Binary factorizations of the matrix of all ones

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Abstract: We consider the factorization problem of the square matrix of all ones where the factors are square matrices and constrained to have all elements equal to 0 or 1. We show that under some conditions on the rank of the factors, these are essentially De Bruijn matrices. More specifically, we are looking for the $n \times n$ binary solutions of $\prod_{i=1}^{m} A_i = \mathbb{I}_n$, where $\mathbb{I}_n$ is the $n \times n$ matrix with all ones and in particular, we are investigating the binary solutions to the equation $A^m = \mathbb{I}_n$. Our main result states that if we impose all factors A_i to be p_i-regular and such that all their products commute, then rank(A_i) $= n/p_i$ if and only if A_i is essentially a De Bruijn matrix. In particular, we prove that if A is a binary m-th root of $\mathbb{I}_n$, then A is p-regular. Moreover, rank$(A) = n/p$ if and only if A is essentially a De Bruijn matrix.