

redundancy, control and collective computation



in network dynamics

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# acknowledgements

## CASCI group



Manuel Marques-Pita



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Artemy Kolchinsky



Ian Wood



Jason Yoder



Rion Brattig Correia



Yizhi Jing



Thomas Parmer



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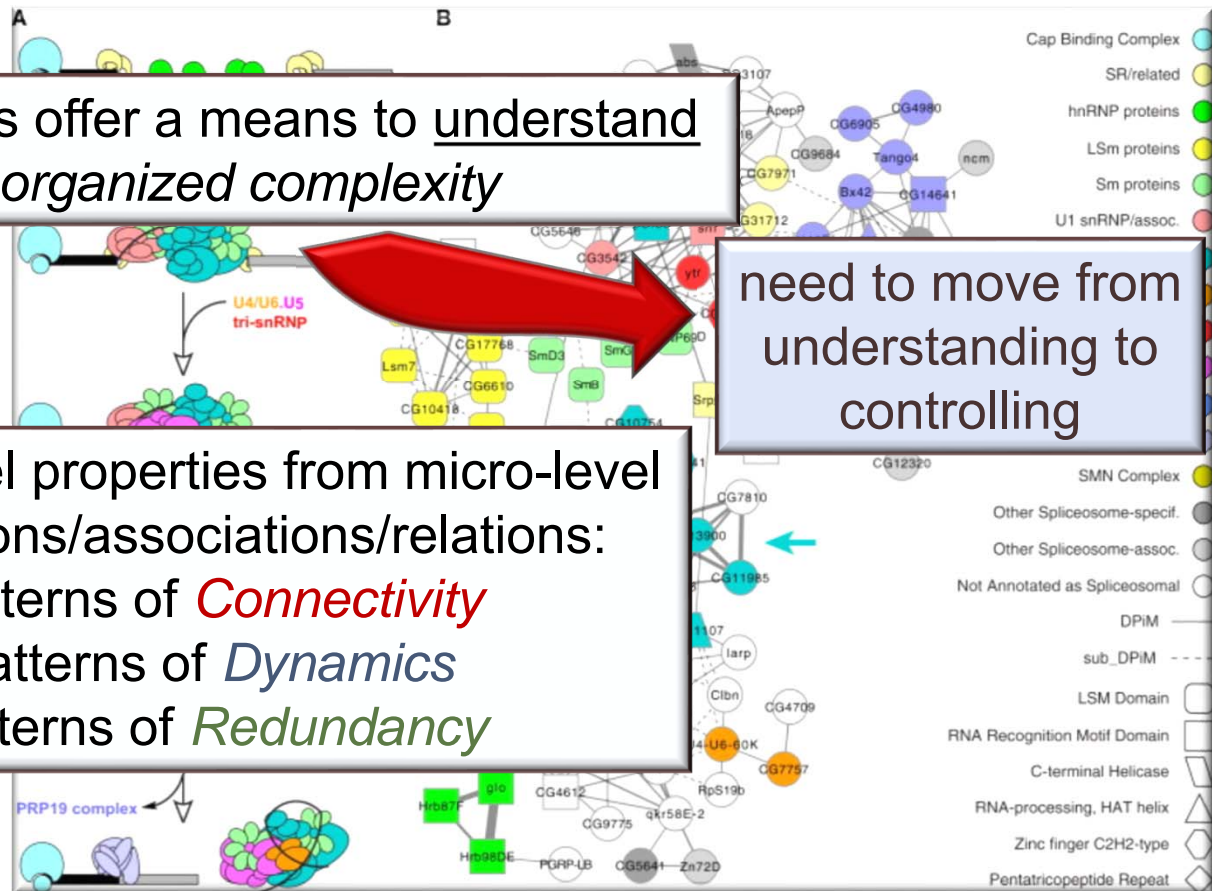
large-scale *drosophila* protein interaction Map (DPiM)

Guruharsha et al [2011]. "A Protein Complex Network of *Drosophila melanogaster*." *Cell*.147(3):690-703.

Networks offer a means to understand *organized complexity*

need to move from understanding to controlling

Macro-level properties from micro-level interactions/associations/relations:  
 Patterns of *Connectivity*  
 Patterns of *Dynamics*  
 Patterns of *Redundancy*



Modularity of the spliceosome subnetwork: 12 well-connected clusters representing interaction of snRNPs with pre-mRNA and other proteins in the process of splicing introns.



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# PATTERNS OF REDUNDANCY IN DYNAMICS

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# the drosophila segment polarity network

## an automata network model built from qualitative data



Based on the ODE model of von Dassow et al. (2000), consists of 4-cell parasegments, each cell with 15 interacting genes and proteins.

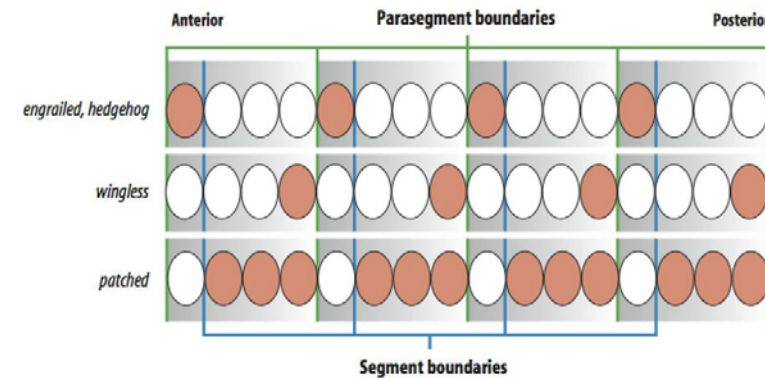
### 2<sup>60</sup> network configurations

Reproduces wild-type and mutant gene expression patterns in development of fruit fly

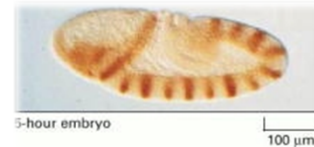
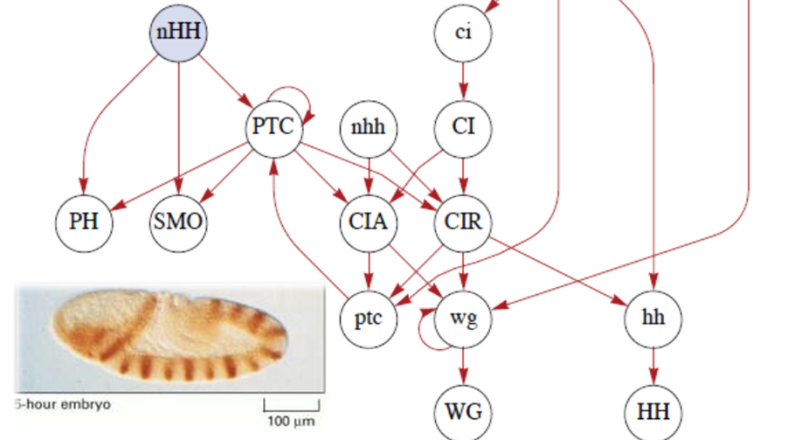
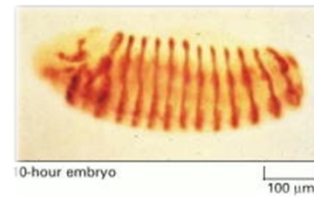
2 intercellular inputs: **nhh** (*hedgehog*), **nWG** (*wingless*)

1 intracellular input: **SLP** (*sloppy paired*)

SLP	x01	x01
wg	x02	$x02 = (x14 \wedge x01 \wedge \neg x15) \vee (x02 \wedge (x14 \vee x01) \wedge \neg x15)$
WG	x03	$x03 = x02$



CI	x13	$x13 = x12$
CIA	x14	$x14 = x13 \wedge (x01 \vee x06_{i-1} \vee x06_{i+1})$
CIR	x15	$x15 = x13 \wedge \neg x11 \wedge \neg x06_{i-1} \wedge \neg x06_{i+1}$



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# quantifying micro-level canalization

## input redundancy, effective connectivity and input symmetry

A Look-Up Table		B Wildcard Schemata	
$f_1$	0 0 1 0 0 1 : 1	$f'_1$	# 0 1 # 0 1 : 1
$f_2$	0 0 1 0 1 0 : 1	$f'_2$	# 0 1 # 1 0 : 1
$f_3$	0 0 1 0 1 1 : 1	$f'_3$	# 0 1 0 # 1 : 1
$f_4$	0 0 1 1 0 0 : 1	$f'_4$	# 0 1 0 1 # : 1
$f_5$	0 0 1 1 0 1 : 1	$f'_5$	# 0 1 1 # 0 : 1
$f_6$	0 0 1 1 1 0 : 1	$f'_6$	# 0 1 1 0 # : 1
$f_7$	0 0 1 1 1 1 : 1	$f'_7$	0 0 1 # # 1 : 1
$f_8$	1 0 1 0 0 0 : 1	$f'_8$	0 0 1 # 1 # : 1
$f_9$	1 0 1 0 0 1 : 1	$f'_9$	0 0 1 1 # # : 1
$f_{10}$	1 0 1 0 1 0 : 1	$f'_{10}$	1 0 1 # # 0 : 1
$f_{11}$	1 0 1 0 1 1 : 1	$f'_{11}$	1 0 1 # 0 # : 1
$f_{12}$	1 0 1 1 0 0 : 1	$f'_{12}$	1 0 1 0 # # : 1
$f_{13}$	1 0 1 1 0 1 : 1		
$f_{14}$	1 0 1 1 1 0 : 1		

$\Theta_1$  (rows  $f_1$  to  $f_7$ )  
 $\Upsilon_9$  (rows  $f_4$  to  $f_7$ )  
 $F'$  (rows  $f'_1$  to  $f'_{12}$ )  
 $F''$  (row  $f''_1$ )

C Wildcard & Position Free Schemata

$f''_1$	# 0 1 # 0 1 : 1
---------	-----------------

$F''$

$$k(x) = 6$$

$$k_r(x) = \frac{\sum_{f_\alpha \in F} \max_{\theta | f_\alpha \in \Theta_\theta} (n_\theta^\#)}{2^k}$$

$$k_e(x) = k(x) - k_r(x)$$

$$k_s(x) = \frac{\sum_{f_\alpha \in F} \min_{\theta | f_\alpha \in \Theta_\theta} |n_\theta^o|}{2^k}$$

- Measuring two forms of canalization
  - $K_r = 2$
  - $K_e = 6 - 2 = 4$
  - $K_s = 4$

Prime Implicants (Quine-McCluskey) plus group invariance

# per-node schema redescription

In biological Boolean network models

- extracting micro-level canalization
  - drosophila segment polarity genes network

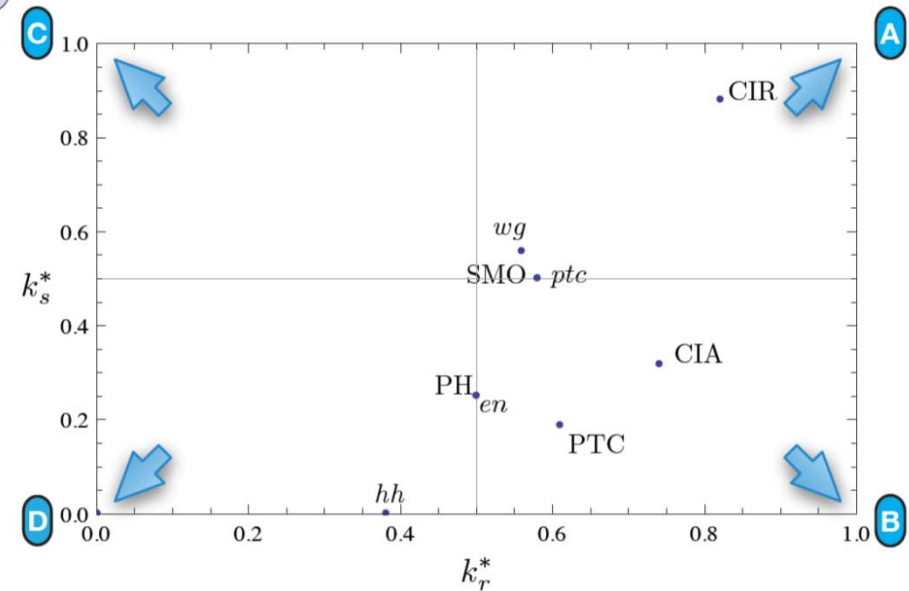
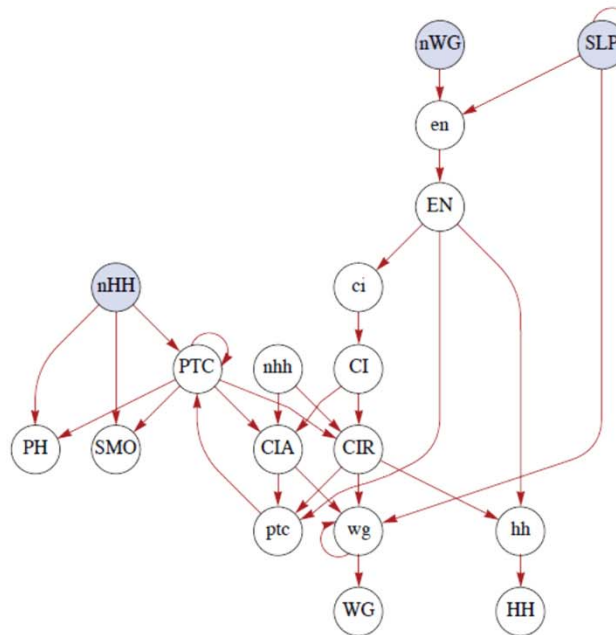
node	inhibition	expression	$k$	$k_e$	$k_r$	$k_s$	$k_r^*$	$k_s^*$
<i>wg</i>	$f''_{2:1}$ $f''_{2:2}$	$f''_{2:3}$	4	1.75	2.25	2.25	0.56	0.56
PTC	$f''_{9:1}$ $f''_{9:2}$	$f''_{9:3}$ $f''_{9:4}$	4	1.56	2.44	0.75	0.61	0.19
PH	$f''_{10:1}$ $f''_{10:2}$	$f''_{10:3}$	3	1.5	1.5	0.75	0.5	0.25
SMO	$f''_{11:1}$	$f''_{11:2}$ $f''_{11:3}$	3	1.25	1.75	1.5	0.58	0.5
<i>ci</i>	$f''_{12:1}$	$f''_{12:2}$	1	1	0	0	0	0
CI	$f''_{13:1}$	$f''_{13:2}$	1	1	0	0	0	0
CIA	$f''_{14:1}$ $f''_{14:2}$	$f''_{14:3}$ $f''_{14:4}$	6	1.55	4.45	1.875	0.74	0.32
CIR	$f''_{15:1}$ $f''_{15:2}$	$f''_{15:3}$	6	1.08	4.92	5.25	0.82	0.88

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# per-node schema redescription

In biological Boolean network models

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node	inhibition	expression	$k$	$k_e$	$k_r$	$k_s$	$k_r^*$	$k_s^*$																																																								
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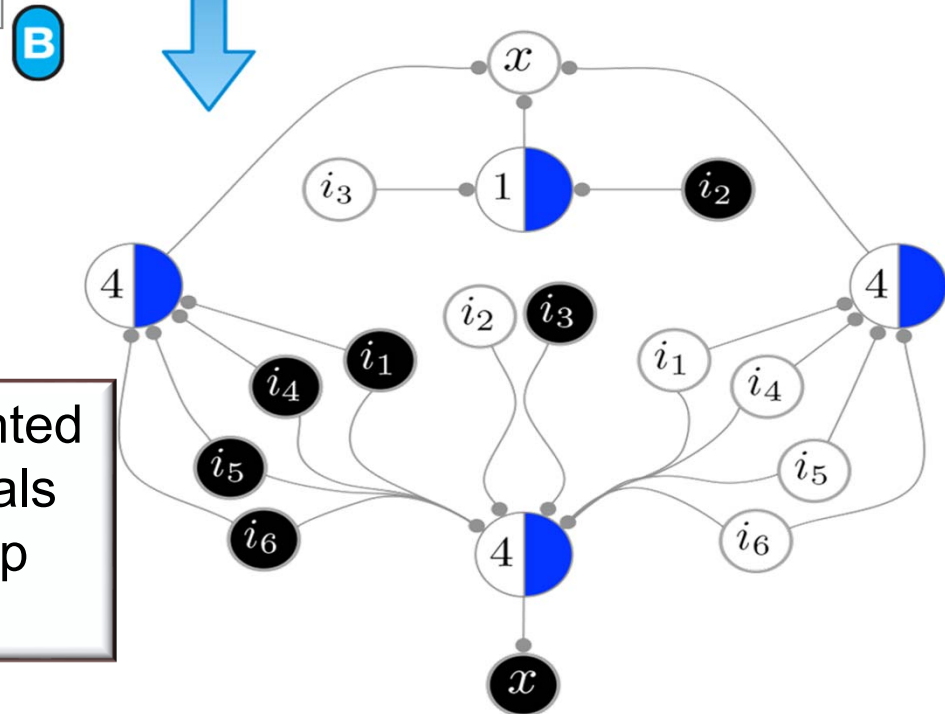
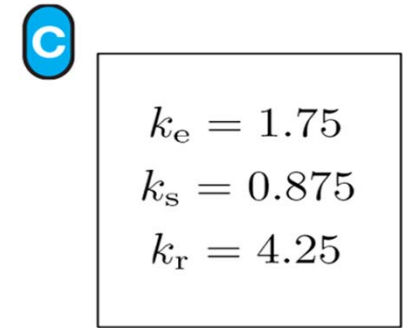
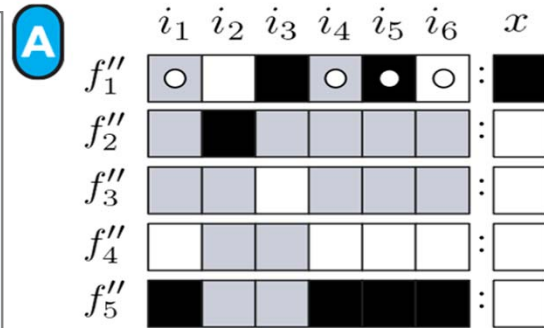
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# canalization map as minimal control

## two-symbol schemata as threshold networks

- understanding natural “computation”
  - minimal wiring (control) of micro-level



Each schema represented by conjunction of literals and symmetric group constraints

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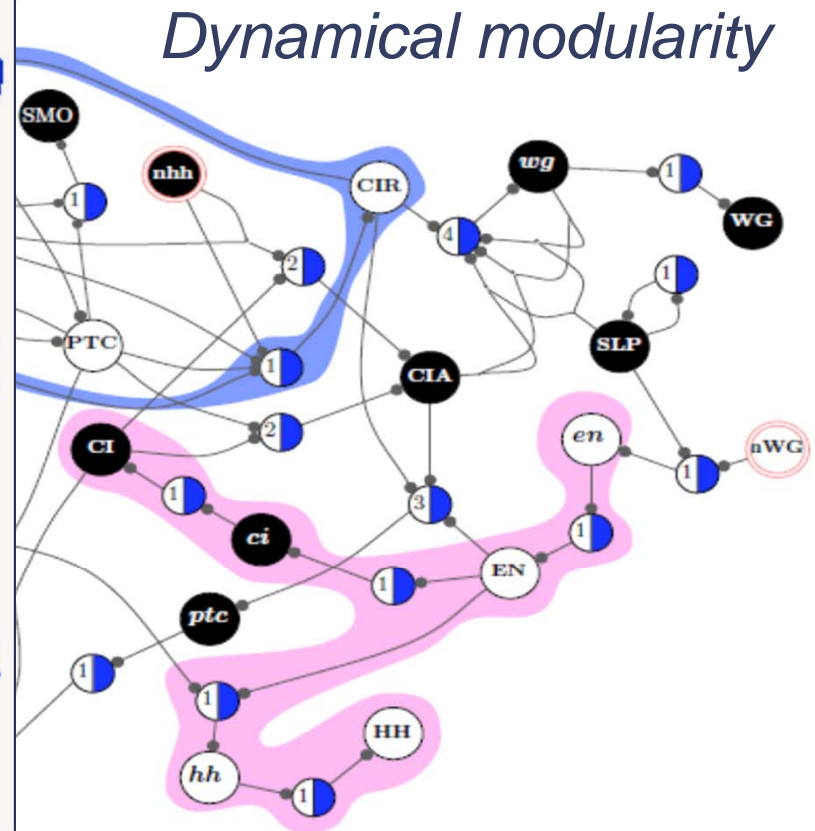
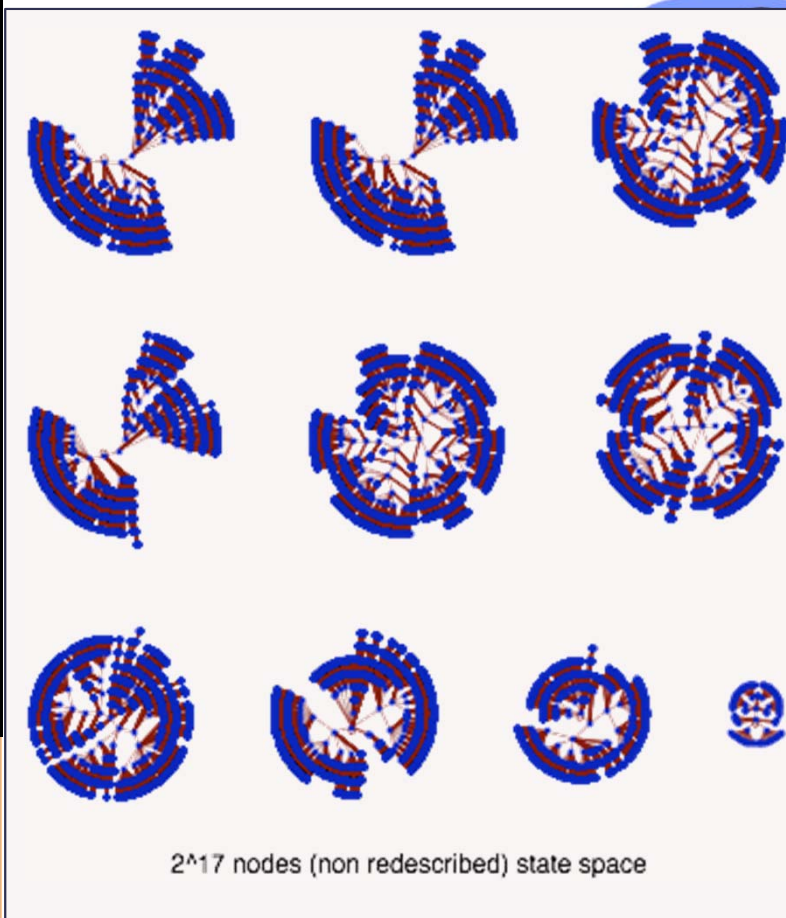


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## (macro-level) dynamics canalization map from per-node schemata redescription

- Full dynamics (of single-cell model) captured by threshold network of  $2*N+M$  nodes





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# CONTROL FROM PATTERNS OF REDUNDANCY IN DYNAMICS

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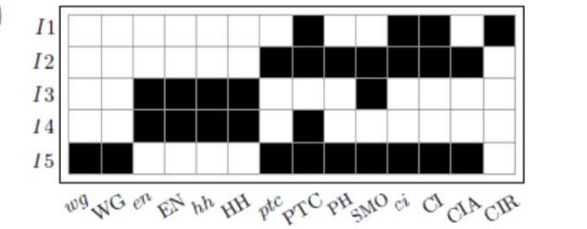
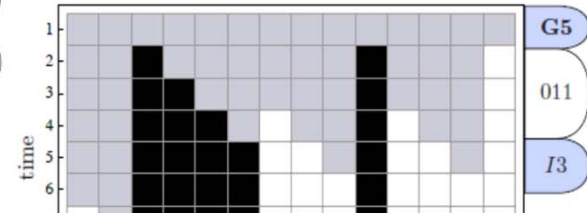
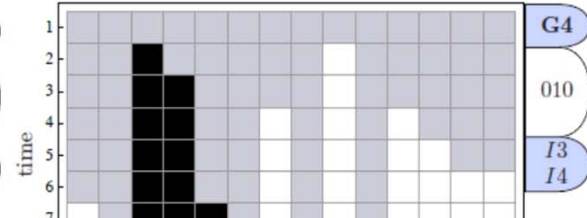
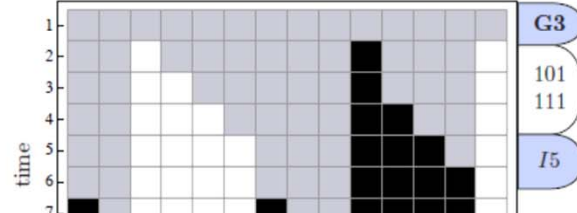
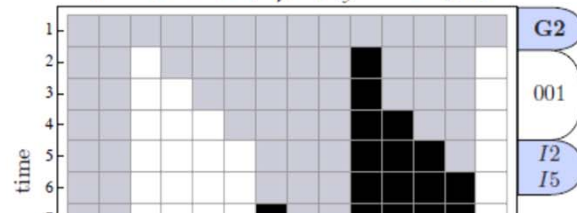
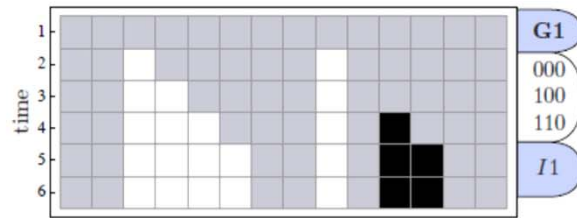
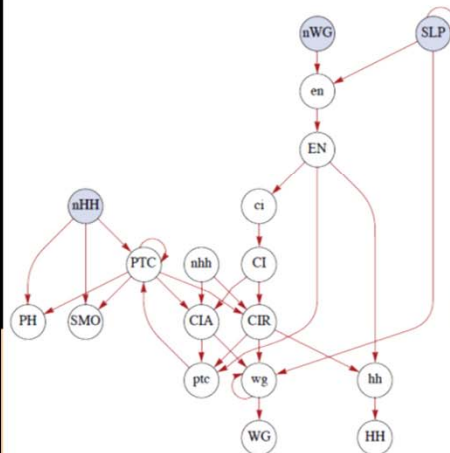
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# (macro-level) control

## Dynamical unfolding from partial information

- inputs in drosophila segment polarity net: SLP, nWG, nhh

How much control certain nodes have on network dynamics.

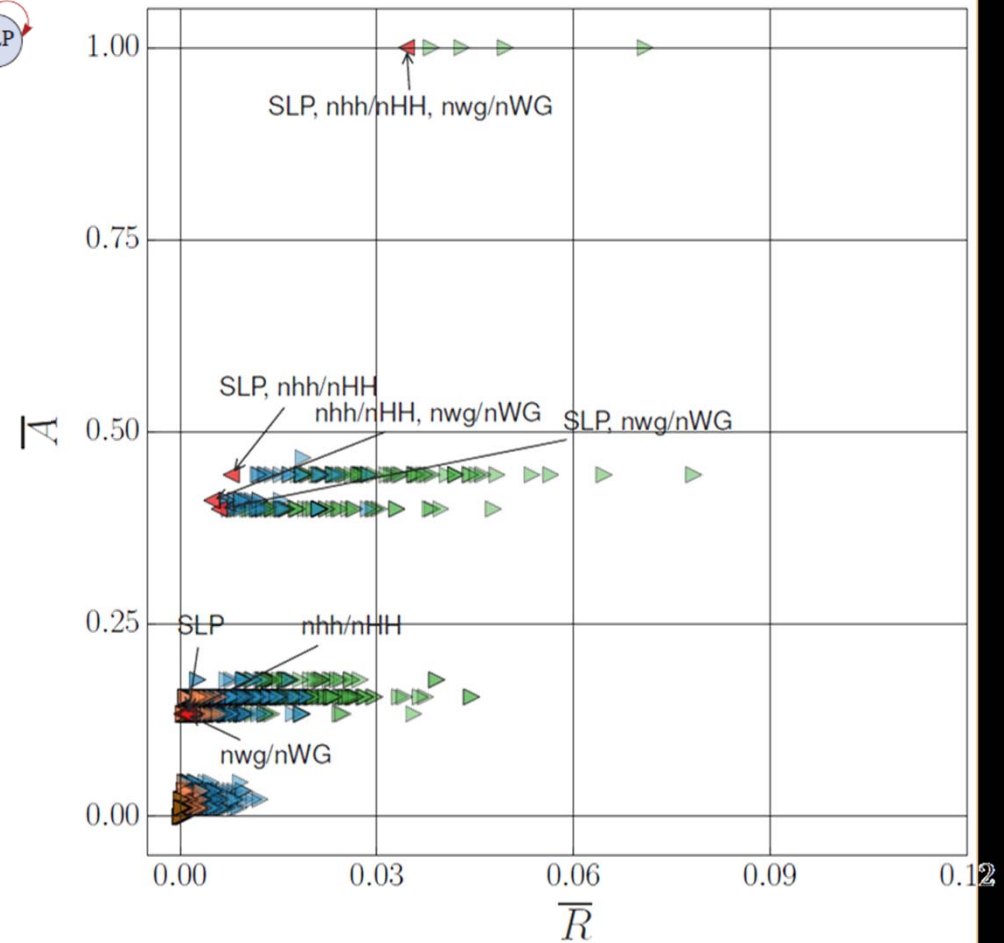
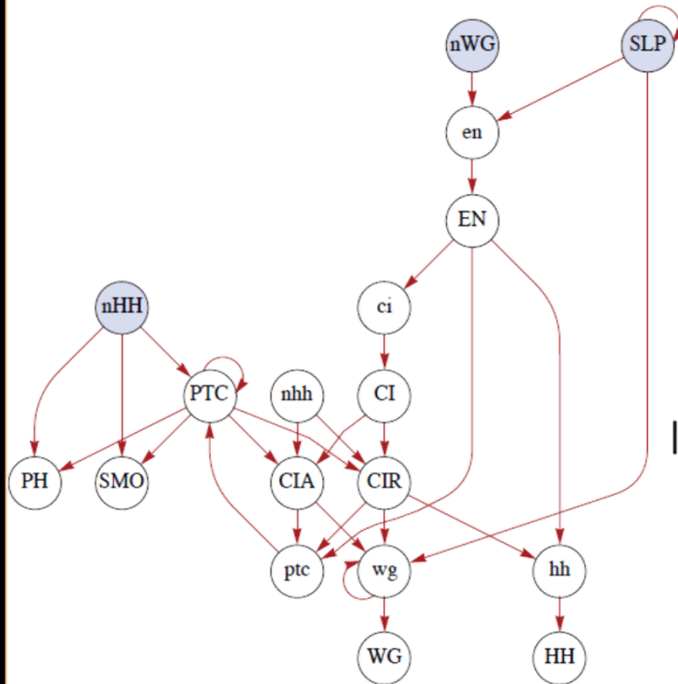


Marques-Pita & Rocha, [2013]. *PLoS ONE*, 8(3): e55946.

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# Can structural controllability uncover control?

## Drosophila model (Albert et al)



4 nodes predicted by structural control:

- $\{SLP, nWG, nhh/nHH, PH\}$ ,
- $\{SLP, nWG, nhh/nHH, SMO\}$ ,
- $\{SLP, nWG, nhh/nHH, CIR\}$ ,
- $\{SLP, nWG, nhh/nHH, CIA\}$

Gates & Rocha [2014]. *ALIFE 2014*: 429-430.

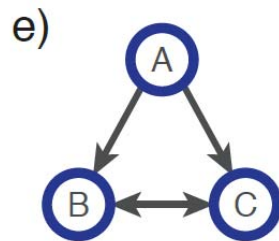
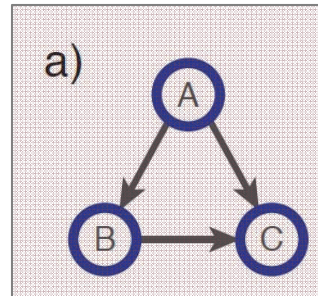
Gates & Rocha [2014]. *Submitted*.

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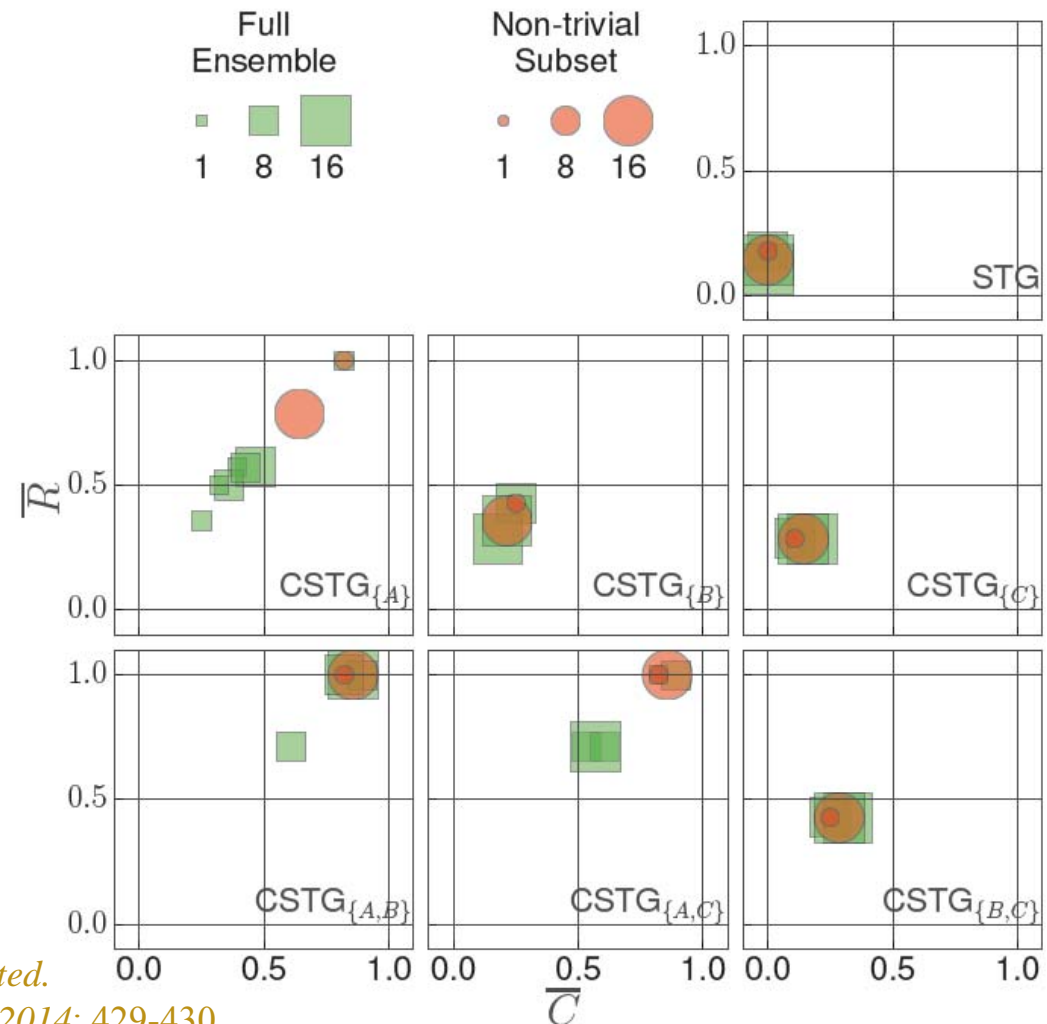
# Can structural controlability uncover control?

In presence of dynamics

Consider small network motifs:



The larger the **canalization**, the less predictable is structural control



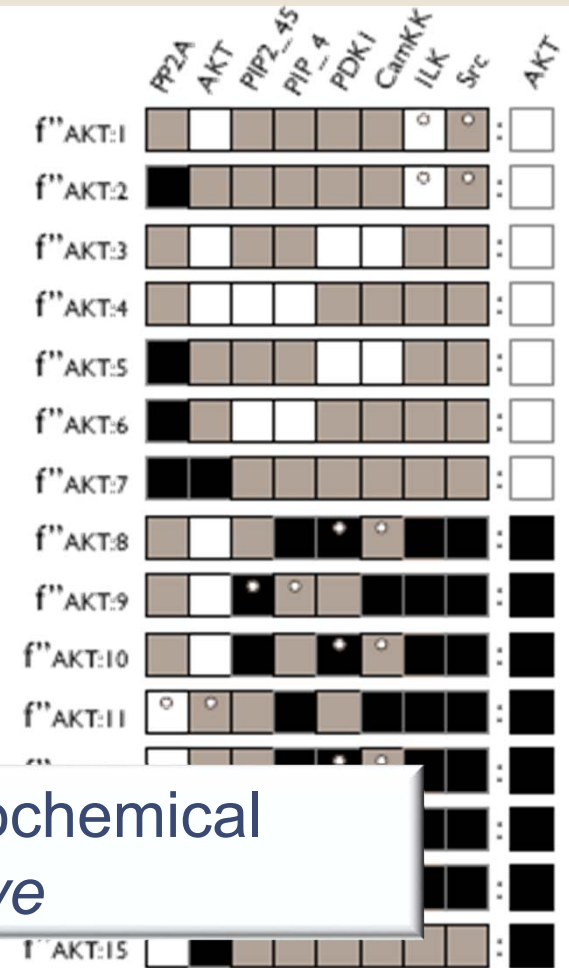
Gates & Rocha [2014]. *Submitted*.  
Gates & Rocha [2014]. *ALIFE 2014*: 429-430.



# redundancy in intracellular signaling networks

## canalization

- Activation of AKT in generic fibroblasts (130 node BN)
  - LUT of  $2^8=256$  entries redescribed by only 15 schemata
    - Large amount of canalization
  - Very few actual inputs need to be known to determine state-transition



**Upcoming work:** analysis of biochemical models in the entire *cell collective*



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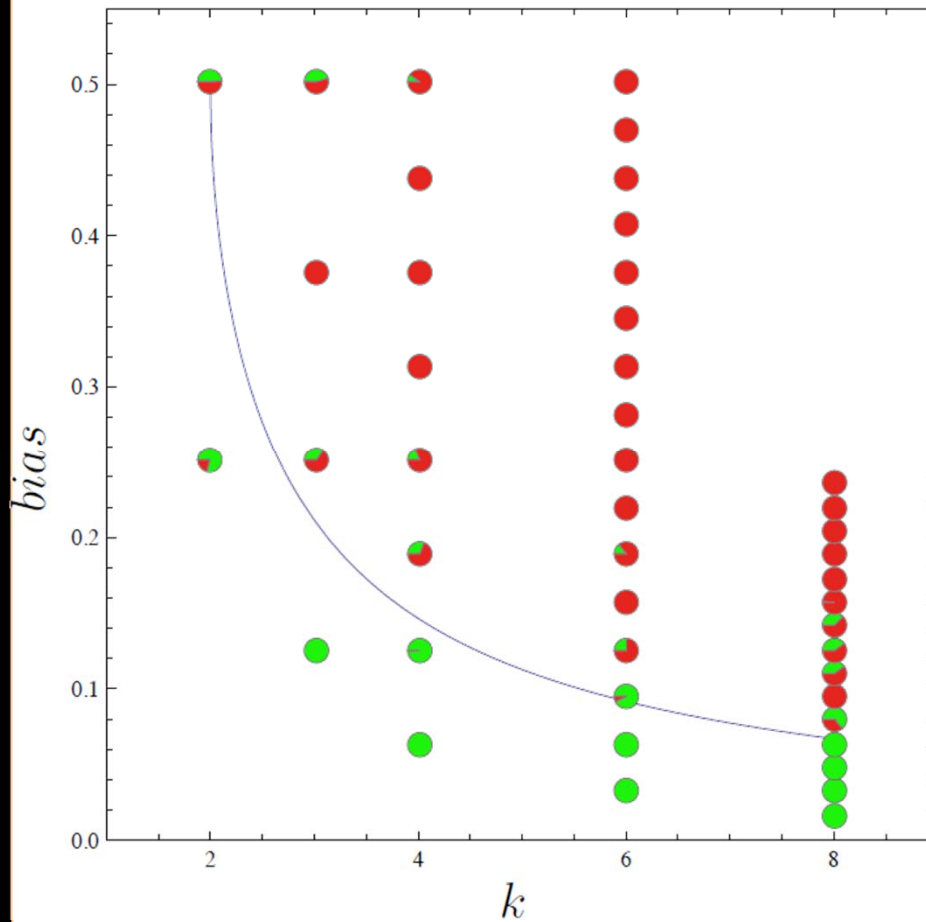


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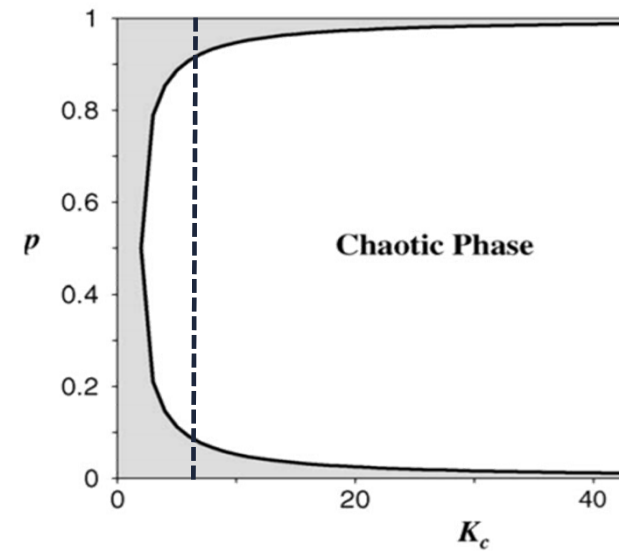
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## Current theory



$$p = \frac{1}{2} \left( 1 - \sqrt{1 - \frac{2}{k}} \right)$$

Aldana, M. [2003]. *Physica D*. **185**: 45–66



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# criticality in the presence of canalization

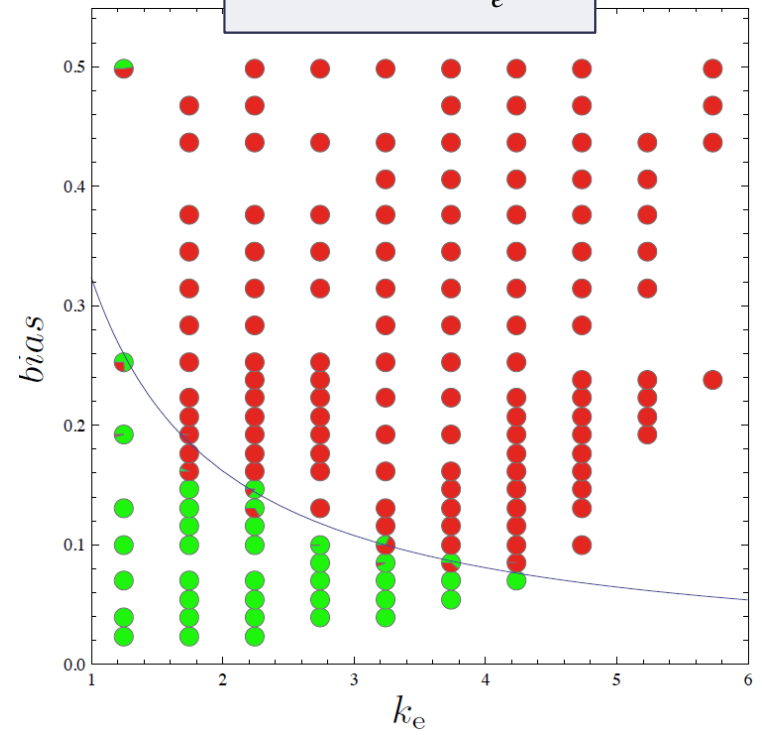
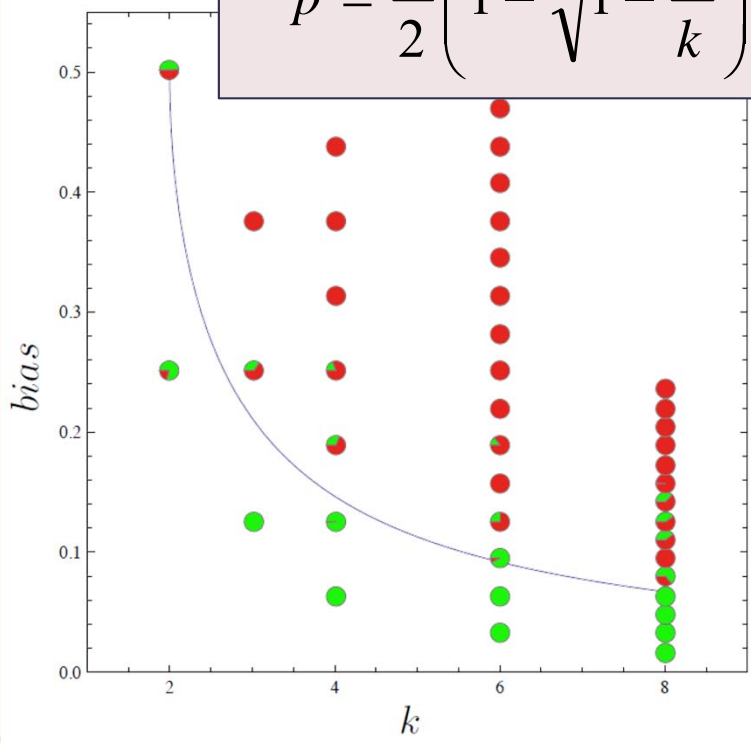
input redundancy, effective connectivity

$$k_r(x) = \frac{\sum_{f_\alpha \in F} \max_{\theta | f_\alpha \in \Theta_\theta} (n_\theta^\#)}{2^k}$$

$$k_e(x) = k(x) - k_r(x)$$

$$p = \frac{1}{2} \left( 1 - \sqrt{1 - \frac{2}{k}} \right)$$

$$p_{crit} = \frac{0.32}{k_e}$$



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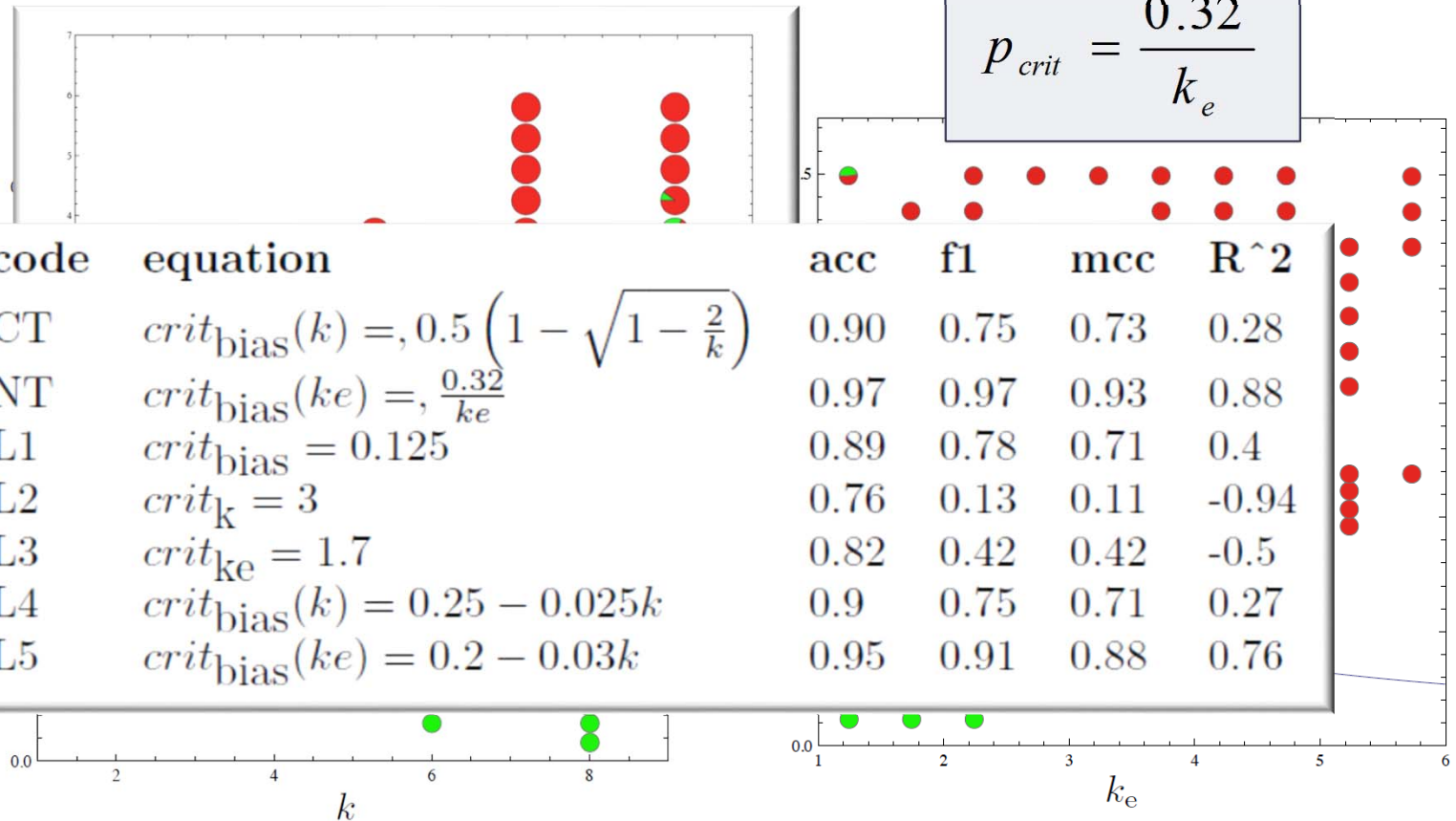
# criticality in the presence of canalization

input redundancy, effective connectivity

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