Playing Clobber on a Cycle[1]

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Abstract *We present the combinatorial* 2*-player game clobber and its* 1*-player version solitaire clobber. We present some old and recent results on both games.*

1 What is Clobber?

The combinatorial game clobber was introduced by Albert, Grossman, Nowakowski and Wolfe in 2002 [1]. It is played with black and white pawns as in the game of Draughts. White begins and moves a white pawn onto an adjacent black pawn and removes this black pawn from the game. The place formerly occupied by the white pawn is then empty and cannot be occupied anymore. Then Black plays similarly with the black pawns, etc. The player making the last move wins the game. As this game admits no draw and only a bounded number of moves, it can be proved that for any initial position there exists a winning strategy for either White or Black. The following initial position (where pawns lie on a line) is for instance winning for White ○●○●○●○●, and the first play could be ○●○●○● ○.
Clobber is usually played on $n \times m$ grids with alternated ○ and ●, but it can be generalized on any graph, locating the the pawns on the nodes. A 1-player version of the game was proposed in [2], Solitaire clobber (see also [3, 4]). In this game, the goal is to remove as many pawns as possible by moving black or white pawns in the same way as in the usual clobber but without necessarily alternating black moves and white moves. We analyze clobber and solitaire clobber in the particular case where pawns lie on a cycle.

2 Previous and new results on Clobber

Consider first the solitaire game. The *reducibility value* of an initial conformation is the minimal possible number of remaining pawns at the end of the game [2, 3]. A conformation allowing one to remove all pawns but one has for example a reducibility value of one. The case where pawns lie on a line has already been studied relatively deeply. It has been proved that the reducibility value is at most $\lceil n/2 \rceil$ (n being the initial number of pawns), and an initial conformation has been found where this bound is tight [2]. We focus on another particular graph, the cycle. It has been recently asked whether the maximal reducibility value on the cycle is $\simeq n/4$[1]. This value is obtained on a cycle of length $n = 4k$ by taking k repetitions of the pattern ○●○● as initial conformation. Indeed, an optimal strategy for the solitaire on this conformation can be proved to be the following: reduce each pattern ○●○● to a simple ○ by first moving the leftmost pawn to the right, then the rightmost pawn to the left, and finally the leftmost one to the right. We answer the question negatively by presenting a conformation on the cycle for which the reducibility value is $k = n/3$, and consisting in k repetitions of the pattern ○○●. Moreover we show that this is the the worst conformation, closing the question of general bounds on the reducibility value on the cycle.
We also analyze the original (two players) clobber game. Some initial conformations have already been studied, many of which leading to interesting open questions It has for instance been conjectured that the line $(○●)^k$ (that is, k repetitions of ○●) always admits a winning strategy for the first player except for $k = 3$. We propose a general strategy for 2-players clobber that leads to victory in an infinite class of initial conformations. Roughly speaking, one puts the game in a symmetric conformation so that for any move of the opposite player, there exists a symmetric move restoring this symmetry. Therefore, the opposite player can only lose since every one of its moves is followed by another one.This strategy is applied to show that the second player can always win a game on the cycle if the original conformation is $2k+1$ repetitions of the pattern ○●.

Acknowledgment: The authors wish to thank Eric Duchêne for having made them discover this interesting topic and for his suggestions of open issues.

References

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[1] This research was supported by the Concerted Research Action (ARC) "Large Graphs and Networks" of the French Community of Belgium and by the Belgian Programme on Interuniversity Attraction Poles initiated by the Belgian Federal Science Policy Office. The scientific responsibility rests with its authors. J.M. Hendrickx and R. Jungers hold a FNRS fellowship (Belgian Fund for Scientific Research)

[1] E-mail communication with Eric Duchêne