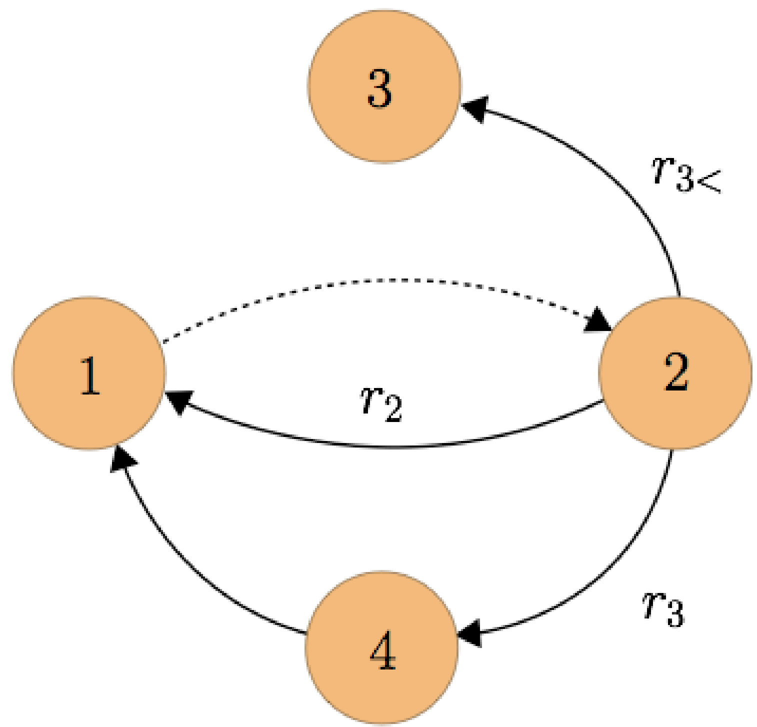


Figure S5: Illustration of the memory model. After performing a jump along the dashed link, a walker can either perform a return step, with probability r_2, a triangular step, with probability r_3, or an exploratory step, with probability $r_{3<}$.

We model/categorize transitions into 3 classes

Model for memory networks

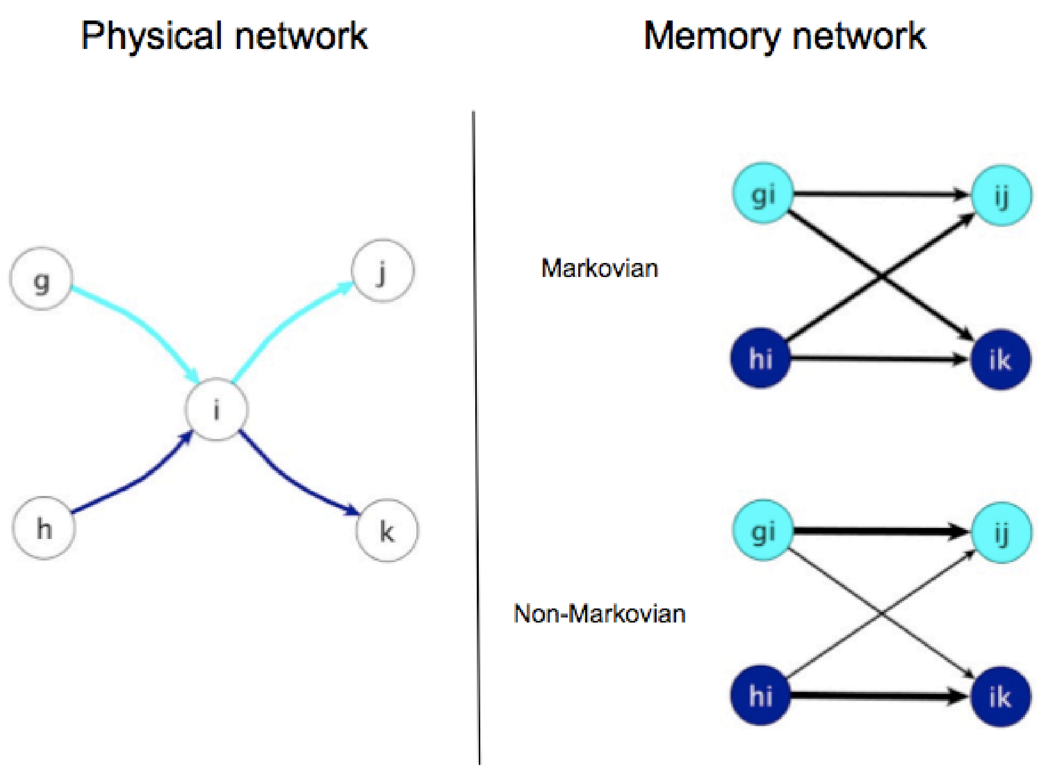
Network	$r_2(\%)$	$r_3(\%)$	$r_{3<}(\%)$
Airports	93 (93-93)	2.8 (2.8-2.8)	4.5 (4.5-4.5)
Cities	91 (91-91)	3.2 (3.2-3.2)	5.6 (5.6-5.7)
Journals	67 (67-67)	25 (25-25)	7.6 (7.2-7.6)
Patients	86 (85-90)	10 (7.4-11)	3.4 (2.8-3.5)
Taxis	17 (16-17)	66 (65-66)	17 (17-18)
Emails	89 (88-89)	8.9 (8.4-9.8)	1.9 (1.8-2.2)

Fitting by minimizing the Kullback−Leibler divergence

V. Dynamics

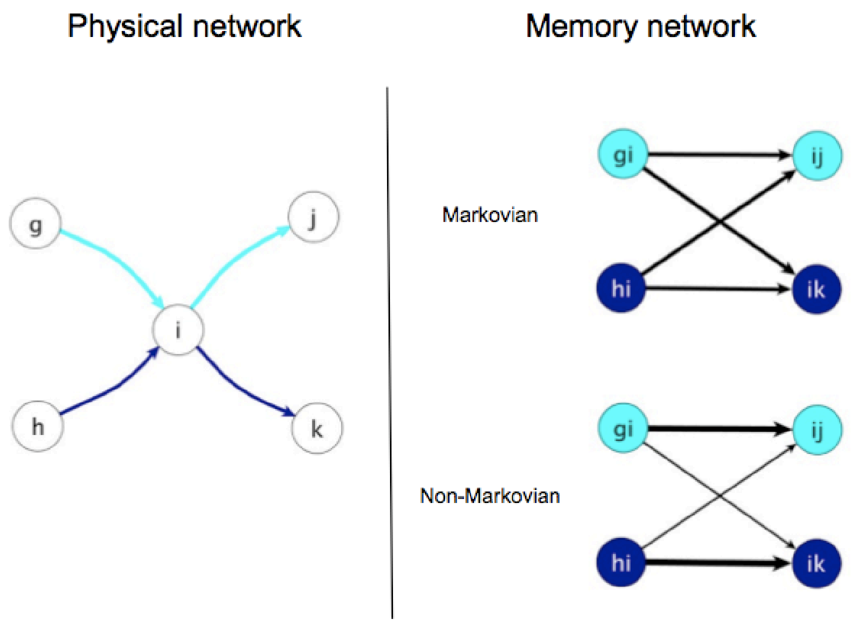
Memory affects mixing time

Second-order Markov: transitions from directed edges to directed edge (memory node)
Memory may induce biases in the transition between memory nodes



Effect of Memory on the Dynamics of Random Walks on Networks, R Lambiotte, V Salnikov, M Rosvall, Journal of Complex Networks 2014

Memory affects mixing time



Random walk on the memory network

$$P(\beta;t+1) = \sum_{\alpha} P(\alpha;t) T_{\alpha\beta}$$

If the dynamics is memoryless, uniform transition:

$$T^M_{\alpha\beta} = \begin{cases} 1/k^{\text{out}}_{\alpha} & \text{for } \beta \in \sigma^{\text{out}}_{\alpha}, \\ 0 & \text{otherwise}, \end{cases}$$

(Left and right) eigenvectors of the spectral gap associated to the best bi-partition of the network (Fiedler)

$$\mathrm{uT} = \lambda_2 \mathrm{u}$$

Small deviation to the Markovian case and perturbation analysis:

$$\boxed{T = T^M + \Delta T}$$

Memory affects mixing time

$$\boxed{\Delta\lambda_2 = -\frac{\sum_{\alpha\beta} u_\alpha^M \Delta T_{\alpha\beta} v_\beta^M}{\sum_\alpha u_\alpha^M v_\alpha^M}}$$

Interplay between memory and the (dominant) bi-modular structures:

- if memory enhances flows inside communities => slowing down of diffusion

- if memory enhances flows across communities => acceleration of diffusion

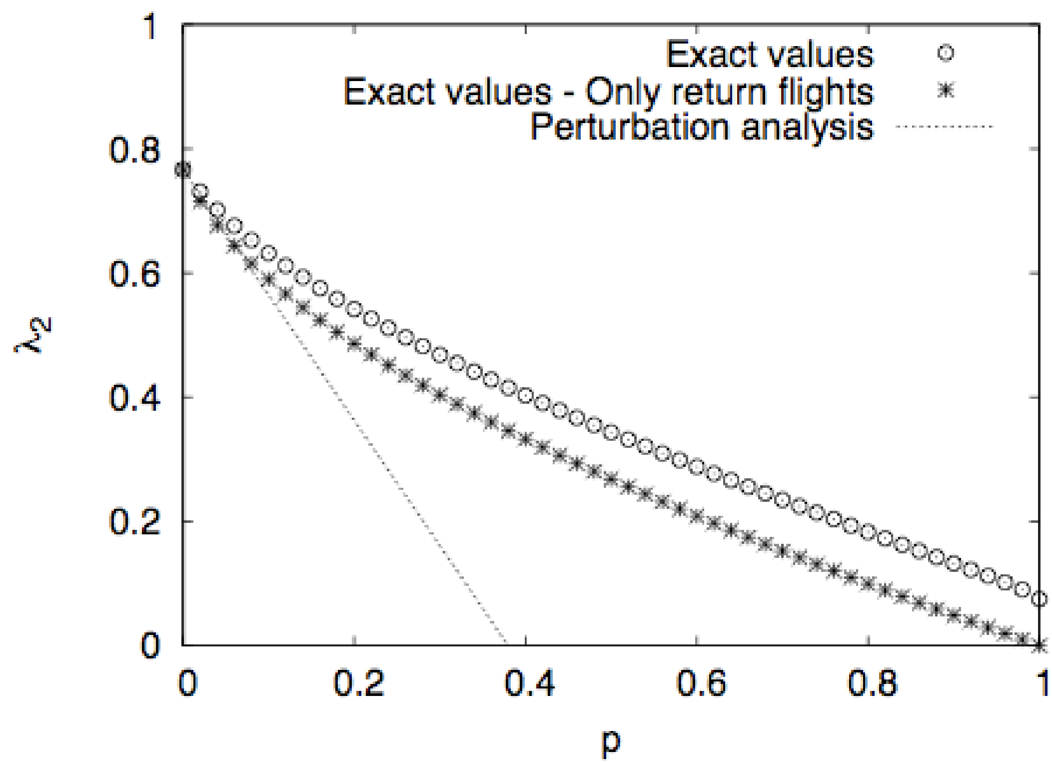
Memory affects mixing time

Random process driven by the tunable transition matrix

$$T^{(p)} = pT + (1-p)T^M$$

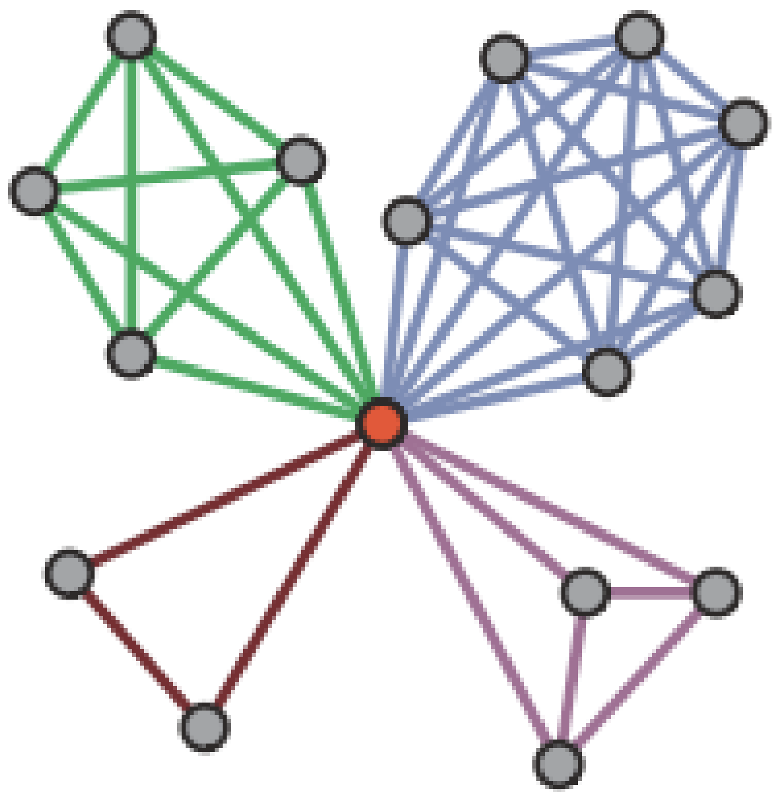
First-order Markov process with probability (1-p), and second-order Markov process with probability p.

This hybrid process models the diffusion of an item, e.g. a virus or a bank note, which travels with passengers and changes owner inside cities with probability p.



The effective size of the system is multiplied by 10 => the network is topologically small **but dynamically large.**

Diffusion in Multiplex Networks



Overlapping community structure of social networks

Information spreads differently in different circles

Diffusion on temporal networks with correlations

Correlations between edge activations lead to non-random pathways, modelled by higher-order Markov processes

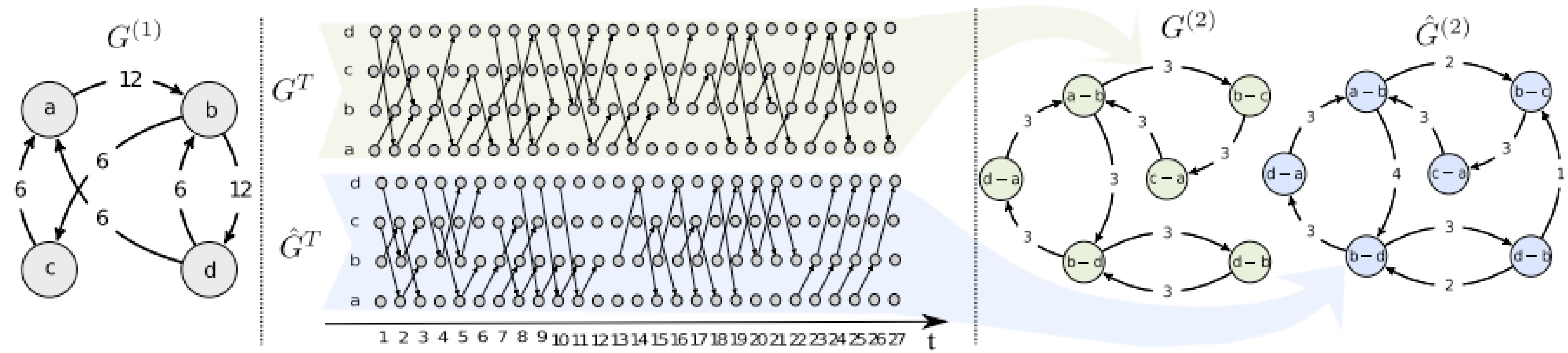


Figure 1: Two temporal networks G^T (top) and $\hat{G}^T$ (bottom) giving rise to the same weighted, aggregate network $G^{(1)}$ (left panel). In the right panel, two second-order aggregate networks $G^{(2)}$ (left) and $\hat{G}^{(2)}$ (right) are shown that correspond to the two temporal networks G^T and $\hat{G}^T$. Both second-order aggregate networks are consistent with $G^{(1)}$.

Slow-Down vs. Speed-Up of Information Diffusion in Non-Markovian Temporal Networks, I Scholtes, N Wider, R Pfitzner, A Garas, C Juan Tessone and F Schweitzer, arXiv:1307.4030

Toolbox for pathways

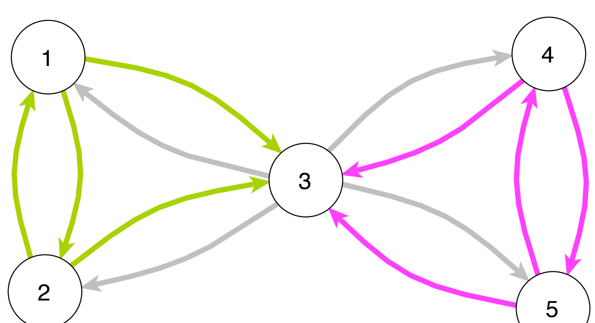
In several empirical systems: memory constraints on flow are statistically significant and temporal correlations strongly modify flows of probability.

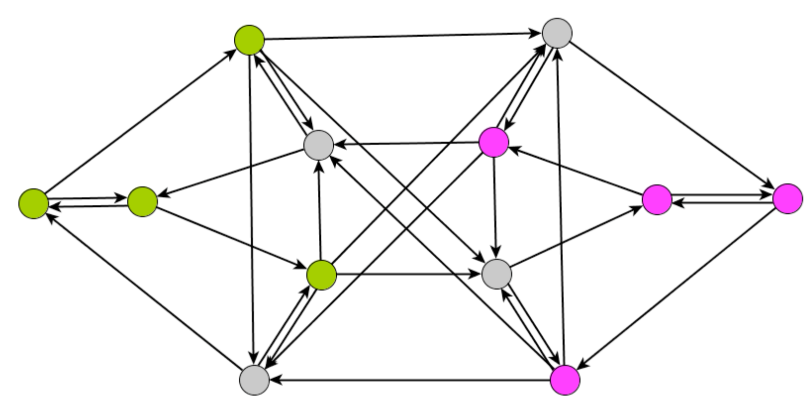
Memory networks are networks, but

- they are much, much larger => computational complexity

- they have a very different structure => require new tools

First insight on community detection, ranking, spreading, but their is a need for appropriate visualisation, statistical and algorithmic tools.





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Till Hoffmann and Mason Porter (Oxford): temporal networks

Martin Rosvall (Umea) and Andrea Lancichinetti (Northwestern): pathways

J.C. Delvenne (Louvain), Luis Rocha (Namur), Vsevolod Salnikov (Namur) and Lionel Tabourier (Namur): diffusion

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