# Opinion dynamic models: on the $2R$ conjecture.[1]

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**Abstract** *We consider an opinion dynamic model and study its convergence and the properties of the equilibria to which it converges. We attempt to explain why the observed distances between opinions at equilibrium are typically larger than what one could expect, an observation captured in the so-called 2R conjecture.*

## 1 Krause opinion dynamic model

We consider a simple dynamical model of agents distributed on the real line. The agents have a limited vision range $R$ and they synchronously update their positions by moving to the average position of the agents that are within their vision range (i.e. those those that are separated from them by a distance smaller than or equal to $R$) This dynamical model was initially introduced in the social science literature as an opinion dynamic model and is known there as the "Krause model" [1].

## 2 Observations, conjectures and results

The model gives rise to surprising and partly unexplained dynamics. One of the central observations is the $2R$ conjecture: when sufficiently many agents are distributed on the real line and have their position evolve according to the above dynamics, the agents eventually merge into clusters that have inter-cluster distances roughly equal to $2R$ as shown in Figure 1. This observation is supported by extensive numerical evidence and is robust under various modifications of the model. It is easy to see that clusters need to be separated by at least $R$ [2, 3]. On the other hand, the unproved bound $2R$ that is observed in practice can probably only be obtained by taking into account the specifics of the model's dynamics. We study these dynamics and consider a number of issues related to the $2R$ conjecture that explicitly uses the model dynamic. In particular, we provide bounds for the vision range that lead all agents to merge into only one cluster, we analyze the relations between agents on finite and infinite intervals, and we introduce a notion of equilibrium stability for which clusters of equal weights need to be separated by at least $2R$ to be stable. These results, however, do not prove the conjecture. To understand the system behavior for a large agent density, we also consider a version of the model that involves a continuum of agents. We study properties of this continuous model and of its equilibria, and investigate the connections between the discrete and continuous models.

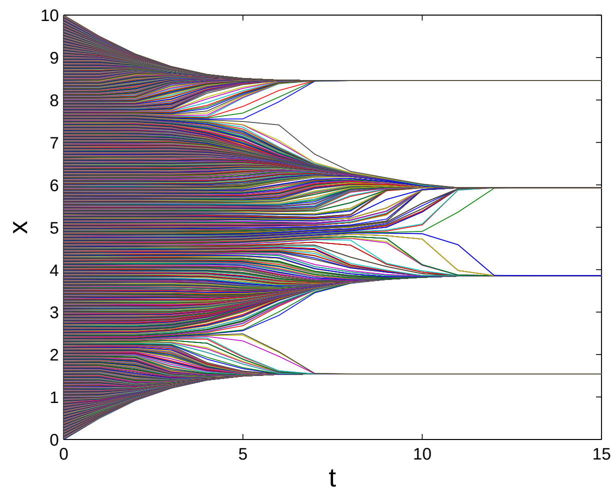


**Figure 1:** Evolution for 15 time steps of 1000 agent opinions initially equidistantly located on an interval of length 10. The observed inter-cluster distances are much larger than the vision range ($R = 1$) of the agents.

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