Interpolating Multi-Fiber Models by Gaussian Mixture Simplification

ISBI 2012, Barcelona, Spain

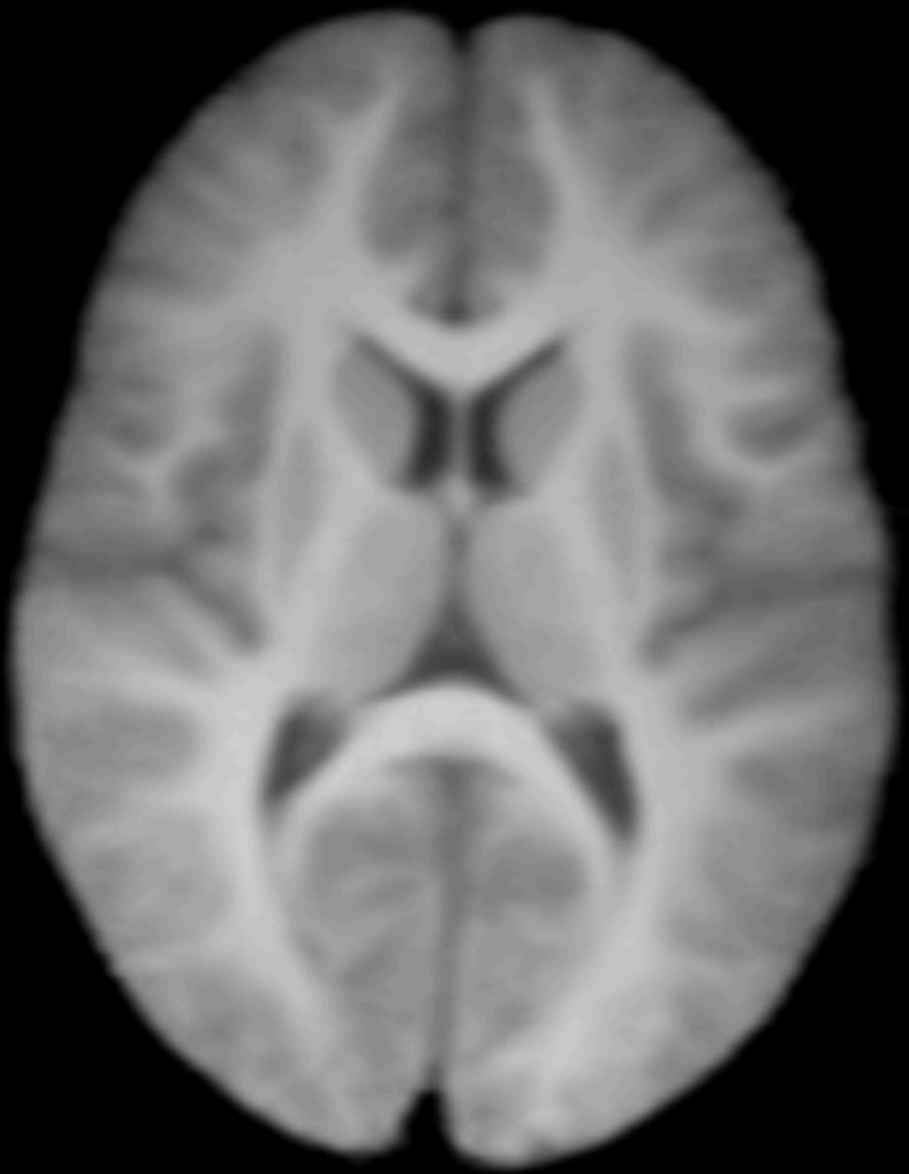
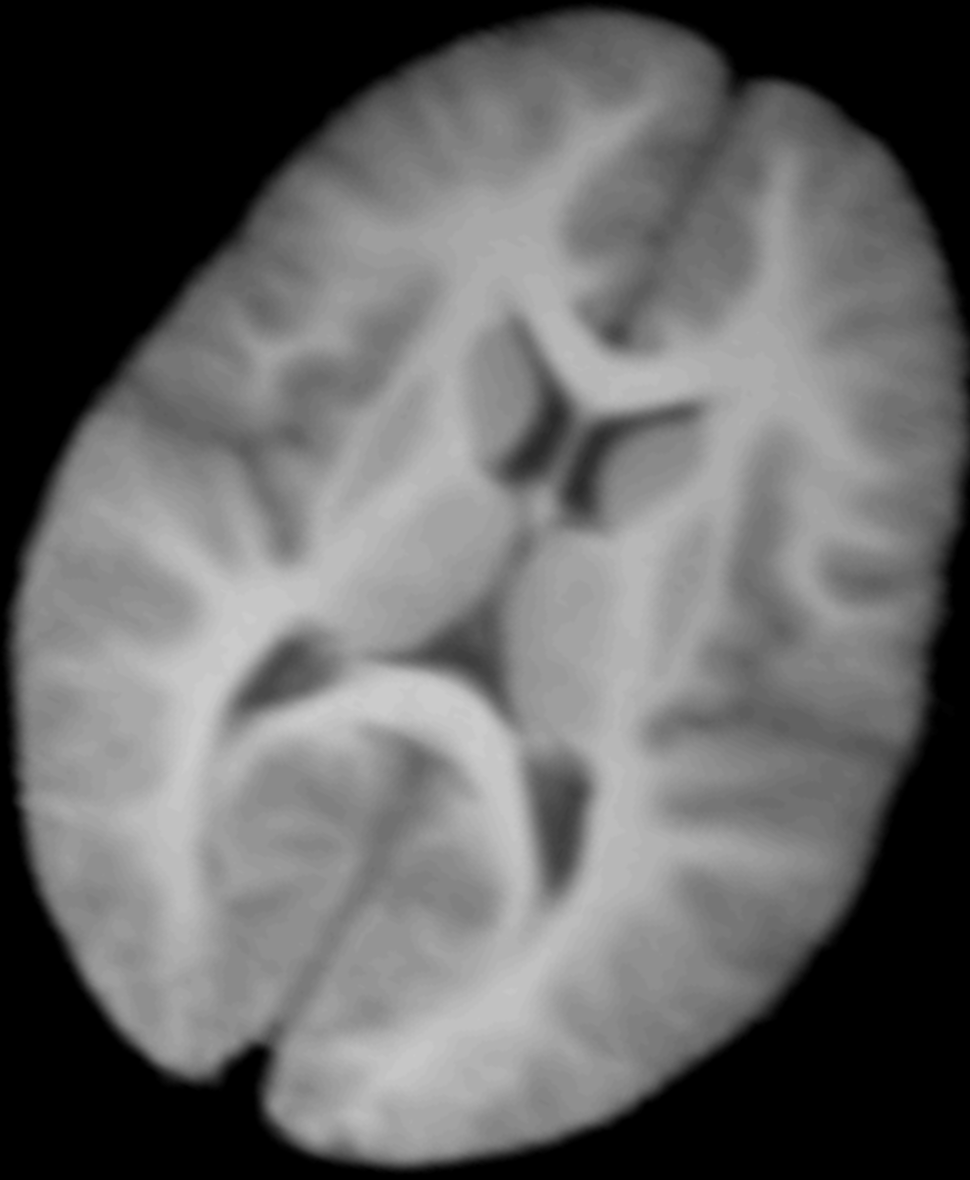
Maxime Taquet,
Benoît Scherrer,
Christopher Benjamin,
Sanjay Prabhu,
Benoît Macq,
Simon K. Warfield

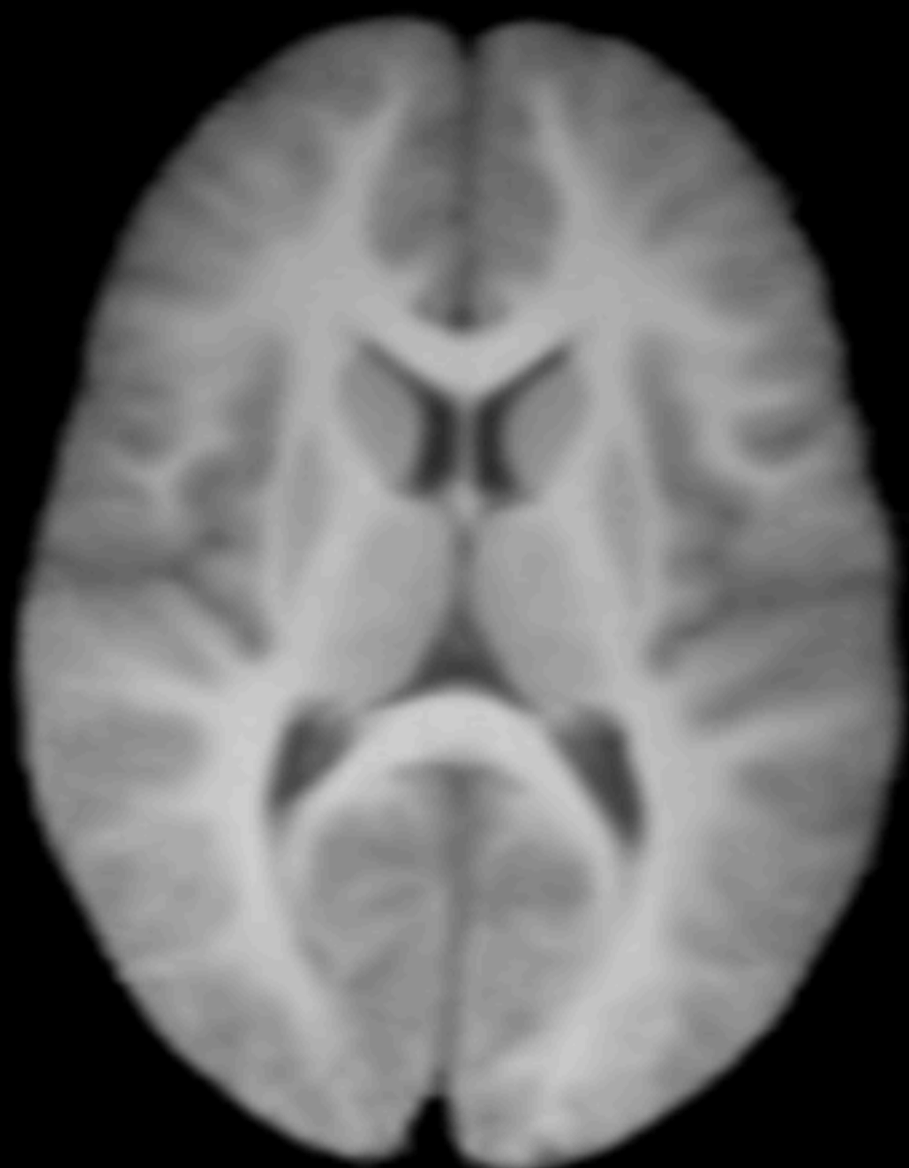
Source: crgis.ndc.nasa.gov

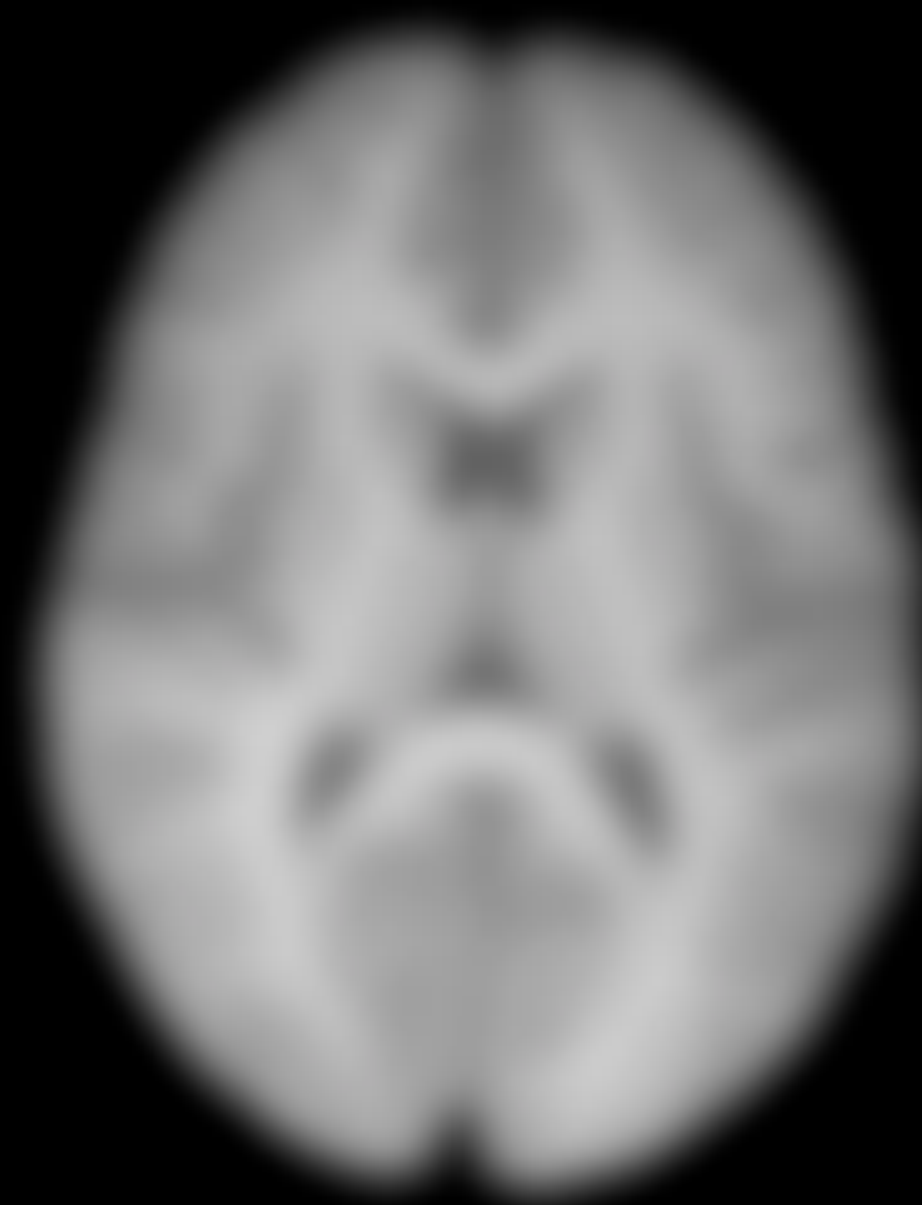
Interpolating Multi-Fiber Models by Gaussian Mixture Simplification

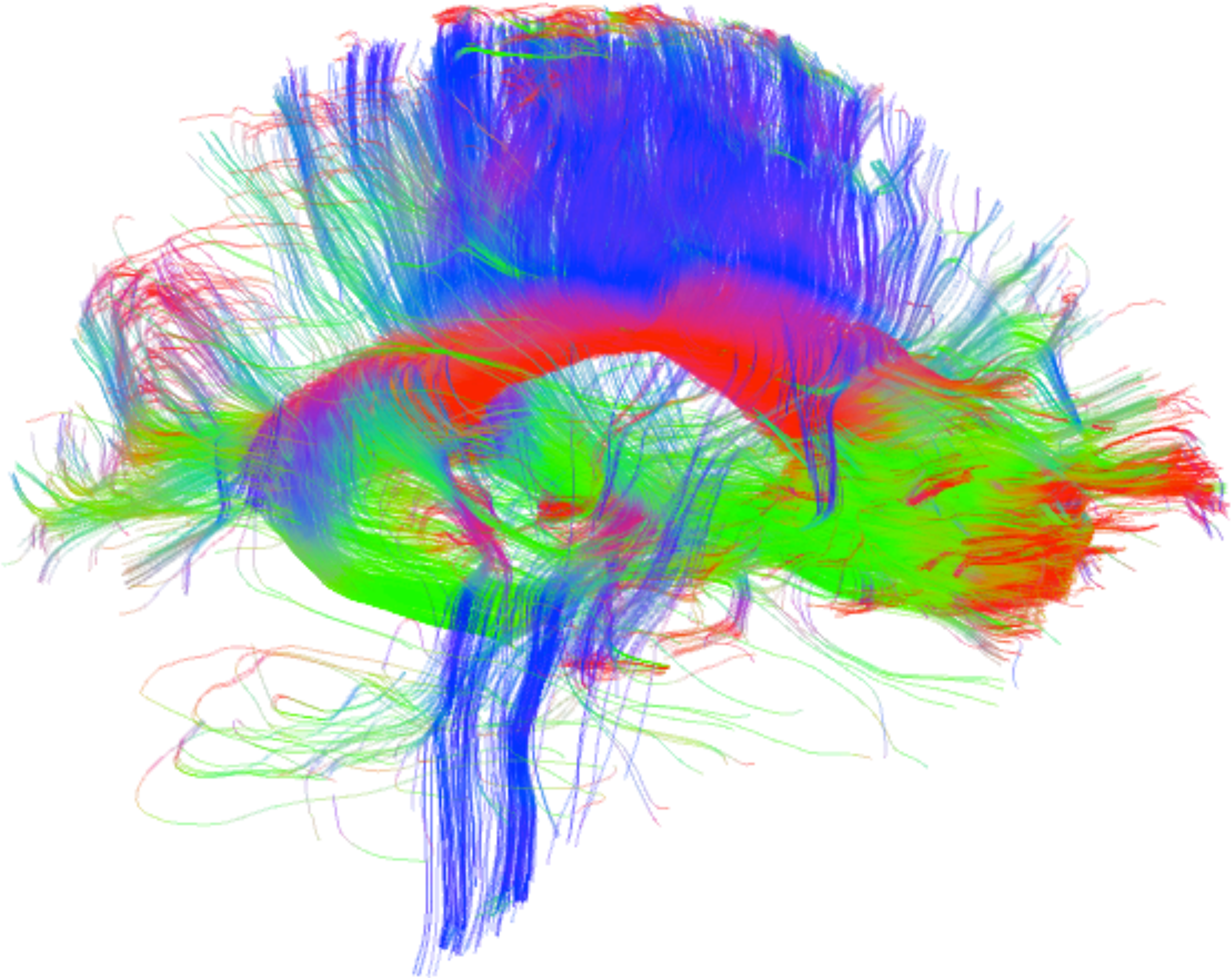
ISBI 2012, Barcelona, Spain

Maxime Taquet,
Benoît Scherrer,
Christopher Benjamin,
Sanjay Prabhu,
Benoît Macq,
Simon K. Warfield

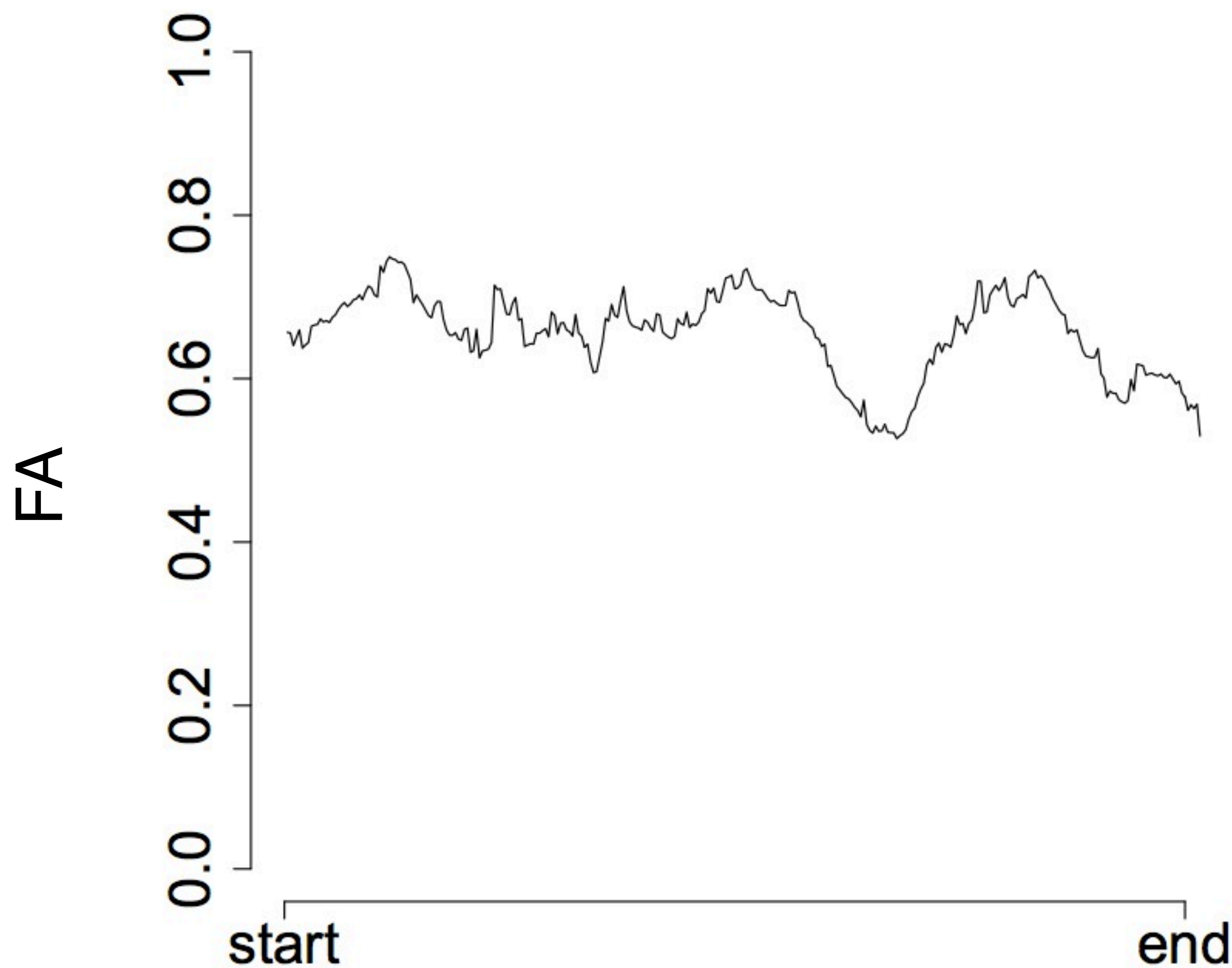









Scalar characteristic along a tract



Heuristics fail

a global approach is required

The problem is well known

as a Gaussian Mixture Model Simplification

It can be efficiently solved

in an expectation-maximization scheme

Heuristics fail

a global approach is required

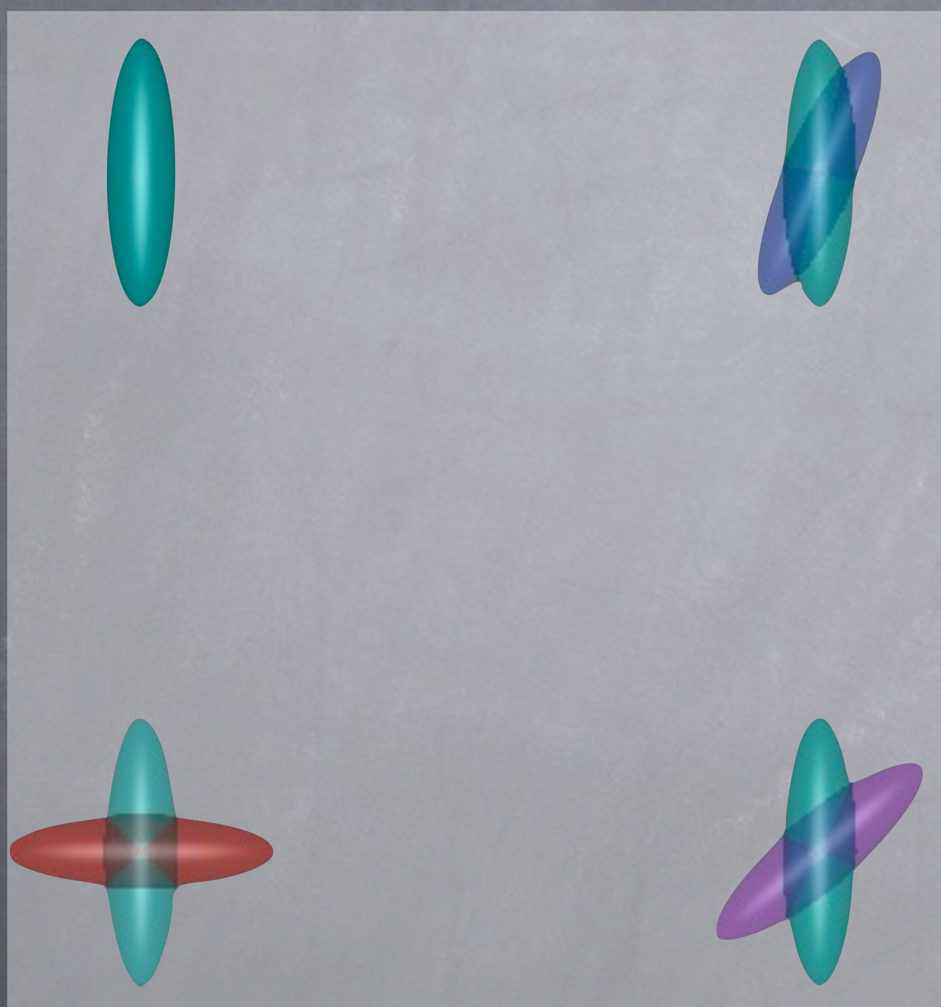
The problem is well known

as a Gaussian Mixture Model Simplification

It can be efficiently solved

in an expectation-maximization scheme

A multi-fiber model represents fibers independently



$$S = \sum_{i=1}^{N} f_i S_i$$

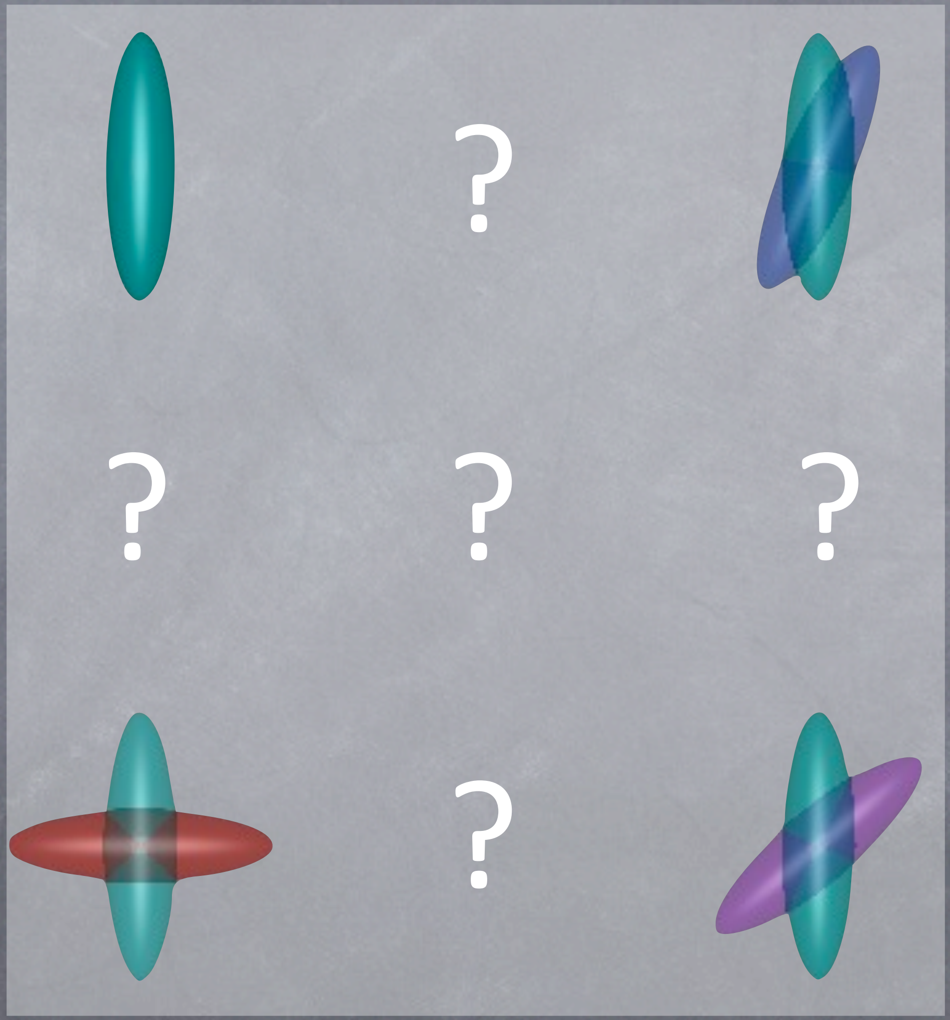
Single-fiber model

Volumetric fractions

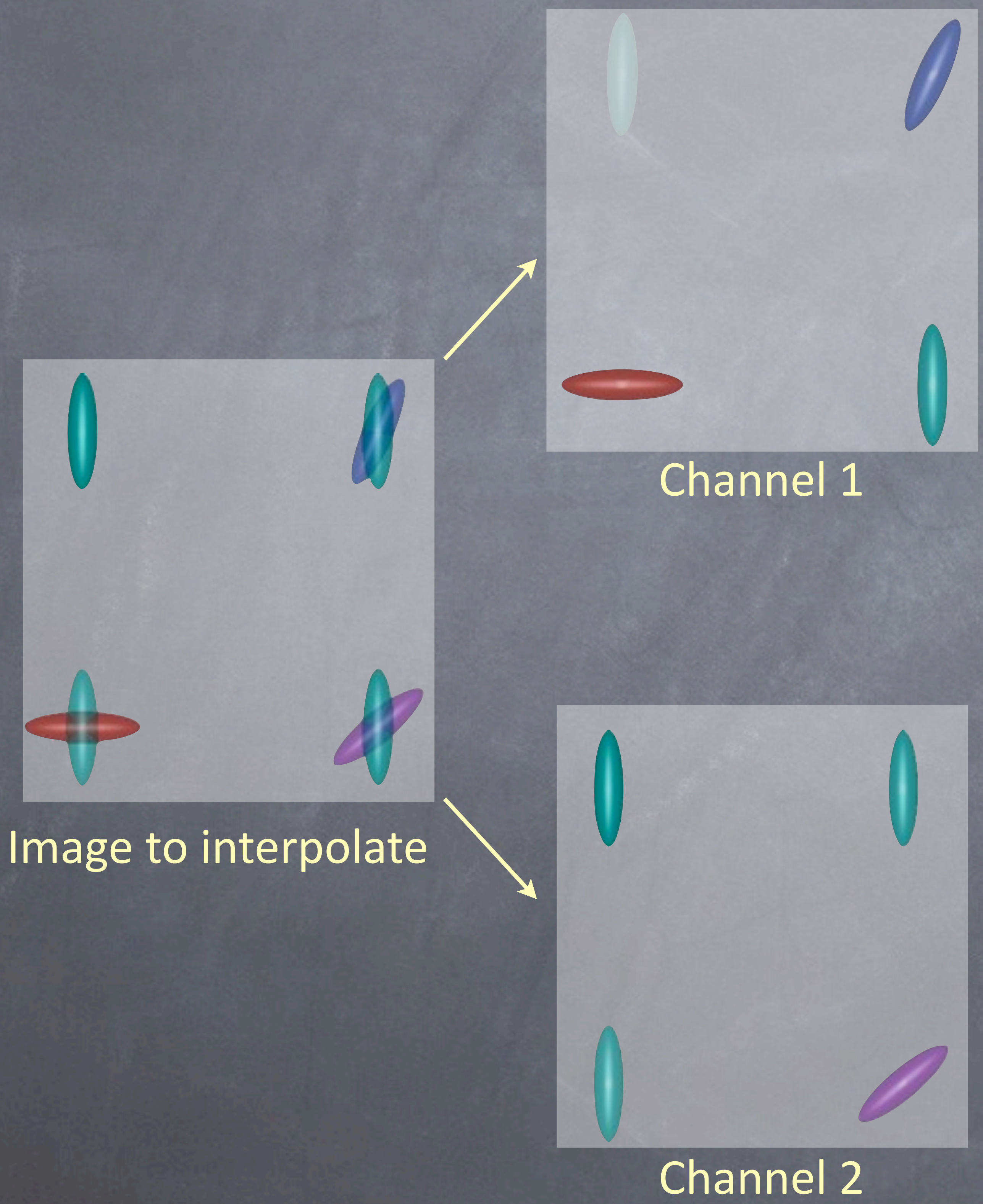
Particular case: the multi-tensor model

$$S = S_0 \sum_{i=1}^{N} f_i e^{-b\mathbf{g}^T D_i \mathbf{g}}$$

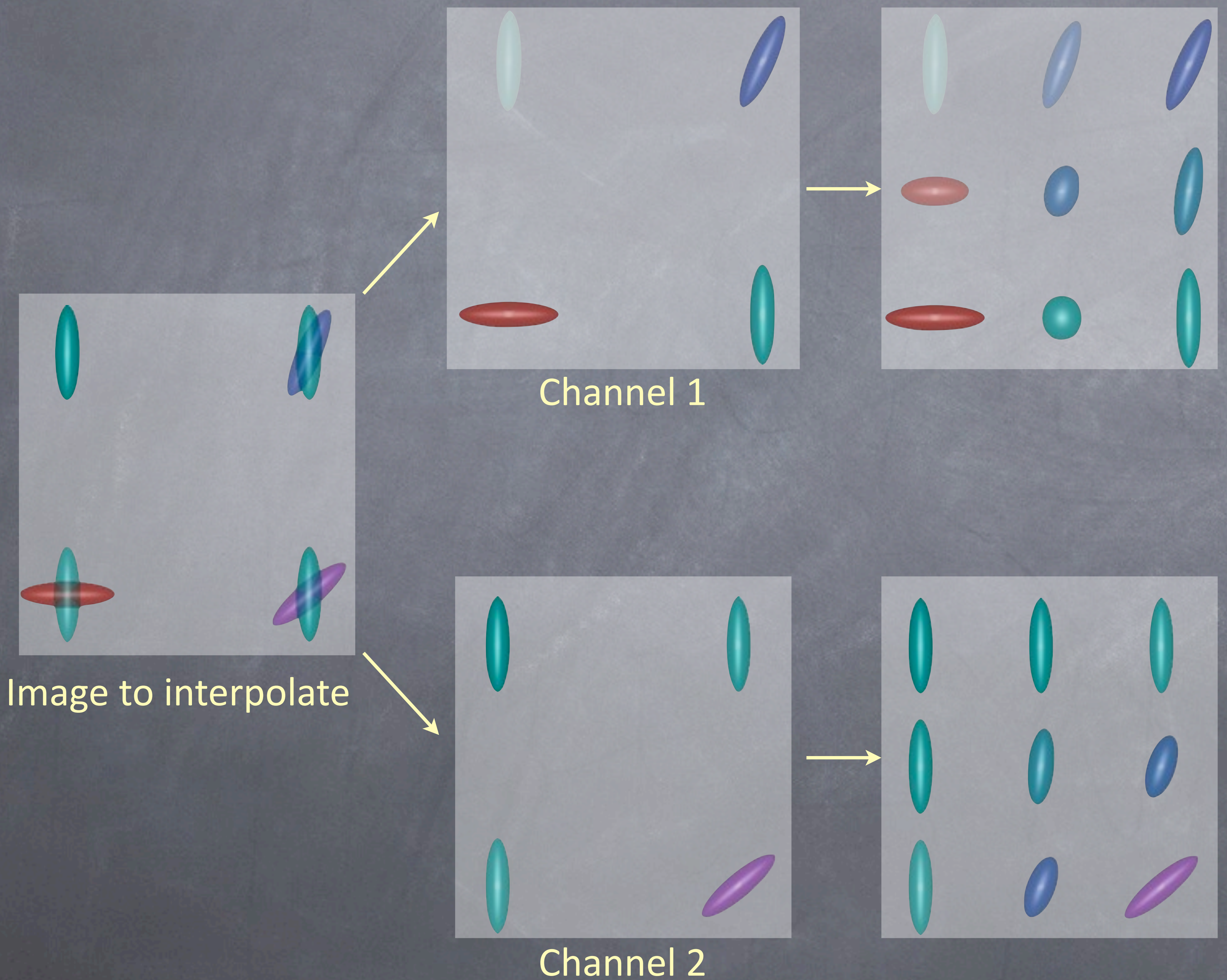
Interpolating a multi-fiber model is challenging



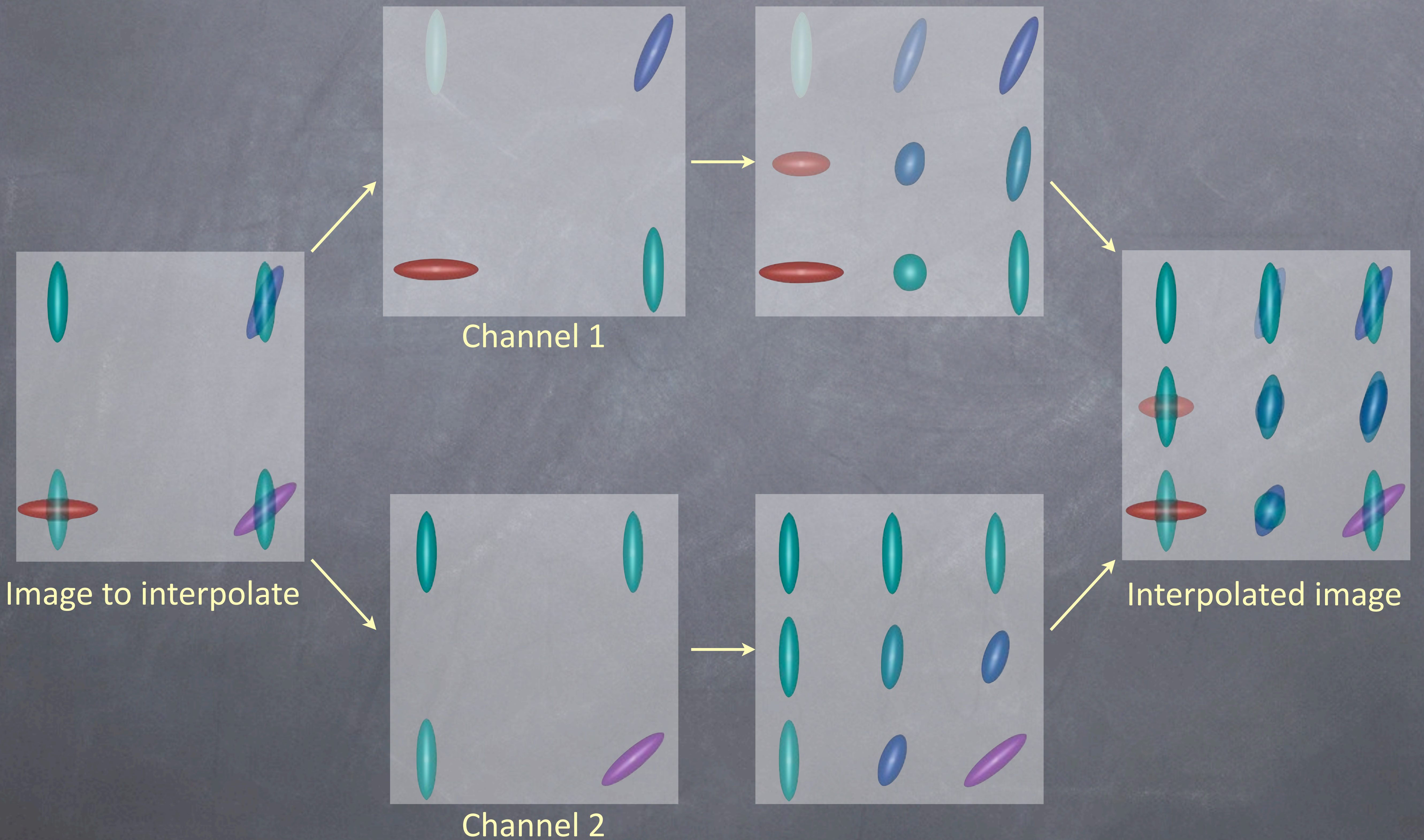
Channel based interpolation does not work



Channel based interpolation does not work



Channel based interpolation does not work



Channel based interpolation does not work

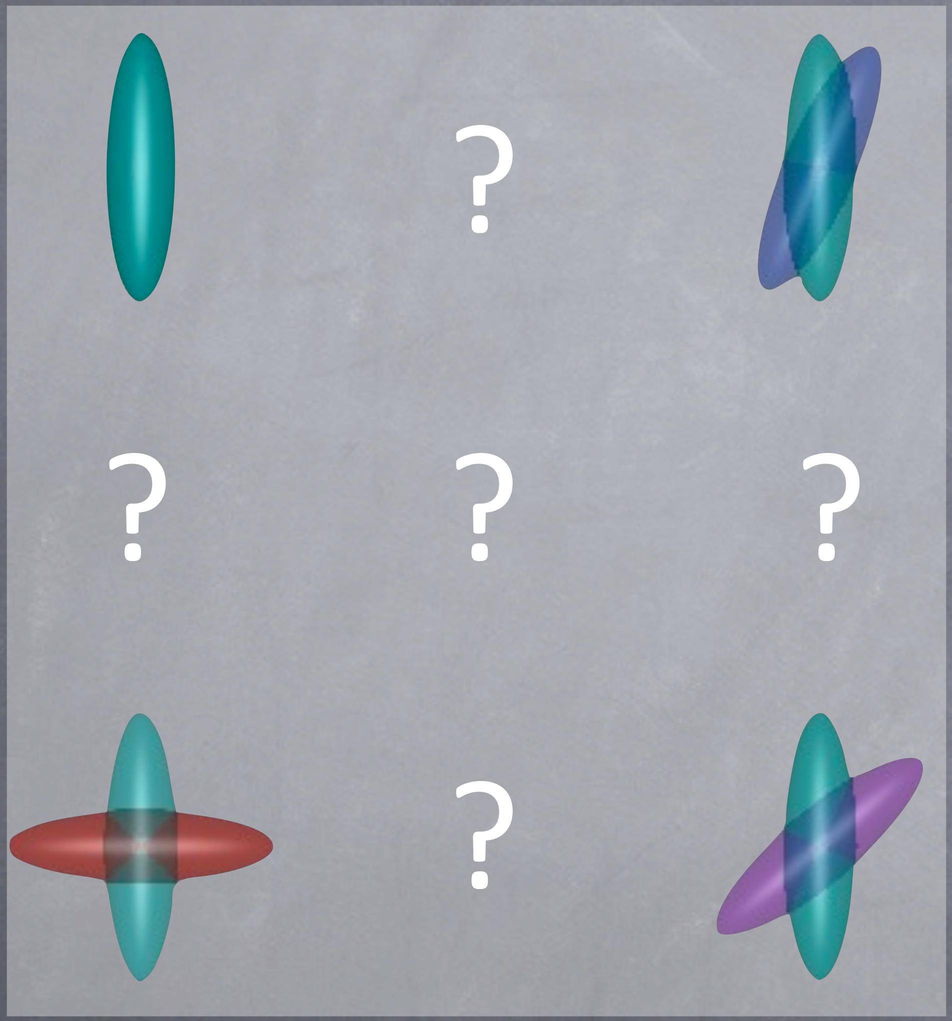
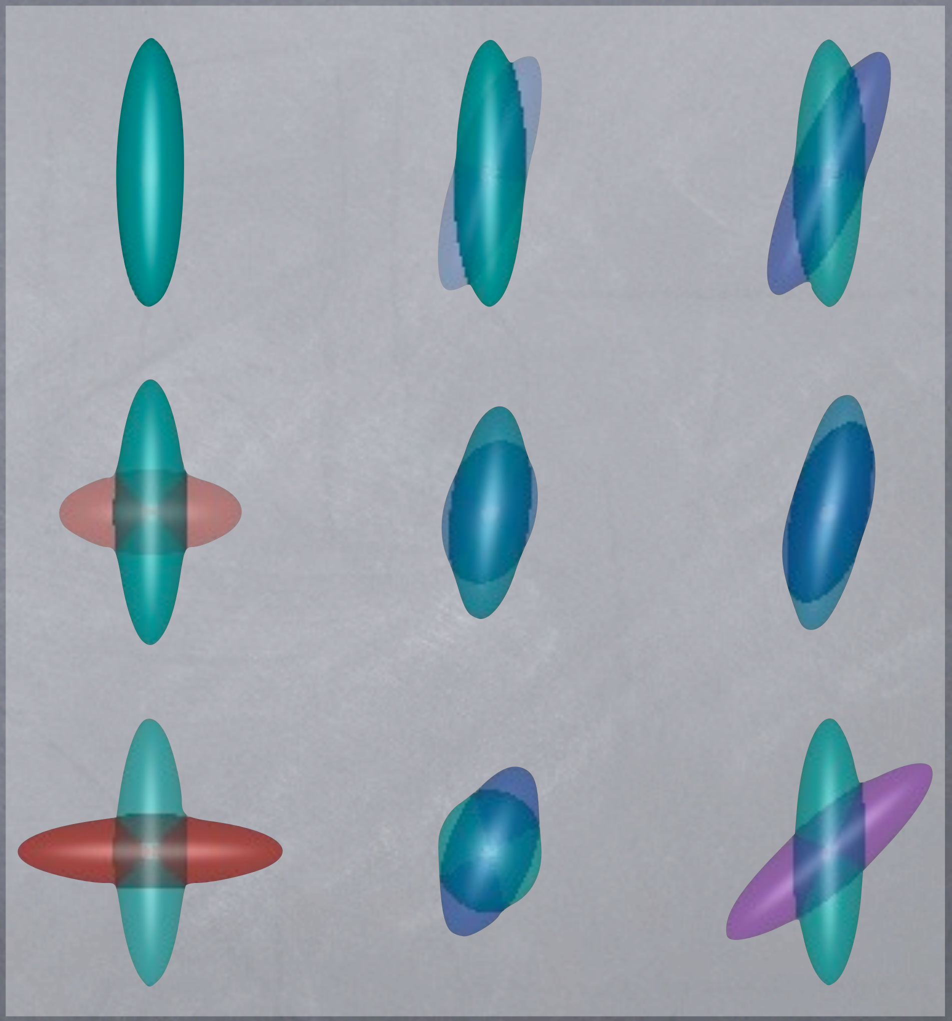


Image to interpolate



channel-based interpolation

Orientation-based interpolation does not work

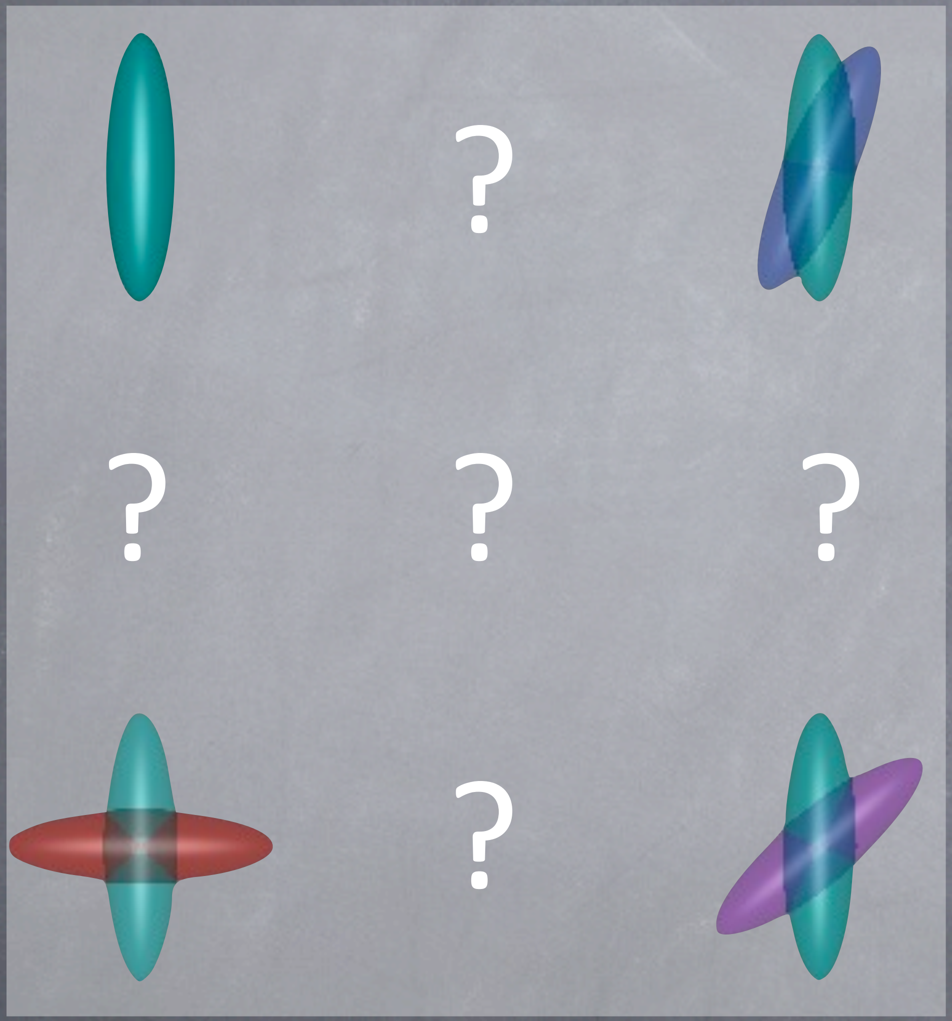
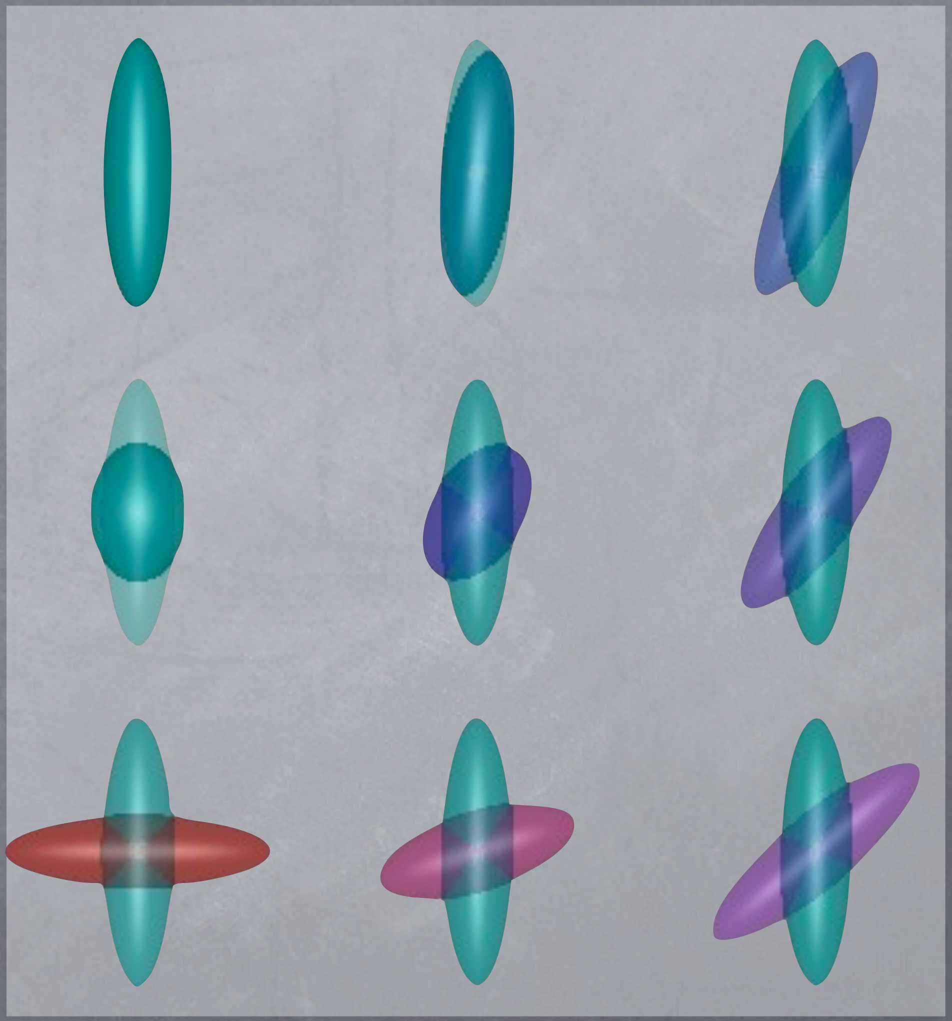


Image to interpolate



orientation-based interpolation

The multiple fibers are paired in two groups based on their orientation.

A global approach is required

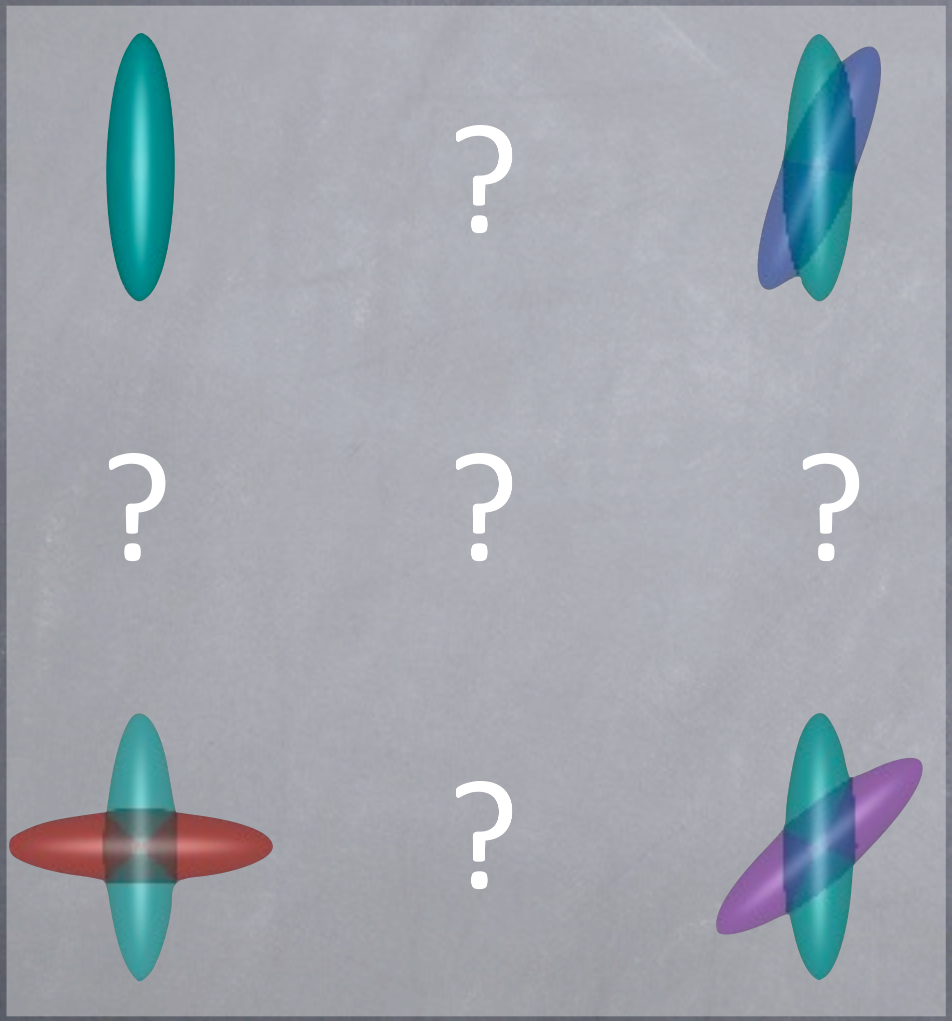
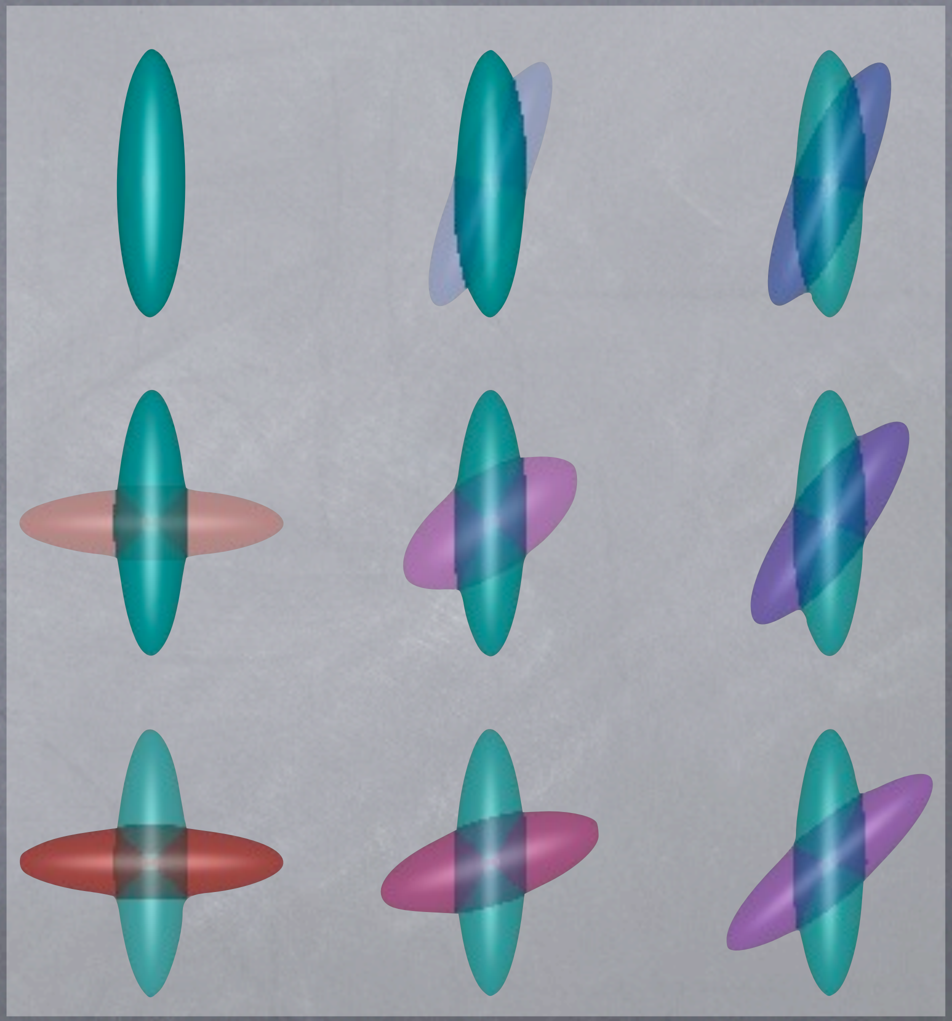
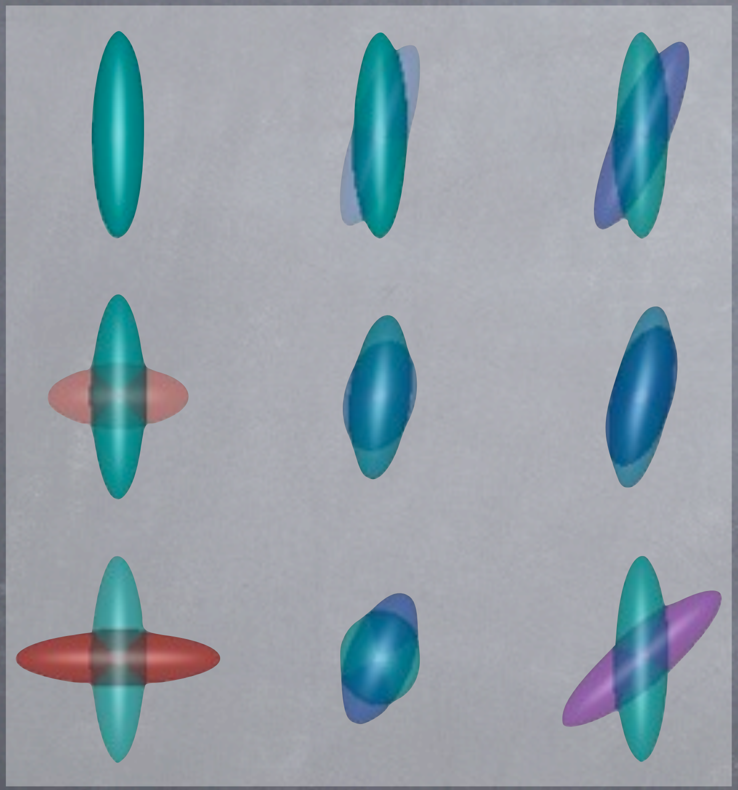


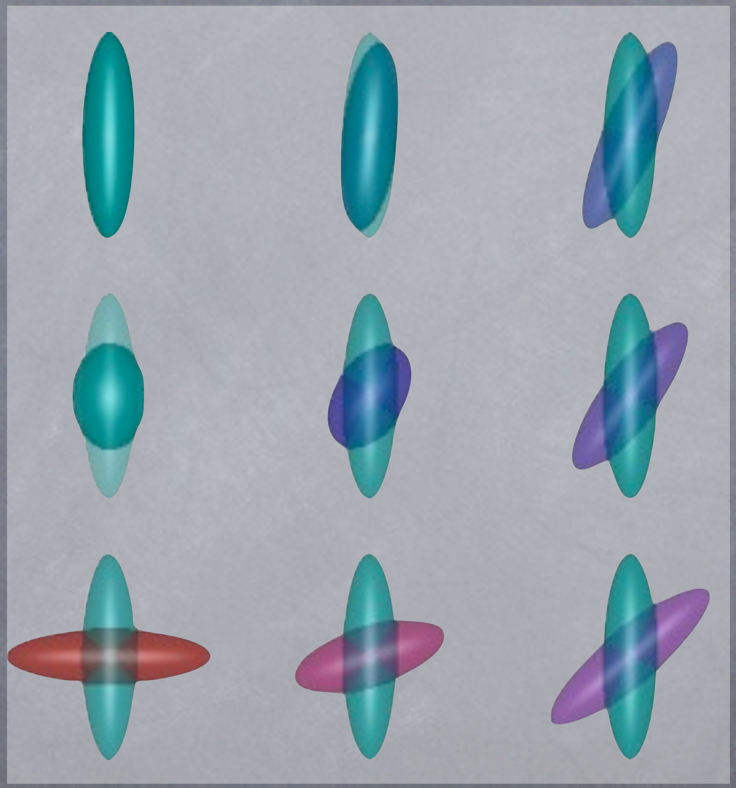
Image to interpolate


Proposed method

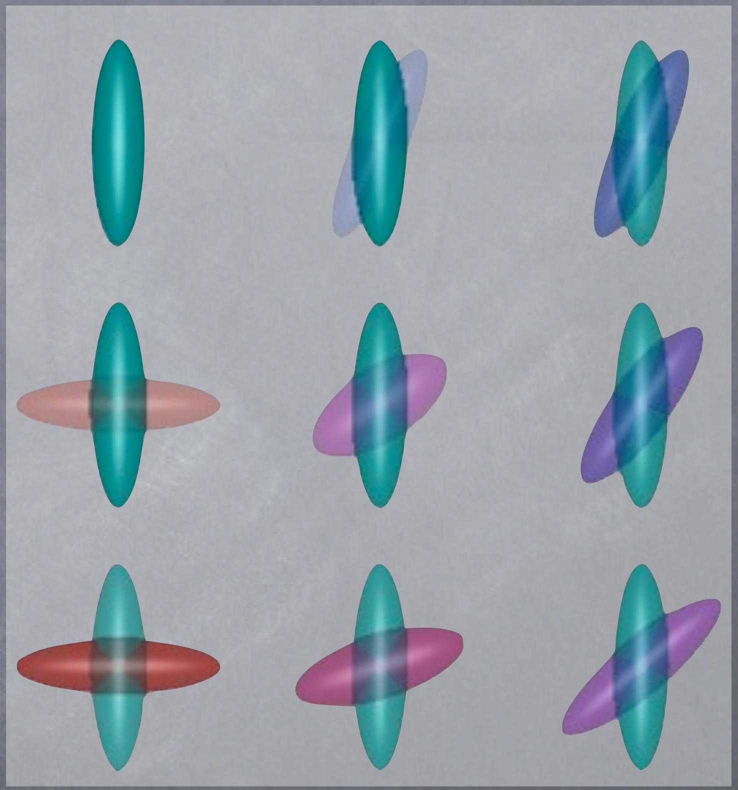
A global approach is required



channel-based



orientation-based



proposed method

Heuristics fail

a global approach is required

The problem is well known

as a Gaussian Mixture Model Simplification

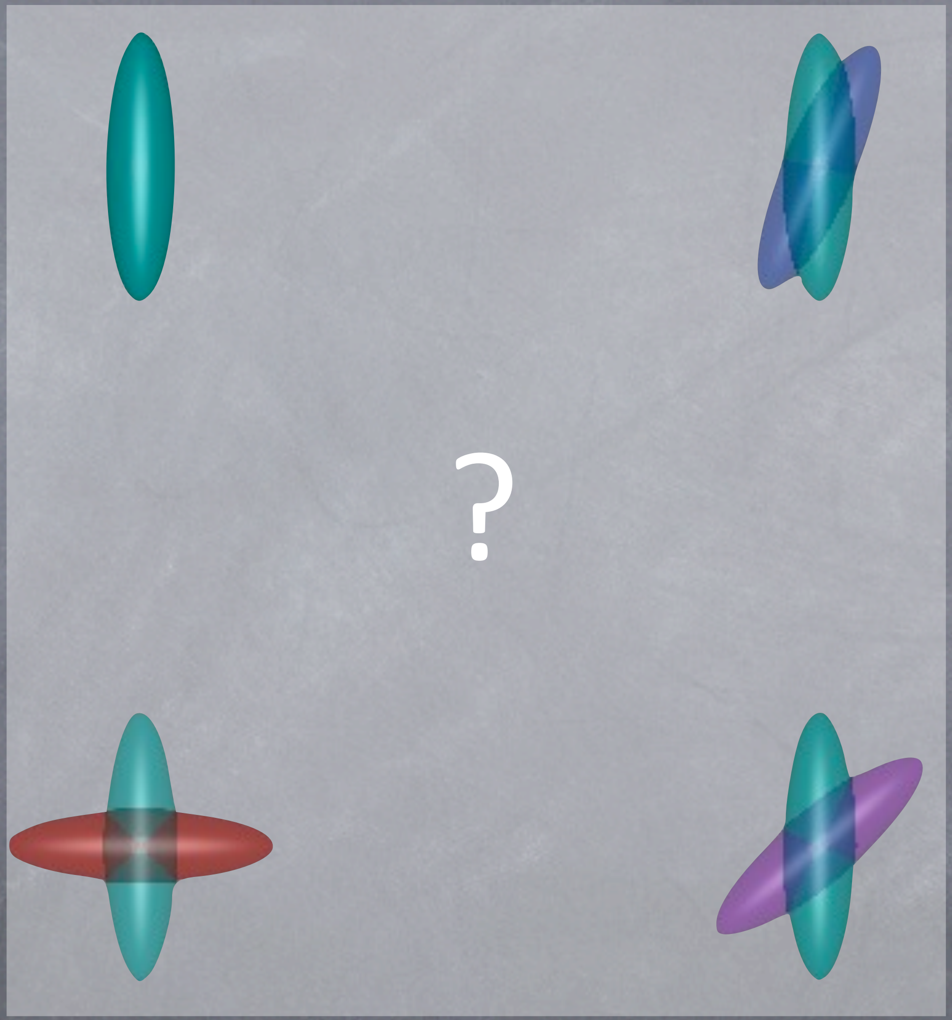
It can be efficiently solved

in an expectation-maximization scheme

Mathematically, we mix mixtures

$$p_1(\mathbf{x}) = \sum_i \alpha_{1,i} G_{1,i}$$

$$p_2(\mathbf{x}) = \sum_i \alpha_{2,i} G_{2,i}$$



$$p_3(\mathbf{x}) = \sum_i \alpha_{3,i} G_{3,i}$$

$$p_4(\mathbf{x}) = \sum_i \alpha_{4,i} G_{4,i}$$

Mathematically, we mix mixtures

$$p_1(\mathbf{x}) = \sum_i \alpha_{1,i} G_{1,i}$$

$$p_2(\mathbf{x}) = \sum_i \alpha_{2,i} G_{2,i}$$

Scalar interpolation coefficients

$$p(\mathbf{x}) = \sum_{j=1}^{4} a_j \sum_i \alpha_{j,i} G_{j,i}$$

All mixtures

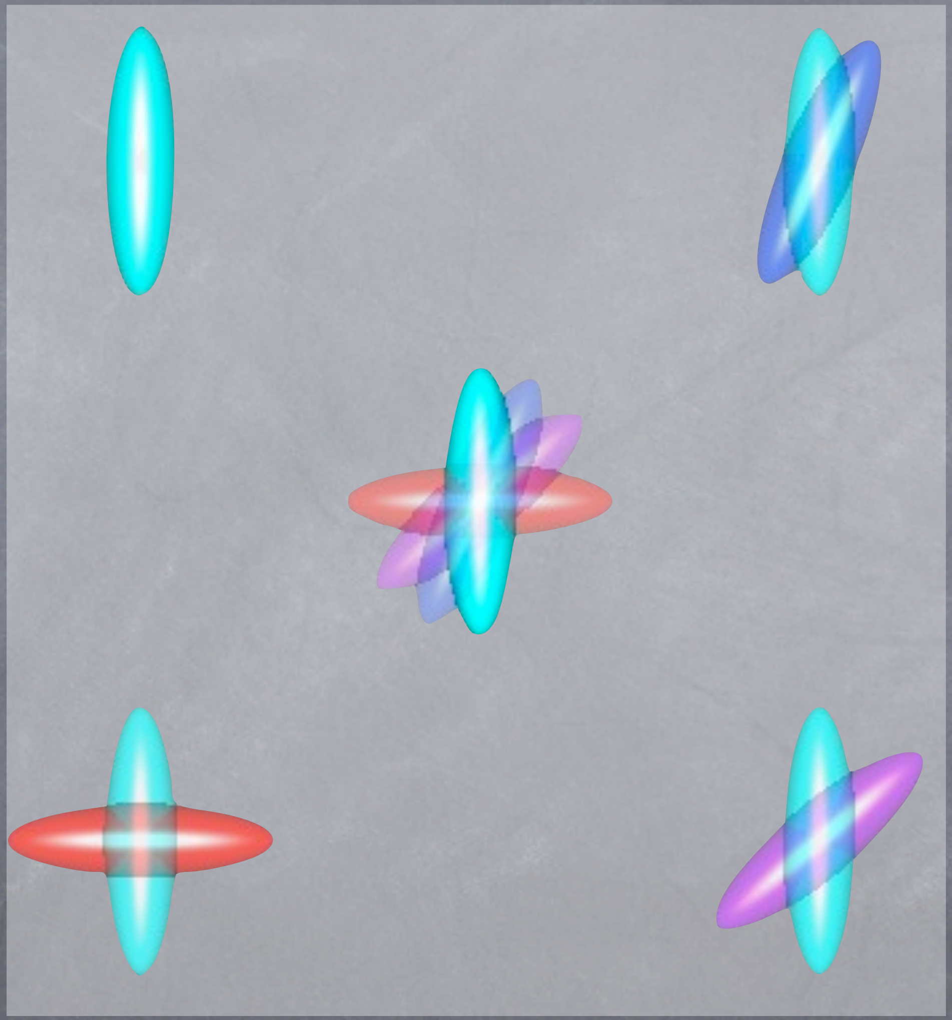
$$p_3(\mathbf{x}) = \sum_i \alpha_{3,i} G_{3,i}$$

$$p_4(\mathbf{x}) = \sum_i \alpha_{4,i} G_{4,i}$$

The result is a high order mixture model

$$p_1(\mathbf{x}) = \sum_i \alpha_{1,i} G_{1,i}$$

$$p_2(\mathbf{x}) = \sum_i \alpha_{2,i} G_{2,i}$$



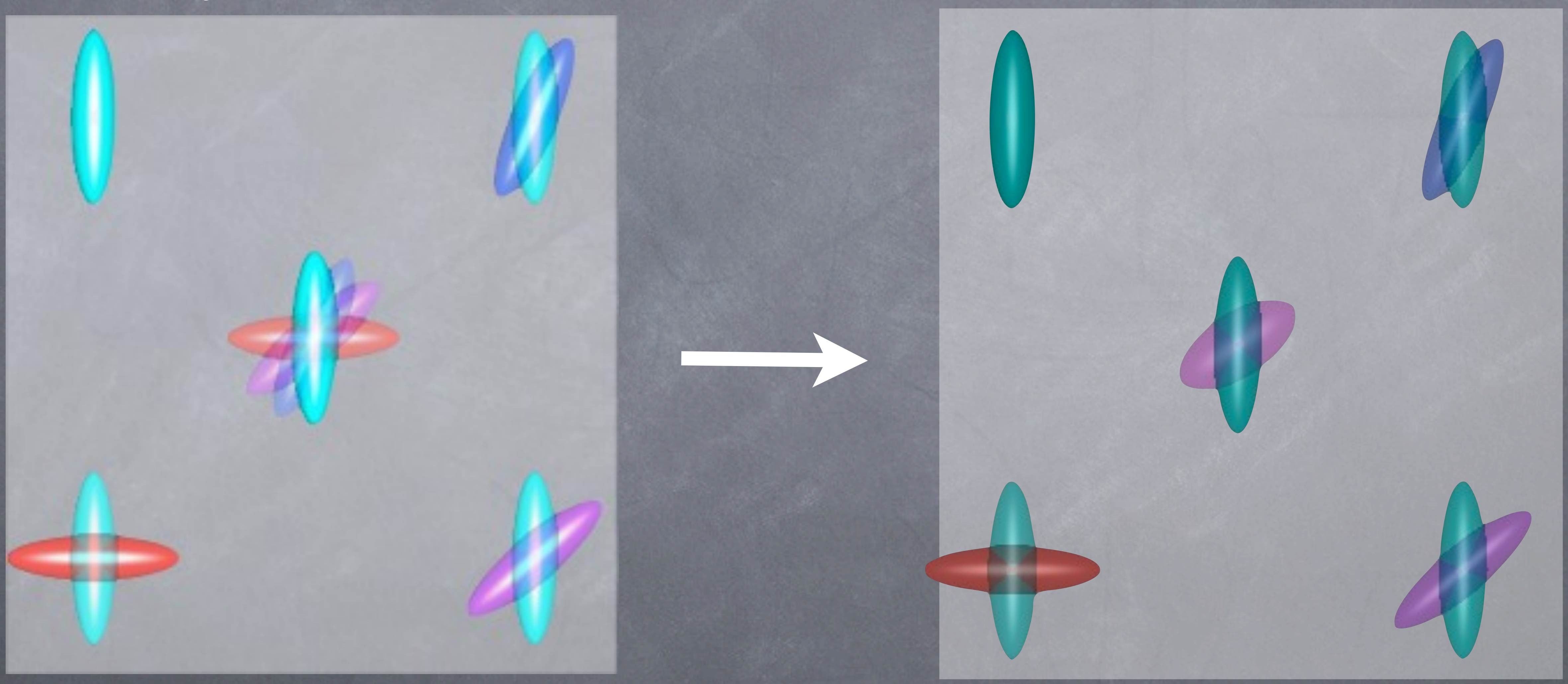
$$p_3(\mathbf{x}) = \sum_i \alpha_{3,i} G_{3,i}$$

$$p_4(\mathbf{x}) = \sum_i \alpha_{4,i} G_{4,i}$$

The complete mixture needs be simplified

$$p(\mathbf{x}) = \sum_{j=1}^{4} a_j \sum_i \alpha_{j,i} G_{j,i}$$

$$p_S(\mathbf{x}) = \sum_i f_i R_i$$



The complete mixture needs to be simplified

$$p(\mathbf{x}) = \sum_{i=1}^{N} \alpha_i G_i \quad \xrightarrow{N > K} \quad p_S(\mathbf{x}) = \sum_{j=1}^{K} f_i R_i$$

The complete mixture needs to be simplified

$$p(\mathbf{x}) = \sum_{i=1}^{N} \alpha_i G_i \xrightarrow{N > K} p_S(\mathbf{x}) = \sum_{j=1}^{K} f_i R_i$$

$$p_S^* = \arg\min_{p_S} D(p, p_S)$$

Minimize some discrepancy measure

The complete mixture needs to be simplified

$$p(\mathbf{x}) = \sum_{i=1}^{N} \alpha_i G_i \quad \xrightarrow{N > K} \quad p_S(\mathbf{x}) = \sum_{j=1}^{K} f_i R_i$$

$$p_S^* = \arg\min_{p_S} D(p, p_S)$$

Minimize some discrepancy measure

$$D(p, p_S) = \sum_j \sum_{i:\pi_i = j} \alpha_i D(G_i || R_j)$$

Cumulative Differential Relative Entropy

Latent variables indicating the grouping of components

Heuristics fail

a global approach is required

The problem is well known

as a Gaussian Mixture Model Simplification

It can be efficiently solved

in an expectation-maximization scheme

Simplifying Gaussian mixtures is guaranteed to converge

Computation time:
100µs/voxel
(< 10 iterations)

$$\mathbf{R}^* = \arg\min_{\mathbf{R}} \sum_j \sum_{i:\pi_i=j} \alpha_i D(G_i||R_j)$$

E-step

Grouping is known
R_j optimized

$$T_j = \frac{\sum_{i:\pi_i=j} \alpha_i \Sigma_i}{\sum_{i:\pi_i=j} \alpha_i}$$

M-step

Grouping optimized
R_j are known

$$\pi_i = \arg\min_j D(G_i||R_j)$$

Both steps can be carried out in closed form (Davis and Dihillon, 2007)

Heuristics fail

a global approach is required

The problem is well known

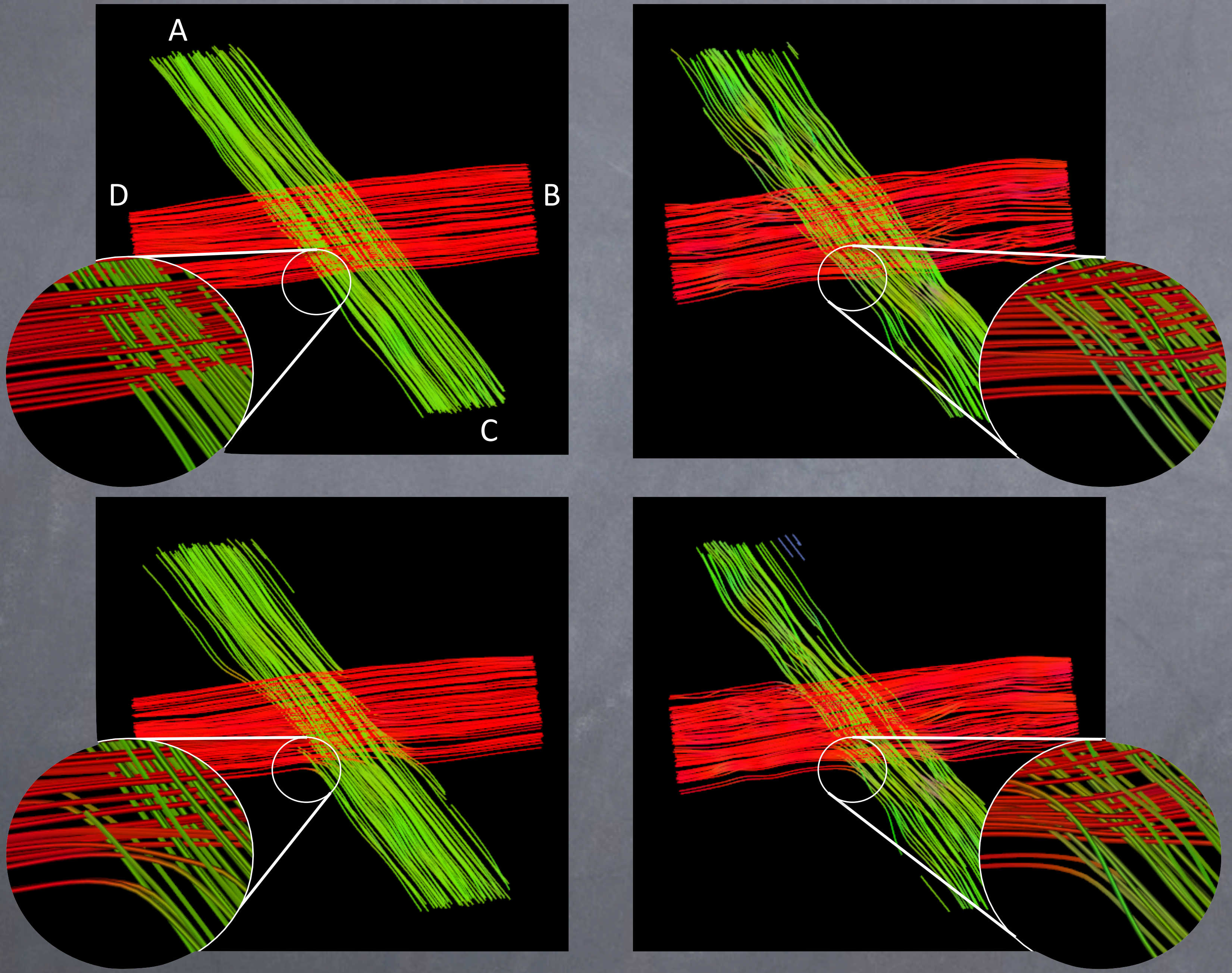
as a Gaussian Mixture Model Simplification

It can be efficiently solved

in an expectation-maximization scheme

How useful is it in practice?

Using GMS prevents confounding white matter fascicles



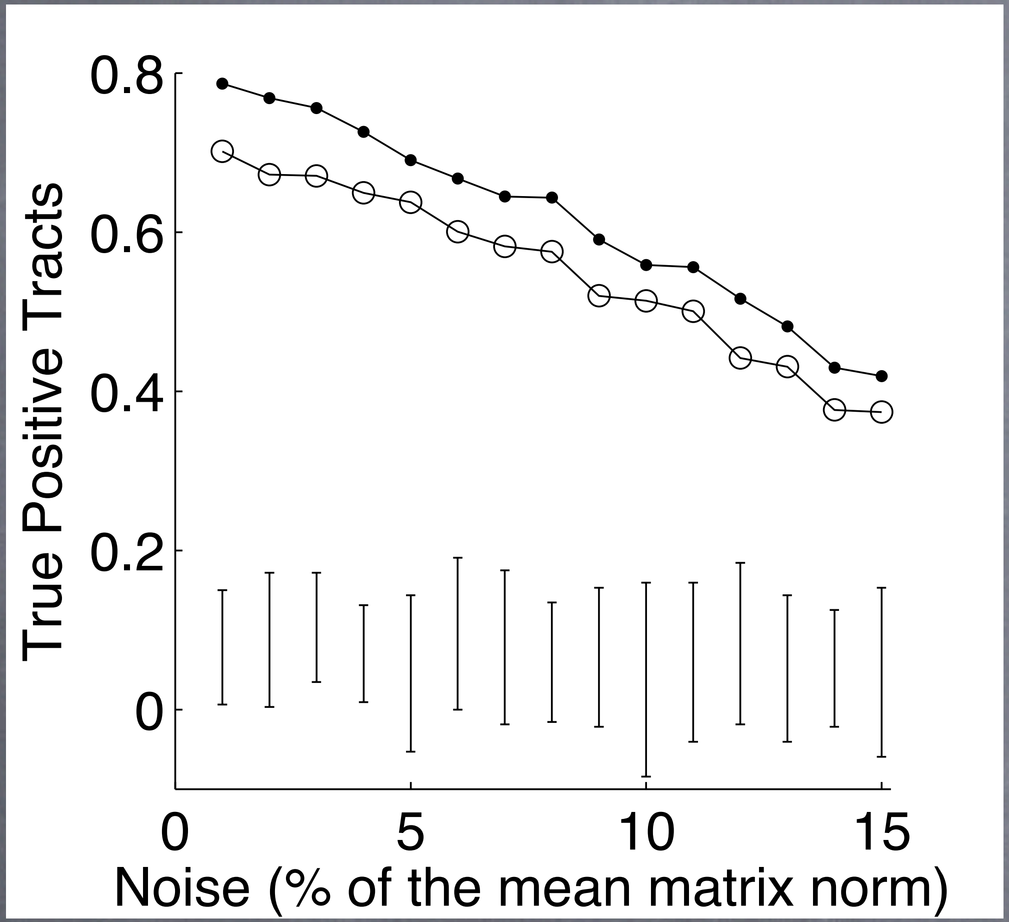
GMS (our method)

Heuristic (orientation-based)

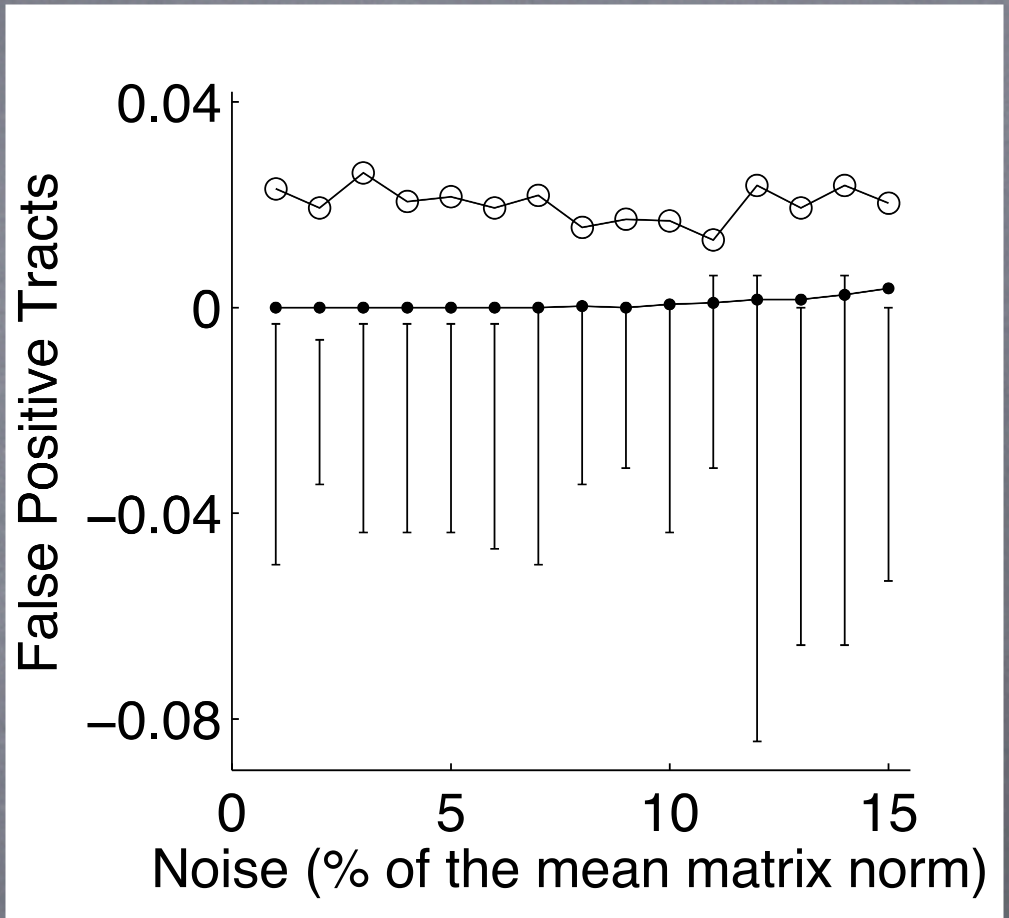
5% Noise

15% Noise

Using GMS prevents confounding white matter fascicles



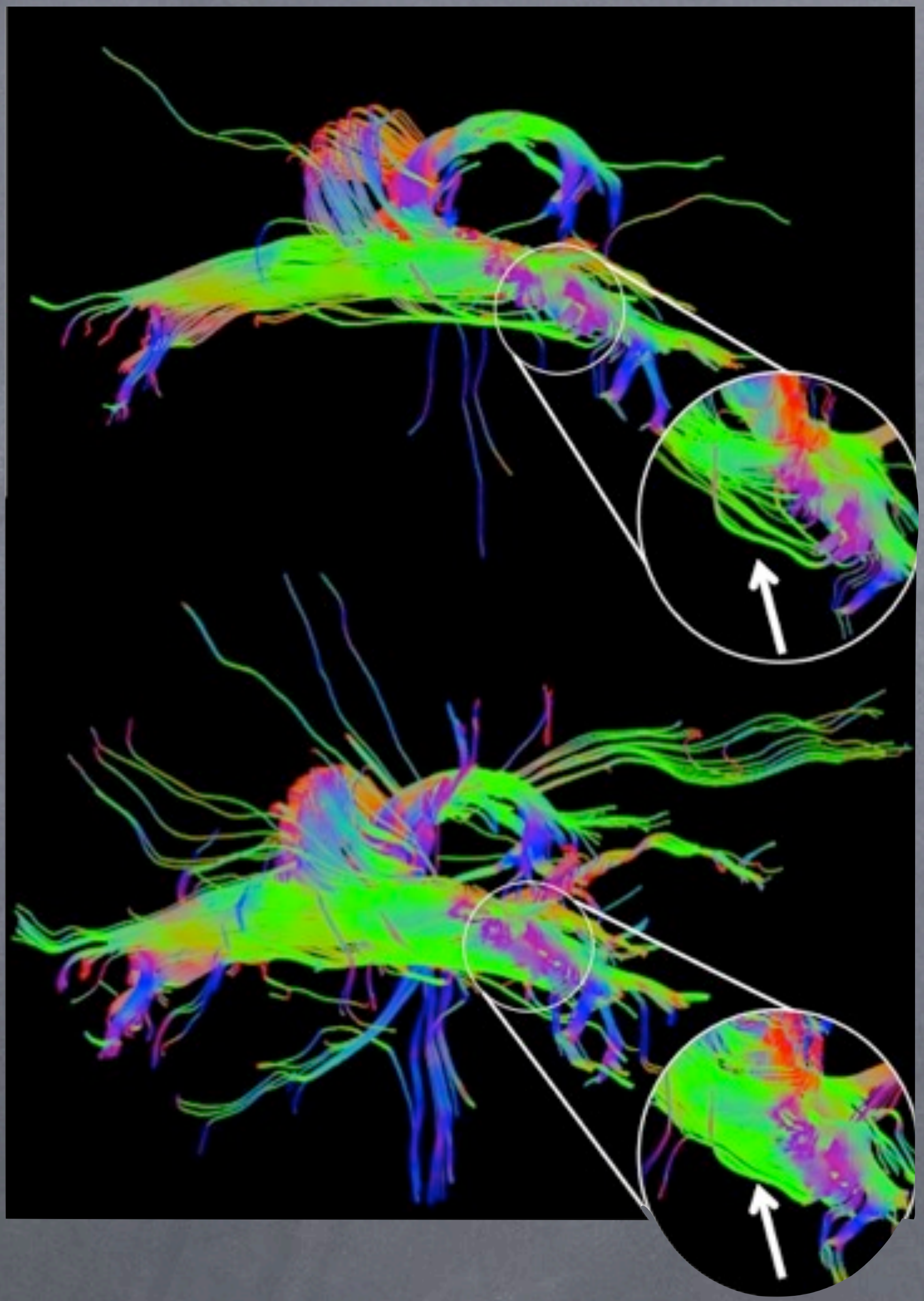
• GMS
o Heuristic



GMS consistently

- increases the true positive rate
- decreases the false positive rate

This advantage may have clinical implications

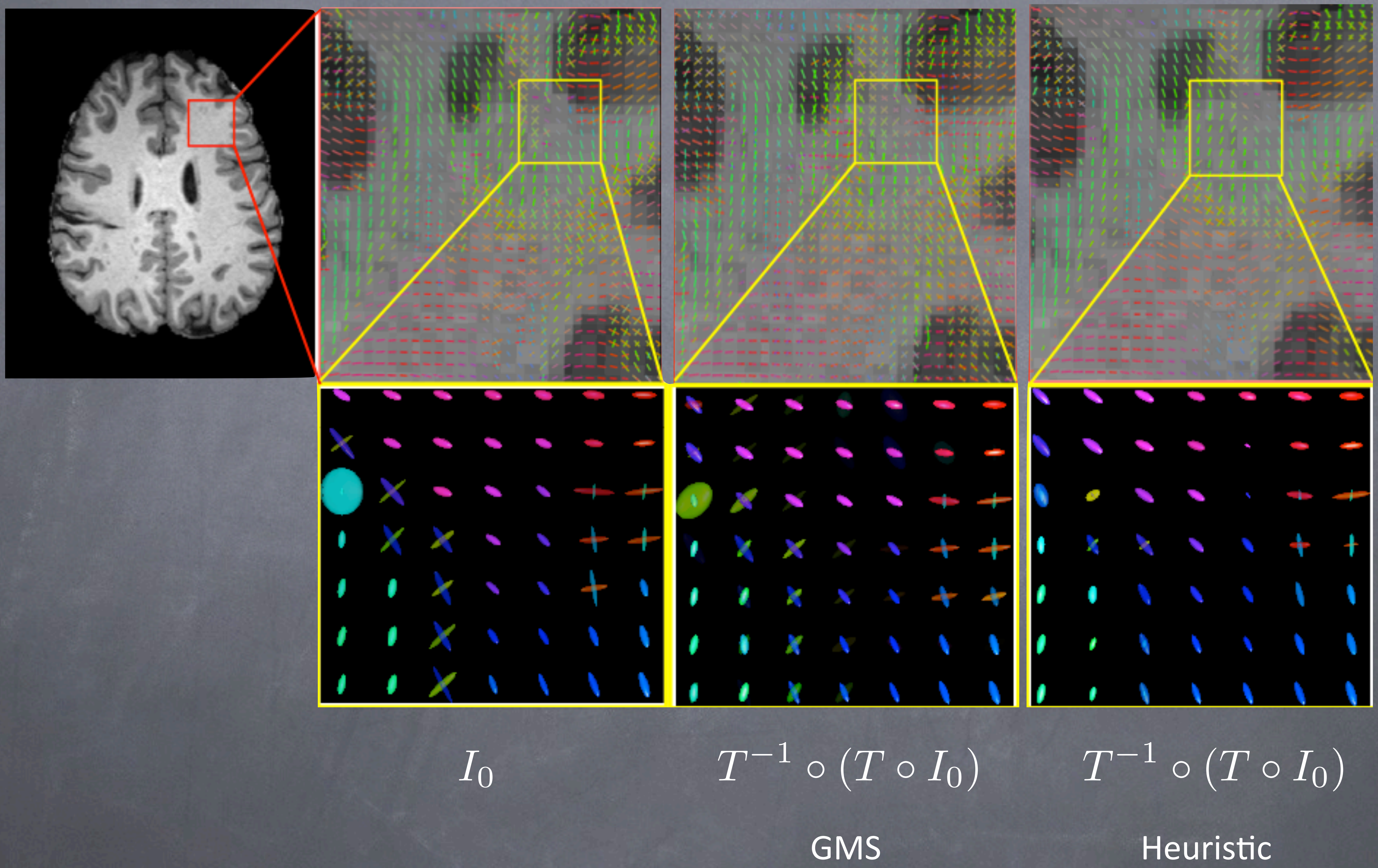


GMS (our method)

Heuristic (orientation-based)

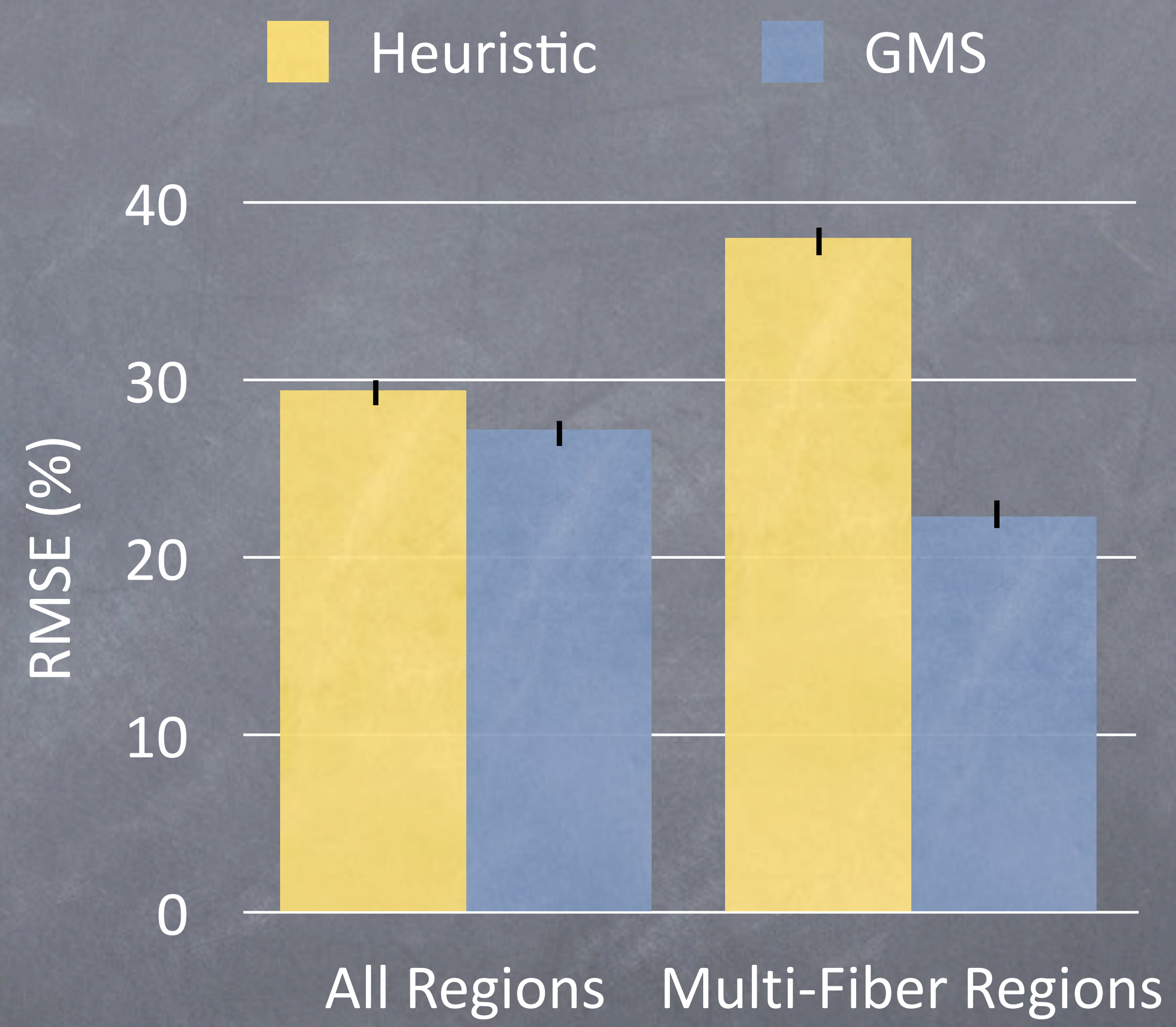
The Meyer's loop is more properly identified
There are fewer spurious tracts

In spatial normalization, GMS preserves more information



In spatial normalization, GMS introduces less error

- 10 multi-fiber DTI
- 1.8 x 1.8 x 2.4 mm^3
- CUSP-45 Acquisition (ISBI 2011)



GMS reduces interpolation errors in all regions and more strikingly in multi-fiber regions

Heuristics fail

a global approach is required

The problem is well known

as a Gaussian Mixture Model Simplification

It can be efficiently solved

in an expectation-maximization scheme

Thank you

Maxime Taquet,
Benoît Scherrer,
Christopher Benjamin,
Sanjay Prabhu,
Benoît Macq,
Simon K. Warfield

Source: crgis.ndc.nasa.gov