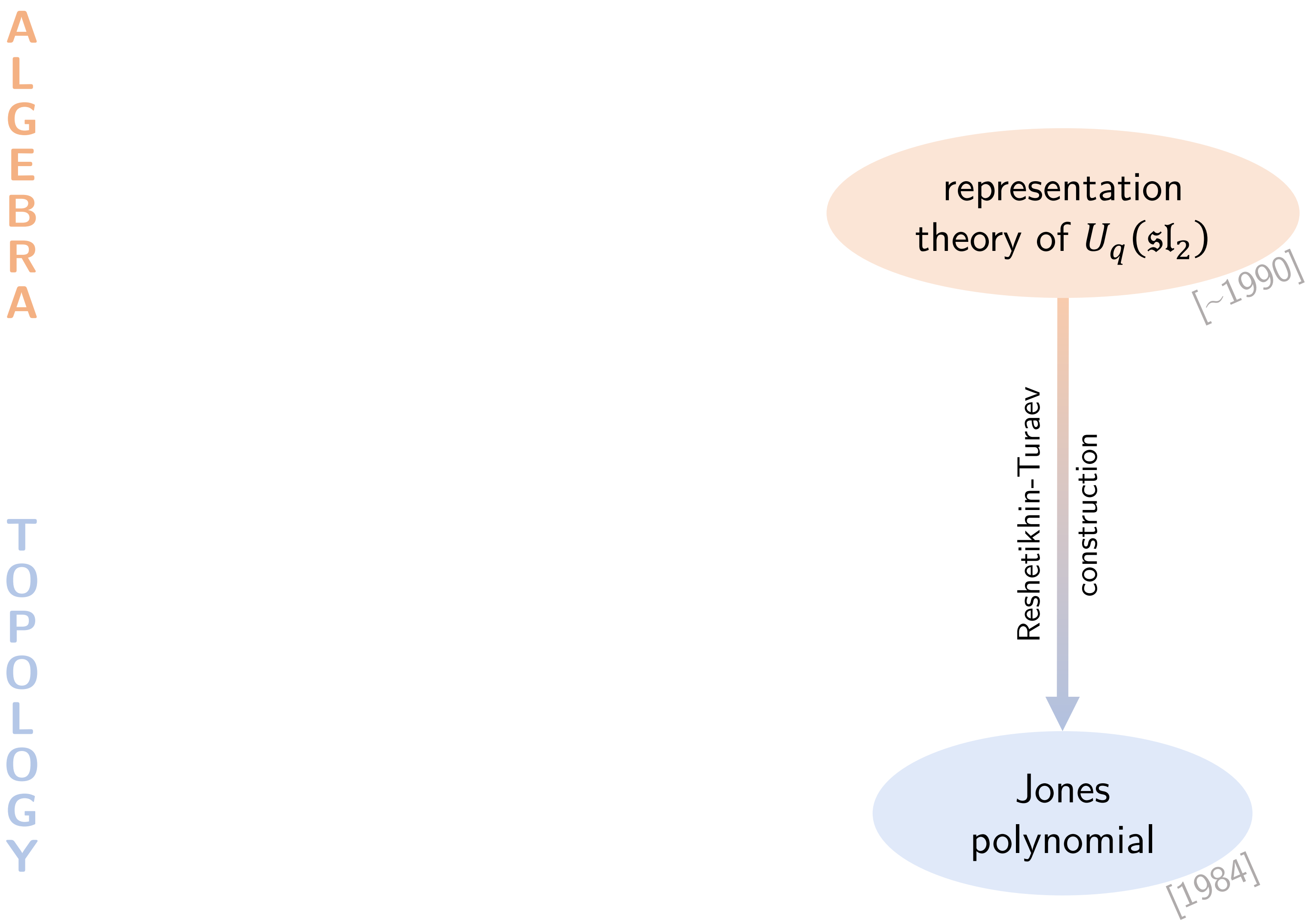
Odd Khovanov homology and 2-supercategories

Léo Schelstraete

Seminar on Quantum groups, Hopf algebras and monoidal categories

May 2nd 2022

THE BIG PICTURE



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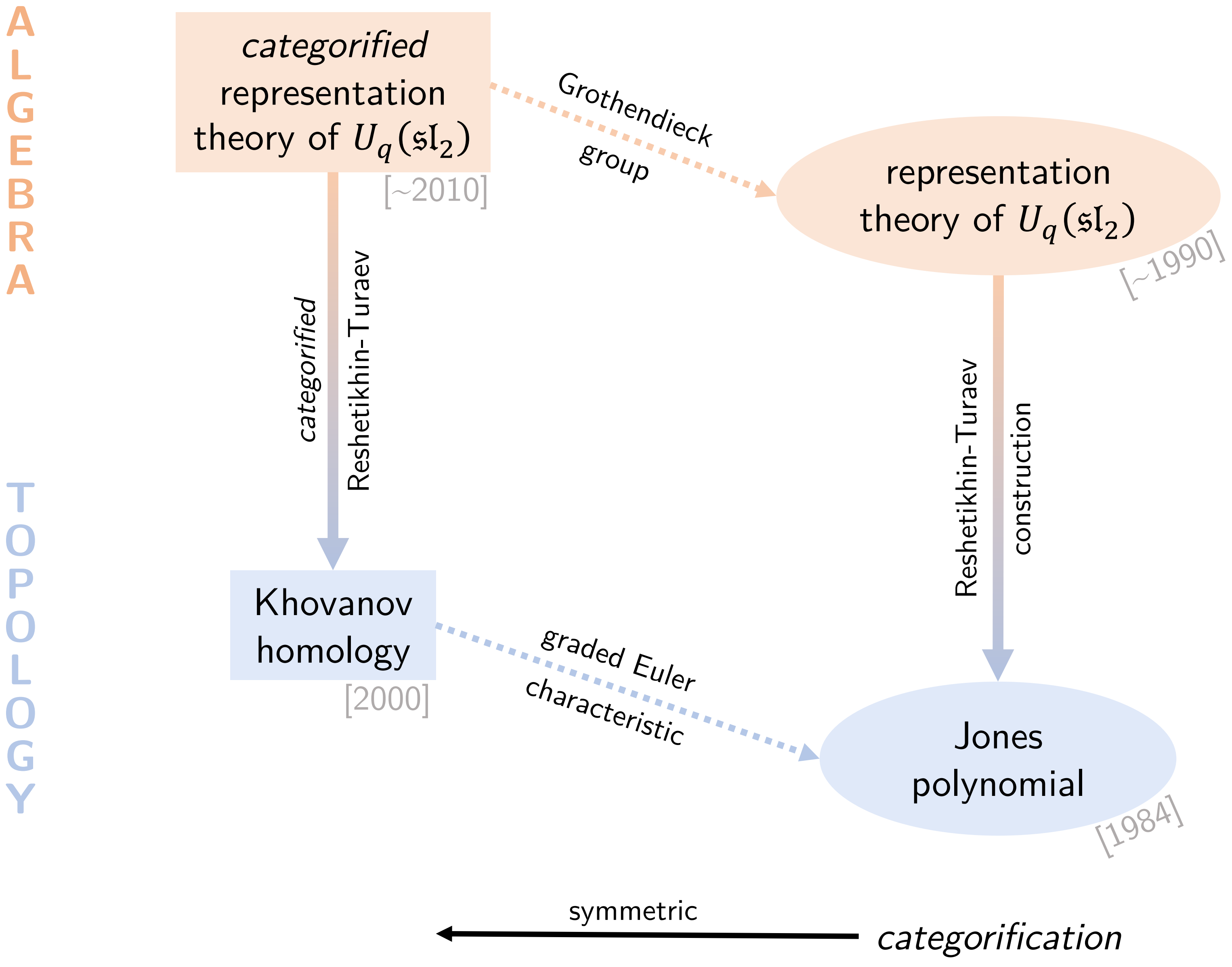

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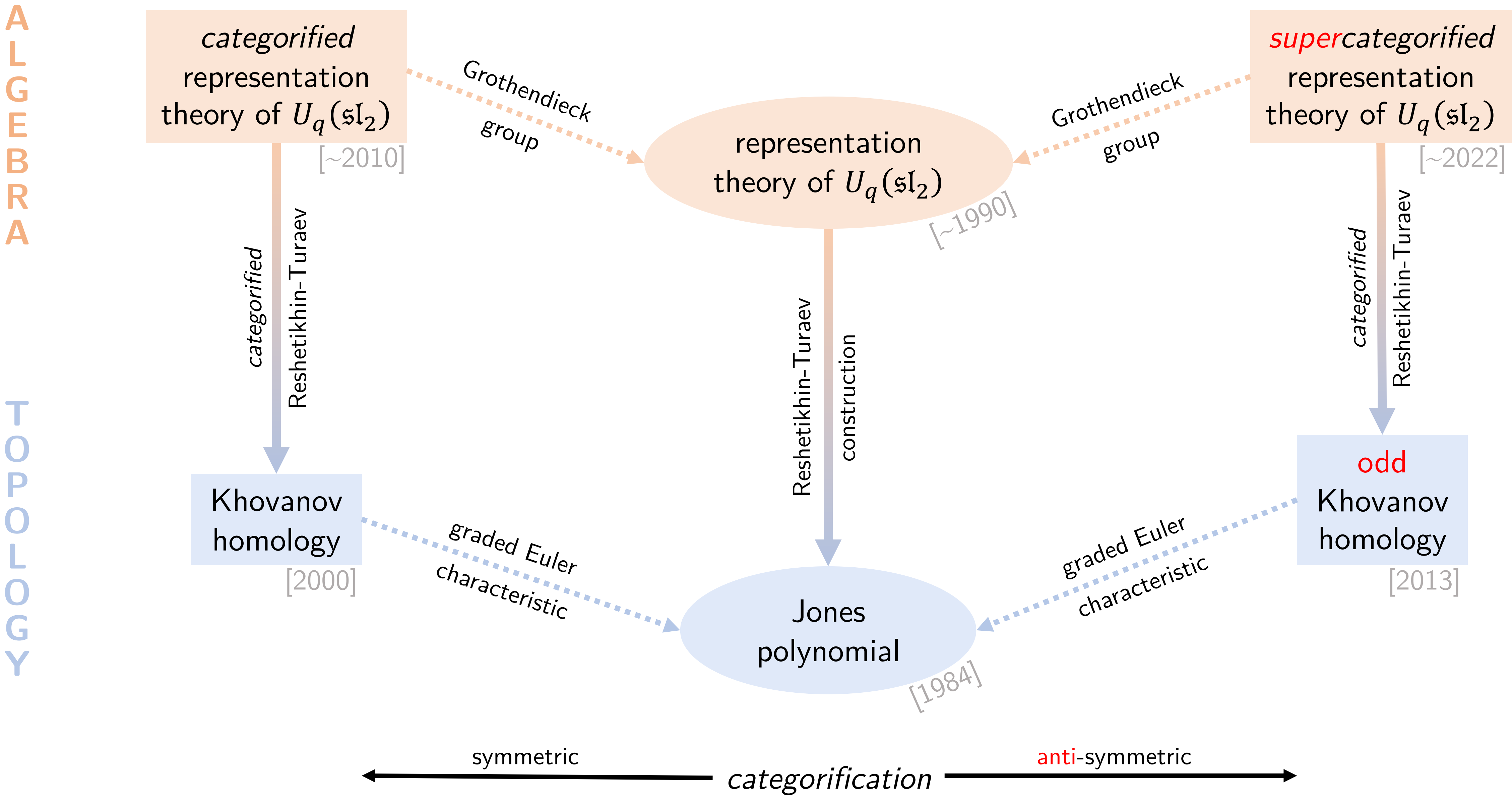


1 | CATEGORIFYING THE JONES POLYNOMIAL

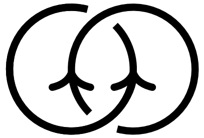
or the topological part of the story

1. CATEGORIFYING JONES | EXAMPLE

Khovanov homology

$$\chi_q\left(Kh^{\bullet,\bullet}\right) = \sum_{j\in\mathbb{Z}} (-1)^j \,\mathrm{qdim}\left(Kh^{\bullet,j}\right)$$

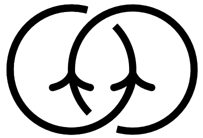
Jones polynomial

Kh () =

homological grading j	0	1	2
$Kh^{\bullet,j}$	$\mathbb{Q}\oplus\mathbb{Q}[2]$	0	$\mathbb{Q}[4]\oplus\mathbb{Q}[6]$

$(-1)^0(1+q^2)$

$(-1)^2(q^4+q^6)$

J () $= 1+q^2+q^4+q^6$

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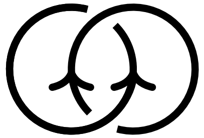
Kh () =

homological grading j	0	1	2
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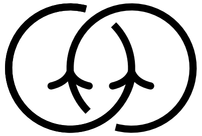
shift in q-grading

$(-1)^0(1+q^2)$

$(-1)^2(q^4+q^6)$

J () $= 1+q^2+q^4+q^6$

Khovanov homology is graded!

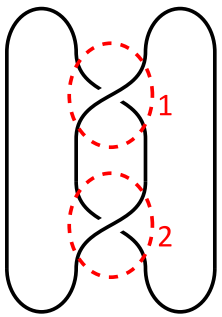
$$Kh^{\bullet,\bullet} = \bigoplus_{i,j\in\mathbb{Z}} Kh^{i,j}$$

quantum grading

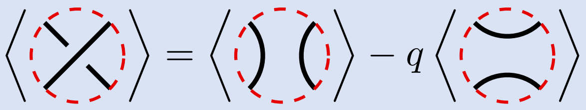
homological grading

$$\mathrm{qdim}\left(Kh^{\bullet,j}\right) = \sum_{i\in\mathbb{Z}} q^i \dim\left(Kh^{i,j}\right)$$

1. CATEGORIFYING JONES | BACK TO KAUFFMAN

$D_L =$ 

Kauffman bracket*

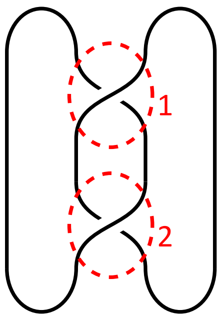


$\left\langle \bigcirc \amalg D \right\rangle = (q + q^{-1})\langle D \rangle$ for any diagram D

*I disregard normalization issues!

1. CATEGORIFYING JONES | BACK TO KAUFFMAN

$D_L =$



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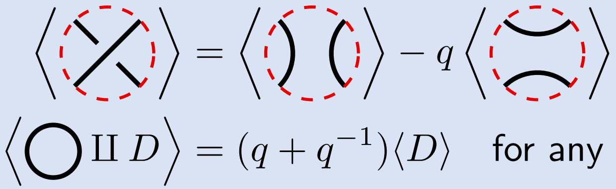

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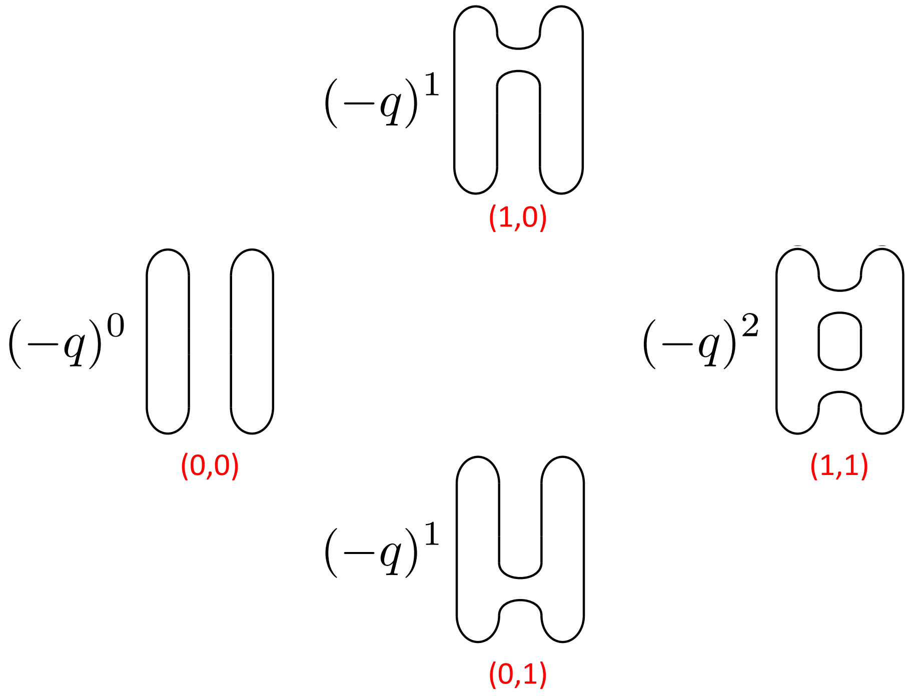


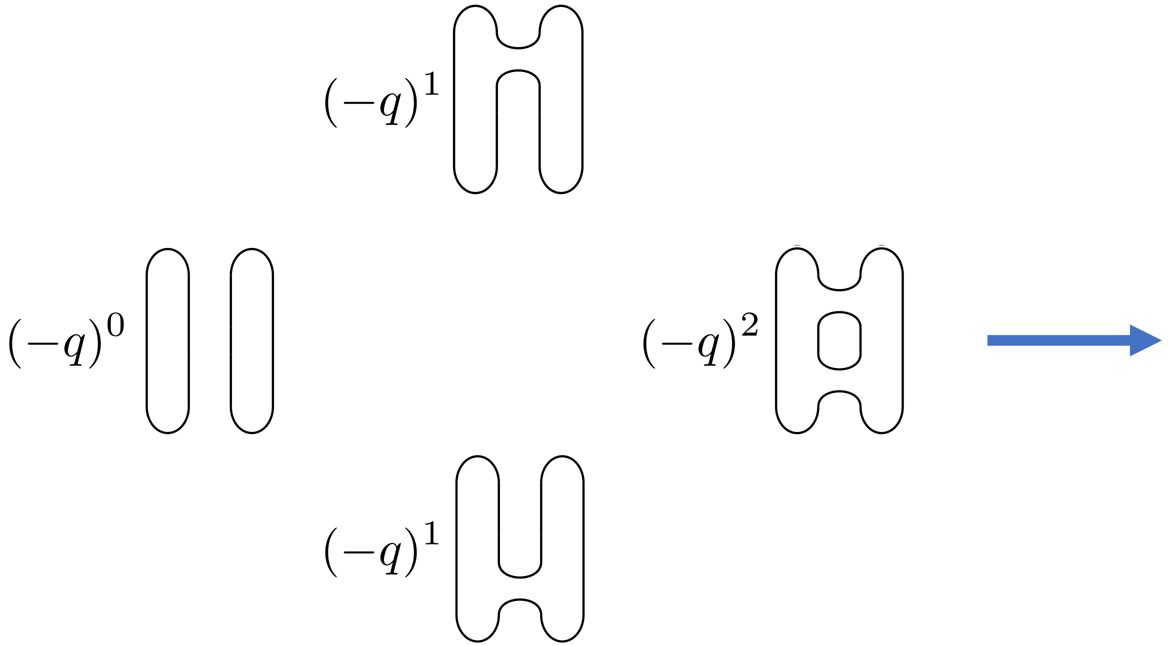
$$(-q)^1(q+q^{-1})$$

$$(-q)^0(q+q^{-1})^2 \quad + \quad (-q)^2(q+q^{-1})^2$$

$$(-q)^1(q+q^{-1})$$

$$\langle D_L\rangle = (-q)^0(q+q^{-1})^2 + 2(-q)^1(q+q^{-1}) + (-q)^2(q+q^{-1})^2$$

1. CATEGORIFYING JONES | MAIN INGREDIENTS



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main ingredients

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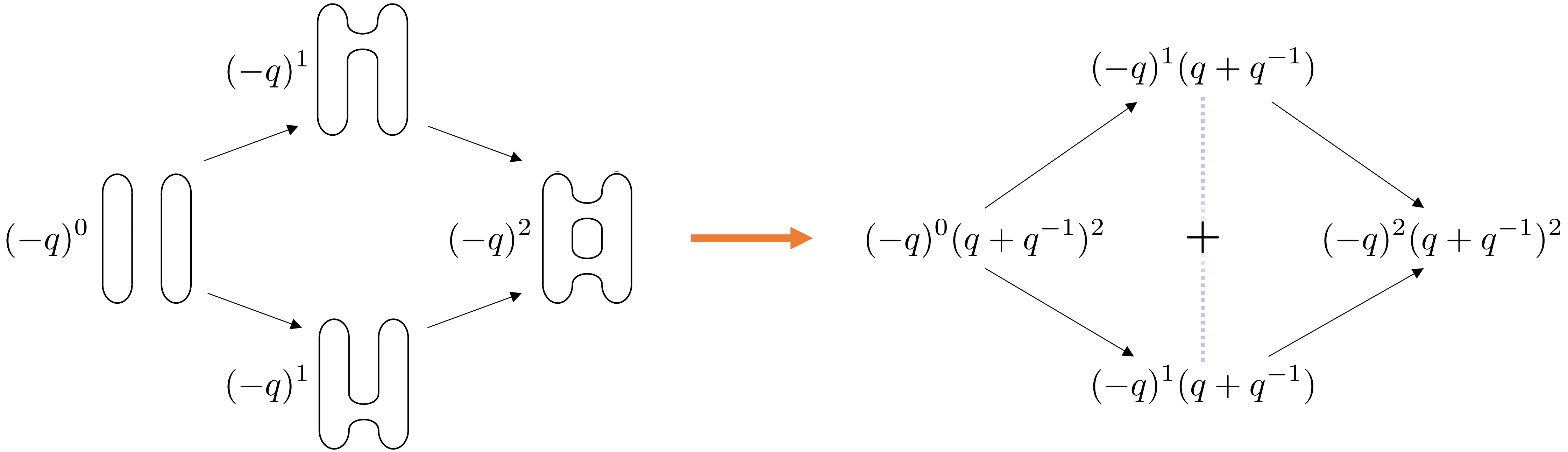

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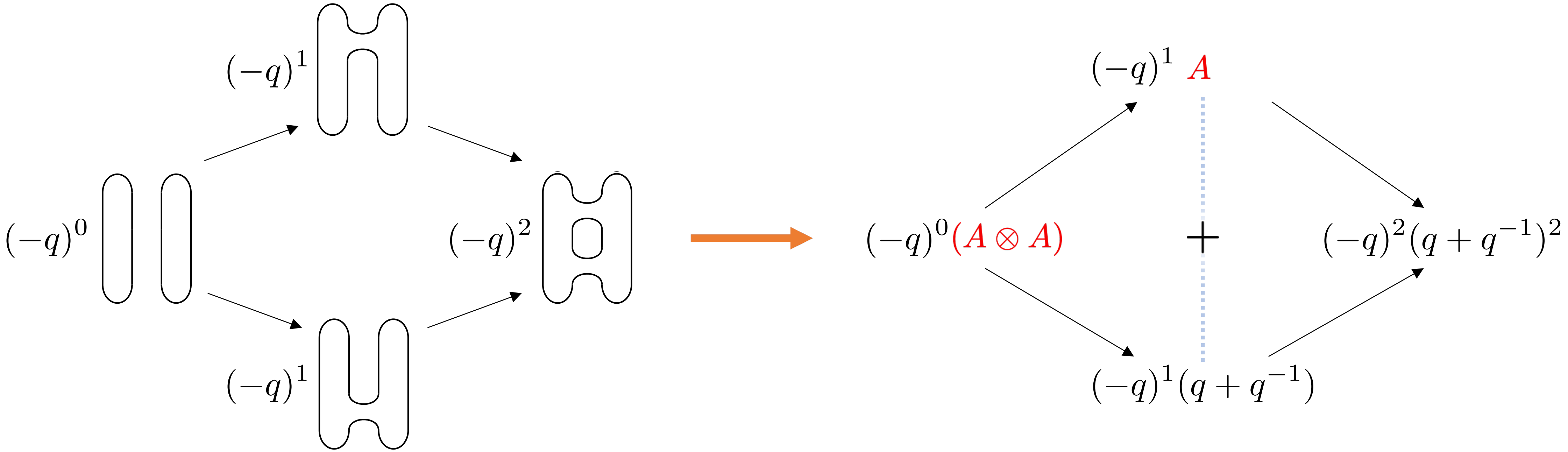

main ingredients

$\bigcirc \mapsto A,$ where A is such that $\mathrm{qdim}(A) = q + q^{-1}$

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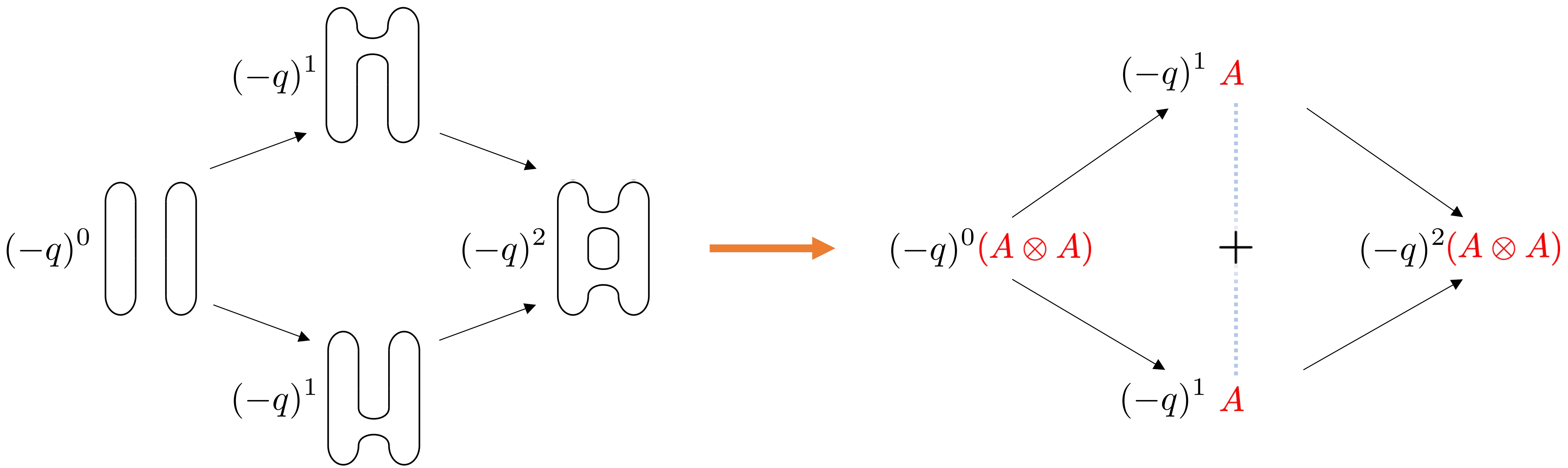

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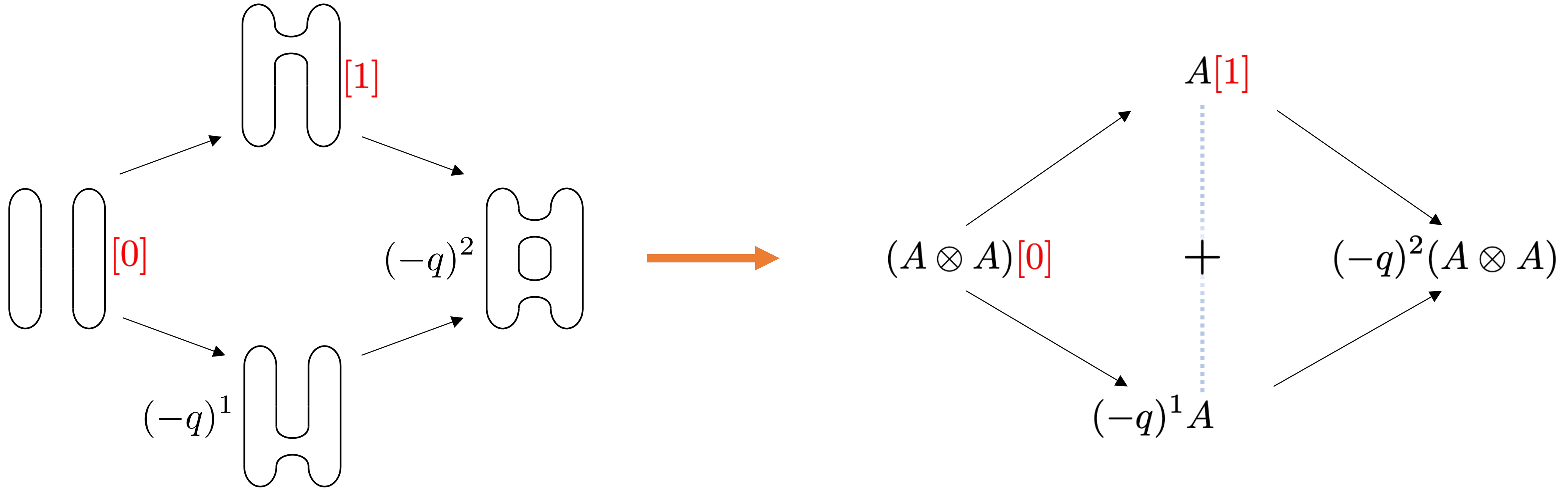

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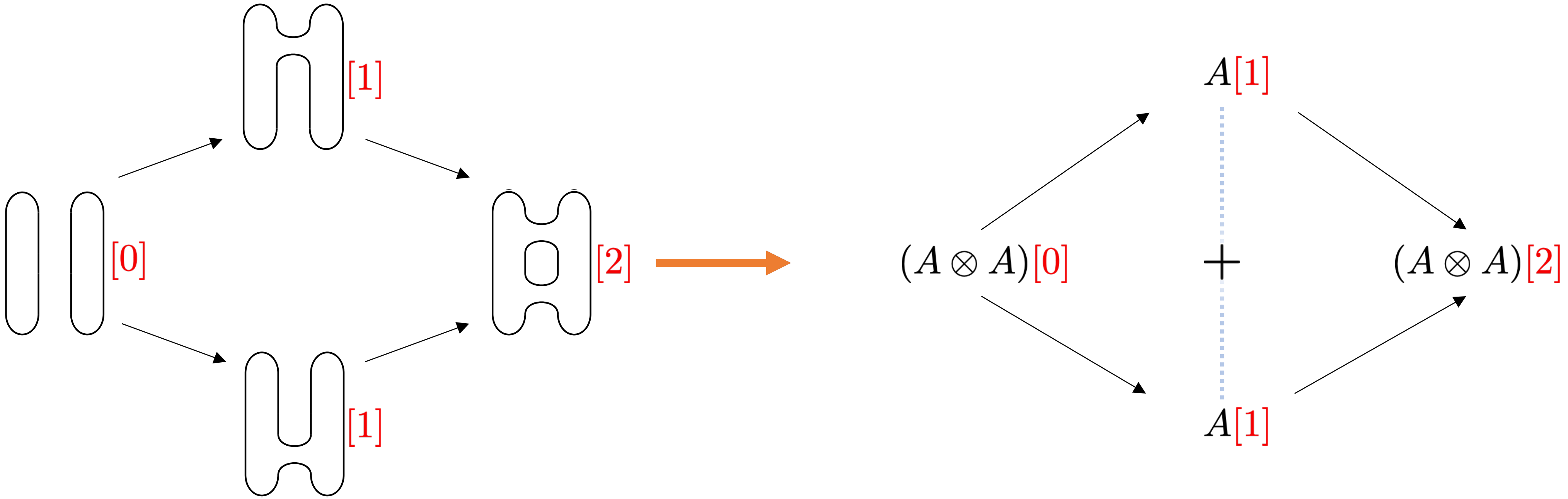

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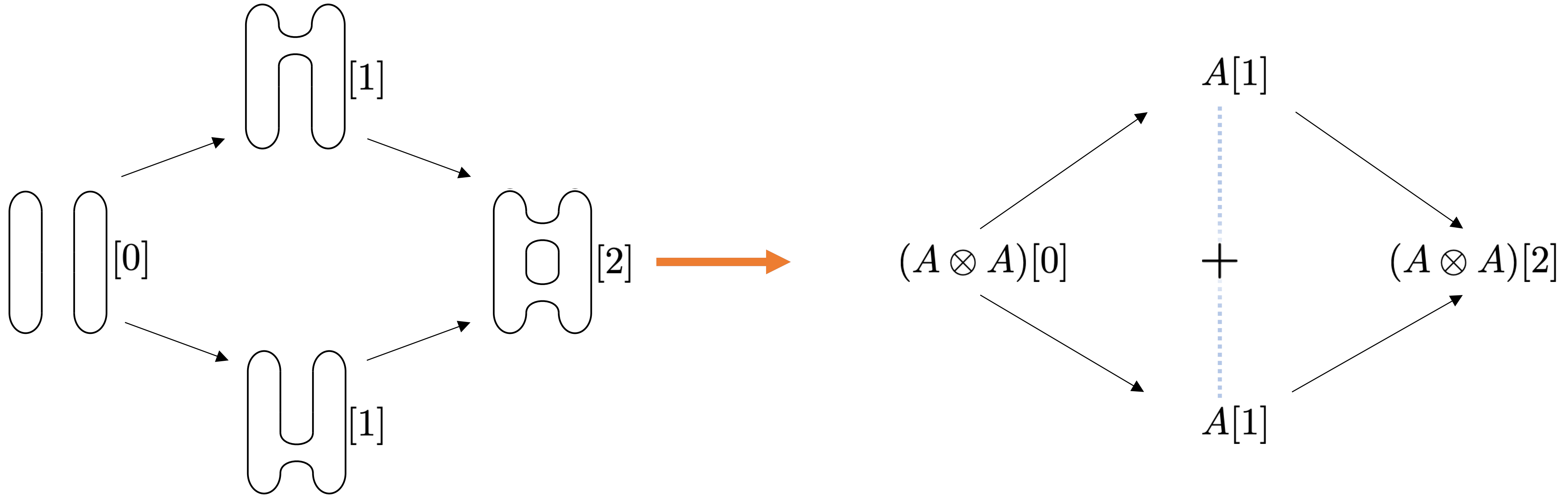

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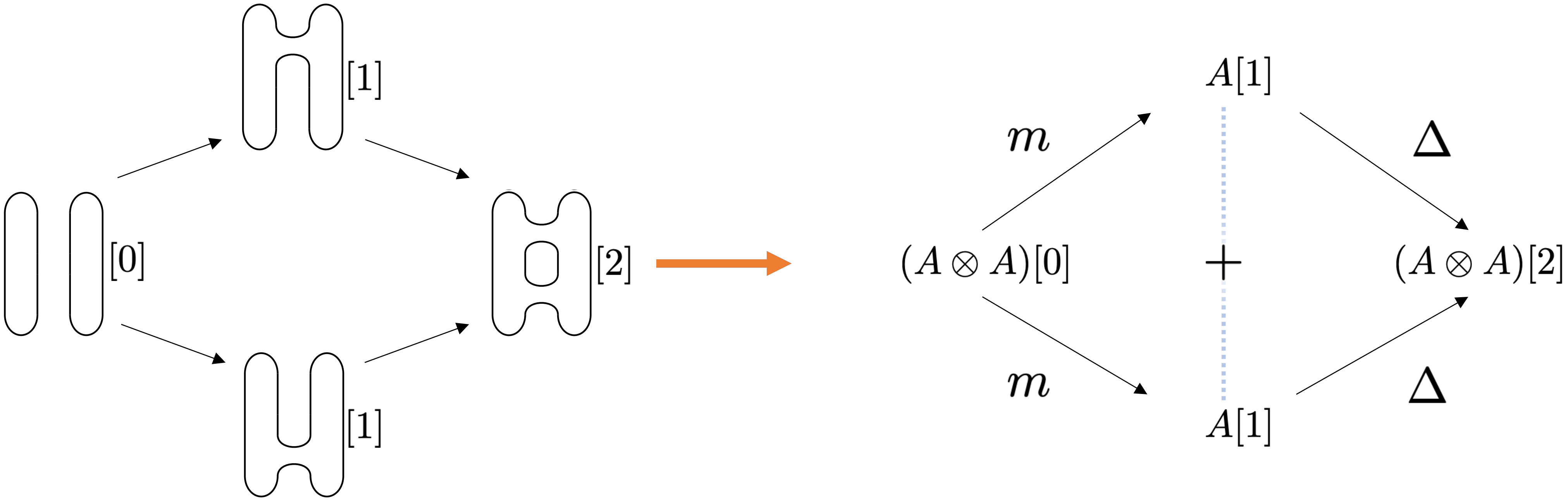

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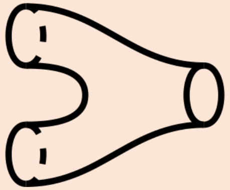
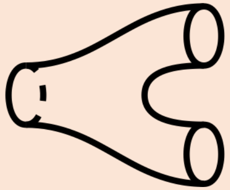
 $\mapsto \; m\colon A\otimes A \to A$ and  $\mapsto \; \Delta\colon A \to A\otimes A$

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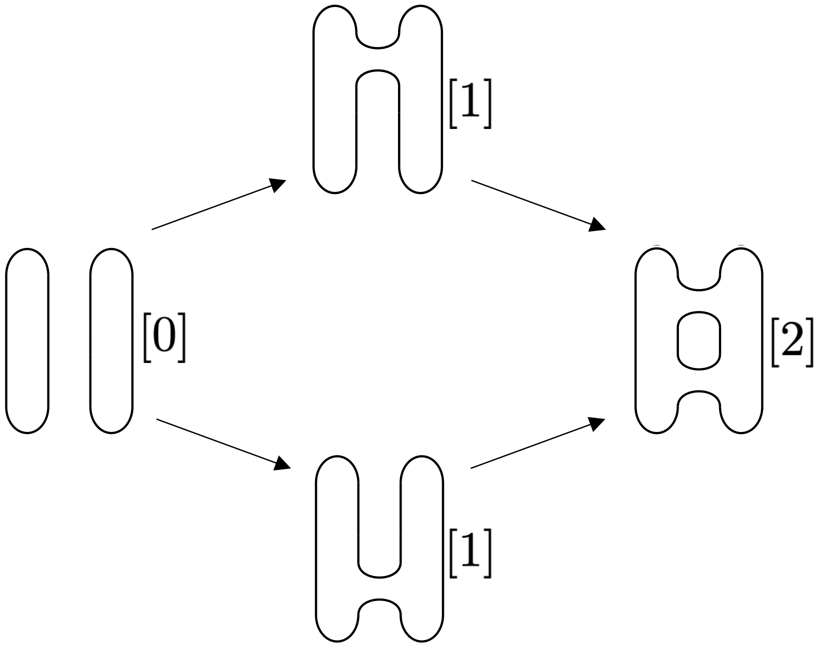

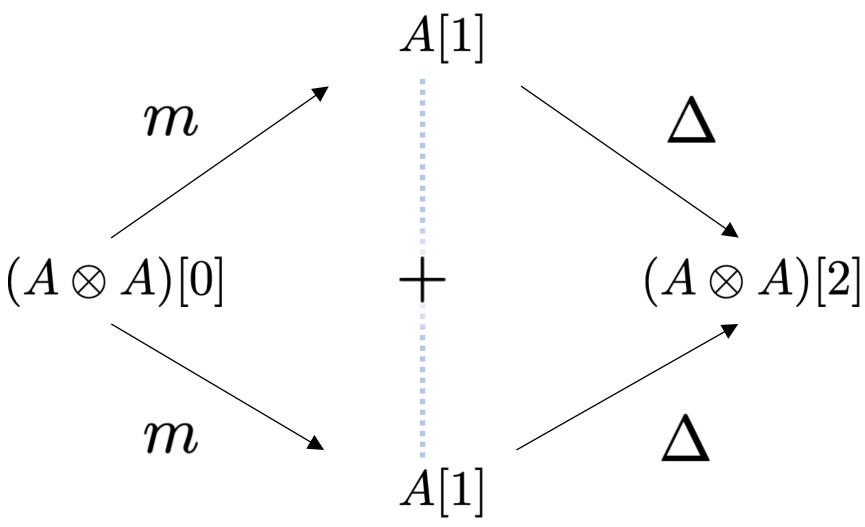

main ingredients

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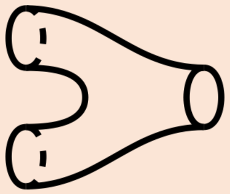
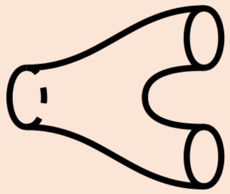
 $\mapsto A,$ where A is such that $\mathrm{qdim}(A) = q + q^{-1}$

 $\mapsto \; m\colon A \otimes A \to A$ and  $\mapsto \; \Delta\colon A \to A \otimes A$

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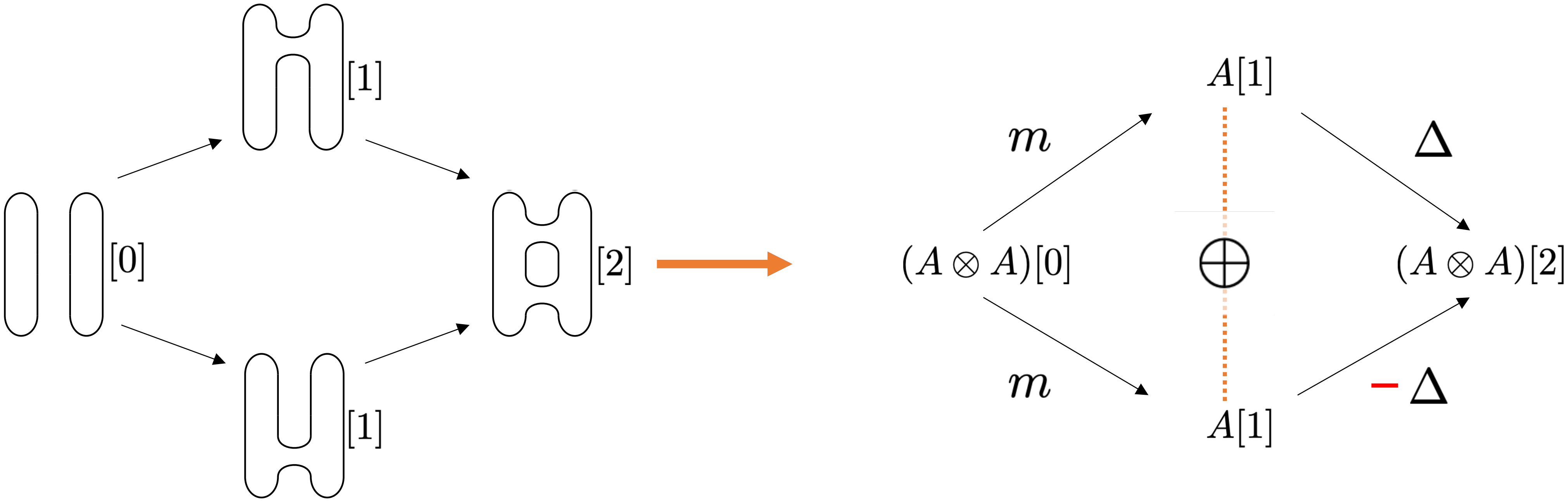

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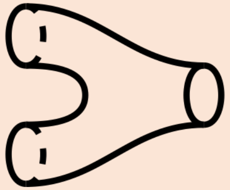
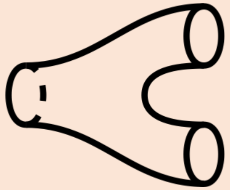
 $\mapsto \; m\colon A \otimes A \to A$ and  $\mapsto \; \Delta\colon A \to A \otimes A$

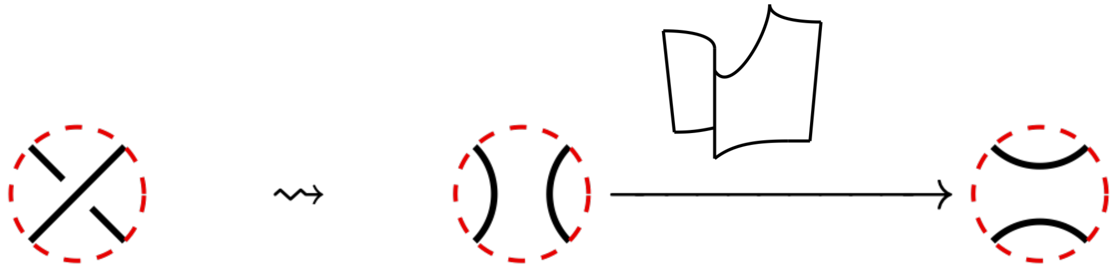
2 SIGNS

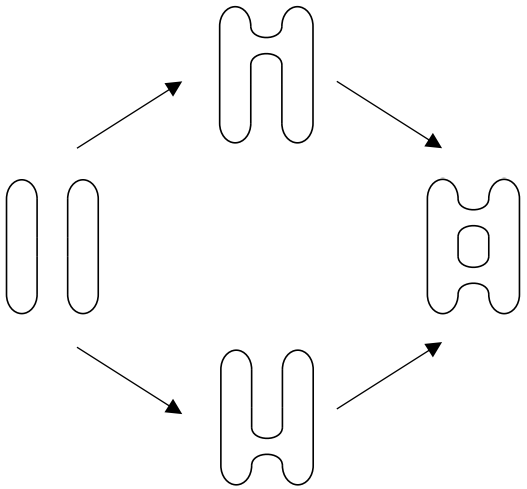
Fix signs so that all squares anti-commute ⇒ chain complex

1. CATEGORIFYING JONES | RECAP

0 HYPERCUBE OF RESOLUTIONS

diagram with n crossings ⇒ hypercube of dimension n





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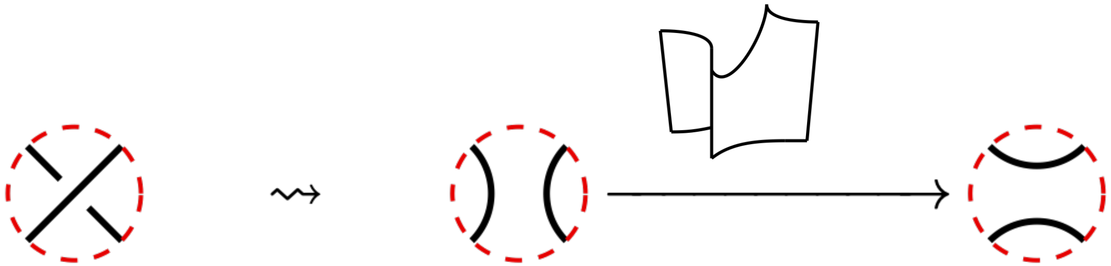

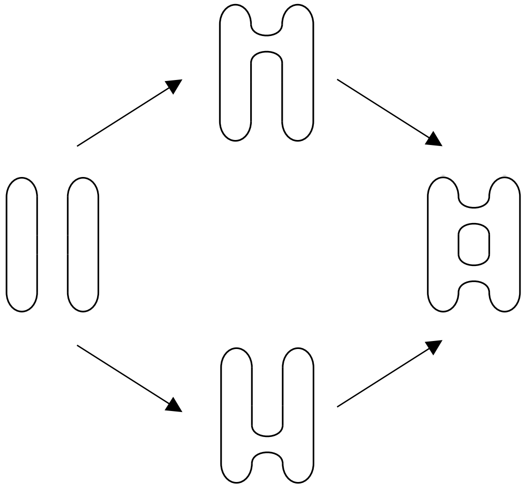

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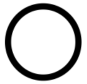
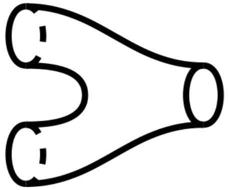
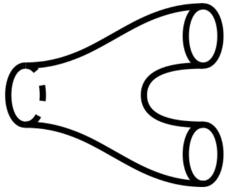
 $\mapsto A$, where A is such that $\mathrm{qdim}(A) = q + q^{-1}$

 $\mapsto \; m\colon A \otimes A \to A$ and  $\mapsto \; \Delta\colon A \to A \otimes A$

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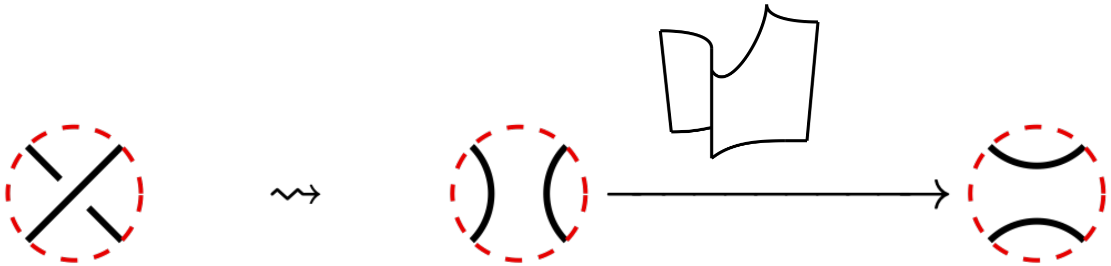

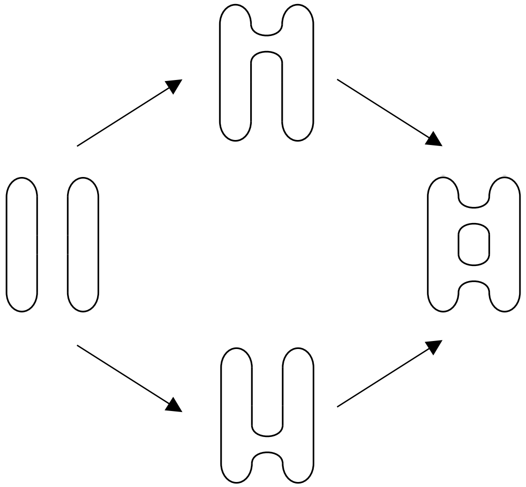

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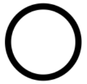
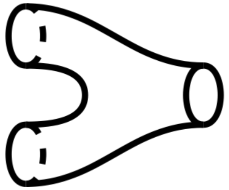
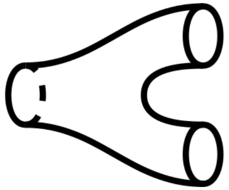
 $\mapsto A$, where A is such that $\mathrm{qdim}(A) = q + q^{-1}$

 $\mapsto \; m\colon A \otimes A \to A$ and  $\mapsto \; \Delta\colon A \to A \otimes A$

2 SIGNS

Fix signs so that all squares anti-commute $\Rightarrow$ chain complex

1. CATEGORIFYING JONES | KHOVANOV HOMOLOGY

1 TQFT

$A = {}^{\mathbb{k}[x]}\!/\!_{x^2}$ and more generally: $A^{\otimes n} = {}^{\mathbb{k}[x_1,\ldots,x_n]}\!/\!_{x_1^2,\ldots,x_n^2}$

$m\colon {}^{\mathbb{k}[x_1,x_2]}\!/\!_{x_1^2,x_2^2} \to {}^{\mathbb{k}[x]}\!/\!_{x^2}$

$f \mapsto f|_{x=x_1=x_2}$

$\Delta\colon {}^{\mathbb{k}[x]}\!/\!_{x^2} \to {}^{\mathbb{k}[x_1,x_2]}\!/\!_{x_1^2,x_2^2}$

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$$m\colon \mathbb{k}[x_1,x_2]/x_1^2,x_2^2 \to \mathbb{k}[x]/x^2$$
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$$\Delta\colon \mathbb{k}[x]/x^2 \to \mathbb{k}[x_1,x_2]/x_1^2,x_2^2$$
$$f \mapsto (x_1+x_2)f|_{x_1=x}$$

2 SIGNS

use **Koszul rule**: turn all commutative squares into anti-commutative squares

NB: non-canonical choice, but every choice gives an isomorphic chain complex

1. CATEGORIFYING JONES | ODD KHOVANOV HOMOLOGY

1 projective TQFT

$$A = \bigwedge(x) \qquad \text{and more generally:} \quad A^{\otimes n} = \bigwedge(x_1, \ldots, x_n)$$

$$f \wedge g = (-1)^{|f||g|} g \wedge f$$

$$m\colon \bigwedge(x_1, x_2) \to \bigwedge(x) \qquad \Delta\colon \bigwedge(x) \to \bigwedge(x_1, x_2)$$

$$f \mapsto f|_{x=x_1=x_2} \qquad f \mapsto (x_1 - x_2) \wedge f|_{x_1=x}$$

non-canonical sign!

1. CATEGORIFYING JONES | ODD KHOVANOV HOMOLOGY

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non-canonical sign!

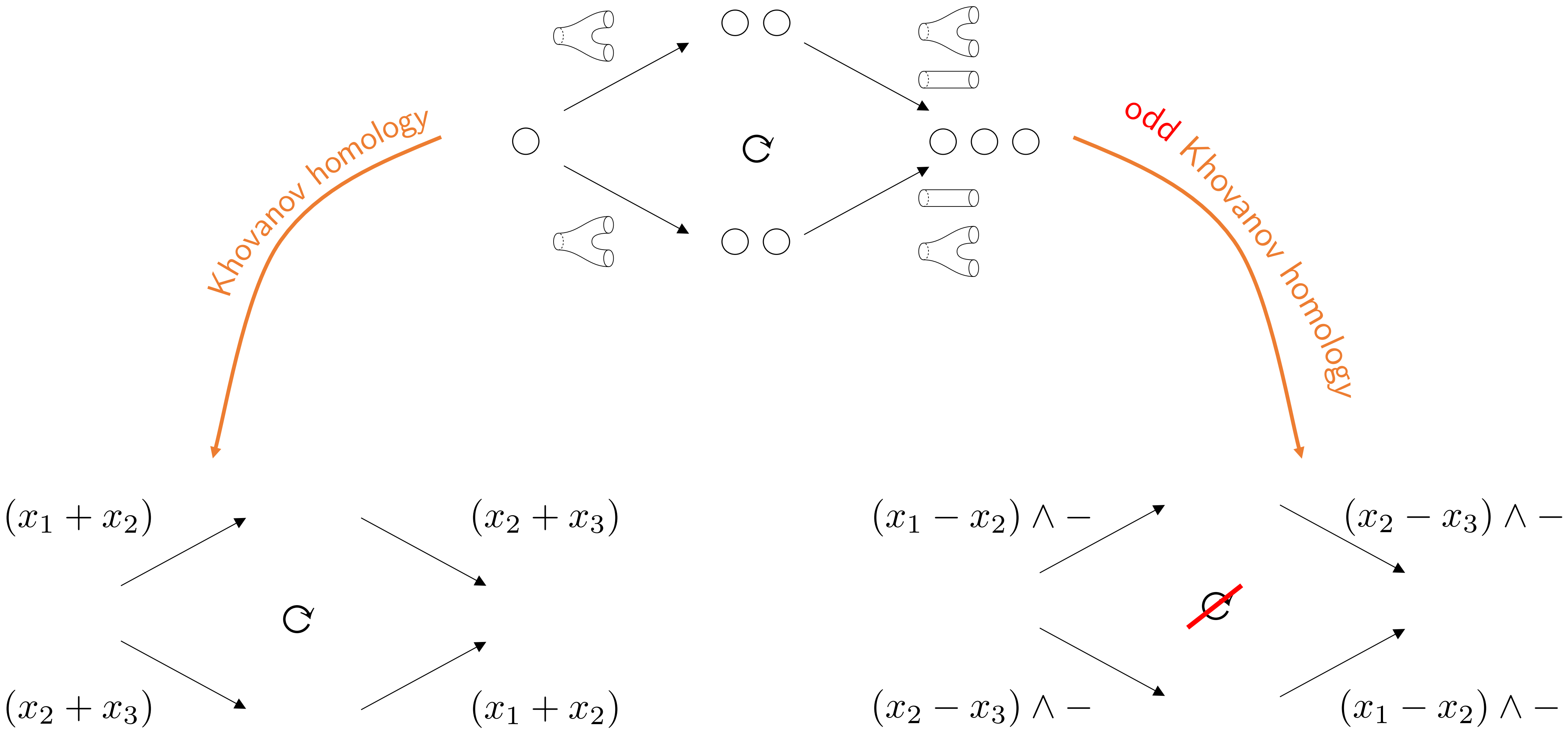
2 SIGNS

some squares are already anti-commutative!

use **super Koszul rule**: turn all ~~commutative~~ squares into anti-commutative squares

NB: non-canonical choice, but every choice gives an isomorphic chain complex

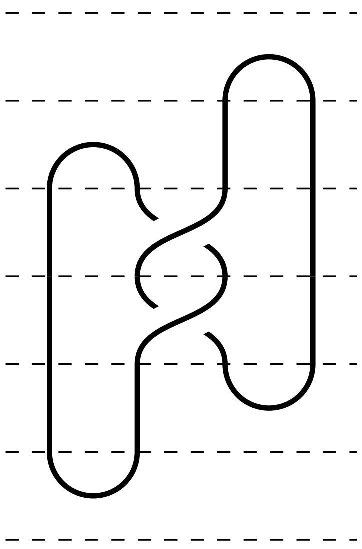
1. CATEGORIFYING JONES | TROUBLE WITH SIGNS



2 | CATEGORIFYING RESHETIKHIN-TURAEV

or the representation theoretical part of the story

2. CATEGORIFYING R-T | EXTENDING TO TANGLES

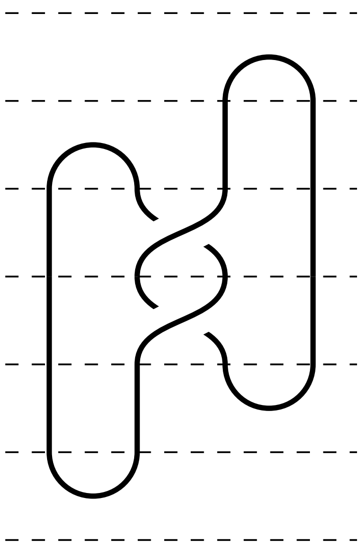


closed tangle diagram

2. CATEGORIFYING R-T | EXTENDING TO TANGLES

Given a quantum group $U_q(\mathfrak{g})$ and a representation V:

Reshetikhin-Turaev



$\mapsto$

$$\begin{array}{c}
\mathbb{C}(q) \\
\uparrow n \\
\mathbb{C}(q) \otimes V \otimes V \\
\uparrow n \\
V \otimes V \otimes V \otimes V \\
\uparrow R \\
V \otimes V \otimes V \otimes V \\
\uparrow R \\
V \otimes V \otimes V \otimes V \\
\uparrow u \\
V \otimes V \otimes \mathbb{C}(q) \\
\uparrow u \\
\mathbb{C}(q)
\end{array}$$

closed tangle diagram

polynomial

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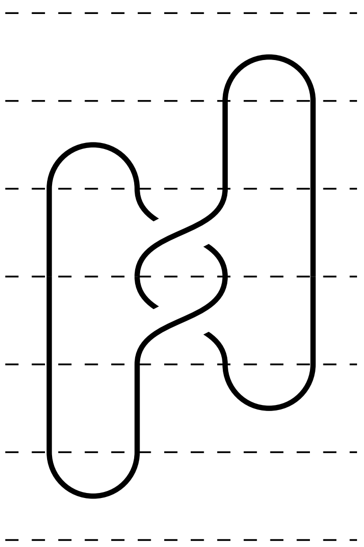


closed tangle diagram

$\mapsto$

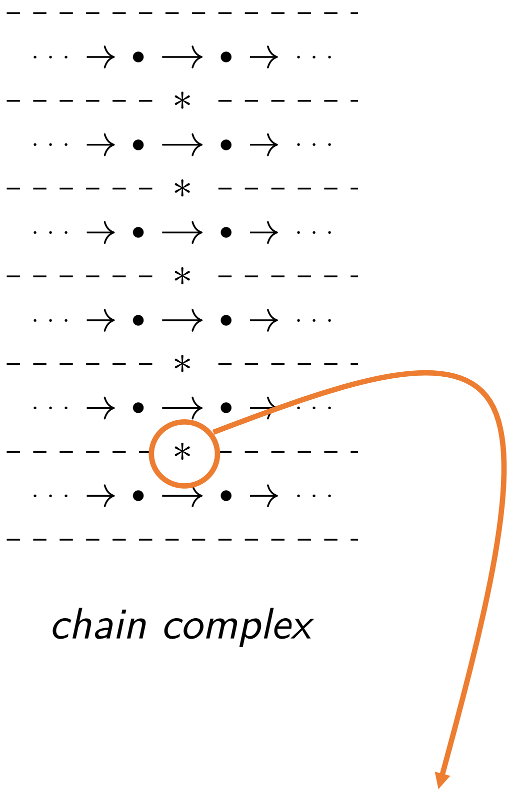
Reshetikhin-Turaev

$$\begin{array}{c} \mathbb{C}(q) \\ \uparrow n \\ \mathbb{C}(q) \otimes V \otimes V \\ \uparrow n \\ V \otimes V \otimes V \otimes V \\ \uparrow R \\ V \otimes V \otimes V \otimes V \\ \uparrow R \\ V \otimes V \otimes V \otimes V \\ \uparrow u \\ V \otimes V \otimes \mathbb{C}(q) \\ \uparrow u \\ \mathbb{C}(q) \end{array}$$

polynomial

or

categorified Reshetikhin-Turaev



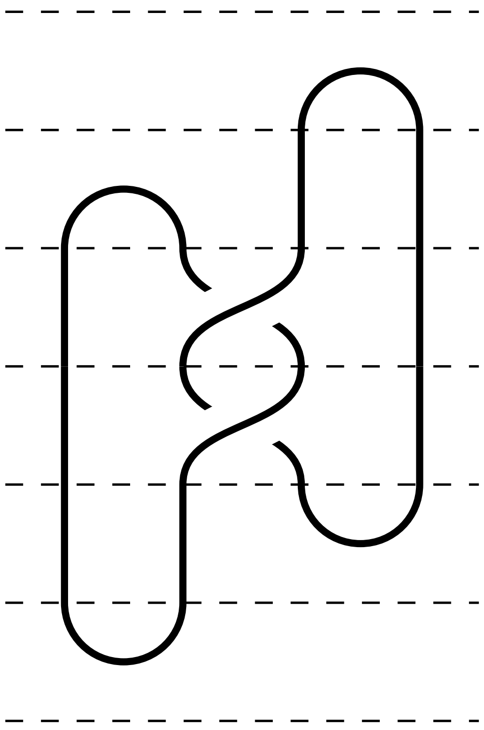
chain complex

we need maps between intertwiners!

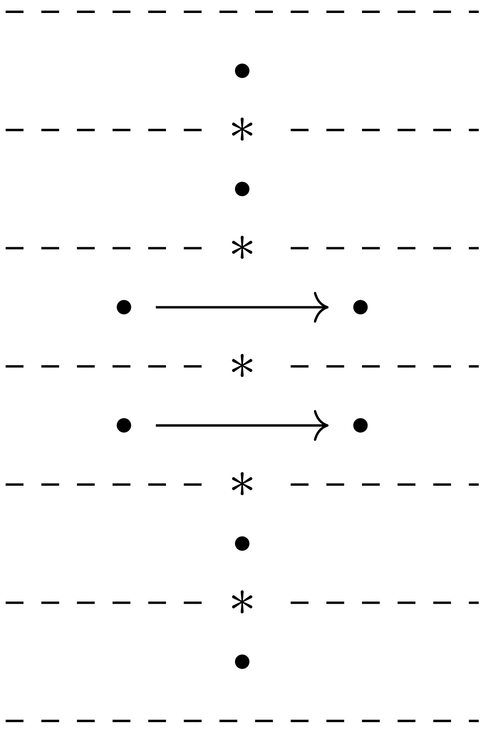
"composition of complexes"

2. CATEGORIFYING R-T | JONES' CASE

For invariants categorifying the Jones polynomial:



$\mapsto$



closed tangle diagram

chain complex **= hypercube!**

2. CATEGORIFYING R-T | 2-CATEGORIES

- - - - - - - - - - - - - - - -

$\cdots \to \bullet \longrightarrow \bullet \to \cdots$

- - - - - - - $*$ - - - - - - -

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two types of composition

$\Rightarrow$ 2-category!

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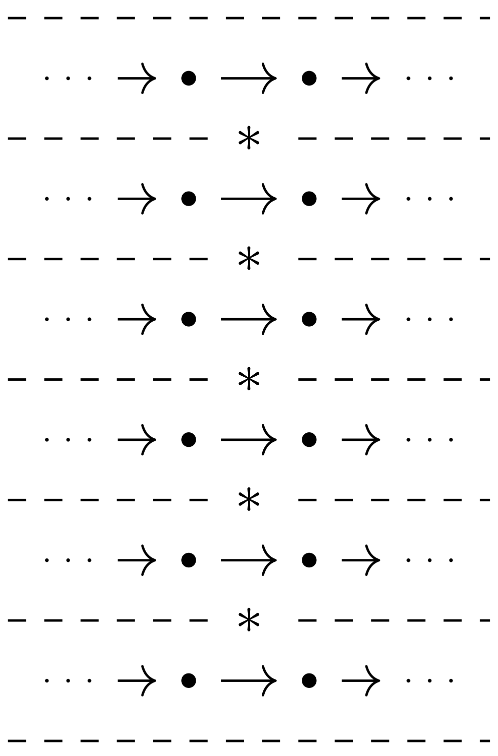

two types of composition
⇒ 2-category!

2-categories

A 2-category is a category with additional "morphisms between morphisms", called *2-morphisms*. They admit two compositions:

- *vertical composition* denoted $\circ$
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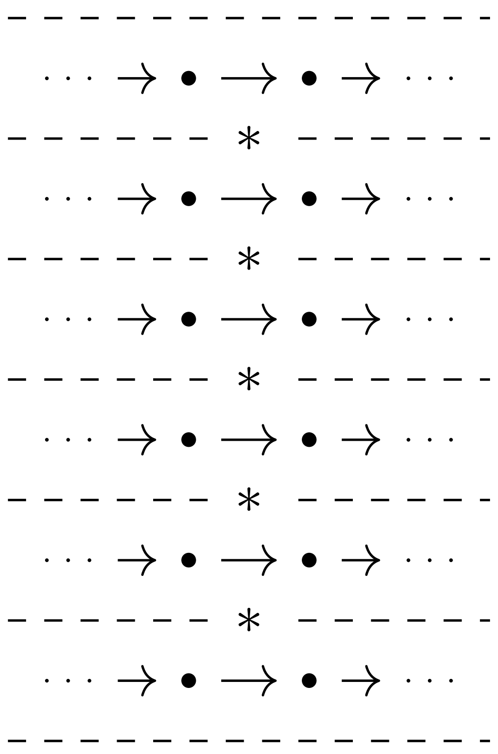
2. CATEGORIFYING R-T | 2-CATEGORIES



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Examples:

- small categories, functors and natural transformations
- (strict) monoidal categories = one-object 2-categories ($\otimes = *$)

2. CATEGORIFYING R-T | GENERAL STRATEGY

A general strategy to categorify the Reshetikhin-Turaev construction:

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- Find a 2-category that "categorifies" the category $Rep\big(U_q(\mathfrak{g}), V\big)$ of representations of $U_q(\mathfrak{g})$ generated by V.

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- To each elementary tangle diagram, assign a chain complex in this 2-category. By composition of chain complexes, this assigns a chain complex to any tangle diagram.

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- Show that the homotopy type of the complex is a tangle invariant.

Composition of complexes must preserve the homotopy type:

$$A^\bullet \simeq B^\bullet \text{ and } C^\bullet \simeq D^\bullet \quad \Rightarrow \quad A^\bullet * C^\bullet \simeq B^\bullet * D^\bullet$$

3 | 2-SUPERCATEGORIES

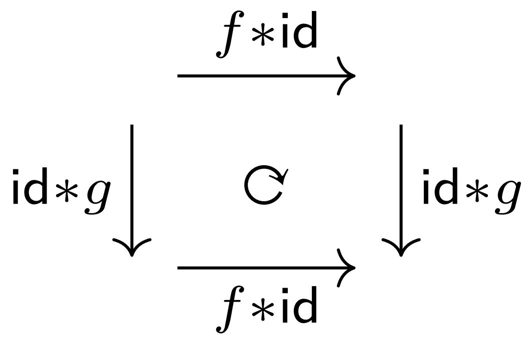
or the categorical part of the story

3. 2-SUPERCATEGORIES | MOTIVATION

2-categories:

$$(\mathrm{id} * g) \circ (f * \mathrm{id}) = (f * \mathrm{id}) \circ (\mathrm{id} * g)$$

interchange law



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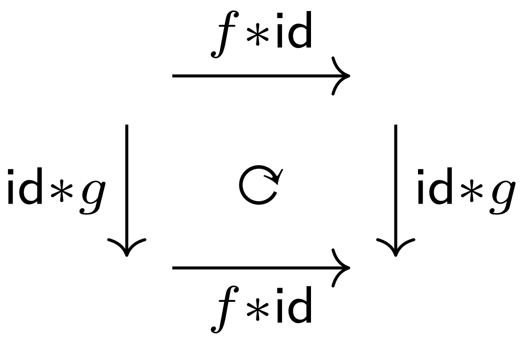

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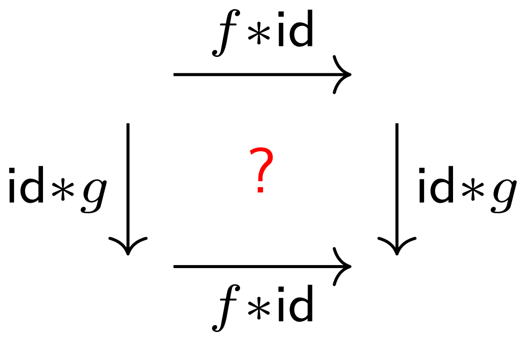
2-supercategories:

$$(\mathrm{id} * g) \circ (f * \mathrm{id}) = (-1)^{|f||g|}(f * \mathrm{id}) \circ (\mathrm{id} * g)$$

super interchange law



⇒ suited for odd Khovanov homology

3. 2-SUPERCATEGORIES | SUPERSPACE

A *superspace* is a $\mathbb{Z}/2\mathbb{Z}$-graded vector space:

$$V = V_0 \oplus V_1, \qquad |v| := \text{grading of } v \ (0 \text{ or } 1)$$

- $End(V,V)$ inherits a superspace structure:
  - *even maps*: maps preserving parity
  - *odd maps*: maps exchanging parity

- super tensor product:

$$(V \otimes W)_0 = (V_0 \otimes W_0) \oplus (V_1 \otimes W_1) \quad \text{and} \quad (V \otimes W)_1 = (V_0 \otimes W_1) \oplus (V_1 \otimes W_0)$$

$$(f \otimes g) \circ (h \otimes k) = (-1)^{|g||h|} (f \circ h) \otimes (g \circ k)$$

- We denote $\mathcal{SV}ec$ the category of superspaces, and $\underline{\mathcal{SV}ec}$ the subcategory restricting to even linear maps.

3. 2-SUPERCATEGORIES | SUPER STRUCTURES

A *supercategory* is a $\mathcal{SV}\mathrm{ec}$-enriched category:

- Each Hom-set is a superspace
- Parities and composition are compatible: $|f \circ g| = |f| + |g|$
- We denote $\mathcal{SC}at$ the category of small supercategories (and functors preserving parities)

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A *monoidal supercategory* is a supercategory with a super tensor product:

$$(f \otimes g) \circ (h \otimes k) = (-1)^{|g||h|}(f \circ h) \otimes (g \circ k)$$

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A *2-supercategory* is a $\mathcal{SC}at$-enriched category:

- Each Hom-set is a supercategory
- Parities and compositions are compatible: $|f \circ g| = |f| + |g|$ and $|f * g| = |f| + |g|$
- Compositions are compatible through the super interchange law:

$$(f * g) \circ (h * k) = (-1)^{|g||h|}(f \circ h) * (g \circ k)$$

⚠ a 2-supercategory is (in general) *not* a 2-category!

3. 2-SUPERCATEGORIES | HOMOLOGY

Theorem (S. 2020)*

Composition of complexes must preserve the homotopy type:

$$A^\bullet \simeq B^\bullet \text{ and } C^\bullet \simeq D^\bullet \quad \Rightarrow \quad A^\bullet * C^\bullet \simeq B^\bullet * D^\bullet$$

*must restrict to complexes that factor through *homogeneous* complexes (each differential in the complex is either even or odd)

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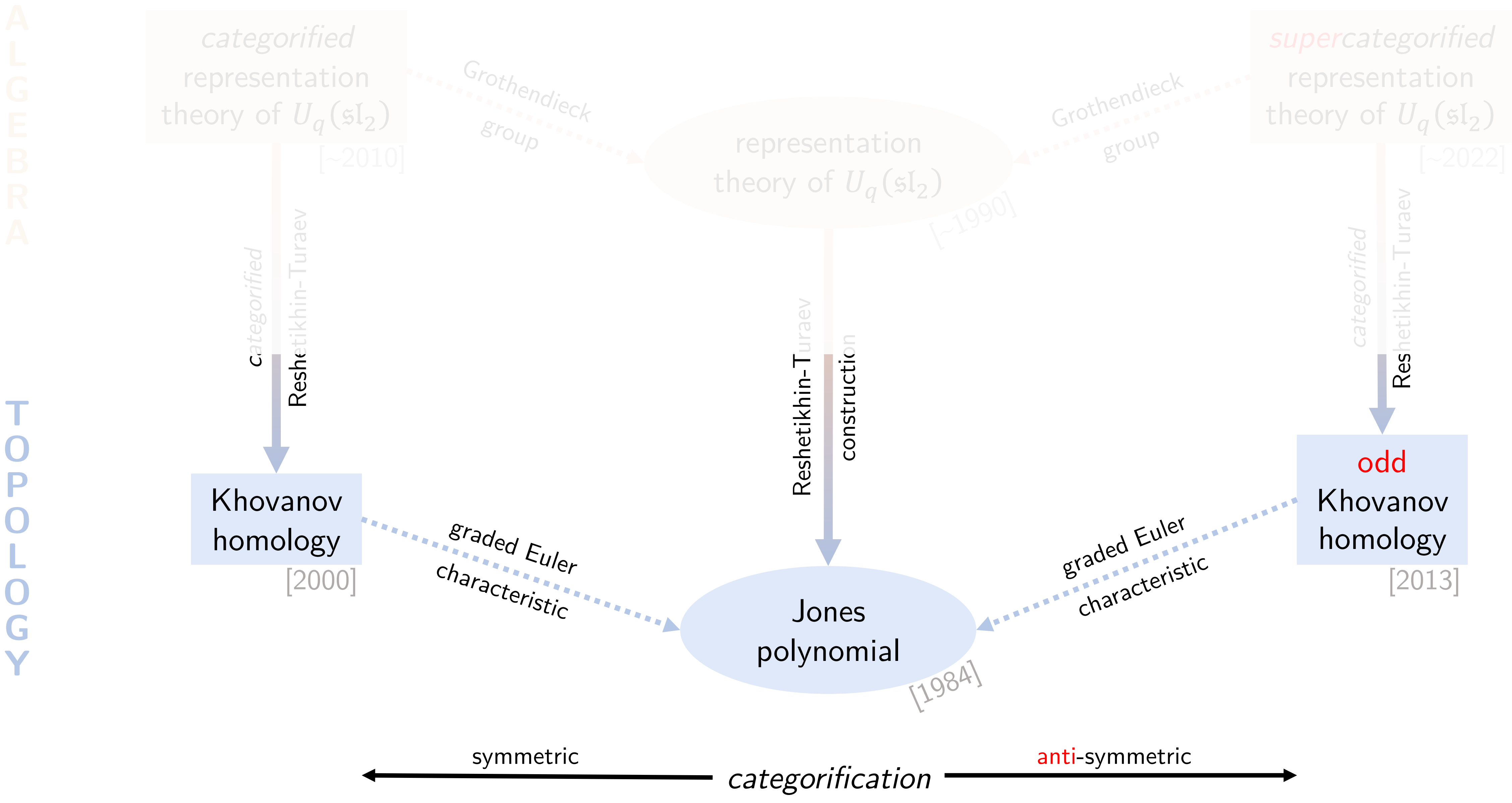
<u>Sketch of proof:</u>

- If $A_1^\bullet$ and $A_2^\bullet$ are chain complexes, find a definition for $A_1^\bullet * A_2^\bullet$
- If f_1 and f_2 are chain maps, find a definition for $f_1 * f_2$
- If h_1 and h_2 are homotopies, find a definition for $h_1 * h_2$

CONCLUSION

or how to combine everything

CONCLUSION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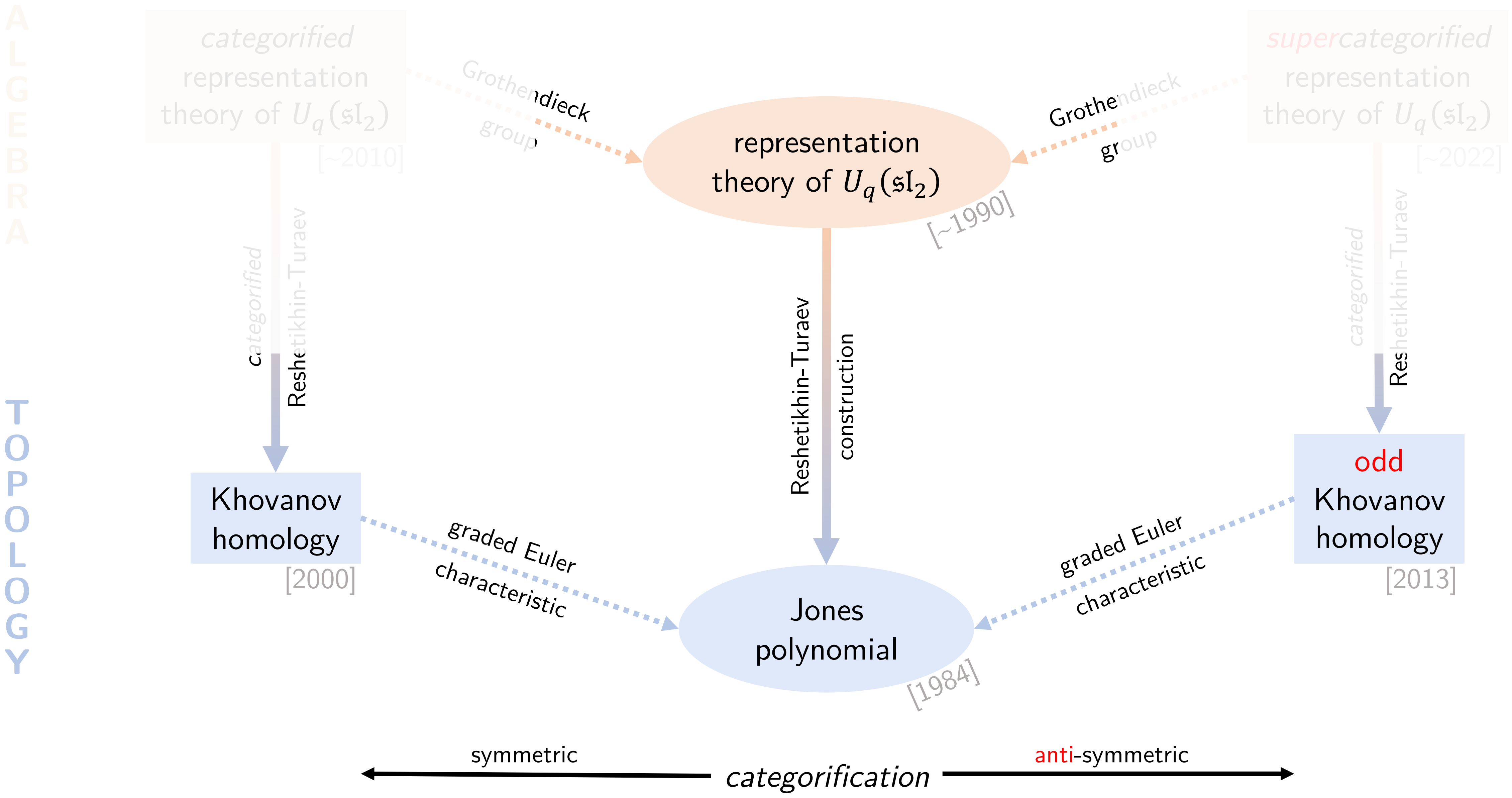
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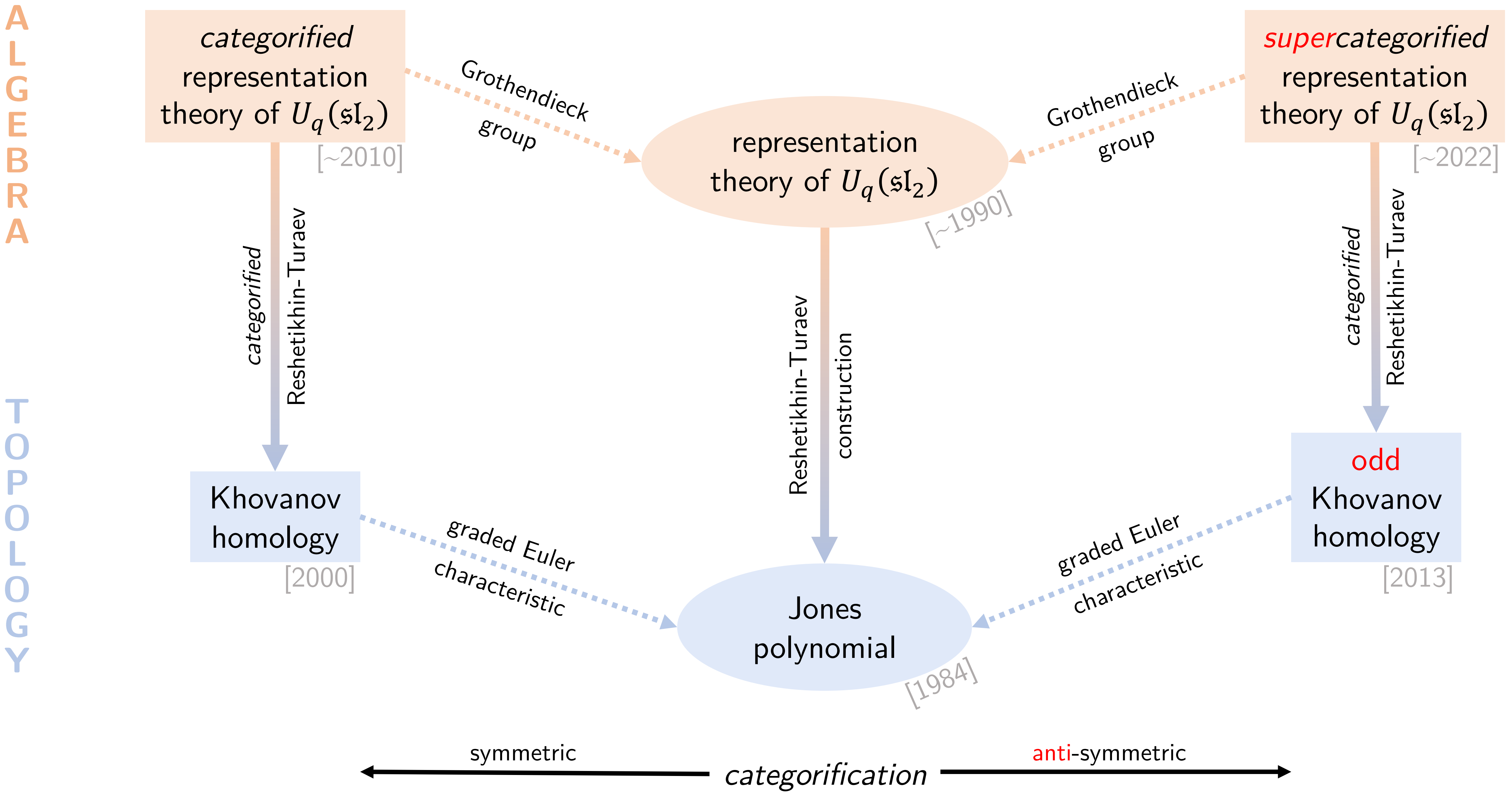
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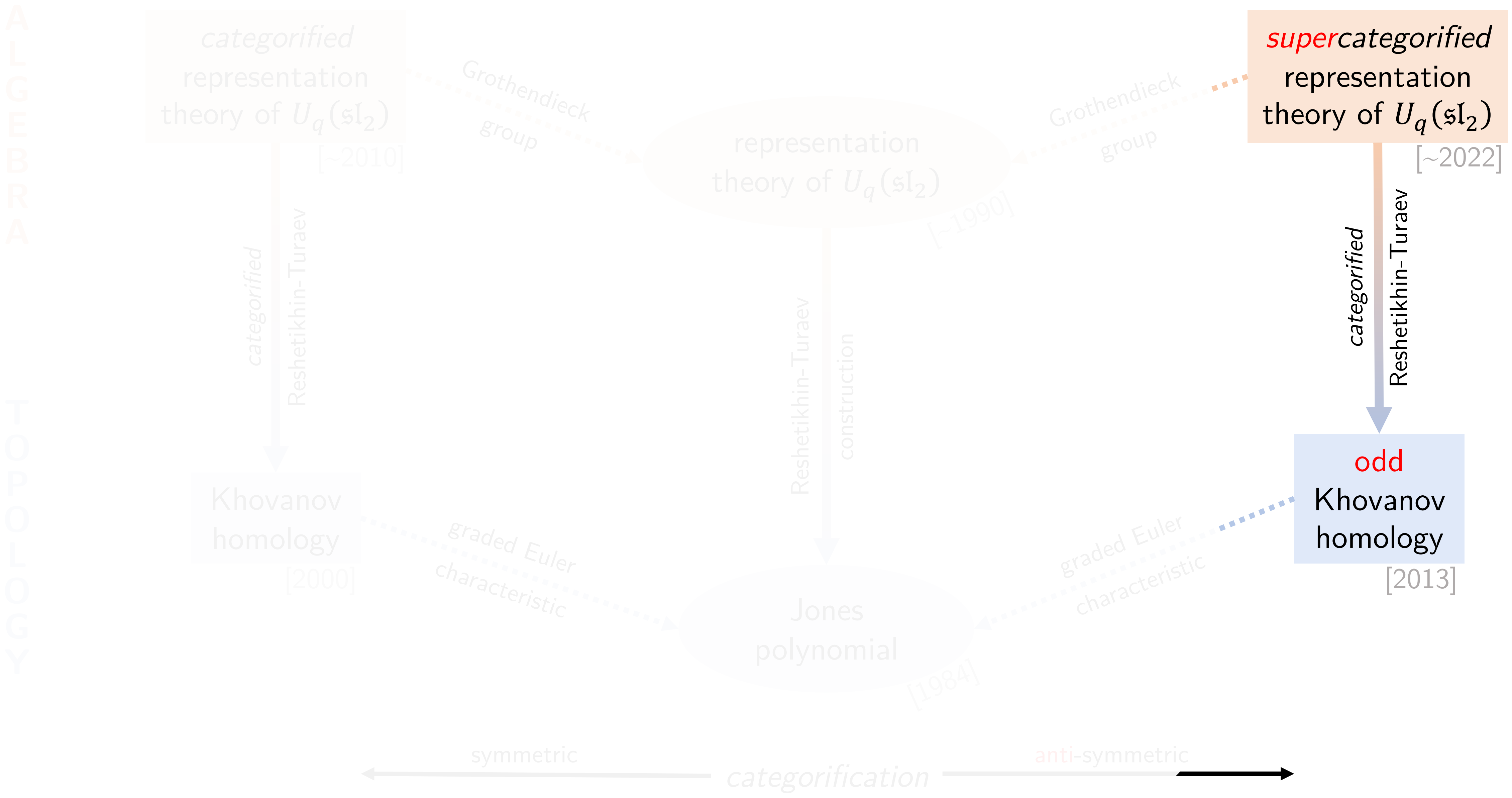
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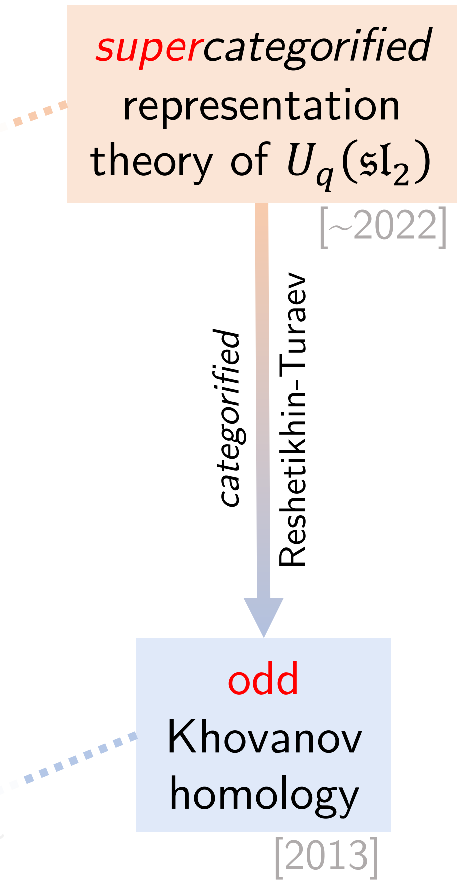
Theorem (Vaz 20; S. and Vaz 2022)

- There exists a categorification of some representation category of $U_q(\mathfrak{gl}_2)$ into a 2-supercategory.
- Using this 2-supercategory, one can construction an homological invariant of tangles.
- This invariant coincide with odd Khovanov homology in the case of links.



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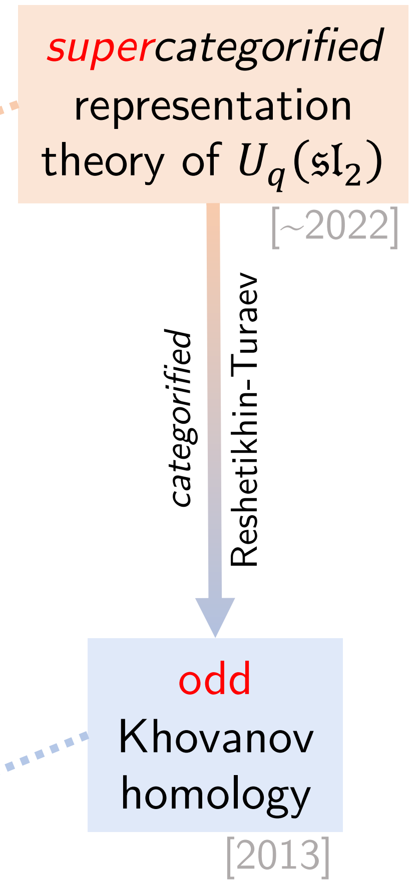

Further questions:

- Can we use this to show functoriality of odd Khovanov homology?
- "How far" can we push the supercategorification program? Eg, can we find a supercategorified homological invariant for *every* choice of $(U_q(\mathfrak{g}), V)$?