A Generalized Correlation Coefficient: Application to DTI and Multi-Fiber DTI.

MMBIA 2012, Breckenridge

Maxime Taquet
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Simon K. Warfield


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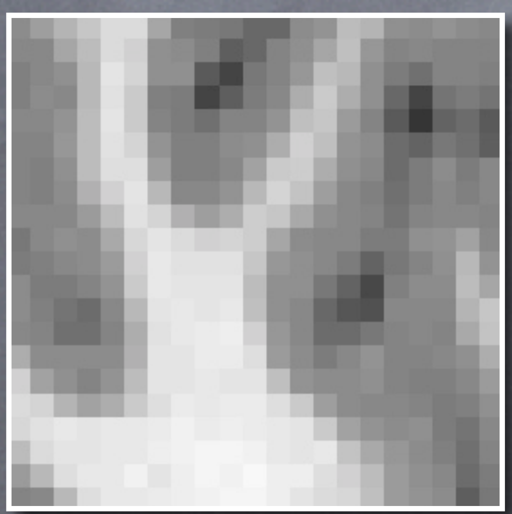
A Generalized Correlation Coefficient

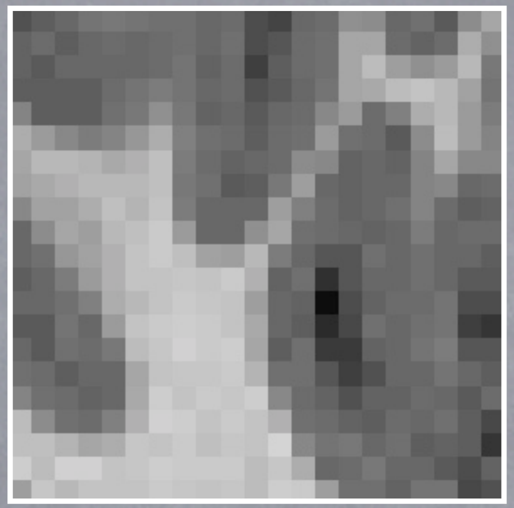
- The correlation coefficient is interesting for matching.
- The **generalized** correlation coefficient embraces new invariance properties.
- It can be applied to DTI and Multi-Tensor DTI.
- It improves the white matter registration accuracy.

Sometimes, the SSD is just not the right criterion.

Target block



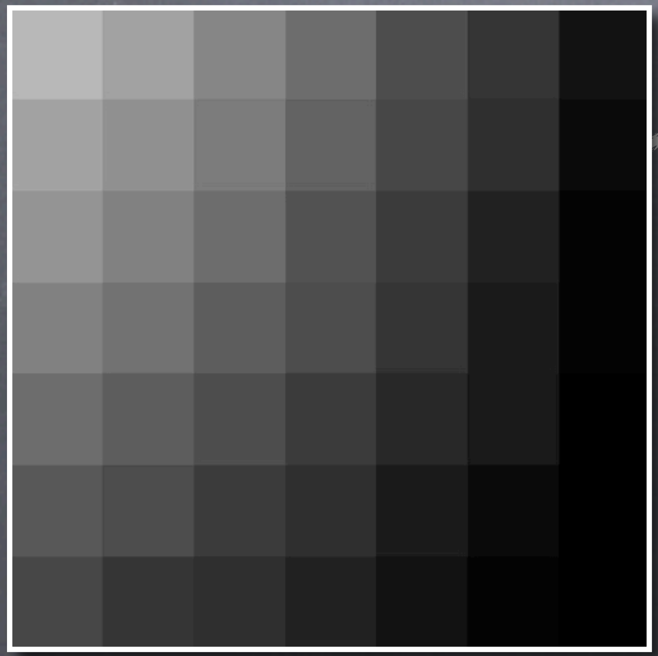
Two candidates

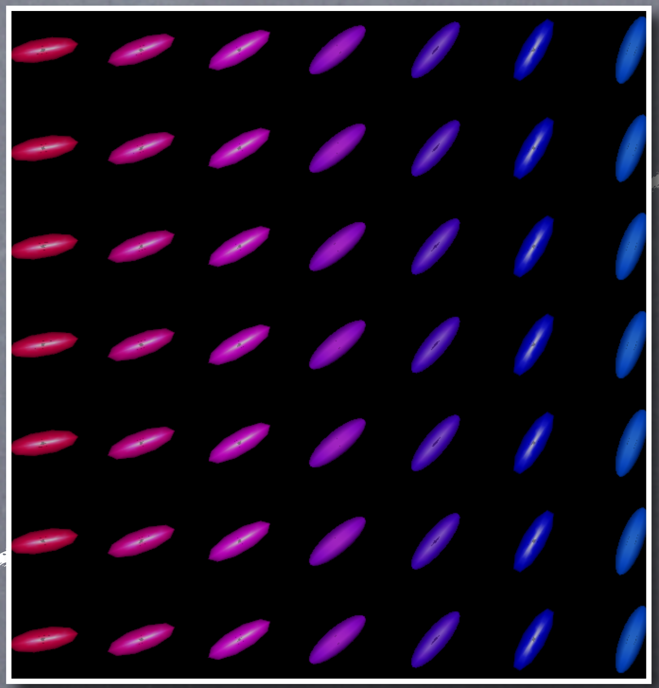


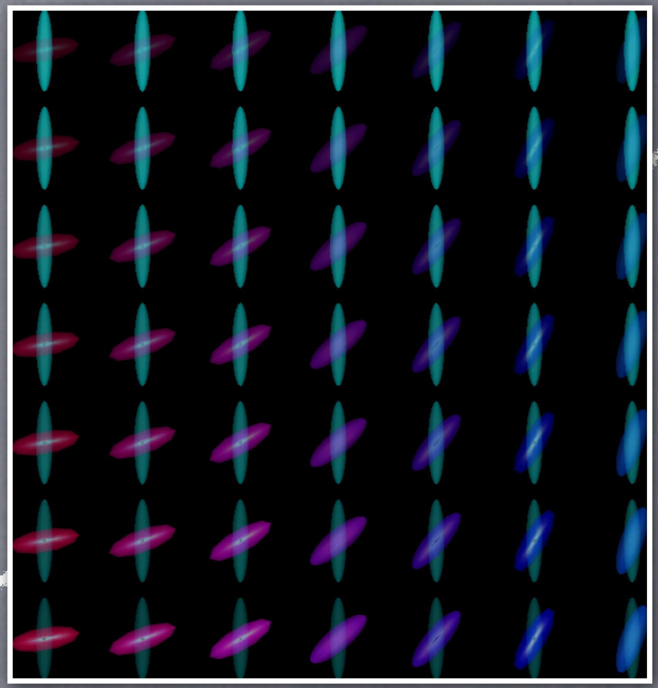
Better in terms of CC

Better in terms of SSD

DTI reveal more structures in the white matter...



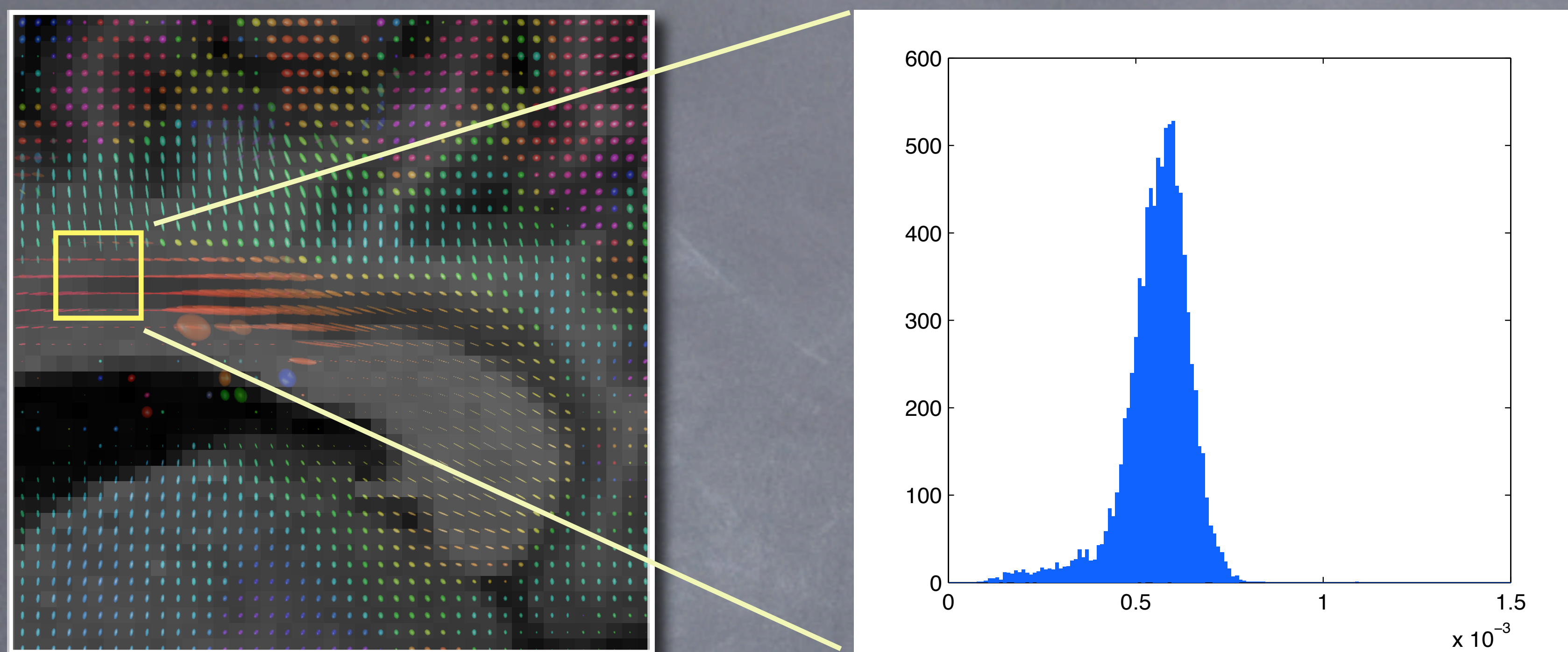




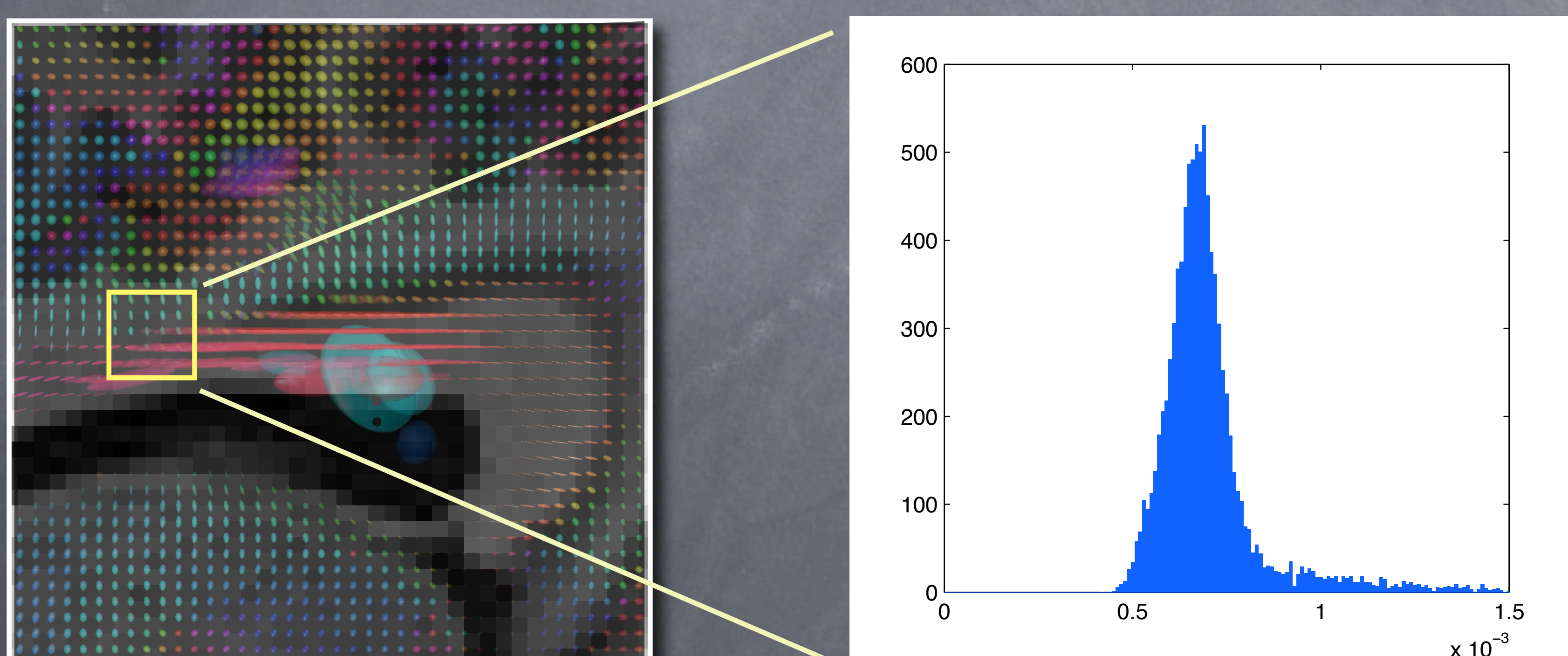
...and multi-tensor DTI account for crossing fibers.

Why would we want any invariance property for DTI metrics?

The tensor eigenvalues are not relevant as such.



First eigenvalue
(after global normalization)



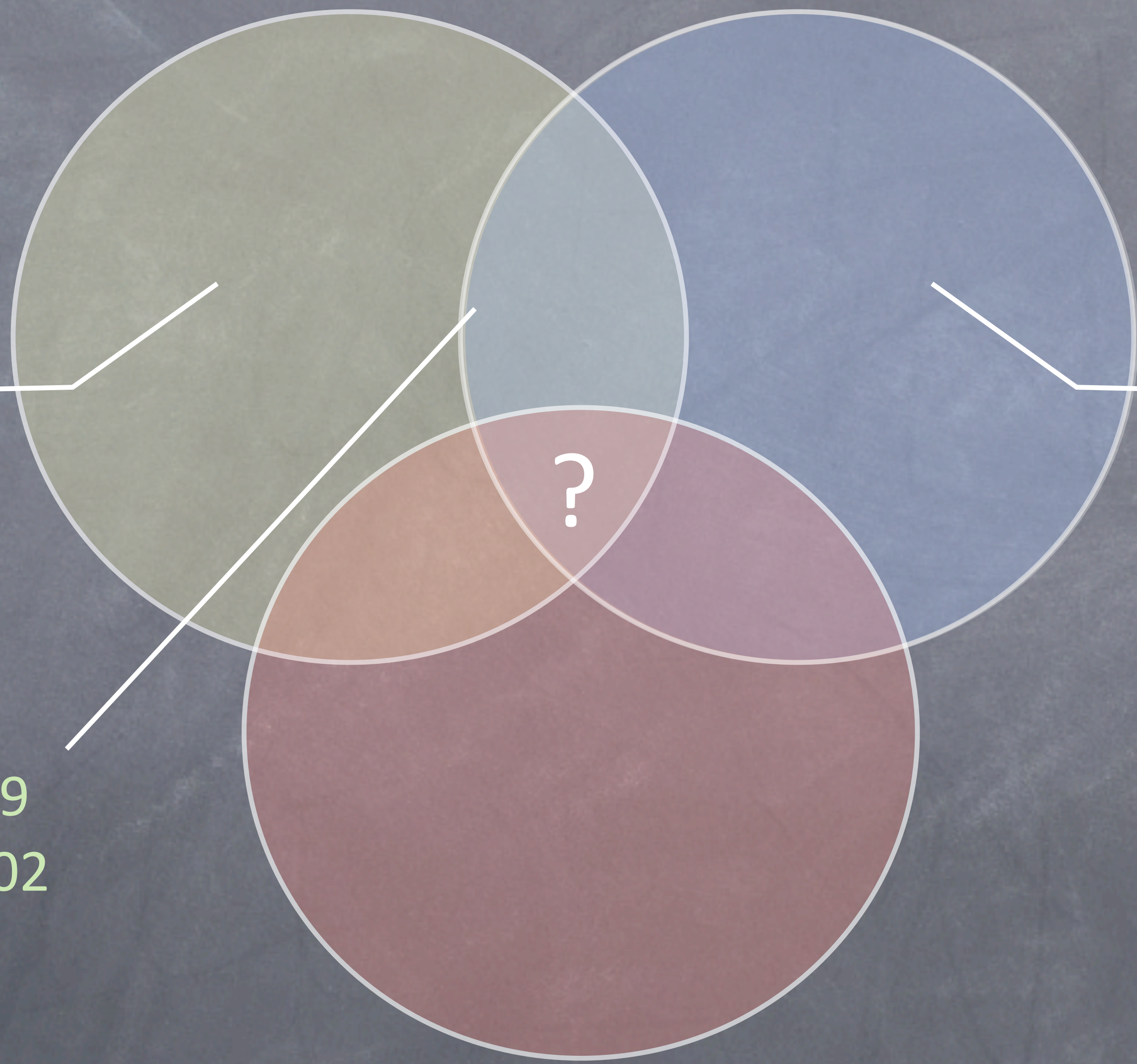
The histograms do not match!

There is a need for a more general invariance property.

Use the whole tensor

Respect the invariance requirement

- Alexander et al., 2000
- Cao et al., 2006
- Yeo et al., 2009
- Chiang et al., 2008

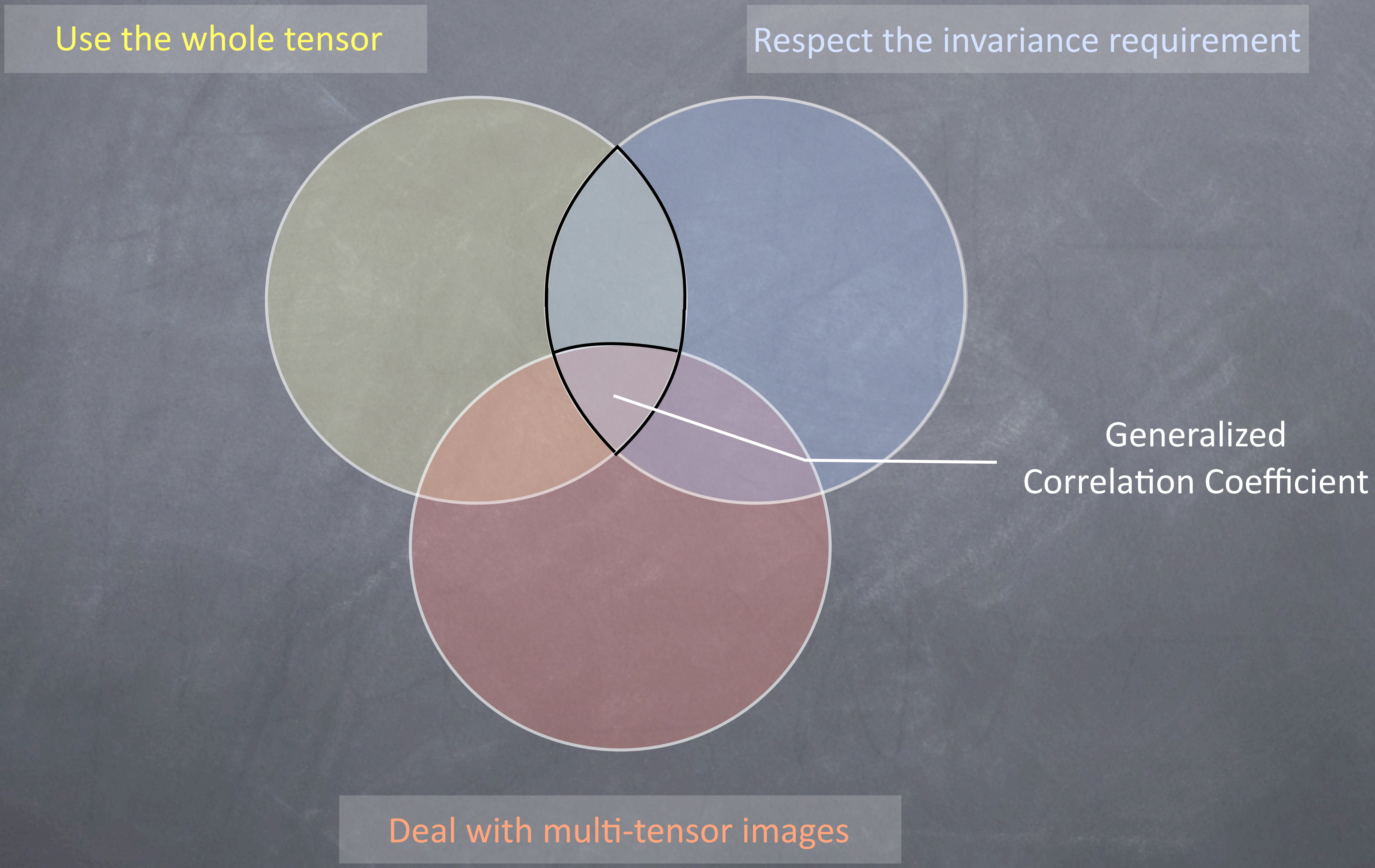


- Guimond et al., 2002
- Park et al., 2003
- Ziyan et al., 2007

- Alexander et al., 1999
- Ruiz-Alzola et al., 2002
- Zhang et al., 2006

Deal with multi-tensor images

There is a need for a more general invariance property.



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First, the mean of the block can be generalized.

$$\rho(R,S) = \left\langle \frac{R-\mu_R}{||R-\mu_R||}, \frac{S-\mu_S}{||S-\mu_R||} \right\rangle$$

$$\mu_R = \langle R, T \rangle \, T$$

← Generalization

$$R \rightarrow aR + bT \Rightarrow \rho \rightarrow \rho$$

← Novel invariance

Second, the scalar product can be generalized.

$$\rho(R,S) = \left\langle \frac{R - \langle R,T\rangle T}{||R - \langle R,T\rangle T||}, \frac{S - \langle S,T\rangle T}{||S - \langle S,T\rangle T||} \right\rangle$$

Does $\langle \cdot, \cdot \rangle$ really need to be a scalar product?

Let us replace it by a more general scalar mapping m(.,.).

$$\downarrow$$

$$\rho(R,S) = m\left(\frac{R - m(R,T)T}{n_m(R - m(R,T)T)}, \frac{S - m(S,T)T}{n_m(S - m(S,T)T)} \right)$$

Not all scalar mappings lead to valid similarity metrics.

$$\rho(R,S) = m\left(\frac{R - m(R,T)T}{n_m(R - m(R,T)T)}, \frac{S - m(S,T)T}{n_m(S - m(S,T)T)}\right)$$

Do all scalar mappings lead to valid similarity metrics ?

No!

We want the to preserve

- the invariance property
- the interpretability of the metric

Valid scalar mappings need to respect a four properties.

1) Symmetry

$$m(R, S) = m(S, R)$$

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2) Cauchy-Schwarz inequality

$$m(R, S)^2 \leq m(R, R)m(S, S)$$

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3) Linearity with respect to the constant image

$$m(aR + bT, T) = am(R, T) + bm(T, T)$$

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$$m(aR + bT, T) = am(R, T) + bm(T, T)$$

4) Constant multiplication of the norm

$$m(aR, aR) = a^2 m(R, R)$$

Any correlation coefficient is generated by choosing m and T.

$$\rho(R,S) = \boxed{m}\left(\frac{R - m(R,T)\boxed{T}}{n_m(R - m(R,T)T)}, \frac{S - m(S,T)T}{n_m(S - m(S,T)T)}\right)$$

Build your own:

1) choose one particular constant image T.

2) choose one particular mapping m respecting the four properties

3) check the new invariance property.

$$R \rightarrow aR + bT$$

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The GCC is invariant under log-linear transformation of the diffusivities.

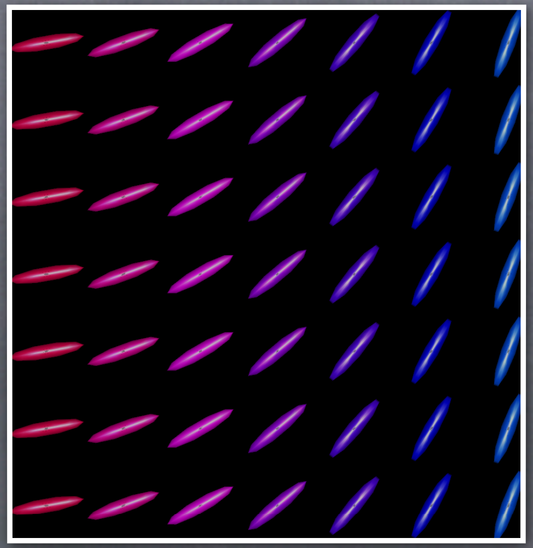
$$T(x) = I_3$$

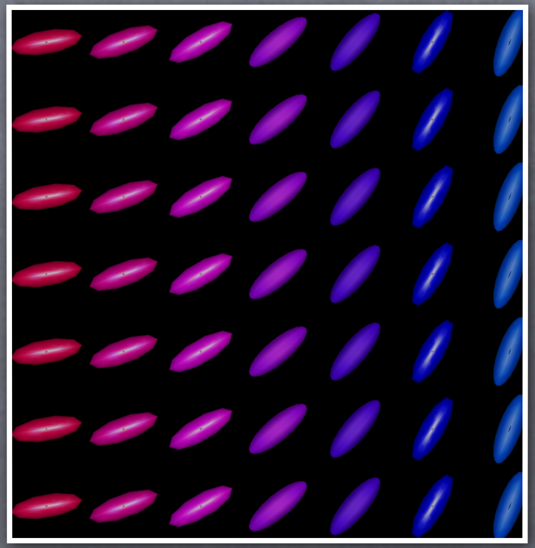
and

$$m(R, S) = \sum_{x \in \Omega} \langle \log R(x), \log S(x) \rangle$$

$$\downarrow$$

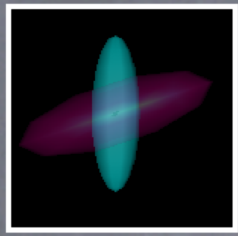
$$\lambda_i \to K\lambda_i^a \Rightarrow \rho \to \rho$$





The correlation coefficient between these two blocks is 1.

A similar invariance is achieved with multi-tensor images.



Two tensors R_1, R_2 with volume fractions f_1 and f_2

$$T(x) = (0.5, I_3, 0.5, I_3)$$

and

$$d_1 = f_1 g_1 \langle R_1, S_1 \rangle + f_2 g_2 \langle R_2, S_2 \rangle$$
$$d_2 = f_1 g_2 \langle R_1, S_2 \rangle + f_2 g_1 \langle R_2, S_1 \rangle .$$

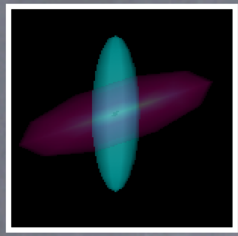
← Construction of a scalar mapping for multi-tensor DTI

$$m_l(R, S) = \arg\max_{d_i} |d_i|$$

↓

$$\lambda_i \to K\lambda_i^a \Rightarrow \rho \to \rho$$

The GCC is invariant under log-linear transformation of the diffusivities.



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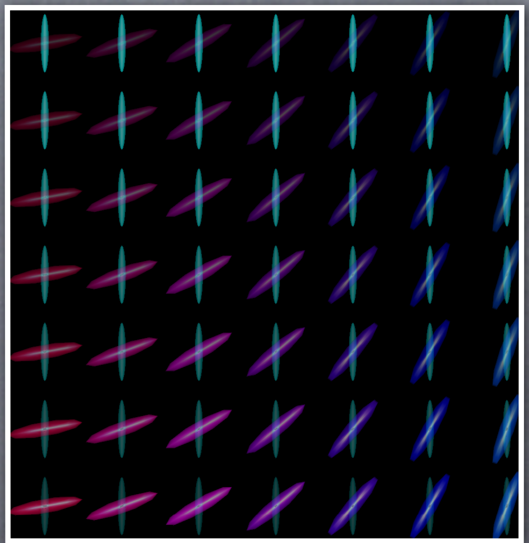

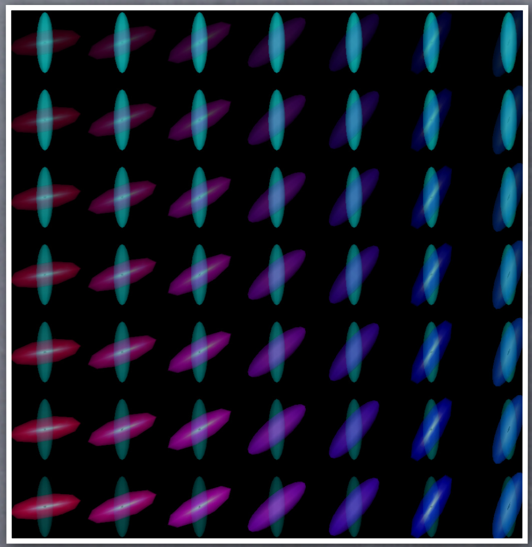

The correlation coefficient between these two blocks is one.

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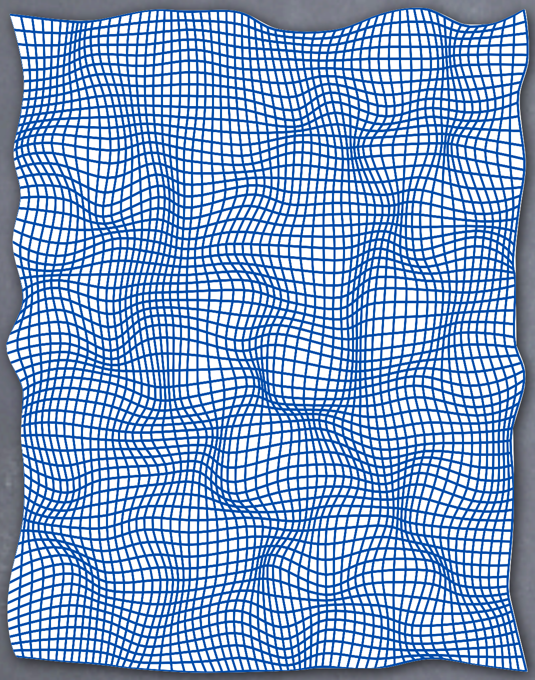
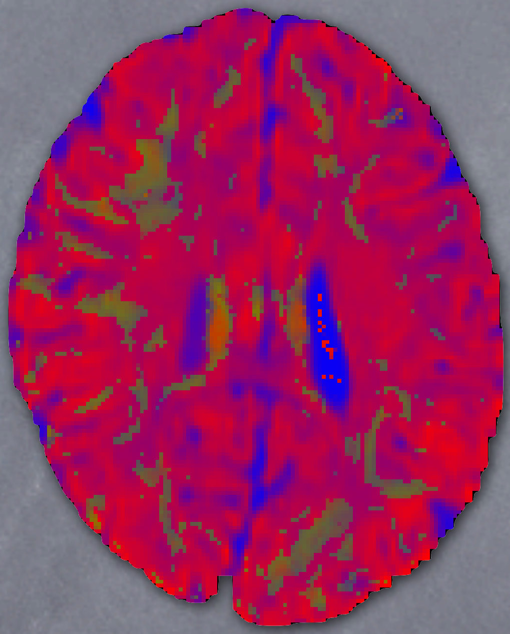
We applied a random field to a DTI and multi-tensor DTI.



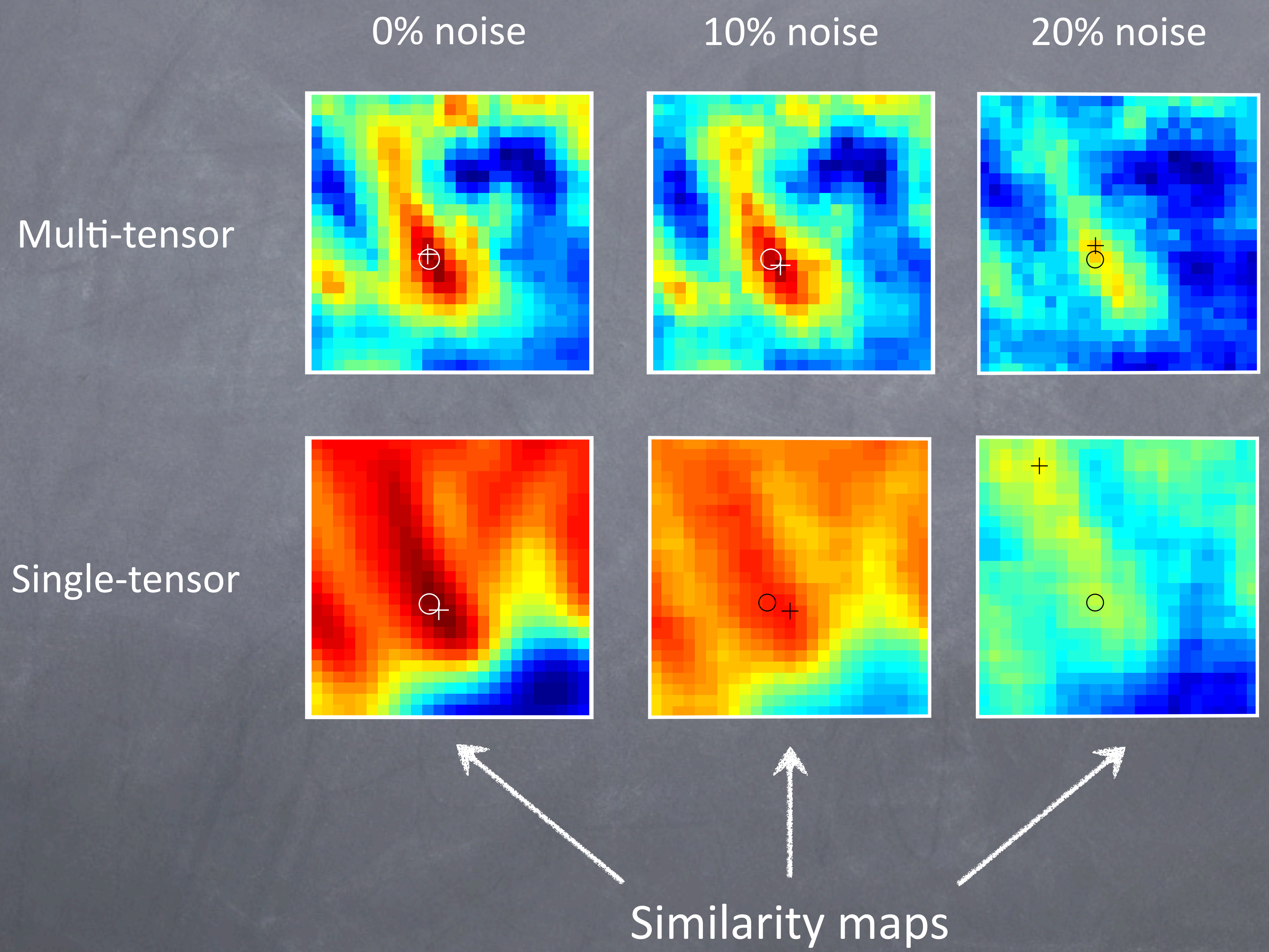
→



→ 400 anatomical landmarks and their correspondences

Processed with different levels of noise: 0% - 20%

Multi-fiber DTI yields more robust and more salient matches.



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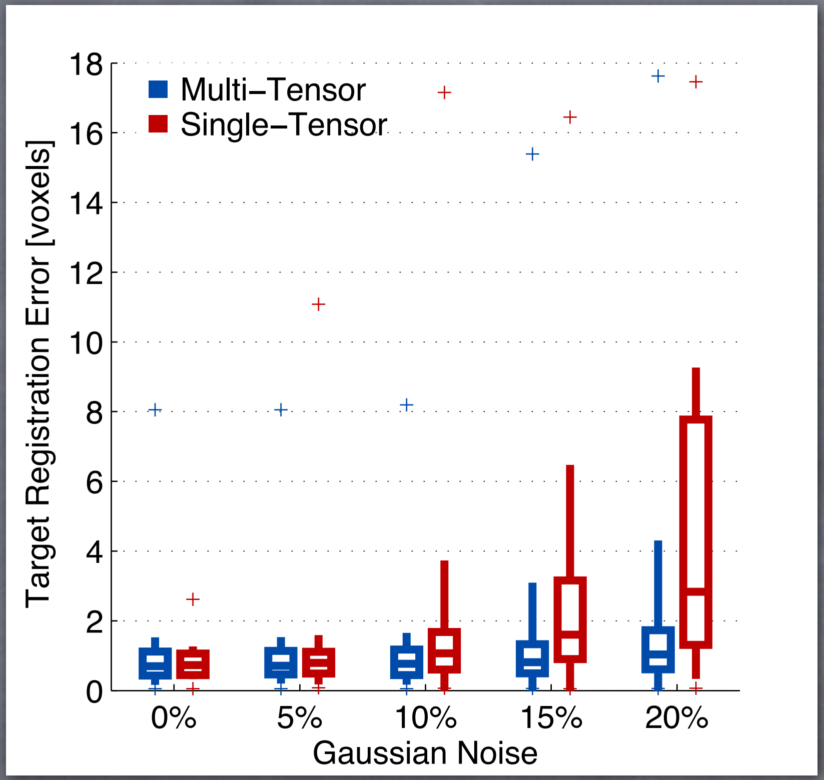
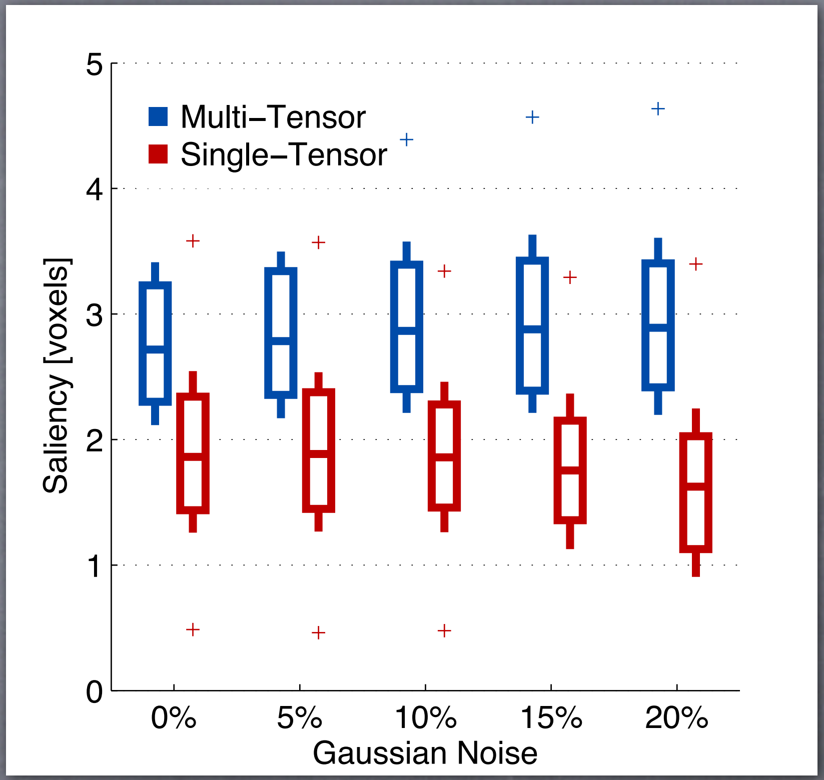


Accuracy



Saliency

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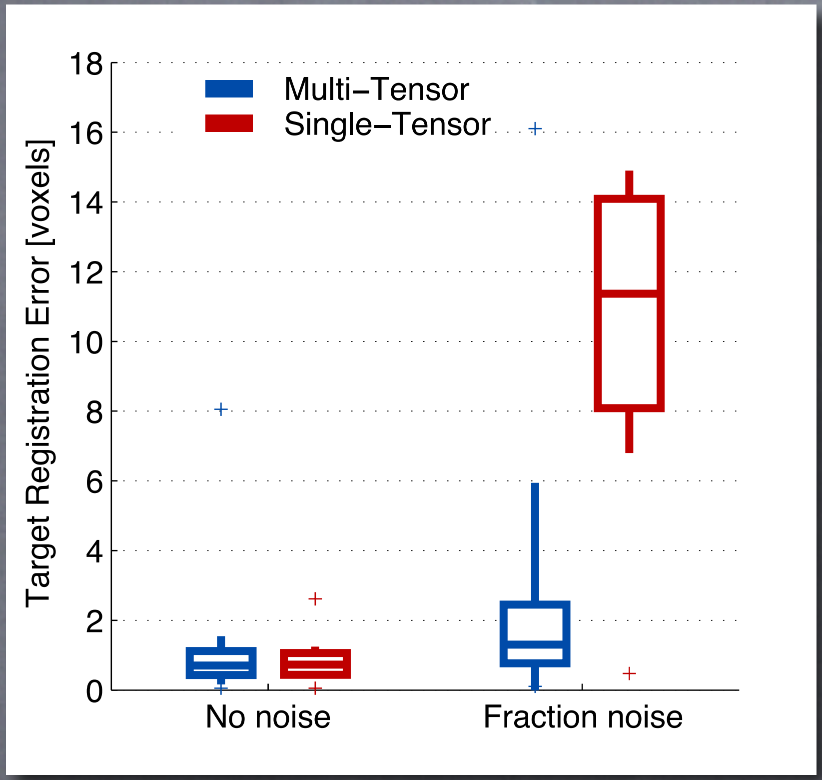
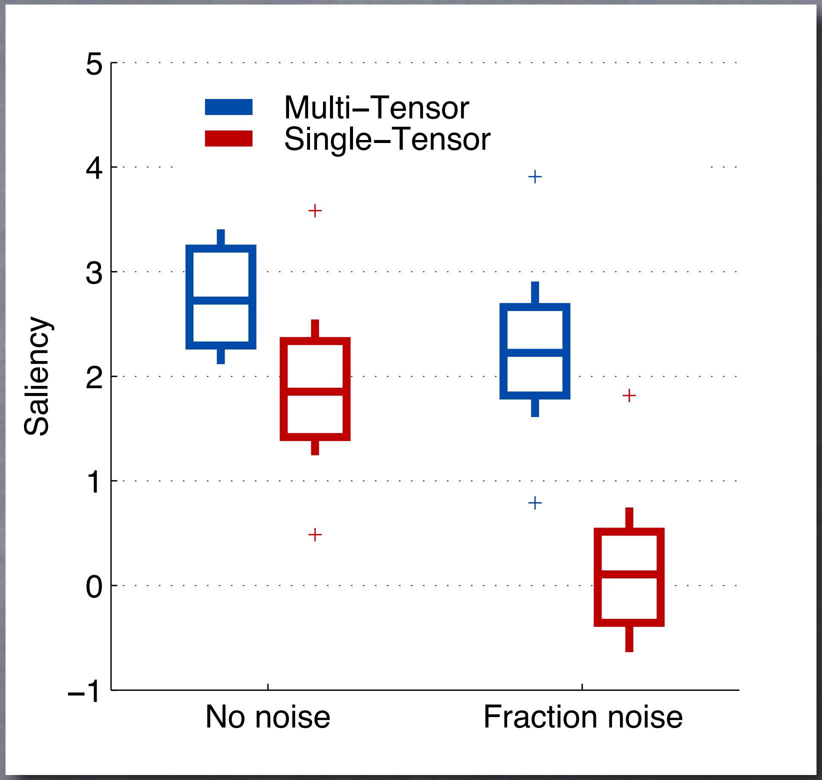


Accuracy



Saliency

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Thank You!