Link Streams

Work in progress...

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http://complexnetworks.fr
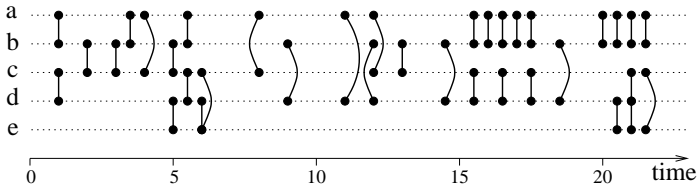
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Link Streams

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Preliminaries

Density

Temporal
paths

Communities

Conclusion

Introducing *link streams*

interactions over time

$$L = (t_i, u_i, v_i)_{i=1..n}$$

$t_i \in [\alpha, \omega]$: time
$u_i, v_i \in V$: nodes

ex: emails, traffic, payments, contacts, calls, ...
$\hookrightarrow$ already much studied

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Link streams = graphs+time?

relations, structure: $\{(a, b)\}$

> ↪ **graph theory / network science**
> density, degrees, clustering,
> paths, diameter, distances, etc

events, time series: $\{(t, x)\}$

> ↪ **signal processing / discrete event theory**
> frequency, speed, inter-event times,
> acceleration, self-similarity, periodicity, etc

interactions, structure+time: $\{(t, a, b)\}$

> ↪ **link streams**
> ??? ??? ???

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Our goal

**define a language for link streams
like graph theory for networks**

to deal *directly* with link streams

describe them, model them, etc

$\hookrightarrow$ density, clustering coefficients,
cliques, paths, communities, etc

some already exist!

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Δ-density

Graphs: proba two random nodes are linked

**Link streams: given Δ,
proba two random nodes are linked**
during a random time interval of duration Δ

$$\delta_\Delta(L) = 1 - \frac{2 \cdot \sum_{u,v \in V, u \neq v} \sum_{t \in \tau(u,v)} \max(0, t - \Delta)}{|V| \cdot (|V| - 1) \cdot \max(0, \omega - \alpha - \Delta)}$$

Δ: given duration, $\tau(u, v)$: inter-contact times

Notes: $\Delta = \omega - \alpha \longrightarrow$ graph density

Clustering coefficient

Graphs: density of neighborhood

to what point all neighbors
are linked together

Link streams: the same!

to what point all neighbors
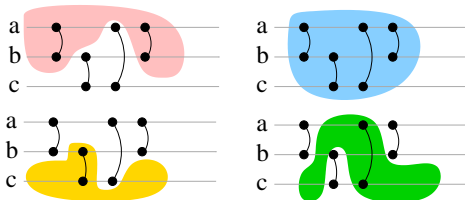*interact all the time*

much larger than global density

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(Maximal) cliques

Graphs: (maximal) sub-graph of density 1

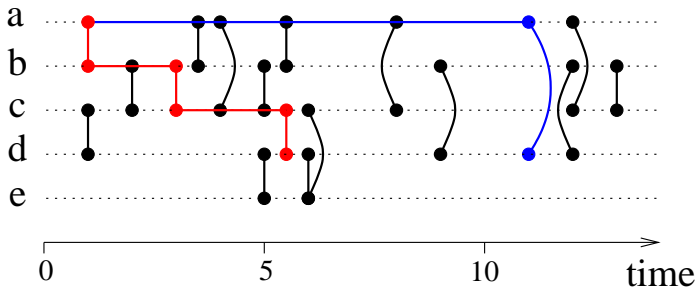*all nodes are linked together*

Link streams: the same: (maximal) sub-stream of
Δ-density 1

all nodes interact at least every Δ

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Paths

↪ length vs duration
↪ trees, spreading
↪ (strong) connectedness

already "classical" from mobile networks and spreading phenomena

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k-closure

k-closure of (t, a, b):

time until a and b at distance $\leq k$
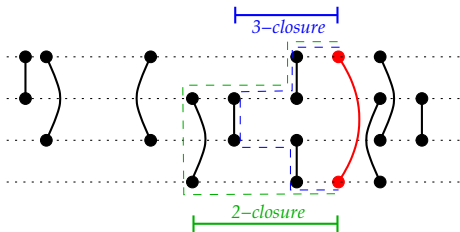
Notes:

$k = 1 \longrightarrow$ inter-contact times

$k = 2 \longrightarrow$ clustering coefficient

mix of time and structure

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Communities in graphs

dense sub-graphs poorly interconnected

in dynamic graphs: evolution of graph communities

t   t+1   t+2
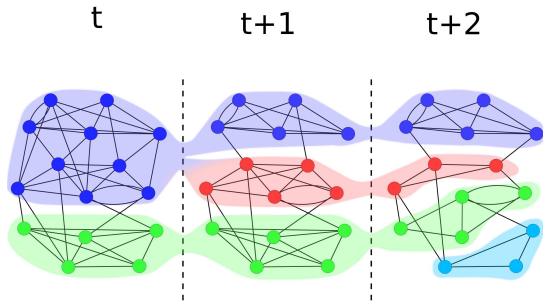
ex: groups of friends evolving over time

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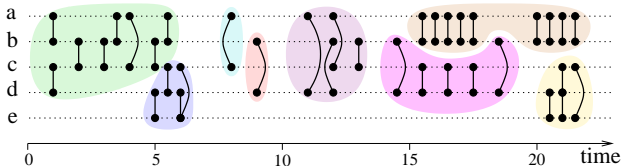
Communities in link streams

dense sub-streams poorly interconnected

i.e. temporally and structurally dense series of interactions

ex: discussions, meetings, sessions, ...

link streams $\neq$ dynamic graphs

In progress:
ground truth (threads in mailing-lists)
+ detection (quality functions)

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Conclusion

link streams model interactions
link streams $\neq$ dynamic graphs

- **Done:** basic notions like density, paths, clustering coefficients, communities, etc

- **In progress:** actual communities, clique algorithmics, event and community detection, relations with TVG

- **Case studies:** mailing-lists (Debian), phone calls (D4D), network traffic (Mawi, companies), mobility/contacts (crawdad, sociopaterns), financial transactions (bitcoins, on-line shopping), etc

- **Extensions:** strength, duration, direction, etc of interactions $\rightarrow$ weighted, bipartite, directed, etc link streams